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HEDGES IN MATHEMATICS TALK: LINGUISTIC POINTERS TO UNCERTAINTY

ABSTRACT. Analysis of transcripts of interviews with children aged 10 to 12, focused on a mathematical task designed to provoke prediction and generalisation, reveals a category of words (called hedges) associated with uncertainty. It is argued that these words – examples include *about, around, maybe, think* – are frequently deployed as a 'Shield' against accusation of error. The analysis draws on linguistic frameworks for categorising types of hedge, and for a theoretical account of how they might succeed in conveying uncertainty to listeners.

1. INTRODUCTION

Mathematics, viewed as a field of human endeavour, as opposed to an inert body of knowledge, offers considerable potential for intellectual risk-taking. The subject holds an invidious reputation within the school curriculum, being associated with fear of error and consequent public humiliation. One common perception is that mathematical propositions are either right or wrong; that the questions teachers ask are not searchlights focused to reveal truth, but traps set to expose ignorance. Janet Ainley, studying children's perceptions of the purposes of teachers' questions, calls such uses "testing questions":

> Because testing questions are so common, particularly in mathematics where answers are seen as being clearly 'right' or 'wrong', there is a danger that pupils may perceive all teacher questions in this way. Such a perception would inevitably be detrimental to attempts to encourage discussion, investigative work or problem solving in mathematics: pupils will feel that the teacher always knows the 'right' answers to any questions she asks, and furthermore that the teacher is always judging pupils by the answers they give. It is not surprising that pupils are reluctant to risk giving 'wrong' answers in these circumstances (Ainley, 1988, 93–94).

From a broader perspective of social discourse, Labov (1970) commented that a question is normally deemed appropriate only when the enquirer meets certain sincerity conditions: that s/he doesn't know the answer, would like to know it, and has reason to believe the hearer is able to supply it. Labov shows that questions in classroom situations are exempted

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from these rules, and that the conditions governing appropriateness in the answers to such questions differ accordingly.

Laurie Buxton has addressed the right/wrong issue, not so much from the viewpoint of what mathematics is and what mathematical activity could be, as from the emotional perspective of the learner sensing the prospect of failure:

> Most classroom maths sets tasks, often with very clearly defined goals; whether they have been reached or not is seldom in doubt ... This clarity tends to enhance the sharpness of emotional response. There is a nakedness about the success or failure in reaching a goal that evokes clearly defined emotions whose nature one cannot disguise to oneself (Buxton, 1981, p. 59).

Whilst Buxton speaks of nakedness, the word that comes to me is vulnerability. Extrinsic and intrinsic sanctions are associated with being wrong; it follows that there is a high premium attached to being right, with insufficient acknowledgement by pupils and teachers that *uncertainty* is a valid, indeed an honest and honourable, state to be in. One could go further, and insist that uncertainty is a productive state, and a necessary precondition for learning. For once we believe we "know", we are no longer open to the possibility of further knowing. When mathematics is coming into being in the awareness of an individual, uncertainty is to be anticipated and expected. Writing about the place of conjecturing in mathematical activity, John Mason describes the qualities of what he calls a 'conjecturing atmosphere'. He advises

> ... let it be the group task to encourage those who are *unsure* to be the ones to speak first ... every utterance is treated as a *modifiable conjecture!* (Mason, 1988, p. 9; his emphasis).

Thus in the making and learning of mathematics, uncertainty is to be expected, acknowledged and explicit.

In this paper I shall describe and analyse some ways in which uncertainty is coded in language, with reference to a mathematical study carried out with children aged between 10 and 12. I chose to work with them on a task which required them (amongst other things) to predict, generalise and explain. Prediction can be viewed as a specialised form of generalisation. Each feeds on the other, each is both parent and child, although predictions are (generally) more straightforward to articulate since they require fewer quantifiers. But the articulation, the assertion of the predicting or generalising insight – making public what it is that one has "seen" – can be shot through with uncertainty until it meets with approval.

In this paper, a class – called hedges – of linguistic pointers to such moments of uncertainty will be identified. My aim is to draw attention to the presence and to analyse the meaning of such hedges in pupils' mathematical discourse.

2. METHOD

The study described arose from my interest in the language that children (specifically those in the 9 to 12 age range) use to invoke, describe and engage with the mathematical process of generalisation. In an earlier article (Rowland, 1992) I have reported how one articulate 9-year-old girl, Susie, deployed the pronoun *it* for deictic purposes, to refer to concepts and generalisations for which she had no name, or for which no received name existed. In this study I moved from extended case study to replication of an "experiment" with a number of children. Intending to diminish the part I played in the discussion, I worked with ten pairs of children aged 10 or 11, for 30–40 minutes with each pair, and encouraged some peer interaction on a common mathematical task. I did, however, remain as a participant, as opposed to a passive observer, originally so as to maximise their engagement with the mathematical task. Contrary to my original intentions, I ended up reflecting on the pupil-teacher interaction too. Cobb *et al.* (1992) provide a brief but pertinent discussion on the teacher's role in classroom conversations in which he or she participates, appearing inevitably as an authority figure, yet having come to that interaction with commitment to a constructivist theory of knowing.

> One feature of the teacher's active and demanding role is therefore to facilitate mathematical discussions between students while at the same time acting as a participant who can legitimise certain aspects of their mathematical activity and sanction others. In doing so, the teacher ideally provides a running commentary on the students' constructive activities from his or her vantage point as an accultured member of the wider community ... in a communicative context that involves the explicit negotiation of mathematical meanings (1992, 102).

The mathematical encounter with each pair of children was standardised to the extent that I initiated each interview with the same combinatorial problem, which I call "Make Ten". This task will be familiar to elementary school teachers, although its potential as a starter for generalisation may not. It begins with consideration of the number of ways that 10 can be "made" as a sum of two "numbers". I did not work from a "script", but the following opening (with two boys, Jubair and Shofiqur) is typical.

Tim: Jubair, I'd like you to give me any two numbers that add up to ten.
Jubair: Six add four.
Tim: Six add four. Shofiqur?
Shofiqur: Eight add two.
Jubair: Five add five.
Tim: OK, so you get the idea. Now what I want you to decide between you is how many ways is it possible to do that?

The next three exchanges were not at all typical, however.

Shofiqur: [almost instantly] Nine ways.
Tim: Nine ways.
Jubair: No, ten ways.

It was more usual for the children to list the possible sums, orally or on paper, and then to count them. Then I would say something like

Now just as you eventually decided about that question for ten, I'd like you to decide between you how many different ways are there of doing that for twenty?

The first phase of the interview continued with similar examples of listing sums and counting how many had been found. I would propose the numbers to be "made" in this way, my choice depending on the children's earlier responses to my questions about "making" 10 and 20 – in particular, on the facility they displayed and whether reversals such as $2+8$ and $8+2$ were both counted [note 1 summarises the mathematical consequences of such choices]. The next phase would then involve my proposing a further target number – say 30, 50 or even 100 – slightly out of the range of those already counted, and inviting a *prediction* of the number of ways this number could be made.

Tim: OK. Now a funny question. Suppose the number now that I'm interested in is twenty-three? So you've got twenty-one add two and so on. How many ways?
Caroline: Twelve, would it?
Anna: Yeah, probably.
Caroline: Twelve.
Tim: What makes you think twelve Caroline?

Subsequent episodes, contingent on preceding ones, would involve my probing for the thinking behind this prediction (and possibly others) and discussion of perceived "rules" – conjectures about what might happen with "any" number. For example:

Tim: Right, OK. Is there a kind of rule that you could state generally, I mean supposing I now picked out any number ... you know like five hundred and thirty-seven or something ... and said how many ways can you make that from adding two numbers. How would you know what the answer was?
Alan: Just take away one, and then you'll know how many you can get. It's the same here, ten, there was nine possibilities, twenty, there was nineteen possibilities, thirty-seven there was thirty-six possibilities.

In some cases we continued to test the generality of such conjectures, and tried to see why they might be true "in general". In practice, such proofs were always founded on the possibility of "seeing the general in the particular" (Mason and Pimm, 1984), producing confident awareness of how things would be for any other particular, as it were. An example of this (a lengthier quotation from Alan and his partner Harry) is given at the end of this article.

The interview technique deployed is a variant of Piaget's clinical method which Ginsburg (1981) labels 'contingent questioning', and which Ginsburg proposes as appropriate for the purposes of identifying and describing (and hopefully explicating) complex cognitive processes. In this case, such processes potentially include prediction, generalisation, explanation. The technique is a clinical interview procedure which

- employs a task to channel the subject's activity;
- demands reflection;
- is such that the interviewer's questions are dependent on the child's responses;
- employes basic features on the experimental method;
- has some degree of standardisation.

The conversations were audio-taped and transcribed, providing a corpus of some 24,000 words (both the children's and mine).

3. 'MAKE TEN': FRANCES AND ISHKA

The following extracts give a fuller picture of how conversations arising from the task developed. They are from an interview with two girls, Frances and Ishka, both about $10\frac{1}{2}$ years old. The interview transcript separates into five episodes A to E, corresponding to the phases described in the previous section. The first three episodes are summarised and illustrated by the following extracts.

EPISODE A [$7\frac{1}{2}$ minutes] In this phase I introduce the problem, that of making 10. The children list and decide five ways, allowing no reversals (decided by Frances).

Frances: Four and six, five and five, six and ... oh that's the same.
Ishka: Five ways?
Frances: Maybe.
Ishka: Mm, maybe ... I think ... [...]
Frances: Shall we just say five ways?
Ishka: There's about five.

I then ask about making 20. They list and decide ten ways. Finally, I ask about making 13, Ishka lists and decides six ways.

EPISODE B [2 minutes] I begin by asking them to recall results so far. Then I ask about making 30, inviting an initial prediction. Frances predicts 15, Ishka agrees, and explains how her prediction relates to the earlier results. There is an air of plausibility rather than certainty in their attitude to Frances' prediction.

> Ishka: I think there'll be around ...
> Frances: Fifteen?
> Ishka: Yup.
> Frances: Maybe? [...]
> Ishka: Most of them are half or just about one away from ... [...]
> Tim: ... let's just go back to what Ishka was saying. She was saying that in most cases it's about half.
> Ishka: Well, yes, 'cause ten was five. [...]
> { Frances: and thirteen was about six.
> { Ishka: but, erm thirteen was six.
> Tim: OK.
> Ishka: Although that isn't exactly half.

I proceed to ask about making 100. Frances instantly answers "Fifty?", Ishka agrees in a vague way, but conveys considerable uncertainty when pressed to commit herself to Frances' prediction.

> Frances: Fifty?
> Ishka: About fifty yeah.
> Tim: About fifty [...] do you really think it is fifty?
> Ishka: Well maybe not exactly, but it's around fifty basically. [...]
> Frances: Maybe around fifty.

EPISODE C [$3\frac{1}{2}$ minutes] I return to making 30 – how sure are they that there are 15 ways?

> Ishka: It's fifteen or around.
> Frances: Yes.
> Ishka: 'cause we can't be exactly sure until we've tried it, but ...

The girls list and count 15 ways.

[The remaining Episodes D and E, seeking rules and explanations, last $5\frac{1}{2}$ minutes.]

4. HEDGES AND UNCERTAINTY

In time, and through discussion with others, I have become convinced of the significance of one particular surface-level feature of the data. This is the children's use of a category of words called (by the linguist George

Lakoff) **hedges** – examples of which include *about, around, maybe, think, normally, suppose, (not) sure, (not) exactly*. The extracts from the interview with Frances and Ishka give an authentic and reasonably sequential picture of when and how they are used. Such words convey a sense of vagueness, at times of uncertainty – a state of mind which, as I have already observed, one would expect to prevail in a conjecturing moment. I shall also have something to say about the way that I too (as teacher/interviewer) use hedges in the discourse.

Whilst the focus of this paper is on the use of hedges to convey uncertainty in prediction and conjecturing activity, the use of "rounders" (a subset of hedges usually associated with lack of precision) such as 'about', 'approximately' and so on is to be expected as a linguistic feature of *estimation*, as a device to indicate that the speaker is providing as much accuracy as is possible or appropriate in a given situation (Channell, 1994). Indeed, Freudenthal (1978, pp. 259–60) points out the need for pupils to acquire judgement to distinguish between two worlds: "the world where precision is a virtue, and the other where it is a vice, and ... to be at home in both of them." For a study providing data and insight into the growth of that judgement and its associated language, see Rowland (unpublished). I am not saying that hedging in estimation activity is always mature and desirable, but do wish to claim that in prediction and generalisation it may be evidence of undesirable anxiety. My point is that estimation, prediction and generalisation are all mathematical processes which, to a degree, involve some element of uncertainty (in fact Clayton, 1992, has studied estimation as a "risk taking" activity) and that children may convey this uncertainty with various degrees of subtlety, and with various pragmatic purposes, through the use of hedges. Whether or not fear, anxiety and so on are present in those situations must depend on the spirit in which the mathematics learning takes place. This is to some extent determined and controlled by the teacher – by the way that s/he responds (language of word and body) to pupil's contributions. The willingness of schoolchildren to expose their thinking will depend on whether or not teacher and pupil share a belief or explicit agreement that they are working in a 'conjecturing atmosphere'.

5. A TAXONOMY OF HEDGES

The study of hedges as a linguistic phenomenon has given rise to a literature which, though not extensive, is illuminating. Lakoff (1972) defined a hedge to be "a word or phrase whose job it is to make things fuzzier", and took on the challenge of exact semantic interpretation of hedges. His solution was to apply the (then) novel logic of fuzzy sets (Zadeh, 1965) to develop a corresponding 'fuzzy semantics' allowing degrees of truth between 0 and 1. More recently, a number of other writers (see Channell 1990, 1994, for details) have approached the analysis of meaning of vague language using the linguistic tools associated with Pragmatics – the study of language from the user's point of view. Such an analysis of hedges, and how they work, will be more concerned with the goals and purposes which lie behind vague utterances and the meanings inferred by hearers than with their classical semantic content as assured by conventional logic.

It is important to be aware that not all hedges do the same job. The case for this observation was initially made in a study (Prince *et al.*, 1982) of paediatric clinicians, whose spoken language in case-conferences turned out to be unusually rich in hedging; for example

There is evidence that's been presented that makes me think that it might be a little risky (1982, 85).

To elucidate the ways (identified by Prince *et al.*) that different hedges work, I shall introduce some categories of hedges and illustrate them by reference to some examples of my own using mathematical language, including examples from the Make Ten transcript data.

The first major type of hedge – a **SHIELD** – is identifiable as a fuzzy prelude, such as *I think that.* The essential characteristic of a Shield is that it lies outside the proposition which follows it, which may be unequivocal. For example the sentence

[C1] Maybe the quadrilateral ABCD is a square [see note 2]

invests all the vagueness in the speaker's uncertainty, as opposed to any possible degree of lack of square-ness of the quadrilateral. The speaker is asserting a proposition (call it S):

The quadrilateral ABCD is a square (S)

S is thus made available to others, who may then consider whether or not it is true, and may act on it – to claim symmetry, for example. The effect of the hedged assertion

Maybe S

is to distance the speaker from S without modifying S itself. In mathematical discourse, such a hedge presents a mathematical assertion in the form of a conjecture, and implicitly invites comment on that conjecture.

Prince *et al.* subdivide Shields into two kinds. The first of these is termed a **Plausibility Shield**, typified by *I think, maybe* and *probably*, as in this selection from the episodic overview of the Frances/Ishka Make Ten interview.

[M1] Ishka: Five ways?
[M2] Frances: *Maybe.*
[M3] Ishka: Mm, *maybe ... I think ...*

A Plausibility Shield 'implicates' (i.e. infers, by a mechanism to be discussed later in this paper) a position held, a belief to be considered – as well as indicating some doubt that it will be fulfilled by events, or stand up to evidential scrutiny.

The second kind, an **Attribution Shield**, implicates some degree, or quality, of knowledge to a *third party*. Examples include *"According to N, S"* (where N is a third party and S a proposition). In the Make Ten data there are relatively few Attribution Shields, and these tend to be used by me rather than the children, as a teacher-like device for meta-comment (Pimm, 1992) on the activity. Thus, with Kerry and Runa, I use Attribution *("says Runa")* as a ploy for being non-committal about the contribution of one child, in order to obscure my evaluation of her answer, and to encourage the participation of the other.

[M4] Tim: OK. Um, how many ways would there be than for twenty-four? [...]
Runa: Um, nineteen. Nineteen ways.
Tim: Nineteen ways, *says Runa.*
Runa: I just guessed.
Tim: Kerry's still thinking.
Kerry: Ten.

A second major category of hedges (**APPROXIMATORS**) includes *about* and *a little bit.* In contrast to Shields, these Approximator-hedges are located inside the proposition itself. The effect is to modify (as opposed to comment on) the proposition, making it more vague. For example, a speaker could insert the Approximator *sort of* into the proposition S above, which then becomes

[C2] The quadrilateral ABCD is a sort of square.

It is both amusing and potentially instructive to consider (in the spirit of Lakoff, 1972) how one might judge, for a given ABCD, the extent to which this sentence **C2** is "true".

Speakers make propositions vague in this way to fulfil all kinds of purposes (see Chapter 8 of Channell, 1994, for details), one of which is "giving the right amount of information" i.e. as much as is needed in the context of utterance. Thus I might tell you that the time is half past three (incidentally concealing the Approximator *about*) when my watch says 15:28:22. *About, around,* and *approximately* are examples of **Rounders**, which constitute the first subcategory of Approximator. Rounders are common in the domain of measurements, of quantitative data. They frequently occur, in the Make Ten corpus, to qualify combinatorial prediction, as in Episode A with Frances and Ishka:

[M5] Frances: Shall we just say five ways?
Ishka: There's *about* five.

and again, in Episode B

[M6] Ishka: I think there'll be *around* ...
Frances: Fifteen?
Ishka: Yup.

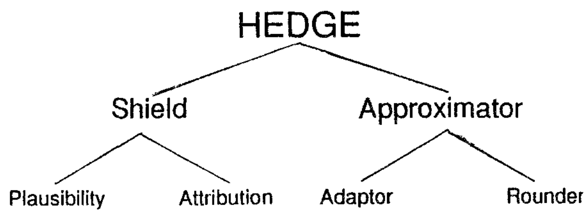
The second type of Approximator is called an **Adaptor**. These words or phrases such as *a little bit, somewhat, fairly*, attach vagueness to nouns, verbs or adjectives. The following examples are from the Make Ten interview with Jubair and Shofiqur. Shofiqur has just indicated what a list of ways of "making" 20 would look like, and predicts 21 different ways.

Shofiqur: It's *just a bit* the same, like this [indicating the list for making 10]
Tim: So Shofiqur is *pretty* convinced that it's twenty-one. Right! Are you persuaded by his argument?
Jubair: Not really.
Tim: Have a go at – I'm *fairly* convinced by what you said Shofiqur, have a go at convincing Jubair that there are twenty-one ways. I mean, take it slowly.

These Adaptors exemplify the hedges which are the subject of Lakoff's semantic work on fuzzy language, where the issue is class membership. Adaptors suggest, but do not define, the extension of categories, concepts and so on (see how I just did it with 'and so on'). Thus Shofiqur uses an Adaptor phrase *just a bit* with respect to same(ness); I use two Adaptors, *pretty* and *fairly*, to suggest, first that Shofiqur's conviction, then mine, is not simple and unreserved, but of a fuzzy kind.

A sift of the transcripts suggests that it is I, rather than the children, who make most use of Adaptors. Like Attribution Shields, and for similar reasons, I use them as a means of commenting on the children's contributions. Specifically, to make indirect comments on their predictions, generalisations and explanations.

The analysis of Prince *et al.* can be summarised as follows, in a taxonomy of hedges in the form of a binary tree:



Now we are ready to consider when and why hedges are deployed when people talk about mathematics.

6. CONVERSATIONAL IMPLICATURE

The taxonomy provides a setting for studying the significance of the hedges used in my Make Ten interviews. The framework is useful in making distinctions and providing starting points. Whilst the four categories of hedges are complete (in the sense that they embrace the hedges in my data), they are not disjoint. It is easy to fall into the trap of superficial, over-simplified attribution of purposes to speakers on the basis of vocabulary alone, as though it were possible for words to *mean* with consistency, divorced from context. For example, it seems at first that *about* is always an Approximator, as in

he must weigh about 180 pounds

and therefore that one can set up mechanical word-searches for Approximators *about, approximately, around, n or so,* and so on. The naivety of such a programme is exposed by

[M7] Tim: I would like you two to decide *about* that, I'm happy to go along with whatever you decide.

in reply to Harry's enquiry as to whether reversals of sums are "allowed". My use of *about* in **M7** is not an Approximator at all, nor indeed any other kind of hedge.

There is a good case, I believe, for speaking of Shielding and Approximating, to emphasis the effect of hedges in the context of use, as opposed to the identification of some rigid lexical categories (such as "*about* is always a Rounder").

With this in mind, I shall draw on the notion of conversational implicature, due to the philosopher Paul Grice, in order to give a possible theoretical account for how hedging succeeds (in these mathematical conversations) in conveying various kinds of uncertainty. Since our understanding of the relation between thought and the production of language is, to put it mildly, partial, we are forced into surmising about thinking, which we cannot perceive, by inference from language, which we can. How, then, are our inferential processes to be guided?

Grice (1975, 1989) has proposed that ordinary conversation is posited on a **cooperative principle** and four **maxims of conversation**. These maxims specify what participants need to do in order to converse rationally and cooperatively. The requirements are, essentially:

- maxim of Quality: let your contribution be truthful;
- maxim of Quantity: let your contribution be informative but not too informative;
- maxim of Manner: let your contribution be clearly expressed – e.g. be brief, orderly, unambiguous;
- maxim of Relevance: let your contribution be relevant to the matter in hand.

Now it is evidently *not* the case that all participants in all conversations observe all of these four maxims in all contributions. More often than not, this has nothing to do with intention to lie or mislead (in which case the participation could not be deemed cooperative). How, then, do hearers make sense of speech which is, for example, superficially false or irrelevant? Viewed from the other side of the coin, how do speakers successfully violate the maxims in order to communicate fine nuances of meaning, to enable the hearer to read "between the lines" as it were?

The genius of Grice's theory is the following recognition. Whilst speakers do not always observe the maxims at the surface level, nevertheless, hearers interpret the contributions of other participants in conversation as *if they were intended to* observe the maxims at some level of meaning other than that contained in the semantic content of the utterance. This has proved to be a robust theory, concise yet surprisingly complete, with a wide field of application, finding resonance with common sense and experience. For Grice describes and explains what we all know – that "communication involves the publication and recognition of intentions" (Sperber and Wilson, 1988, p. 24).

Such a view of communication underpins a means of pragmatic inference identified by Grice, and which he named 'conversational implicature'. I shall show this means of inference in action in mathematical conversation. First, to clarify the meaning and process of implicature, consider the following exchange, adapted from Levinson (1983, p. 126).

[C3] Teacher: Why haven't you brought your calculator to my lesson?
[C4] Pupil: My brother has a maths exam today.

The pupil's reply, taken literally, is irrelevant to the teacher's question. Indeed, the pupil appears to be flouting the maxim of Relevance. We could interpret the pupil's contribution to be simply non-cooperative, failing to address the teacher's enquiry about the missing calculator. In practice, we interpret the exchange as cooperative at some level, albeit not a superficial one, and so we infer, from the ostensibly irrelevant **C4** that

[C5] my brother has my calculator, because he needs it for his exam.

The inference is an example of a (conversational) *implicature*, and we say that **C4** implicates the conclusion **C5**. Thus, the human tendency is to accept Grice's theory as an accurate insight, one that exposes and codifies that which we "knew", but had not yet isolated.

It is not my intention, here, to discuss the mechanics of pragmatic inference i.e. how implicatures might be "calculated". Suffice to remark that the hearer brings his or her *cognitive environment* (Sperber and Wilson, 1986) to bear on the situation, for the purpose of interpretation. This 'environment' consists of a (very large) set of facts which are *manifest (ibid)* to individuals by knowledge or by assumption (including, for example, the materials needed by candidates in a maths exam). What matters here is that pragmatic implicature is different from logical implication, in that the inferred conclusion – **C5** in the example – *cannot be obtained solely from what has actually been uttered* by application of a syllogism, or other process of logical deduction.

For a further example, immediately relevant to the theme of this paper, consider the sentence

[C6] Maybe I'll come and visit you next week

which flouts the maxim of Quantity (also that of Manner) and thereby implicates

[C7] I may fail to come to see you next week

because, if I were firm in my desire and intent to come, I would have said so.

In practice, speakers tend to preface contributions like **C4** with a discourse particle such as *Well*, to indicate some sort of insufficiency in the answer to be given (Lakoff, 1973). In effect, *well* then acts as "maxim hedge", in this case a relevance hedge (Brockway, 1981). That is to say, this prefatory use of *well*, considered against the backdrop of Grice's maxims, can usually be argued to attach some vagueness to the speaker's compliance with one or more of the maxims. The speaker is serving notice to the hearer that the contribution about to come will fall short of the normal cooperative standards. Indeed, this device is not uncommon in my Make Ten data:

Tim: Thirty-nine ... why, how do you know that?
Susan: *Well,* you've got the, you've got your, let me see, nineteen ways, and then you've got another set of nineteen ways going the other way.

It could be argued that Susan can foresee rather a rambling account ahead, likely to violate the maxim of Manner ... See also the Fraces/Ishka Episode B, and also look ahead to **M27** for further evidence.

7. APPLICATION: VAGUE MATHS-TALK

The children whom I interviewed were aged 10 or 11, and were being invited to make mathematical predictions and generalisations. With reference to the Make Ten transcript data, my central claim will be that when they hedge, it is more often than not in order to implicate uncertainty of one kind of another. In other words, their hedges are nearly all Plausibility Shields first and foremost, although they may at the same time have other relevant functions. Later I shall suggest that Shields are deployed at significant and identifiable stages in the interviews. Furthermore I will show that the teacher/interviewer (me) also hedges, but for different purposes. These teacher-like purposes – to which intentions, in this case, I have direct access – will be considered from time to time.

For the sake of maintaining coherence in the argument, whilst sampling from the data, I shall examine when and how particular hedges, or small groups of hedges, are used.

7.1. *'maybe', 'think'*

I have already observed that *maybe* and *think* are stereotypic Plausibility Shields (bearing in mind my earlier semantic *caveat,* distinguishing Shield from Shielding) which can successfully convey a speaker's lack of full commitment to a proposition under consideration. It is necessary here to give more detail from Episode A of the Frances/Ishka interview, for immediate and future reference. I had asked the two girls to come to an agreement about the number of ways of making 10. Their discussion proceeds:

Frances: There's one and nine.
Ishka: Yeah.
Frances: So that's one. Two and eight ... and then there's

Frances and Ishka: Three and seven.
Frances: Four and six, five and five, six and ... oh that's the same.
[M1] Ishka: Five ways?
[M2] Frances: Maybe.
[M3] Ishka: Mm, maybe ... I think ...
Frances: What do you think?
Ishka: We haven't had five have we?
Frances: We have!
Ishka: Oh, OK, erm ...
[M8] Frances: The others are like, if you do six four, we've already done four six.
Ishka: Mm [sighs]
Frances: Shall we just say five ways?
[M9] Ishka: There's about five.
Tim: Erm, I'd like you to be more convinced Ishka. I mean if it's about five then it's four or six or seven or whatever ... the number's sufficiently small that I think you should be sure one way or another.
Frances: I think it's five ways.
Ishka: But I'm sure.
Tim: You are sure.
Frances: Me too.

Having enumerated five ways, Frances begins to repeat herself ("oh ... that's the same"). Rather, she offers me (and Ishka) the first insight into what *sameness* means to her in this context. She has an implicit criterion, which surfaces when she withdraws "six and ...". Ishka shares or accepts the view that reversals will not count separately, and she makes the tentative claim **M1**, hedged with rising intonation. Frances (**M2**) perhaps responds to Ishka's uncertainty; or perhaps she may feel that Ishka's answer is offered prematurely, before she has exhausted all the pairs she can bring to mind. In any case the pair now seem to have an understanding that it will be productive to assert their uncertainty, and reconsider the "five ways" claim. Ishka effectively conveys this (**M3**) in the form of two Shields without a substantive proposition. Frances encourages here ("What do you think?") to articulate her position – this is typical of a number of instances of apparent teacher-like behaviour by Frances in Episode A, at which phase of the interview she projects herself as the dominant, more confident partner. However, having encouraged Ishka, she is impatient at Ishka's next contribution, which only suggests that Ishka has forgotten what has already been listed. In fact, Frances indicates (**M8**) that she is now satisfied that no further possibilities have been overlooked [note 3]. There follows an apparent reversal of the earlier roles (**M1, M2**) of Ishka and Frances in relation to the claim that there are five ways. At the time, however ("I'd like you to be more convinced, Ishka") I inferred that Frances was fully committed to the claim, and was seeking Ishka's assent to it. I had, after all, introduced the dialogue cited above with a clear request for common consent:

Tim: I'd like you two to agree between you ... incidentally we'll adjust that [microphone] Frances so it's not quite so close, right, um I'd like you just to – yours is fine Ishka – I'd like you two to agree between you, how many different ways there are of doing that. Right? Two numbers that add to ten, and I'll just be quiet for a moment.

My repeated request for agreement is complied with by Frances and Ishka to a remarkable degree, certainly in comparison with most of the pairs I interviewed for Make Ten. Ishka is not yet, however, prepared to concede unqualified agreement (**M9**). The function of her chosen hedge, *about*, will be considered later in this paper. In any case she has successfully implicated the fragility of her commitment, borne out by the fact that I press her quite explicitly on the matter of being (more) convinced, urging that she "should be sure". Frances responds with an apparently hedged (but see discussion of the ambiguity of *I think* later) indication of where she stands. Ishka's response is unhedged, fully committed – but is it genuine, or have I blackmailed her into renouncing doubt in order to please me? After all, what I have demanded is not "the answer" but for Ishka to be "more convinced". Of course, I really wanted both. I am at this stage wanting valid instances of a generalisation-in-waiting. That there are five ways of making ten is such an instance. I readily accept Ishka's assurance that she is "sure" without comment as to whether or not she is right.

7.1.1. *Maybe*

Is a modal from which seems to be user-friendly, in that it is favoured by the children in comparison with the apparently synonymous *perhaps* and *possibly*, which occur not once in the whole corpus. The following transcript data illustrates the appearance of *maybe* within hedged **predictive** statements:

Tim: Alright um, supposing ... we've done, we've done ten, twenty, thirty, sixteen [...] I'd just like you to sort of say how many you think there would be, say for the number twenty-four. [...]
Rebecca: No, that was ten, twenty-two? No, not twenty-two ways. Twelve ways? [...] On the twenty there were more ways than the sixteen, so twenty-four ... must be more than the twenty, because that was less, because it was a lower number.
Tim: Right, how many more?
Rebecca: I'm not sure. [pause]
Runi: [whispers] Eleven and twelve, [inaudible, presumably "forty"] ways.
Rebecca: Not forty, fourteen.
Runi: Yeah, that's what I was going to say
[M10] Tim: Let's just see. Runi thinks **maybe** fourteen ways, and I think you suggested twelve Rebecca, yeah?
Rebecca: Yeah.

Tim: Um, what was your reason for suggesting twelve?

[M11] Rebecca: Well it was four off than twenty, and then twenty-two [?twenty ... two] was two less than four so you've twelve. Have twelve because, if you had ... twenty had ten ways and twenty-four was four more than twenty, then **maybe** it would be twelve because it's um ... half way in between.

Tim: 'cos it's half way in between. OK. And what do you think Runi, are you saying it's four more, so it's four more ways?

Runi: Yeah, that's what I was thinking of.

When Rebecca explains her reason for suggesting twelve (as I put it), she seems to be reasoning that what happened in the increase from sixteen to twenty might happen again with a further increase to twenty-four. But she signals an awareness that she might be jumping to conclusions by hedging (**M11**) "maybe it would be twelve". It is an honest and straightforward expression of doubt, as to the validity of the reasoning and the conclusion. By contrast, I double-hedge in (**M10**) "Runi thinks maybe ...' as a device to cast doubt on Runi's unhedged – and incorrect – contribution ("fourteen ways") which is beginning to take over from Rebecca's interrupted – but correct – train of thought. We are some way into the interview, this is the fifth example I've asked them to consider, and I'm getting impatient. My reaction to Runi's off-course prediciton is to undermine it by attributing doubt where there may have been one. Thus (M10)

Runi *thinks* ...

is intended to implicate "but that's only what Runi thinks" and furthermore

Runi thinks *maybe* fourteen

was intended (now I think about it) to convey "even though Runi said fourteen, she wasn't really sure about it, and you shouldn't be either".

7.1.2. *Think*

(usually *I think*) is, in some respects, nice and straightforward; it appears to be the most frequently-deployed hedge in my transcripts. For example, in this extract, Alex rejects my prompt to list ways of making twelve, and goes straight for a prediction. When I appear to question it, she affirms, hedges, then revises.

Tim: Any ideas about how many ways there would be say for twelve? For twelve you could have twelve ...

Alex: [instantly] Six.

Tim: Six ways?

Alex: Yeah, I think so. Seven. Twelve add zero as well.

There is, however, a potential ambiguity (Stubbs, 1986) associated with this, and with other 'private' verbs such as *believe, suppose* and so on. Let me offer an example:

[C8] You could use calculate to find the minimum, but I think that completing the square would be more elegant.

The ambiguity here concerns whether *think* is being used to implicate:

(a) an uncertainty concerning the validity of the substantive statement (whether or not completing the square would be more elegant);
(b) a firmly held position (completing the square would be more elegant), arrived at after consideration along with other tenable positions. That is, an assertion of what *I* judge to be the case.

In speech the intended force may be more evident by intonation i.e. "I *think* that ..." as opposed to "*I* think that ...".

The extracts which follow are from my interview with Anthony and Sam. Italics are used to highlight occurrences, not necessarily to indicate emphasis.

Tim: What I want you to do is to talk to each other and come to an agreement about how many different ways you can do it. OK? [...] And I'll just listen for a moment. How many different ways can you do that?
[M12] Anthony: Er, let's have a *think* ... Halves, um ...

[end of first extract]

[M13] Tim: Eleven. Is that all the ways do you *think*, or are there any more? [long pause] What do you *think* Sam? Do you *think* there's any more ways, or do you *think* that's all the possible ways of doing ...
Sam: There's more.
[M14] Tim: You *think* there's more? OK. What would, give me an idea of what another one might be, or what it might look like.
Anthony: What about ... you can put quarters into ten parts and like that can't you.
Tim: Mm ...
Anthony: Well if we put them in about nine parts ... if it, all the way, keep doing that, you might end up to number ten.
Tim: Are, right, um ...
Anthony: If you started at one.
Tim: Yeah.
Anthony: And I got a bit less, bit less, bit less, bit less, about um, slowly get to number two. Keeps going round. Yeah? Would that work?
[M15] Tim: Right, I *think* I get the idea. I mean, can you sort of get us started, and we'll try and *think* it through together. [pause] Mm hm?
[M16] Anthony: Let's have a *think*. [pause]
Tim: Did you say we were dividing it up into ten?
Anthony: Yeah.
Tim: Right. So, what could one number be?
Anthony: You could have it into eighths as well.
Tim: Uh huh.

Anthony: I've heard of eighths, um ...

The interview with these two boys was unique in failing to develop a scenario for combinatorial generalisation. Matters were not helped by the fact that Anthony had been diagnosed as having an aphasic language disorder; over the years he had become expert in manipulating adults by diverting questions or topics which he was unable to understand or cope with. The problem was compounded by the fact that I was *encouraging* the children to control the agenda (contingent questioning).

In **M12** and **M16** Anthony is 'simply' stating his intention to engage with a problem (**M12**) or task (**M16**). Actually, on the evidence of the data, this is not at all typically childlike (to announce an intention to think). It is very much the mark of the confident adult, willing to think on his or her feet, and in public. It is the response of the person – a lecturer, for example, holding forth within their specialist field – to a question for which s/he does not have an instant answer. The announcement "I shall have to think about that" has the effect of

- flattering the one who asked the question: it suggests that the answer is non-trivial;
- making space for the speaker to arrive at a response, either by recalling some information or by the exercise of reason upon available information;
- implicating uncertainty without undue discomfort.

With Anthony, however, what came across (and was later supported by information gleaned about his social strategies for coping with aphasia) was a plausible and well-used device for mimicking cooperative intellectual effort, with the intention of retaining, or even gaining, the teacher's goodwill.

7.2. *'about', 'around'*

I shall examine here the use of *about* by three different children, in two extracts from the data. The first is with Harry and Alan.

Tim: So how many ways is it Alan?
Alan: Nine.
Tim: Nine, right. [pause] What if instead of saying two numbers adding up to ten I said two numbers adding up to twenty?
[M17] Harry: That would be about, yeah I think ... that would be eighteen.
[M18] Alan: [simultaneously] Eighteen ways. About eighteen, probably.

The second, from Frances/Ishka Episode B, includes use of *around*. In fact the pragmatic analysis of *about* which follows could be applied equally well to *around*, and be illustrated from this extract and elsewhere in the corpus.

Frances: Fifty?
[M19] Ishka: About fifty yeah.
[M20] Tim: About fifty [...] do you really think it is fifty?
Ishka: Well maybe not exactly, but it's around fifty basically? [...]
Frances: Maybe around fifty.

In each case a prediction is being made – the number of ways of making 20 (**M17, M18**) and 10 (**M19**) – and each time the hedge is as an Approximator (a Rounder, in fact) at the surface level. So the boys predict that the number of ways to make 20 is in the region of eighteen, maybe more, maybe less. I suggest, however, that the deep level purpose and function of the hedge is *Shielding* against possible error in the cognitive basis of their prediction. This suggestion is supported by closer inspection of the data in context. Harry and Alan have already listed ways of making 10, and decided on nine integer-possibilities, allowing reversals but not including zero as a summand. On being presented with the second problem (making 20) it was more common for children to list and count again, as Frances and Ishka do in Episode A. Harry, however, is a confident boy. He is a risk-taker, and goes straight for a prediction for making 20, avoiding the tedium of listing and counting. The basis of Harry's prediction seems to be proportional reasoning (doubling) – there are 9 ways for making 10, so there are 18 for 20. For fuller insight into Harry's thinking, his next contribution (following **M18** above) is

Harry: No, I think nineteen.

The conversation continues:

Tim: Eighteen, nineteen?
Harry: I should write that again. [laughs]
Alan: What's that?
Harry: Up to twenty.
[Harry begins a list 10 + 10, 9 + 11, 8 + 12]

Later, and before the list is complete, he ventures

Harry: It think that'll be nineteen.

From the outset, then, Harry is uncertain as to whether the "answer" to my question is 18 or 19. We just don't know how he arrives at these two possibilities. If his prediction is an extension of his experience of making ten, then (as already noted) doubling would produce Harry's first prediction. A more detailed awareness of the nature of his list of ways of making ten (which I tried to prompt in the later episodes of some Make Ten interviews) could have led to the second prediction. The fact that he articulates it ("No, I think nineteen") is all the more remarkable because his first, incorrect prediction is confirmed by Alan, albeit with something less than total commitment (**M18**). It seems, then, that Harry may be entertaining these two different predictions from the moment I ask about making twenty, and he seems (**M17**) to be testing out the first possibility, not just for my consideration (and possibly Alan's) but also (perhaps especially) for his own:

Harry: That *would be about, yeah I think* ... that would be eighteen.

The effect of the initial hedging is to allow himself some space for further consideration, and to declare uncertainty in the assertion which completes the sentence. In the end he feels the need to resort to listing and counting, presumably since he lacks sufficient confidence in either of his predictions to choose between them when I ask him to do so. ("Eighteen, nineteen?").

My conclusion is that the hedge *about*, although classified as an Approximator, is being used by Harry in **M17** principally to serve Shield-like ends, reinforced by the prototypic Shield *I think*. Ishka has the same end in mind in **M19**, an inferential premise which is supported by my next turn in the conversation:

[M20]Tim: About fifty [...] do you really think that it is fifty?

What I infer from Ishka's "about" (**M19**) is not that she has rounded the actual number of ways to the "round" number 50, but that she is in possession of a generalisation, a conjecture which would lead to exactly 50 as prediction. Incidentally, it is normal practice to use round numbers as vague numerical reference points (see discussion in Channell, 1980, elaborated further in Channell, 1994); indeed a round number on its own may serve as a rounder (i.e. without a prefix like "about" or "approximately"), as in, for instance

a suit like that would cost you £300

The fact that round numbers are normally chosen with numerical Rounders is further evidence in support of my suggestion that Harry and Alan (**M17, M18**) are deploying 'about' as a Shield, and not as a Rounder.

My contribution, then, in **M20** is designed to test out Ishka's commitment to 50, asking "do you really think it is". Again, my use of *think* here is in the sense of *believe*, and I strengthen the probe by the adverb *really*. Ishka's reply indicates her discomfort; she skilfully sidesteps my demand for commitment with a reply which is a miniature masterclass in hedging:

Ishka: Well maybe not exactly, but it's around fifty basically.
Frances: Maybe around fifty.

Even Frances, who at that stage is displaying more confidence (and less hedging) double-hedges her response.

7.3. *'basically'*

This is an interesting and unusual hedge, used by only 3 of the 21 children, and only on this one occasion by Frances. Unlike its use by adults as a "bottom line" underpinning, as in

John's problem is that he is basically lazy,

it seems to have the effect, as used by the children, of qualifying the content of what is being said or claimed; thus it acts as an Approximator. The following extract is from an earlier, loose-framed conversation with Simon (aged $12\frac{3}{4}$), which turned out to be a forerunner of the Make Ten Task. Simon rapidly moved on from positive integer pairs to decimals. After a while I intervened:

Tim: What if I gave you one of the numbers, one point three recurring, what would the other number be?
Simon: Em, eight point six recurring.
Tim: Why?
[M21] Simon: Because one point three recurring is basically a third ...
Tim: You mean the point three ...
[M22] Simon: ... point three recurring is basically a third, so you need ... well, the one, that's one, so to make it up to nine you add on eight, then you need another two thirds, which is point six recurring.
[M23] Tim: If you have, um, point three and point six recurring, and you add them up, what do you get?
[M24] Simon: Point nine recurring. Mmm – nearly one.
Tim: Nearly one.
Simon: Yes.
Tim: Why nearly one?
[M25] Simon: Because it's not, because point three isn't, it's just nearly a third. It doesn't quite get to the third.
Tim: When it's point three recurring.
Simon: Yeah.
[M26] Tim: Oh, so point three recurring isn't really a third at all?
[M27] Simon: Well, it's very nearly a third.
Tim: Very nearly a third.
Simon: Yeah.

Simon's statement (**M21, M22**) that "point three recurring is basically a third" is not in fact an assertion of a fundamental (*basic*, so to speak) property of point three recurring. I shall claim that "basically" is being deployed as a hedge, a Rounder in fact, so that the force of the statement is much the same as that of "point three recurring is *approximately* a third", or perhaps "point three recurring *is as good as* a third", much as one would say "97% is as good as full marks". It is as near as makes no difference.

Trace now the course of the above exchanges as the force of Simon's "basically" is revealed in the questioning. My analysis goes like this: as it stands, **M21** is "incorrect" – not that the true/false dichotomy is very meaningful when applied to hedged assertions! (Lakoff, 1972) – although the intention is clear to me. In **M22** Simon responds to my prompt to correct, or perhaps to clarify his statement in **M21**. In fact he interrupts my prompt to self-correct and (re)state that "point three recurring is basically a third". On the other hand he completes the arithmetic in **M22** with "you need another two thirds, which is point six recurring". No "basically" this time. I (in my role as interviewer) am aware that confusion about the value of infinite decimals is commonplace with students – of all ages. This is not intended to be a patronising remark, given the range of foundational positions which might underpin any attitude to the matter. The issue here, however, is the usual psychological and notational difficulties associated with equating an infinite series (the decimal) with its sum (the fraction). My strategy, to ascertain where Simon stands in relation to these two recurring decimals – determined "on the hoof" as the "um" (**M23**) indicates – is to ask him about their sum. As I expect, his reply conveys his belief that the sum falls short of one. Asked to explain, Simon is more explicit in **M25**

because point three isn't, it's just nearly a third. It doesn't quite get to the third.

I press the conclusion in **M26**, the "Oh" attempting to convey some neutrality, some surprise, so as not to put the words into his mouth. But he remains uncertain, and unable to agree without qualification to the bald statement that "point three recurring isn't really a third at all". His reluctance to concede is marked by the maxim hedge "Well", (**M27**) as he flouts the maxim of Manner, and arguably others besides! The whole exchange is marked by Simon's desire to be cooperative, yet true to himself, his beliefs, and his uncertainties.

8. SUMMARY AND CONCLUSION

In this paper I have outlined and exemplified a classification (that of Prince *et al.* 1982) of hedges into functional categories, and offered an interpretive framework which can be applied to account for some uses of vague language as it occurs in a mathematics conversational setting where children are being provoked into predicting and generalising. I have noted that:

- I (as interviewer) use Attribution Shields and Adaptors, usually for teacher-like purposes;

whereas

- the children typically use Rounders and Plausibility Shields, and nearly always to implicate uncertainty, to insert some space between conviction and proposition.

I suggest that that space, between what we believe and what we are willing to assert, deserves a name: I propose the 'zone of conjectural neutrality' (ZCN). Even Rounders, such as *about*, which in their form attach some fuzziness to the proposition itself, are deployed by the children to achieve Shield-like ends. This, and the forms of linguistic Shielding which I have discussed, have the effect of reifying the ZCN and thus distancing the speaker from the assertion that he or she makes. Whilst truth and falsity may be decided in the ZCN, a person may articulate a proposition without necessarily being committed to its truth. In such a cognitive and affective *milieu*, it is the proposition that is on trial, not the person.

A brief and tentative remark be appropriate here, concerning the suspicion of use of hedges by speakers to 'mark time', to continue to command the attention of their audience whilst they assemble their thoughts. Such behaviour need not be associated with uncertainty, and it is perhaps tempting to dismiss some hedging behaviour as nothing more than prevarication. On the other hand such a judgement may be precipitate, given the extensive analysis by linguists of the pragmatic content of members of the class of semantically-vacuous 'discourse markers' such as *well, oh* and *y'know* (Brockway, 1981; Schiffrin, 1987).

Channell (1985) has identified a number of goals which speakers achieve by the use of vague expressions. Amongst these are:

- giving the right amount of information;
- saying what you don't know how to say;
- covering for lack of specific information;
- expressing politeness, especially deference;
- protecting oneself against making mistakes.

There is evidence of each of these purposes in adult-child maths talk, in the data I have collected, and I have chosen to focus on the last of these. Given the prevailing school-culture (maths is about right and wrong answers, and it is much better to be right), the use of hedging is evidently deployed by many children as a Shield against being "wrong". These Shields could be seen to act as linguistic pointers to intellectual "risks", with attendant vulnerability. In principle, of course, it would be preferable for students to believe that being unsure is a genuine and creative option available to them. For not only is uncertainty an intellectually tenable position, but the assertion of uncertainty draws the attention of the teacher to the existence of a ZCN, and thus opens up the possibility that s/he might provide for the student some cognitive 'scaffolding' (Wood *et al.*, 1976) to support, and perhaps transform that state. This seems to be what's happening to Harry here, in a final extract:

Tim: How do you know there are forty-nine Harry?
Harry: **Well, I am not completely certain actually,** but I would expect it because if you start off with fifty and you do forty-nine add one, forty-eight add one, but then you'd end up with one add forty-eight wouldn't you, so they always change ... [...] Forty-eight add two I mean.
Tim: OK. And the last one in that list would be:
Harry: One add forty-nine, so they'd all be ... [interrupted by Alan sneezing]
Tim: How do you know that there's forty-nine different ways that you've listed? You started with forty-nine add one and you ended up with one add forty-nine. Now how do you know that there are forty-nine pairs in that list?
Harry: Well there's fifty numbers, and you just, there's lots of ways because you just go forty-nine add one, forty-eight add two all the way down 'til you get to the one, but you can't do fifty add nought, so that will take away one which will make you with forty-nine. **I'm quite certain about that.**

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NOTES

[1] It may be helpful to summarise some of the "answers" to this task, corresponding to the principal ways that the children chose to interpret it. Let n be a positive integer and f(n) be the number of pairs (a, b) such that a+b=n, where a, b belong to a set A of "numbers". If (b, a) is taken to be distinct from (a, b) (unless a=b) and A is the set $\mathbf{N} = \{1,2,3,\ldots\}$ of natural numbers, then f(n)=n-1; if A also includes zero then f(n)=n+1. If, however, (a, b) is always identified with (b, a), and A=**N**, then $f(n)=\frac{1}{2}n$ when n is even, and $\frac{1}{2}(n-1)$ when n is odd. With zero included in A these become $\frac{1}{2}n+1$ and $\frac{1}{2}(n+1)$. Of course, if A includes the set of integers, then f(n) is not finite.

[2] This paper uses the following conventions to distinguish between different sorts of data:

[Cn] contrived to illustrate an argument, whilst being intuitively plausible utterances.
[Mn] from the corpus of mathematical conversation collected by the author.

[3] The use of a linguistic formula such as "like, if you do" to refer to a general relation or a general process – in this instance additive commutativity, or symbolic reversal – by means of an *instance* of that relation/process, is commonplace. It is an instance of the power of

the 'generic example' (Mason and Pimm, 1984, Balacheff, 1988) to evoke well-founded confidence in a related generality.

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