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MATHEMATICS ACHIEVEMENT AND GENDER: A LONGITUDINAL STUDY OF SELECTED COGNITIVE AND AFFECTIVE VARIABLES [GRADES 6–12]

ABSTRACT. This study examined the relationship of selected cognitive and affective variables to mathematics achievement for a random sample of 60 students as they progressed through 6th, 8th, 10th and 12th grades. A consistent gender difference was found for stereotyping mathematics as a male domain. No consistent significant gender difference between means was found for spatial skills, verbal skill or mathematics achievement. Confidence, verbal skill and spatial visualization were each consistently positively correlated with mathematics achievement for both males and females. No gender difference was found for these correlations. However, spatial skills alone were found to be consistent significant predictors of mathematics achievement for females each year of the study, but not for males. Verbal skill was a consistent significant predictor of mathematics achievement for males, but not for females. The results of this study could lead to a reevaluation of the hypothesis that spatial skills help boys achieve in mathematics.

Everyone should have the same opportunity to learn and achieve in mathematics. Studies have shown that, on average, girls do not score as high as boys do on mathematics tests, especially if those tests involve higher level cognitive tasks (Leder, 1990). Females are also under-represented in advanced mathematics courses, college majors and careers that involve mathematics (Armstrong, 1985).

Adolescence is a period of great change. The relationships among skills, talents, attitudes and achievement as students progress through middle school and high school are not clearly understood. Some gender differences in mathematics, perhaps not present at younger ages, have been reported to appear as the students become adolescents (Hyde, Fennema and Lamon, 1990). Some of these studies have reported that beginning at puberty boys tend to outscore girls on tests of spatial skills (McGee, 1979). In addition, Maccoby and Jacklin (1974), in their classic book about sex differences, stated that at "about age 10 or 11...girls begin to come into their own in verbal performance. From this age through the high school and college years we find them outscoring boys at a variety of verbal skills" (p. 84). However, Hyde and Linn (1988), in a meta-analysis of studies of gender differences in verbal ability, concluded that there was no gender difference in verbal skill.

Gender differences related to mathematics pose complex but urgent questions. Important among them are – what factors are related to mathe-

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matics achievement for boys and girls and do these relationships change during the critical period of adolescence?

Since early in this century researchers have been interested in discovering and isolating specific skills and talents which might relate to mathematics achievement. Aiken (1971) indicated that verbal skill correlated well with mathematics achievement. Spatial skills also have been found to be related to mathematics achievement (Tartre, 1990). Although spatial and verbal skills are not the only cognitive factors which correlate significantly with mathematics achievement, they have been cited consistently as skills which might help explain the gender-related differences often found in mathematics achievement. What needs to be identified more clearly is how, if at all, verbal and spatial skills contribute to mathematics achievement in general, and how, if at all, those skills relate to possible gender differences in mathematics achievement.

Affective variables also have been examined as they relate to mathematics achievement. For example, several studies have found that confidence in one's ability to learn mathematics is positively correlated with mathematics achievement (Meyer and Koehler, 1990). Fennema and Sherman (1977) found confidence in learning mathematics almost as strongly related to achievement as verbal and spatial visualization skills. Since girls have tended to be less confident in their ability to do mathematics than boys are (Meyer and Koehler, 1990), it is reasonable to hypothesize that confidence is an important variable to investigate.

Many other affective variables have been examined for their possible relationship to mathematics achievement. Among those affective factors which have been found to be related to mathematics learning and in which gender differences have been found are: how useful mathematics is perceived to be, the perceived role of the teacher in learning mathematics, and whether or not the student enjoys studying and learning mathematics (Fennema and Sherman, 1977). Another affective variable which has been hypothesized as important to understanding gender differences in mathematics is whether studying mathematics is considered to be gender appropriate for the student (Fennema and Sherman, 1977).

These variables have been studied extensively, often with one age group, or in cross-sectional studies of students of various ages. However, only through longitudinal studies, which investigate the relationships of several cognitive and affective variables at several points of time, are we able to build a more complete picture of patterns of the contributions of skills and attitudes to learning mathematics for girls and boys.

The purpose of this longitudinal study was to examine the relationships of selected cognitive and affective variables to mathematics achievement as students move through middle and high school. We were interested in examining the following questions:

1. Are there consistent patterns of gender differences for the cognitive and/or affective variables?
2. Is there a pattern of cognitive and/or affective variables which predict mathematics achievement for each gender? Is the pattern the same for each gender?

1. METHOD

1.1. *Sample*

Data were collected from the entire 6th grade population of four 6th-through- 8th- grade middle schools in a Midwestern city in the United States. A random sample of those students was tested in 8th grade and retested in 10th and 12th grades. Complete data exist for 60 students (32 females and 28 males) as they progressed through middle school and high school.

1.1.1. *Instruments*

Each student was tested on several cognitive and affective factors related to performance in mathematics. The cognitive measures included mathematics achievement, spatial visualization, spatial orientation and verbal skill. Mathematics achievement was measured in 6th grade by the Mathematics Concepts Test (Naslund, Thorpe and LaFever, 1971), in 8th and 12th grades by the Sequential Test of Educational Progress (Educational Testing Service, 1979) and in 10th grade by the Test of Academic Progress (Houghton Mifflin Co., 1971). The test identified as the 12th grade mathematics achievement measure was given system-wide by the school district in the spring of the students' 11th grade year. It was assumed that the results from this test would not change substantially between the second half of the students' 11th grade year and the first half of their 12th grade year when the rest of the measures were administered.

Spatial visualization skill was measured each year by the Space Relations subtest of the Differential Aptitude Test (Bennett, Seashore and Wesman, 1973). Spatial orientation skill, measured by the Gestalt Completion Test (Ekstrom, French and Harmon, 1976), was tested only in 10th and 12th grades. Verbal skill was measured by the vocabulary subtests at each grade level from the Cognitive Abilities Test, Verbal Battery (Thorndike and Hagen, 1975).

TABLE I

Means (standard deviations) and z-values for the 6th grade population and the random sample

	Population[a]	Sample[b]	z-Value
Mathematics	23.70	24.71	1.23
Achievement	(6.34)	(6.76)	
Verbal	16.66	16.53	−0.23
Skill	(4.34)	(4.76)	
Spatial	24.71	25.90	1.05
Visualization	(8.77)	(9.20)	
Confidence	45.61	47.63	1.87
	(8.38)	(7.00)	
Perceived	42.37	44.05	1.50
Usefulness	(8.66)	(7.54)	
Teacher	47.19	46.97	−0.23
	(7.47)	(6.50)	
Male Domain	49.28	49.58	0.28
	(8.19)	(7.47)	

[a]N = 667. [b]N = 60.

The affective factors were measured using the Fennema-Sherman Mathematics Attitude Scales (Fennema and Sherman, 1976). Those scales included confidence in learning mathematics, perceived usefulness of mathematics, the perception of mathematics as a male domain and the effect of the teacher on the learning of mathematics.

2. RESULTS

To test whether the group that completed the study was different from the original 6th grade population, z-statistics were computed. Table I shows the means, standard deviations and z-values for the population and the sample for the 6th grade measures. No significant difference was found. From this we can infer that the sample that completed all testings in the study was probably not an unlikely sample from the sampling distribution of the population in 6th grade.

1. Are there consistent patterns of gender differences for the cognitive and/or affective variables?

To answer this question, t-tests were done to test for gender differences for all variables (see Tables II and III). Although the mean for males was higher than the mean for females at each grade level, the mean difference between genders for mathematics achievement was significant only for 8th grade. Similarly for spatial visualization, the trend for the means favored males at each grade level, but no significant gender difference was found.

The only consistent significant gender difference found for the affective variables was for Male Domain, indicating that the males stereotyped mathematics as a male domain to a greater degree than the females did for each year of the study. Males also had a significantly higher standard deviation for this variable for each year of the study. No other consistent pattern of gender differences was found for any of the other cognitive or affective variables.

2. Is there a pattern of cognitive and/or affective variables which predict mathematics achievement for each gender? Is the pattern the same for each gender?

Since the sample size was relatively small, making any one regression equation unreliable, we decided to examine stepwise and backward multiple linear regression equations with all possible combinations of years to look for any consistent patterns. With all the possible combinations of predictors, the separation of each testing by two years, the differences in appropriate mathematics content by grade level and the use of different publishers' mathematics achievement tests, probability favored that no consistent pattern would be found. Table IV summarizes the results of regression analyses done by gender using mathematics achievement at each grade level as the dependent variables (complete regression information is available from the first author upon request). The scores for grade 6 were used as independent variables for predicting mathematics achievement for each of grades 8, 10 and 12 [6–8,6–10 and 6–12 respectively]. The scores for grade 8 were used to predict mathematics achievement for each of grades 10 and 12 [8–10 and 8–12 respectively] and the scores for grade 10 were used to predict mathematics achievement for grade 12 [10–12]. Both stepwise and backward regression were used and the outcomes were consistent. In addition, most of the other possible combinations were done adding each variable last and the results were consistent with the backward and stepwise regressions presented. Entering the entire set of cognitive and affective factors as independent variables in the regression equations accounted for over 50% of the variance for mathematics achievement for each grade level, with a range of 65% to 80% for females and 51% to 76% for males.

Not surprisingly, prior mathematics achievement was the single most consistent and strongest predictor of mathematics achievement for both

TABLE II

Means (standard deviations) and t-values for the random sample by gender for cognitive variables

	Grade	Females (N = 32)	Males (N = 28)	t-Value
Mathematics Achievement[a]	6	23.94 (7.89)	25.61 (5.19)	0.95
	8	37.47 (6.45)	41.82 (5.19)	2.85**
	10	23.44 (9.33)	26.50 (10.66)	1.19
	12	38.78 (6.99)	41.32 (5.81)	1.52
Verbal[a]	6	16.31 (4.80)	16.79 (4.78)	0.38
	8	17.22 (4.01)	17.00 (3.69)	−0.22
	10	16.59 (3.55)	16.89 (3.38)	0.33
	12	15.38 (4.41)	15.21 (4.14)	−0.14
Spatial Visualization[b]	6	25.66 (10.09)	26.18 (8.24)	0.22
	8	35.91 (10.26)	38.32 (9.58)	0.94
	10	39.75 (10.96)	44.43 (10.54)	1.68
	12	42.63 (10.79)	46.89 (8.09)	1.71
Spatial Orientation[b]	10	12.97 (2.92)	13.18 (2.83)	0.28
	12	14.63 (2.60)	14.97 (1.86)*	0.57

*$p < 0.05$, **$p < 0.01$. [a]tests were different each year. [b]the same test was given each year.

genders at any grade level (adjusted R^2 was approximately 0.55). It is with the other variables to be entered in the regression equation that the pattern for each gender differs. For females, spatial visualization was included

TABLE III

Means (standard deviations) and t-values for the random sample by gender for affective variables

	Grade	Females (N = 32)	Males (N = 28)	t-Value
Confidence[b]	6	46.41	49.04	1.47
		(7.34)	(6.44)	
	8	47.81	51.36	1.62
		(8.63)	(8.28)	
	10	47.56	48.32	0.33
		(8.89)	(9.02)	
	12	45.78	46.43	0.27
		(8.74)	(10.05)	
Usefulness[b]	6	43.44	44.75	0.67
		(7.92)	(6.84)	
	8	47.50	51.36	2.02*
		(8.27)	(6.40)	
	10	49.09	49.39	0.14
		(7.90)	(8.17)	
	12	48.13	49.75	0.78
		(8.03)	(4.13)	
Male Domain[b c]	6	52.35	46.64	−3.05**
		(5.71)	(8.21)* [d]	
	8	53.91	46.32	−4.48***
		(3.71)	(8.73)* [d]	
	10	56.03	50.07	−3.98***
		(3.85)	(7.42)* [d]	
	12	55.94	50.89	−3.10**
		(4.16)	(8.06)* [d]	
Teacher[b]	6	46.29	47.75	0.87
		(6.36)	(6.80)	
	8	42.97	42.61	−0.19
		(8.00)	(7.03)	
	10	45.59	44.36	−0.69
		(6.75)	(7.04)	
	12	46.22	40.32	−3.14**
		(8.33)	(6.17)	

*$p<0.05$, **$p<0.01$, ***$p < 0.001$. [b]the same test was given each year. [c]higher score indicates less stereotyping. [d]significant gender difference between corresponding variances.

TABLE IV

Summary of Variables Appearing in Backward Multiple Linear Regression by Gender

	Females			Males		
Mathematics Achievement	6–8[a]					
	6–10	8–10		6–10	8–10	
	6–12	8–12	10–12[b]	6–12	8–12	10–12
Verbal				6–8		
				6–10		
			10–12[c]	6–12		10–12
Spatial Visualization	6–8					
	6–10	8–10				
	6–12	8–12	10–12			
Spatial Orientation[d]			10–12			
Confidence	6–10					
	6–12			6–12	8–12	
Teacher				6–8[c]		
				6–10[c]	8–10	
Usefulness				6–8		
				6–10		
Male Domain						

[a]6–8 means independent variables from 6th and dependent variable is 8th grade mathematics achievement.
[b]10–12 included spatial orientation in analysis.
[c]negative standardized beta coefficient.
[d]independent variable only for grade 10.

as a predictor variable in all backward regression analyses. However for males, spatial visualization was never included in a regression equation. In fact most of the time it was the first variable removed using backward regression analysis, indicating that spatial visualization contributed the least to the regression equation for males. The regression equations for males did tend to include verbal skill, often as the only variable besides mathematics achievement. For females, verbal skill was not included in any regression equation created through the stepwise process. It was removed from all but one equation using the backward process, and often was the first variable removed.

In the regression of grade 10 scores on 12th grade mathematics achievement, spatial orientation was chosen instead of spatial visualization as the variable to be included in the stepwise regression equation after 10th grade mathematics achievement for females. Spatial orientation appeared to account for 6% of the remaining variance for 12th grade mathematics achievement for females after 10th grade mathematics achievement had been included. Analyses showed that spatial visualization would have changed the adjusted R^2 4% for females. Two new regression equations were created to predict 12th grade achievement for males, which first entered 10th grade mathematics achievement and then entered either spatial orientation or spatial visualization. These new analyses showed that either spatial orientation or spatial visualization would have accounted for less than 1% of the remaining variance for males.

All of the affective variables, except Male Domain, were included in at least one regression equation. However, the patterns for each gender were not as consistent as for the cognitive variables. Confidence appeared in regression equations more often than the other affective variables. No other affective variable appeared in regression equations for females. For males, the Teacher and Usefulness scales were significant predictors of mathematics achievement for grades 8 and 10. The 6th grade testing provided the independent variables for most of the cases in which an affective variable was included in regression analyses for males.

It was possible that the presence of mathematics achievement as an independent variable in the regression equation models could have affected the patterns of the regression equations for each gender described above, especially since mathematics achievement appeared to account for a larger share of the variance than any of the other variables. To see if there were consistent patterns of prediction by gender using just the other variables, regression analyses were done with all combination of years without mathematics achievement as an independent variable. It was expected that the removal of mathematics achievement as an independent variable would scramble any previous pattern and perhaps not show any consistent pattern of prediction. Table V summarizes the backward regression analysis done without including previous mathematics achievement for each grade as independent variables. The new models accounted for approximately 50% of the variance. For females the adjusted R^2 ranged from 0.40 to 0.65 (with a mean of 0.51) and for males from 0.44 to 0.65 (with a mean of 0.56). The pattern of the regression models for females to include spatial visualization (and orientation) skill [and not verbal skill] and for males to include verbal skill [and not spatial skill], rather than disappear, was reinforced more consistently with this new analysis. In addition, Confidence was included more consistently in the regression equations for both

TABLE V

Summary of Variables Appearing in Backward Multiple Linear Regression by Gender (without Mathematics Achievement)

	Females			Males		
Verbal		6–8[a]		6–8		
				6–10	8–10	
				6–12	8–12	10–12[b]
Spatial Visualization	6–8					
	6–10	8–10				
	6–12	8–12	10–12			
Spatial Orientation[d]			10–12			
Confidence	6–10	8–10			8–10	
	6–12	8–12	10–12	6–12	8–12	10–12
Teacher				6–8[c]		
				6–10		
Usefulness				6–8		
				6–10		
Male Domain	6–12					

[a] 6–8 means independent variables from 6th and dependent variable is 8th grade mathematics achievement.
[b] 10–12 included spatial orientation in analysis.
[c] negative standardized beta coefficient.
[d] independent variable only for grade 10.

genders. Male Domain was included in the regression equation of grade 6 variables on grade 12 mathematics achievement for females.

The correlation coefficients were also examined for all year combinations to see consistency of patterns. Examination by gender showed strong positive correlations among verbal skill, spatial visualization skill and mathematics achievement for both genders for every year of the study (see Table VI). The exception to this pattern was that the 6th grade spatial visualization test for males was only positively correlated with 6th grade mathematics achievement.

There was no clear trend in the correlations which would have anticipated the consistent pattern of predictions found in the regression analyses. For example the correlations between verbal skill and mathematics for females tended to be as high as those for males. If one compares the correlations of verbal skill and spatial visualization each to mathematics achievement for females for the year combinations involved in the regression analysis

TABLE VI

Correlations by Gender: Mathematics Achievement, Spatial Visualization and Verbal Skill

	Mathematics Achievement							
	Female				Male			
	6th	8th	10th	12th	6th	8th	10th	12th
Verbal								
6th	0.61***	0.60***	0.60***	0.62***	0.61***	0.68***	0.65***	0.70***
8th	0.44*	0.46**	0.61***	0.37*	0.37	0.56**	0.46*	0.40*
10th	0.51**	0.61***	0.65***	0.52**	0.42*	0.41*	0.49**	0.64***
12th	0.61***	0.61***	0.60***	0.49**	0.58***	0.74***	0.82***	0.77***
Spatial Visualization								
6th	0.49**	0.60***	0.64***	0.61***	0.43*	0.31	0.27	0.32
8th	0.59***	0.55**	0.61***	0.62***	0.44*	0.69***	0.52**	0.48**
10th	0.56***	0.57***	0.64***	0.71***	0.41*	0.72***	0.49**	0.59***
12th	0.52**	0.55**	0.60***	0.58***	0.49**	0.69***	0.62***	0.66***
	Spatial Visualization							
	Female				Male			
	6th	8th	10th	12th	6th	8th	10th	12th
Verbal								
6th	0.48**	0.59***	0.57***	0.45*	0.27	0.54**	0.51**	0.66***
8th	0.44*	0.51**	0.40*	0.40*	0.09	0.42*	0.36	0.53**
10th	0.62***	0.62***	0.58***	0.56***	0.27	0.33	0.60***	0.49**
12th	0.51**	0.56***	0.48**	0.47**	0.17	0.56**	0.65***	0.70***

*$p < 0.05$, **$p < 0.01$, ***$p < 0.001$

the difference between the correlations was less than 0.04 in four of the six cases and the direction of any difference was not consistent.

The pattern for spatial orientation was quite different (see Table VII). For males, spatial orientation was not positively correlated with mathematics achievement, verbal or spatial visualization skill for any year of the study. However, for females, the 10th grade spatial orientation test was positively correlated with mathematics achievement grade 6 and 12, spatial visualization grade 10 and verbal skill each year. The 12th grade spatial orientation test was positively correlated with mathematics achievement grade 12 and verbal skill each year for females. Significant gender differences were found for the correlations of spatial orientation (both years) with verbal skill grades 6 and 12. A significant gender difference was also

TABLE VII

Correlations by Gender: Spatial Orientation with Mathematics Achievement, Spatial Visualization and Verbal Skill

	Spatial Orientation			
	Female		Male	
	10th	12th	10th	12th
Mathematics Achievement				
6th	0.37**	0.20	0.04	−0.09
8th	0.33	0.22	0.10	−0.02
10th	0.32	0.17	−0.18	−0.28
12th	0.53**[d]	0.41*	0.00	−0.04
Spatial Visualization				
6th	0.27	0.25	0.29	0.36
8th	0.28	0.26	0.20	0.19
10th	0.43*	0.33	0.27	0.35
12th	0.34	0.19	0.19	0.31
Verbal				
6th	0.52**[d]	0.53**[d]	−0.02	−0.08
8th	0.39*	0.45*	0.08	0.15
10th	0.49**	0.42*	0.21	0.27
12th	0.57***[d]	0.49**[d]	0.02	0.01

*$p < 0.05$, **$p < 0.01$, ***$p < 0.001$
[d]significant ($p < 0.05$) gender difference between corresponding correlations.

found for the correlations for spatial orientation grade 10 and mathematics achievement grade 12.

Examining the correlations for the affective variables with mathematics achievement showed a strong pattern of positive correlations between mathematics achievement and Confidence for both males and females (see Table VIII). Neither gender showed consistent patterns of correlations between mathematics achievement and the effect of the Teacher or the perceived Usefulness of mathematics. However, there appeared to be differences between genders in the pattern of the correlations between mathematics achievement and the stereotyping of mathematics as a Male Domain. The Male Domain scale was not related to mathematics achievement for males for any year of the study. However, for females less stereotyping was positively correlated to mathematics achievement for grades

TABLE VIII

Correlations by Gender: Mathematics Achievement with Affective Scales

	Mathematics Achievement							
	Female				Male			
	6th	8th	10th	12th	6th	8th	10th	12th
Confidence								
6th	0.51**	0.53**	0.65***	0.64***	0.40*	0.19	0.43*	0.43*
8th	0.38*	0.39*	0.45*	0.49**	0.42*	0.40*	0.56**	0.63***
10th	0.63***	0.62***	0.62***	0.65***	0.29	0.48**	0.60***	0.51**
12th	0.32	0.47**	0.47**	0.47**	0.39*	0.22	0.42	0.38
Male Domain								
6th	0.58***	0.53**	0.39*	0.36*	0.27	0.15	0.11	0.22
8th	0.59***	0.47**	0.35	0.25	0.30	0.00	0.18	0.14
10th	0.55**	0.40*	0.13	0.19	0.28	0.14	0.14	0.29
12th	0.61***[d]	0.40*	0.15	0.11	0.15	0.03	0.01	0.01
Teacher								
6th	0.37*	0.30	0.26	0.42*	0.05	0.17	0.45*	0.24
8th	0.29	0.26	0.31	0.51*	−0.16	0.03	0.29	0.10
10th	0.33	0.37*	0.30	0.35	−0.02	0.39*	0.57**	0.32
12th	0.24	0.19	0.17	0.41*	0.18	0.32	0.43*	0.32
Usefulness								
6th	0.40*	0.28	0.31	0.13	0.32	0.11	0.14	0.15
8th	0.32	0.24	0.12	0.10	0.27	0.16	0.13	−0.07
10th	0.45*	0.18	0.13	−0.01	0.46*	0.32	0.20	−0.01
12th	0.37	0.17	0.10	0.11	0.41*	0.31	0.26	−0.04

*$p < 0.05$, **$p < 0.01$, ***$p < 0.001$;
[d]significant ($p < 0.05$) gender difference between corresponding correlations.

6 and 8. In addition, the 6th grade Male Domain scale was positively correlated with mathematics achievement each year for females.

In an attempt to examine change in mathematics achievement over time new variables were created by finding the difference between z-scores for mathematics achievement for each of the six combinations of grade levels used for the regressions [6–8, 6–10, 6–12, 8–10, 8–12, 10–12]. Correlations for these new variables with the cognitive and affective variables by gender revealed few significant relationships. The only significant positive correlation between any of change variables and any of the cognitive or affective variables for females was between the spatial orientation test (grade 10) and the z-score change in mathematics achievement between 10th and 12th grade. This correlation was 0.45 ($p < 0.01$). For males the only significant positive correlation was between z-score change from 8th to 12th grade and the influence of the teacher scale grade 8 ($r=0.40$, $p < 0.05$) [complete information about this analysis is available from the first author].

The validity of the normality assumption was checked for each of the regression models with normal probability plots by gender for each grade level. In the regression analyses, one female was found to be an outlier in three of the equations, all with mathematics achievement as an independent variable and involving twelfth grade mathematics achievement. Removing that student from the analysis did not change the pattern of results. Indeed, results of the Cook's Distance test for each regression analysis were non-significant for each grade level by gender, implying that no outlier was found which would impact the models.

3. DISCUSSION

The overall picture described in this study supports several results found in many other studies over the years.

- Prior mathematics performance is the strongest single predictor of future mathematics achievement.
- Confidence in learning mathematics is the affective variable most consistently related to mathematics achievement.
- Males tend to stereotype mathematics as a male domain more than females do.
- Spatial visualization and verbal skills are related to mathematics achievement.

This study identified consistent gender differences in the roles of spatial and verbal skills in predicting mathematics achievement. Many have believed that male superiority in spatial visualization skill has contributed to the gender differences found in mathematics achievement. Although mathematics achievement and spatial visualization skill were found to be positively correlated for both girls and boys, there was no evidence from this study that spatial visualization skill significantly contributed to mathematics achievement for males.

The approach to the statistical analyses for this study, to create multiple regression equations from several to other points in time was nontraditional. Generally the process of analyzing the results of longitudinal studies has been to create one regression equation from one point to another point in time. There are well identified limitations in attempting to rely upon one regression equation developed this way. However, the gender patterns found for the regression equations to predict mathematics achievement in this study are consistent and intriguing. It is difficult to believe that the consistent pattern of prediction by gender happened completely by chance. In addition, each testing was separated by two years, a long time in the life and experiences of an adolescent. Although mathematics achievement tests of the type used do not vary widely, it is important to note that the four mathematics achievement tests came from three different publishers and were adjusted to be grade level appropriate for each testing. The mathematics content that is included in a standardized test at the 6th grade level is quite different from the mathematics content included at the 12th grade level. The four vocabulary tests were also different by grade level. In addition, there were no significant gender differences in the distributions for spatial visualization or verbal skills, as measured by the means and the variances, or for the correlations among spatial visualization skill, verbal skill and mathematics achievement. Yet this consistent pattern remains: spatial visualization skill was not a part of the predictive regression equations for males for any year of the study, but verbal skill was included. In contrast, spatial visualization appeared to be an important predictive variable for females, since it was consistently included in the regression equations, and verbal skill was not included in the predictive pattern for females. The findings from this study may force the adjustment of our assumptions about the contributions of spatial and verbal skills to mathematics achievement for girls and boys.

The results from this study also suggested that the cognitive variables were more consistently related to mathematics achievement than were the affective factors. However, the only consistent pattern of gender differences between means for all measures given in this study was for stereotyping mathematics as a Male Domain. Males consistently gender stereotyped the study of mathematics more than the females did. The Male Domain scale only correlated with mathematics achievement for females. Since this scale measures the degree to which the study of mathematics is perceived to be appropriate only for males, it is reasonable that the extent of, or lack of sex-role congruency would only relate to females' achievement. There was little change in the means for this scale for either gender between grades 6 and 8 or between grades 10 and 12 (see Table III). However, both groups appeared to stereotyped less in 10th grade than they did in 8th. It is possible that the period of time between grades 8 and 10, when students move from middle to high school, is a critical time for students to modify their views about whether mathematics is as appropriate for girls to learn as it is for boys.

Perhaps surprisingly, since much of the research concerning affective variables has focused on their relationship to mathematics achievement for females, affective variables appeared to be more predictive of mathematics achievement for males than for females. This relationship appeared to be stronger when the students were younger than when they were in the last years of high school.

Spatial orientation skill also appeared to play a role in the gender-difference patterns that emerged from this study. Spatial orientation skill was positively correlated with mathematics achievement only for females. This correlational relationship was reflected in the regression analyses. There were strong correlations among spatial visualization skill, verbal skill and mathematics achievement. Those correlational patterns coupled with the positive correlation found between spatial orientation (grade 10) and the z-score change in mathematics achievement between grades 10 and 12 and the fact that spatial orientation skill was included in the stepwise regression of 10th grade variables on 12th grade mathematics achievement suggest that spatial orientation contributed something unique to the prediction of mathematics achievement for females.

Spatial orientation was positively correlated with verbal skill for females but not males in our study. The spatial orientation test, the Gestalt Completion Test, is not usually described as having much, if any, verbal component. We could conclude that verbal skill, as measured by vocabulary tests, contributes to success on that spatial test for girls; or that the spatial test contributes to learning vocabulary. However, it is also possible that the group who scores low on one test happens to be low on the other test as well. It may be that there is a tendency for those skills to be developed together more for girls than for boys. We do not have enough information to know.

Prediction through regression analyses and correlations do not necessarily imply causation or direct contribution. Large scale multiple choice testing can restrict the types of information gained about groups of people. Taking these limitations into consideration, we can state that, at present, spatial skills may play a role in identifying gender differences in mathematics only in that certain spatial skills predict achievement in mathematics for girls. This study did not find consistent gender differences in levels of mathematics achievement. Spatial skills did not predict achievement in mathematics for boys in this study. Spatial orientation skill was not correlated with mathematics achievement for boys. Other studies have found that boys who score low on spatial tests often do as well or better on mathematics tasks as boys who score high on spatial tests (Tartre, 1990). One compensating skill that low spatial orientation boys demonstrated was making use of verbal hints given to them much more often than the low spatial girls (Tartre, 1990). The positive correlation found for males between z-score change in mathematics achievement (grade 8 to 12) and the influence of the Teacher (grade 8) also may be related to making good use of help.

The content and format of the mathematics tests determined the foundation for identifying the relationships among these cognitive and affective skills and attitudes in this study. Many scholars are engaged in productive discussions about the relative merits of traditional standardized mathematics achievement tests in measuring different kinds of learning outcomes in mathematics. Standardized testing of large groups of people for research purposes has helped to identify the possible contributing factors, very much like a wide angle lens allows us to recognize various elements in a landscape.

Early research identified gender differences in mathematics achievement and spatial skills and correlations between them through mass standardized testing. One result of that research has been a rush to teach spatial activities because of the belief that those skills helped boys achieve in mathematics. Although these changes in the curriculum have been well-intentioned, the foundation for that change needs to be re-thought. Even the "wide angle" approach of the current study has shown that spatial skills are not necessarily related to mathematics achievement for boys. It is premature at best, and perhaps even damaging to people, to indicate that we should teach spatial relationships to help eradicate gender differences in mathematics. That idea is based on the assumption that there are gender differences and helps prepetuate the stereotyping of these activities by gender. In addition, relating success on mathematics tasks to perceived expertise on space-related activities may impact the expectations that teachers and students have about mathematics learning for girls and boys.

Some mathematics relationships are inherently spatial in nature and need to be included in the curriculum because they are important for people to understand. However, many of those skills are not measured on spatial tests. There may not be any gender difference in learning about those spatial mathematics relationships.

Longitudinal studies can provide information about a piece of the puzzle of improving mathematics learning in a way that cross-sectional studies of students at different ages cannot. However, large scale longitudinal studies of the duration of this study are extremely difficult to accomplish, particularly in our increasingly mobile world. And yet, learning about patterns from year to year is necessary to identify the factors that contribute to mathematics. The next step in the chain of inquiry about these cognitive and affective skills and attitudes and mathematics learning is the "zoom lens" approach to examine and understand how these skills and attitudes may be used in doing mathematics.

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