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The Core of an Economy with Multilateral Environmental Externalities

PARKASH CHANDER

Indian Statistical Institute and California Institute of Technology, Sansanwal 7, New Delhi 110016, India

HENRY TULKENS

CORE, Université Catholique de Louvain, 34 Voie du Roman Pays, B-1348 Louvain-la-Neuve, Belgium

Abstract: When environmental externalities are international – i.e. transfrontier – they most often are multilateral and embody public good characteristics. Improving upon inefficient laissez-faire equilibria requires voluntary cooperation for which the game-theoretic core concept provides optimal outcomes that have interesting properties against free riding. To define the core, however, the characteristic function of the game associated with the economy (which specifies the payoff achievable by each possible coalition of players – here, the countries) must also reflect in each case the behavior of the players which are not members of the coalition. This has been for a long time a disputed issue in the theory of the core of economies with externalities.

Among the several assumptions that can be made as to this behaviour, a plausible one is defined in this paper, for which it is shown that the core of the game is nonempty. The proof is constructive in the sense that it exhibits a strategy (specifying an explicit coordinated abatement policy and including financial transfers) that has the desired property of nondomination by any proper coalition of countries, given the assumed behavior of the other countries. This strategy is also shown to have an equilibrium interpretation in the economic model.

1 Introduction

We deal in this paper with an economy with several agents, whose productive activities generate "multilateral" externalities, i.e., externalities that each one of them can both generate and be a recipient of. We have in mind externalitites that are detrimental for the recipients. We also call these externalities "environmental" because we assume that they exhibit public goods (actually public "bads") characteristics in the sense that when they are generated, they affect all agents in the economy. We should also call them "additive" because we further assume that what is received by any recipient is simply the sum of what is emitted by the various generators. For such an economy we call upon the concept of the *core* of a cooperative game associated with it, in order to identify joint actions that would (*a*) improve upon inefficient laissez-faire equilibria, (*b*) achieve Pareto efficiency, and (*c*) induce this outcome in such a way that it deters both individual and coalitional free riding behaviours.

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Being motivated by an interest of long standing[1] for the sources of cooperation between countries on issues of transfrontier pollution (a currently prominent example of multilateral environmental externalities), we shall interpret our model below as one of an international economy, where the countries are also the players in the games associated with it.[2] While other interpretations are conceivable, in our present interpretation the core outcome we are seeking for may be seen as a "strategically stable" result of negotiations on joint pollution abatement.

To define the core, the characteristic function of the game – which specifies the payoff achievable by each possible coalition of players – must also reflect, in the case of games with externalities, the behavior of the players which are *not* members to the coalition. This is a disputed issue, and alternative assumptions in this respect lead to alternative core concepts such as, *e.g.* the α- and β-cores. These are studied and contrasted to each other in various externalities contexts by SCARF 1971, STARRETT 1972 and LAFFONT 1977 (chapter V).[3] However, when using one of these assumptions in analyzing international environmental negotiations by means of his acid rain game, (actually one implying a particular version of the α-core, as we shall see below), MÄLER 1989 comes to the conclusion that the core concept is useless because it results in too large a set of outcomes, actually the whole set of Pareto optima.

In this paper, we adopt an alternative assumption on the behavior of players outside each coalition that to our knowledge has not been explored in the literature, and that we think to be a more plausible one in the context of environmental externalities. We use this assumption to formulate what we call the γ-characteristic function of the game, and for the ensuing γ-core concept we prove nonemptiness. The proof is constructive in the sense that it exhibits a joint strategy (*i.e.* in our pollution interpretation an explicit coordinated abatement policy accompanied by financial transfers) that has the desired property of nondomination by any proper coalition of the players. Furthermore, this strategy is shown to have an equilibrium interpretation which is conceptually analogous to that of *ratio equilibrium* (KANEKO 1977; MAS COLELL and SILVESTRE 1989) in the context of economies with public goods.

Our analysis assumes full information throughout: this is because our primary concern is with a satisfactory definition and an existence proof – which are lacking – of the concept of core allocation for the kind of economy we are

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[1] A first version of the international environmental model alluded to here was given in TULKENS 1979, and elaborated upon from a game theoretic point of view in CHANDER and TULKENS 1991, 1992a and 1992b.

[2] The "acid rain game" of MÄLER 1989 is a similar international environmental model, to which we apply, in CHANDER and TULKENS 1995, the concepts and results developed

[3] The issue also arises in games associated with economies with public goods, as in FOLEY 1970, CHAMPSAUR 1975, MOULIN 1987 and CHANDER 1993.

considering. This should provide firm ground for further work where other information structures are allowed for.

2 The Model of an Economy with Multilateral Environmental Externalities and its Efficiency Characterization

The agents of the economy (countries in our international interpretation) are denoted by the index i, with $N = \{i | i = 1, 2, \ldots, n\}$. Three categories of commodities are considered:

(i) a standard private good, whose quantities are denoted by $x \geq 0$ if they are consumed, and by $y \geq 0$ if they are produced;
(ii) pollutant discharges, the quantities of which are denoted by $p \geq 0$; and
(iii) ambient pollutant quantities, denoted by $z \leq 0$.

Each agent i's preferences are represented by a utility function $u_i(x_i, z)$ satisfying:

Assumption 1: $u_i(x_i \cdot z) = x_i + v_i(z)$ *i.e.* quasi-linearity; and

Assumption 2: $v_i(z)$ concave, differentiable and such that $dv_i/dz \equiv \pi_i(z) > 0$ for all $z \leq 0$.

With each agent i there is furthermore associated a technology, described by the production function $y_i = g_i(p_i)$, satisfying:

Assumption 3: $g_i(p_i)$ strictly concave and differentiable over an interval; and

Assumption 4: There exists $p_i^0 > 0$ such that

$$\frac{dy_i}{dp_i} \equiv \gamma_i(p_i) \begin{cases} > 0 \text{ if } p_i < p_i^0 & \text{(i)} \\ = 0 \text{ if } p_i \geq p_i^0 & \text{(ii)} \\ = \infty \text{ if } p_i = 0. & \text{(iii)} \end{cases}$$

Inputs, which are not explicitly mentioned in the production functions, are subsumed in the functional symbols g_i. Although this amounts to treat them as fixed in the analysis that follows, our results will not rest in an essential way on that assumption, which is made here mostly for expositional convenience.

All possible behaviors within the economy, in terms of the consumption, production and pollutant discharge decisions taken by its agents, as well as in terms of the the resulting values of ambient pollutant are formally described by feasible states:

Definition 1: Feasible states of the economy (or "allocations") are vectors

$$(x,p,z) \equiv (x_1,\ldots,x_n;p_1,\ldots,p_n;z)$$

such that

$$\sum_{i\in N} x_i \leq \sum_{i\in N} g_i(p_i) \tag{1}$$

and

$$z = -\sum_{i\in N} p_i. \tag{2}$$

Among feasible states,

Definition 2: A Pareto efficient state of the economy is a feasible state (x,p,z) such that there exists no other feasible state (x',p',z') for which $u_i(x_i',z') \geq u_i(x_i,z)$ for all $i \in N$ with strict inequality for at least one i.

To characterize efficient states, the usual first order conditions take in this case the form of the following system of equalities:

$$\sum_{j\in N} \pi_j(z) = \gamma_i(p_i), \quad i = 1,2,\ldots,n. \tag{3}$$

Henceforth, we shall always write π_N for $\Sigma_{j\in N}\pi_j$. Existence of (several) Pareto efficient states easily follows from our assumptions and, due to Assumption 4(iii) in particular, one has $p_i > 0$ for all i in any efficient state.

A further property of efficient states in our model, important for our purposes, is the following:

Proposition 1: Assumptions 1–3 imply that in all Pareto efficient states, the vector of emission levels $(p_1,\ldots,p_n)$ is the same.

Proof: Suppose not, and let $(\bar{x},\bar{p},\bar{z})$ and $(\tilde{x},\tilde{p},\tilde{z})$ be two states, both Pareto efficient, with $\bar{p}_i \neq \tilde{p}_i$ for some i. On the one hand, if $\tilde{z} = \bar{z}$, then $\pi_N(\tilde{z}) = \pi_N(\bar{z})$ implies by (3) that $\gamma(\bar{p}_i) = \gamma_i(\tilde{p}_i)\ \forall i \in N$; but this, by strict concavity of all functions $g_i(p_i)$, is only possible if $\tilde{p}_i = \bar{p}_i \forall i$. If, on the other hand, $\tilde{z} \neq \bar{z}$, say $\tilde{z} < \bar{z}$ without loss of generality, the fact that $\pi_N(\tilde{z}) \geq \pi_N(\bar{z})$ now implies by (3) that $\gamma_i(\tilde{p}_i) \geq \gamma_i(\bar{p}_i) \forall i$, which in turn implies by concavity that $\tilde{p}_i \leq \bar{p}_i \forall i$. However, since $\tilde{z} = -\Sigma_{i\in N}\tilde{p}_i$ and $\bar{z} = -\Sigma_{i\in N}\bar{p}_i$, one would have $\tilde{z} \geq \tilde{z}$, which is a contradiction.

3 Noncooperative Games Associated with the Economy

In this section we examine various noncooperative equilibrium concepts – the Nash equilibrium, the strong Nash equilibrium and the coalition proof Nash equilibrium – that might be called upon to describe the state of the economy in the absence of cooperation. We conclude that in the case of detrimental environmental externalities studied here only the Nash equilibrium is appropriate for that purpose.

We consider now each agent i of the economy as a player in an n-person noncooperative game. To that effect, let

$$T_i = \{(x_i, p_i) | 0 \leq p_i \leq p_i^0; 0 \leq x_i \leq g_i(p_i^0)\}, \quad i \in N$$

be the strategy set of each player i, and

$$T(S) = \{(x_i, p_i)_{i \in S} | 0 \leq p_i \leq p_i^0 \forall_{i \in S} \text{ and } 0 \leq \sum_{i \in S} x_i \leq \sum_{i \in S} g_i(p_i^0)\}$$

be the space of joint strategies of players in S. Clearly, $T(S) \supset X_{i \in S} T_i$. Let T denote the space of joint strategies of all players, i.e., $T \equiv T(N)$. Any joint strategy choice $[(x_1, p_1), \ldots, (x_n, p_n)] \in T$ of the players induces a feasible state (x, p, z) of the economy if $z = -\sum p_i$.

Then, if for each $i = 1, \ldots, n$ and any $[(x_1, p_1), \ldots, (x_n, p_n)] \in T$ we choose $u_i(x_i, z) = x_i + v_i(z)$ with $z = -\Sigma_{j \in N} p_j$ as the payoff for player i and write $u = (u_1, \ldots, u_n)$, we have defined a noncooperative game $[N, T, u]$, associated with the economy.

Definition 3: For the noncooperative game $[N, T, u]$, the joint strategy choice $[(\bar{x}_1, \bar{p}_1), \ldots, (\bar{x}_n, \bar{p}_n)]$ is a Nash equilibrium if for all $i \in N$,

$(\bar{x}_i, \bar{p}_i)$ maximizes $x_i + v_i(z)$

subject to

$$x_i \leq g_i(p_i)$$

and

$$p_i + z = -\sum_{\substack{j \in N \\ j \neq i}} \bar{p}_j.$$

Definition 4: For the economy, a disagreement equilibrium is the state $(\bar{x}_1, \ldots, \bar{x}_n; \bar{p}_1, \ldots, \bar{p}_n; \bar{z})$ with $\bar{z} = -\Sigma_{i \in N} \bar{p}_i$ induced by the Nash equilibrium $[(\bar{x}_1, \bar{p}_1), \ldots, (\bar{x}_n, \bar{p}_n)]$ of the game $[N, T, u]$.

To characterize a Nash equilibrium or a disagreement equilibrium, the first order conditions of the maximization problem in Definition 3 yield the well known system of equalities:

$$\pi_i(\bar{z}) = \gamma_i(\bar{p}_i), \quad i = 1, \ldots, n. \tag{4}$$

From the fact that the system (4) differs from (3), the standard statement is derived that a disagreement equilibrium is not an efficient state of the economy. Furthermore, it will prove useful to have the following two properties established:

Proposition 2: For the game $[N, T, u]$ there exists a Nash equilibrium; this equilibrium is unique.

Proof: The existence of a Nash equilibrium follows from standard theorems (see, e.g., FRIEDMAN 1990), since each player's strategy set is compact and convex, and each player's payoff function is concave, and therefore continuous, and bounded. All these conditions are obviously met in our model. To prove uniqueness of the equilibrium $[(\bar{x}_1, \bar{p}_1), \ldots, (\bar{x}_n, \bar{p}_n)]$, suppose contrary to the assertion that there exists another Nash equilibrium

$$[(\hat{x}_1, \hat{p}_1), \ldots, (\hat{x}_n, \hat{p}_n)] \neq [(\bar{x}_1, \bar{p}_1), \ldots, (\bar{x}_n, \bar{p}_n)].$$

Without loss of generality assume that $\Sigma\hat{p}_i \leq \Sigma\bar{p}_i$, entailing $\hat{z} = -\Sigma\hat{p}_i \geq \bar{z} = -\Sigma\bar{p}_i$. From the characterization of the disagreement equilibrium and the concavity of the functions $v_i(z)$, we would then have $\pi_i(\hat{z}) \leq \pi_i(\bar{z})$ as well as $\gamma_i(\hat{p}_i) \leq \gamma_i(\bar{p}_i)$ for each $i \in N$. But given the concavity of the production functions, this last inequality would imply that $\hat{p}_i \geq \bar{p}_i$ for each $i \in N$, which contradicts the assumption that $\Sigma\hat{p}_i \leq \Sigma\bar{p}_i$, and $[(\hat{x}_1, \hat{p}_1), \ldots, (\hat{x}_n, \hat{p}_n)] \neq [(\hat{x}_1, \bar{p}_1), \ldots, (\hat{x}_n, \bar{p}_n)]$. □

Turning to the economy, the existence and uniqueness of a disagreement equilibrium state are also established by Proposition 2.

Let us now examine the relevance for our externalities model of two alternative equilibrium concepts which are offered by the noncooperative game theoretic literature, namely the **strong Nash equilibrium** and the coalition proof Nash equilibrium.

Definition 5: For the noncooperative game $[N, T, u]$, the joint strategy choice $[(\tilde{\tilde{x}}_1, \tilde{\tilde{p}}_1), \ldots, (\tilde{\tilde{x}}_n, \tilde{\tilde{p}}_n)]$ is a strong Nash equilibrium if for all $S \subseteq N$,

$$[(\tilde{\tilde{x}}_i, \tilde{\tilde{p}}_i)_{i \in S}] \text{ maximizes } \sum_{i \in S} [x_i + v_i(z)] \tag{5.a}$$

$$\text{subject to } \sum_{i \in S} x_i \leq \sum_{i \in S} g_i(p_i) \tag{5.b}$$

$$\text{and } \sum_{i \in S} p_i + z = \begin{cases} 0 & \text{if } S = N \\ -\sum_{j \in N \setminus S} \tilde{\tilde{p}}_j & \text{if } S \neq N. \end{cases} \tag{5.c}$$

For the analysis of economies with detrimental environmental externalities, this concept is unfortunately of no use because of the following result.

Proposition 3: For the game $[N, T, u]$, there does not exist a strong Nash equilibrium.

Proof: Definition 5 implies both that a strong Nash equilibrium is also a Nash equilibrium, and that it induces a Pareto efficient state of the economy. However, as was observed earlier, the Nash equilibrium cannot induce such a Pareto efficient state. It follows from this contradiction that a strong Nash equilibrium cannot exist for the game $[N, T, u]$.[4] □

Having noted that strong Nash equilibria almost never exist in general. BERNHEIM, PELEG, and WHINSTON (1987) comment that the strong Nash concept is "too strong". They therefore propose as an alternative the **coalition-proof Nash equilibrium.** For the externalities model we are dealing with here, however, this refinement cannot bring much conceptual progress for the following reason: a coalition-proof Nash equilibrium is always also a Nash equilibrium, and in our $[N, T, u]$ game the Nash equilibrium is unique; therefore, any coalition-proof Nash equilibrium – if one exists at all – cannot be different from the Nash equilibrium characterized above.

The coalition proof Nash equilibrium concept might be used, however, not for describing the state of the economy in the absence of cooperation, as we have done so far, but instead for predicting and/or characterizing the outcome of negotiations. In this perspective, the fact that a coalition-proof equilibrium is not a Pareto optimum in general prevents us to use it, as we are searching for efficient outcomes. Moreover the concept of coalition-proof Nash equilibrium involves, as in the case of strong Nash equilibrium, the assumption that when a coalition deviates, it takes as given the strategies of the complement – an assumption that we do not find justified, as we shall argue in the following section.

4 Cooperative Games Associated with the Economy

4.1 Inappropriateness of the α-Characteristic Function

Turning now to the cooperative part of our analysis, let us associate, with every coalition S, the number $w(S)$, called the "worth" of the coalition S and defined as

[4] MÄLER (1989) shows that for his "acid rain game" also there exists no strong Nash equilibrium.

the highest aggregate payoff $\Sigma_{i\in S}u_i$ that the members of the coalition can achieve using some strategy. Thus, the pair $[N, w(\cdot)]$ consisting of the players set N and the characteristic function $w(\cdot)$ defines a cooperative game (with transferable utilities) associated with our economy.

Stated in full, the characteristic function reads

$$w(S) = \max_{\{(x_i,p_i)_{i\in S}\}} \sum_{i\in S}[x_i + v_i(z)] \tag{6}$$

For the relation between this function and our economy to be meaningful – i.e. to correspond to feasible states, the variable z should satisfy condition (2). However, this equality involves strategic choices made by players who are *not* members of S. Thus, the worth of a coalition in our game is not only a function of actions taken by its members, but also of actions of players outside the coalition.

This typical feature of cooperative games associated with economies with externalities and/or public goods requires the characteristic function to specify explicitly what the actions are both of the members of S and of the other players. A familiar way to get around this problem has been to assume that the players outside the coalition adopt those strategies that are *least favorable* to the coalition. That is, the characteristic function is defined as

$$w^{\alpha}(S) = \max_{\{(x_i,p_i)_{i\in S}\}} \sum_{i\in S}[x_i + v_i(z)] \tag{7.a}$$

$$\text{subject to } \sum_{i\in S} x_i \leq \sum_{i\in S} g_i(p_i) \tag{7.b}$$

$$\sum_{i\in S} p_i + z \begin{cases} = -\sum_{j\in N\setminus S} p_j^0 & \text{if } S \neq N \\ = 0 & \text{if } S = N. \end{cases} \tag{7.c}$$

Games with a characteristic function of this type are used in most of the studies dealing with the core of economies with public goods (see, e.g., FOLEY 1970, CHAMPSAUR 1975, MOULIN 1987 and CHANDER 1993). This is also the case with the game used by SCARF 1971 in his study of cores of economies with externalities: his "α-characteristic function" is similar to the one defined above[5]. And more recently, pushing this "pessimistic" view to the

[5] LAFFONT 1977 shows that the α-and the β-characteristic functions are identical for cooperative games associated with economies exhibiting detrimental externalities of the type dealt with here. Hence, all our subsequent developments on α-cores will also be valid for β-cores.

extreme, MÄLER 1989 assumes in his analysis of the acid rain game that there is no upper bound such as our p_i^0 for the individual pollutant discharges; thus, coalitions can be hurt by up to infinite amounts of pollutants emitted by players outside the coalition.

This extreme form of the assumption behind the α-characteristic function suggests that it is actually ill-adapted to the issues at stake with environmental externalities. Indeed there are at least two grounds for considering that assuming such strategies on the part of the players outside the coalition is questionable. On the one hand, these strategies may not maximize the individual payoffs of these players. On the other hand, they rest on the presumption that when a coalition forms, its payoff is what it would get when members of the complementary coalition act so as to minimax this coalition's payoff. But why should they behave in this extreme fashion, and why should the coalition formed presume that they would follow such strategies?

Searching for alternative assumptions in the spirit of the strong Nash equilibrium or of the coalition proof Nash equilibrium is not satisfactory either: they assume for their part that when a coalition deviates from the equilibrium it takes as given the strategies of its complement at that equilibrium. But as before, this may not maximize the payoff of the complement's members; and in addition, why should their strategies remain unchanged when S forms?

4.2 Equilibrium with Respect to a Coalition

These considerations lead us to introduce here a concept in which it is assumed that when a coalition forms it neither takes as given the strategies of its complement, nor does it assume that the complement would follow minimax strategies: instead, it looks forward to the best reply payoff corresponding to the equilibrium that its actions would induce. More specifically, we propose to assume that when S forms players outside S do not take particular coalitional actions against S, but adopt only individually best reply strategies. This results in a Nash equilibrium between S and the remaining players, with the members of S thus playing their joint best response to the individual strategies of the others.[6] In terms of our economic model and of its associated game, this is formalized as follows.

Definition 6: For the noncooperative game $[N, T, u]$, given a coalition $S \subset N$, a Nash equilibrium with respect to S is the joint strategy choice $[(\tilde{x}_1, \tilde{p}_1), \ldots,$

[6] Our analysis might be extended to the case when $N \backslash S$ may also form, but we do not pursue this possibility in the present paper. It may be noted however that if the players outside S form one or more non-singleton coalitions then the worth of S is generally higher. That players outside S act individually is therefore equivalent to granting S a certain degree of pessimism.

$(\tilde{x}_n, \tilde{p}_n)] \in T$ where

$$\text{(i)} \quad [(\tilde{x}_i, \tilde{p}_i)_{i \in S}] \text{ maximizes } \sum_{i \in S} [x_i + v_i(z)] \tag{9.a}$$

$$\text{subject to } \sum_{i \in S} x_i \leq \sum_{i \in S} g_i(p_i) \tag{9.b}$$

$$\text{and } \sum_{i \in S} p_i + z = -\sum_{j \in N \setminus S} \tilde{p}_j, \tag{9.c}$$

$$\text{(ii)} \quad \forall j \in N \setminus S, (\tilde{x}_j, \tilde{p}_j) \text{ maximizes } x_j + v_j(z) \tag{9.d}$$

$$\text{subject to } x_j \leq g_j(p_j) \tag{9.e}$$

$$\text{and } p_j + z = -\sum_{\substack{i \in N \\ i \neq j}} \tilde{p}_j. \tag{9.f}$$

As suggested by our choice of terminology, this equilibrium amounts to a Nash equilibrium between the coalition S acting as one individual player and the other players acting alone.

Definition 7: For the economy, given a coalition $S \subset N$, a partial agreement equilibrium with respect to S is the state $(\tilde{x}_1, \ldots, \tilde{x}_n; \tilde{p}_1, \ldots, \tilde{p}_n; \tilde{z})$ with $\tilde{z} = -\Sigma_{i \in N} \tilde{p}_i$ induced by $[(\tilde{x}_1, \tilde{p}_1), \ldots, (\tilde{x}_n, \tilde{p}_n)]$, the Nash equilibrium with respect to S of Definition 6.

In terms of first order conditions, a partial agreement equilibrium with respect to S so defined is characterized by the system of n equalities:

$$\sum_{j \in S} \pi_j(\tilde{z}) = \gamma_i(\tilde{p}_i), \quad i \in S$$

and

$$\pi_j(\tilde{z}) = \gamma_j(\tilde{p}_j), \quad j \in N \setminus S.$$

Note that inequality (9.b) in Definition 6 allows for some members of S possibly to consume more, or less, than what they produce. The strategy thus includes the possibility of transfers of private goods among the members of the coalition. The inequality also implies that the algebraic sum of these transfers be zero within the coalition. Further properties of a partial agreement equilibrium with respect to a coalition S that are important for the sequel are collected hereafter.

Proposition 4: For any coalition $S \subset N$,

(i) there exists a partial agreement equilibrium with respect to S;
(ii) the individual emission levels corresponding to this equilibrium are unique;
(iii) the individual emission levels of the players outside of S are not lower than those at the disagreement equilibrium,
(iv) although the total emissions are not higher.

Proof: The existence proof follows from similar arguments as in Proposition 2. The uniqueness of individual emission levels follows from similar arguments as in Proposition 1. We therefore only formally prove here the remaining two parts of the proposition.

Let $\tilde{z} = -\sum \tilde{p}_i$ and $\bar{z} = -\sum \bar{p}_i$, where $(\tilde{p}_1, \ldots, \tilde{p}_n)$ and $(\bar{p}_1, \ldots, \bar{p}_n)$ are the emission levels corresponding to a partial agreement equilibrium with respect to S and to the disagreement equilibrium, respectively. We first show that $\tilde{z} \geq \bar{z}$.

Suppose contrary to the assertion that $\tilde{z} < \bar{z}$. We must then have $\pi_i(\tilde{z}) \geq \pi_i(\bar{z})$ for each $i \in N$. From the characterizations of a partial agreement equilibrium and of the disagreement equilibrium it follows that

$$\sum_{j \in S} \pi_j(\tilde{z}) = \gamma_i(\tilde{p}_i) \geq \gamma_i(\bar{p}_i) = \pi_i(\bar{z}), \quad \forall i \in S$$

and

$$\pi_j(\tilde{z}) = \gamma_j(\tilde{p}_j) \geq \gamma_j(\bar{p}_j) = \pi_j(\bar{z}), \quad \forall j \in N \backslash S.$$

From the strict concavity of each function g_i, it follows that $\tilde{p}_i \leq \bar{p}_i$ for each $i \in N$. But this contradicts our supposition that $\tilde{z} < \bar{z}$. Hence, we must have $\tilde{z} \geq \bar{z}$.

Finally, since $\tilde{z} \geq \bar{z}, \pi_j(\tilde{z}) \leq \pi_j(\bar{z})$ for each $j \in N \backslash S$. The inequalities above and strict concavity of g_j imply that $\tilde{p}_j \geq \bar{p}_j$ for each $j \in N \backslash S$. □

Note that in view of Assumption 2, the total emissions corresponding to a partial agreement equilibrium with respect to a coalition of two or more players are strictly lower than those corresponding to the disagreement equilibrium. This means that as compared to the disagreement equilibrium, the players outside a coalition of two or more players are strictly better-off at a partial agreement equilibrium with respect to that coalition – which is actually a form of free riding on the part of those players.

4.3 The γ-Characteristic Function

We are now equipped to introduce a new characteristic function, that embodies the behavioral assumptions we found desirable at the beginning of the previous subsection. We call it the "partial agreement characteristic function" or, for short, the γ-characteristic function, and denote it by w^γ. It is defined by:

$$w^\gamma(S) = \max_{\{(x_i,p_i)_{i\in S}\}} \sum_{i\in S}[x_i + v_i(z)] \tag{10.a}$$

$$\text{subject to } \sum_{i\in S} x_i \leq \sum_{i\in S} g_i(p_i) \tag{10.b}$$

$$\text{and } \sum_{i\in S} p_i + z = -\sum_{j\in \mathrm{N}\setminus \mathrm{S}} p_j, \tag{10.c}$$

$$\text{where } \forall j \in N\setminus S, (x_j, p_j) \text{maximizes } x_j + v_j(z) \tag{10.d}$$

$$\text{subject to } x_j \leq g_j(p_j) \tag{10.e}$$

$$\text{and } p_j + z = -\sum_{\substack{i\in N\\ i\neq j}} p_i. \tag{10.f}$$

The worth of coalition S is thus determined by an equilibrium concept, namely that of partial agreement equilibrium with respect to S. It is not assumed that the players outside the coalition S do the worst, as with the α-characteristic function; nor is it assumed, as in the concepts of strong and coalition-proof Nash equilibria, that they do not react to the actions of S.

Let us note however that for each S, the value assigned by the partial agreement characteristic function w^γ is at least as much as that assigned by the α-characteristic function, *i.e.* $w^\gamma(S) \geq w^\alpha(S)$ for each $S \subset N$. In fact, examples can be constructed such that $w^\gamma(S) > w^\alpha(S)$ for all $S \subset N$. This implies that the core of the game $[N, w^\gamma]$, *i.e.* the "γ-core", is, if nonempty, contained in the "α-core", and possibly smaller. Moreover, if we had assumed that when a coalition S forms, the complementary coalition $N\setminus S$ also forms and chooses the best response strategy for its members given what S does, then the core of the so-defined game would most likely be smaller than that of the game $[N, w^\gamma]$, and even perhaps empty in which case a stable strategy for the grand coalition N would be impossible to find.[7]

[7] This remark points to the work of CARRARO and SINISCALCO 1993 who in a model with identical agents assume that when S forms and achieves the aggregate payoff $w(S)$, if some $i \in S$ leaves S, the coalition $S\setminus\{i\}$ remains formed.

5 A Strategy in the γ-Core with Linear Pay-off Functions

Let us recall:

Definition 8: For any cooperative game $[N, w]$, a strategy of the all player coalition N is said to belong to the core of the game if for any subset $S \subset N$ the payoff it yields to the members of S is larger than $w(S)$, *i.e.* the payoff that S can achieve by itself.

Emptiness *vs.* nonemptiness of the core typically depends upon the form of the characteristic function, which reflects the power of each coalition in the game. In the presence of externalities, this power is crucially affected by the assumed behavior of the players outside the coalition. Thus, with the α-characteristic function, coalitions are weakened by the presumed minimax behavior, letting hope that the corresponding α-core be nonempty. This is indeed the case in SCARF 1971, as well as in the version given by LAFFONT 1977 of the SHAPLEY and SHUBIK 1969 "garbage game",[8] and also in MÄLER's 1989 acid rain game.

The former two results, however, do not bear on an economy with externalities of the environmental type we are dealing with here, and Mäler's argument is only an informal one. Furthermore, no results are available, to our knowledge, on cores of games with the characteristic function w^γ we have introduced above.

To find out whether or not the core of a cooperative game is nonempty various approach can be used. Two qualitative ones are offered by SHAPLEY 1967 and SHAPLEY 1971 who respectively show nonemptiness if the game is balanced or convex. None of these approaches proved succesful in our case. We therefore turn to another, constructive, approach, which is to exhibit a strategy for which we can show that it satisfies Definition 8.

Specifically, let $(x^*, p^*, z^*) = (x_1^*, \ldots, x_n^*; p_1^*, \ldots, p_n^*; z^*)$ be the Pareto efficient state defined by

$$\begin{aligned} x_i^* &= g_i(\bar{p}_i) - \frac{\pi_j^*}{\pi_N^*}\left(\sum_{j\in N} g_j(\bar{p}) - \sum_{j\in N} g_j(p_j^*)\right), \quad i \in N, \\ z^* &= -\sum p_i^*, \end{aligned} \tag{11}$$

They show that then it may be better for i to leave S. As this advantage grows with the size of coalitions, they conclude that only small coalitions can prevail, and that N will never form. More recently, in a model where coalition formation is endogeneous, RAY and VOHRA 1996 also conclude that in general N cannot form.

[8] This is also the case in the many studies of economies with public goods alluded to above of FOLEY 1970, CHAMPSAUR 1975, MOULIN 1987 and CHANDER 1993, where the unfavorable behavior consists in producing no public good at all; these have typically large cores.

where $\pi_i^* = \pi_i(z^*)$ for each $i \in N$ and $(p_1^*, \ldots, p_n^*)$ and $(\bar{p}_1, \ldots, \bar{p}_n)$ are the (unique) individual emission levels corresponding to the Pareto efficient states and to the disagreement equilibrium, respectively.[9] By Pareto efficiency $\pi_N^* = \gamma_i(p_i^*)$ for each $i \in N$.

As can be easily seen, this state of the economy[10] implies that for each $i \in N$,

(x_i^*, p_i^*, z^*) maximizes $x_i + v_i(z)$

subject to

$$x_i \leq g_i(\bar{p}_i) - \frac{\pi_i^*}{\pi_N^*}\left(\sum_{j \in N} g_j(\bar{p}_j) - \sum_{\substack{j \in N \\ j \neq i}} g_j(p_j^*) - g_i(p_i)\right),$$

$$p_i + z = -\sum_{j \neq i} p_j^*$$

and

$$0 \leq p_i \leq p_i^0.$$

which means that the state $(x_1^*, \ldots, x_n^*; p_1^*, \ldots, p_n^*; z^*)$ is an equilibrium concept. As it can be given an interpretation analogous to that of the ratio equilibrium (see KANEKO 1977, and MAS-COLELL and SILVESTRE 1989), we shall refer to it as the ratio equilibrium with respect to the disagreement equilibrium.[11]

We now prove the nonemptiness of the γ-core by showing that the ratio equilibrium with respect to the disagreement equilibrium belongs to the core of the game $[N, w^\gamma]$.

We first consider a special case of our general model, namely the one where it is assumed that the payoff functions are linear, i.e.:

Assumption 1′: $u_i(x_i, z) = x_i + \bar{\pi}_i z, \quad \bar{\pi}_i > 0.$

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[9] CHANDER 1993 analyzes an instantaneous analog of this cost sharing rule in a public good context.

[10] Expression (11) implies transfers $T_i(> 0$ if received. < 0 if paid) of the private good, such that $\forall i \in N, x_i^* = g_i(p_i^*) + T_i$, where

$$T_i = -(g_i(p_i^*) - g_i(\bar{p}_i)) + \frac{\pi_i^*}{\pi_N^*}\left(\sum_{j \in N} g_j(p_j^*) - \sum_{j \in N} g_j(\bar{p}_j)\right),$$

with the property that $\Sigma_{i \in N} T_i = 0$

[11] Note that one can also consider the Pareto efficient state defined by:

$$x_j^* = g_i(p_i^0) - \frac{\pi_i^*}{\pi_N^*}\left(\sum_{j \in N} g_j(p_j^0) - \sum_{j \in N} g_j(p_j^*)\right), i \in N.$$

But we are unable to establish the same properties for this Pareto efficient state. It seems that the reference point matters.

This is actually the case for which MÄLER 1989 proves the nonexistence of a strong Nash equilibrium. In SHAPLEY and SHUBIK's 1969 garbage game also, the payoff functions are linear (unlike here, however, their externalities are directional and involve no diseconomies of scale).

As can be easily seen from the characterizations of the Nash equilibrium and of a partial agreement equilibrium, an important consequence of the linearity assumption is that in a partial agreement equilibrium with respect to a coalition, the emission levels of the players outside the coalition are the same as in the Nash equilibrium. In fact, under the linearity assumption the Nash equilibrium is a dominant strategy equilibrium.

Theorem 1: Under the linearity Assumption 1′, the strategy $[(x_1^*, p_1^*), \ldots, (x_n^*, p_n^*)]$ of the grand coalition N that induces the ratio equilibrium (x^*, p^*, z^*) of the economy belongs to the core of the game $[N, w^\gamma]$.

Proof: Suppose contrary to the assertion, that the strategy inducing $(x_1^*, \ldots, x_n^*; p_1^*, \ldots, p_n^*; z^*)$ is not in the core of the game $[N, w^\gamma]$. Then there exists a coalition $S \subset N$ and a strategy for S inducing the feasible state $(\tilde{x}_1, \ldots, \tilde{x}_n; \tilde{p}_1, \ldots, \tilde{p}_n; \tilde{z})$ such that $(\tilde{x}_1, \ldots, \tilde{x}_n; \tilde{p}_1, \ldots, \tilde{p}_n; \tilde{z})$ is a partial agreement equilibrium with respect to S, and $\tilde{x}_i + \bar{\pi}_i \tilde{z} > x_i^* + \bar{\pi}_i z^*$ for all $i \in S$. From the characterization of a partial agreement equilibrium with respect to a coalition, it follows that $\tilde{p}_i = \bar{p}_i$ for all $i \in N \backslash S$, that $\tilde{p}_i \geq p_i^*$ for all $i \in S$, and that $\Sigma_{i \in S} \tilde{x}_i = \Sigma_{i \in S} g_i(\tilde{p}_i)$.

Consider now the alternative efficient state $(\hat{x}_1, \ldots, \hat{x}_n; p_1^*, \ldots, p_n^*; z^*)$, defined as:

$$\begin{aligned} \hat{x}_i &= g_i(\tilde{p}_i) - \frac{\bar{\pi}_i}{\bar{\pi}_N} \left(\sum_{i \in N} g_i(\tilde{p}_i) - \sum_{i \in N} g_i(p_i^*) \right), i \in N, \\ z^* &= -\sum_{i \in N} p_i^*. \end{aligned} \tag{12}$$

We show below that, as far as the members of S are concerned, one has

$$\sum_{i \in S} \hat{x}_i + \sum_{i \in S} \bar{\pi}_i z^* > \sum_{i \in S} \tilde{x}_i + \sum_{i \in S} \bar{\pi}_i \tilde{z}, \tag{13}$$

which implies

$$\sum_{i \in S} \hat{x}_i + \sum_{i \in S} \bar{\pi}_i z^* > \sum_{i \in S} x_i^* + \sum_{i \in S} \bar{\pi}_i z^* \tag{14}$$

if S is able to achieve $\tilde{x}_i + \bar{\pi}_i \tilde{z} > x_i^* + \bar{\pi}_i z^*$ for all $i \in S$ as it is supposed to do.

We further show that as far as the other players are concerned,

$$\hat{x}_i + \bar{\pi}_i z^* \geq x_i^* + \bar{\pi}_i z^* \text{ for all } i \in N \backslash S. \tag{15}$$

As together the inequalities (14) and (15) imply that the state $(\hat{x}_1, \ldots, \hat{x}_n; p_1^*, \ldots, p_n^*; z^*)$ is Pareto superior to the Pareto efficient allocation $(x_1^*, \ldots, x_n^*; p_1^*, \ldots, p_n^*; z^*)$, we get an impossibility. Proving (13) and (15) will thus establish the theorem.

To show (13), the definition (12) allows one to write

$$\begin{aligned}\sum_{i\in S} \hat{x}_i + \sum_{i\in S} \bar{\pi}_i z^* &= \sum_{i\in S} g_i(\tilde{p}_i) - \frac{\sum_{i\in S}\bar{\pi}_i}{\bar{\pi}_N}\left(\sum_{i\in N} g_i(\tilde{p}_i) - \sum_{i\in N} g_i(p_i^*)\right) + \sum_{i\in S}\bar{\pi}_i z^* \\ &= \sum_{i\in S}\tilde{x}_i - \frac{\sum_{i\in S}\bar{\pi}_i}{\bar{\pi}_N}\left(\sum_{i\in N} g_i(\tilde{p}_i) - \sum_{i\in N} g_i(p_i^*)\right) + \sum_{i\in S}\bar{\pi}_i z^* - \sum_{i\in S}\bar{\pi}_i\tilde{z} + \sum_{i\in S}\bar{\pi}_i\tilde{z} \\ &= \sum_{i\in S}\tilde{x}_i + \frac{\Sigma_{i\in S}\bar{\pi}_i}{\bar{\pi}_N}\left[\bar{\pi}_N(z^* - \tilde{z}) - \left(\sum_{i\in N} g_i(\tilde{p}_i) - \sum_{i\in N} g_i(p_i^*)\right)\right] + \sum_{i\in S}\bar{\pi}_i\tilde{z}.\end{aligned} \tag{16}$$

From the respective characterizations of a Pareto efficient state, and of a partial agreement equilibrium, we have for all $i \in N$, $\bar{\pi}_N = \gamma_i(p_i^*)$ and $\tilde{p}_i \geq p_i^*$. Hence, the strict concavity of each function g_i implies

$$\bar{\pi}_N > \frac{g_i(\tilde{p}_i) - g_i(p_i^*)}{\tilde{p}_i - p_i^*}, \quad \text{for all } i \in N,$$

which in turn implies that

$$\bar{\pi}_N(z^* - \tilde{z}) > \sum_{i\in N} g_i(\tilde{p}_i) - \sum_{i\in N} g_i(p_i^*).$$

Then (13) follows from (16).

On the other hand, from the respective characterizations of the disagreement equilibrium and of a partial agreement equilibrium with respect to a coalition, we have $\tilde{p}_i \leq \bar{p}_i$, for all $i \in N$, $\tilde{p}_i = \bar{p}_i$ for all $i \in N\backslash S$, and thus $\Sigma_{i\in N} g_i(\tilde{p}_i) \leq \Sigma_{i\in N} g_i(\bar{p}_i)$. Therefore,

$$\begin{aligned}\hat{x}_i &= g_i(\tilde{p}_i) - \frac{\bar{\pi}_i}{\bar{\pi}_N}\left(\sum_{i\in N} g_i(\tilde{p}_i) - \sum_{i\in N} g_i(p_i^*)\right) \\ &\geq g_i(\bar{p}_i) - \frac{\bar{\pi}_i}{\bar{\pi}_N}\left(\sum_{i\in N} g_i(\bar{p}_i) - \sum_{i\in N} g_i(p_i^*)\right) = x_i^* \quad \text{for all } i \in N\backslash S.\end{aligned}$$

These inequalities establish (15). □

Note that the domination used in the proof of Theorem 1 is more demanding than the usual one in that it requires any dominating payoff vector to correspond to an equilibrium strategy combination. Thus, the α-core includes the γ-core, just as the coalition proof Nash equilibria include the strong Nash equilibria if they are both nonempty. In other words, the γ-core seems to satisfy a consistency requirement.[12]

6 γ-Core Property of the Strategy Under Non-linear Payoff Functions

Doing away with the linearity of payoff functions makes the interaction between the strategic variables more complex. To see this, note that the Nash equilibrium is no longer a dominant strategy equilibrium. Moreover, it need not be true that $\tilde{p}_i \leq \bar{p}_i$ for all $i \in S$, where $\tilde{p}_i, i \in N$, are the emission levels corresponding to the partial agreement equilibrium with respect to S. As these inequalities play a crucial role in the proof of Theorem 1, we do extend our result to the more general case by imposing a condition which is sufficient to ensure that these inequalities continue to hold. Consider the following:

Assumption 1″*:* For all $S \subset N, S \neq N, |S| \geq 2, \Sigma_{i \in S}\pi_i(z^*) \geq \pi_j(\bar{z}), j \in S$, where $\bar{z}$ and z^* correspond to the disagreement equilibrium and a Pareto efficient state, respectively.

Clearly, this assumption is satisfied when the payoff functions are linear. In words, the assumption says that the (now non-constant) marginal utilities of z should not fall "too much" between the disagreement equilibrium and the Pareto optimum. It covers a large class of quadratic utility functions, among others.

Proposition 5: Under Assumption 1″, the emission level of each player in the coalition of a partial agreement equilibrium is not higher than the one corresponding to the Nash equilibrium.

Proof: Let $S \subset N$ be some coalition, and let $((\tilde{x}_1, \tilde{p}_1), \ldots, (\tilde{x}_n, \tilde{p}_n))$ be a partial agreement equilibrium with respect to S. Using an argument which is analogous to that in the proof of Proposition 4, we first show that $\Sigma_{i \in N}\tilde{p}_i \geq \Sigma_{i \in N}p_i^*$, where $p_i^*, i \in N$, are the emission levels corresponding to a Pareto optimum.

Indeed, suppose $\Sigma_{i \in N}\tilde{p}_i < \Sigma_{i \in N}p_i^*$ instead, implying $\tilde{z} > z^*$. By concavity of the functions $v_i(z)$, one would have $\pi_j(\tilde{z}) \leq \pi_j(z^*) \forall j \in N$, and from this, together with the first order characterizations of a partial agreement

[12] We are grateful to the referee who made that point.

equilibrium and of an optimum, it would follow that, for each $j \in S$.

$$\sum_{k\in S} \pi_k(\tilde{z}) = \gamma_j(\tilde{p}_j) \leq \sum_{k\in S} \pi_k(z^*) \leq \sum_{k\in N} \pi_k(z^*) = \gamma_j(p_j^*).$$

and for each $j \in N\backslash S$,

$$\pi_j(\tilde{z}) = \gamma_j(\tilde{p}_j) \leq \pi_j(z^*) \leq \gamma_j(p_j^*).$$

Since $\gamma_j(p_j)$ is decreasing in p_j, these two expressions imply that $\tilde{p}_j \geq p_j^* \forall j \in N$, contradicting the supposition.

Now, since $\tilde{z} \leq z^*$, concavity again implies that $\Sigma_{i\in S}\pi_i(\tilde{z}) \geq \Sigma_{i\in S}\pi_i(z^*)$ for all $S \subset N$. From the characterizations of the Nash equilibrium and of the partial agreement equilibrium with respect to S, it then follows that $\gamma_j(\tilde{p}_j) \geq \gamma_j(\bar{p}_j)$ for all $j \in S$, that is $\tilde{p}_i \leq \bar{p}_j$ for all $j \in S$.

We can now state: □

Theorem 2: Under Assumption 1″, the strategy $[(x_1^*, p_1^*), \ldots, (x_n^*, p_n^*)]$ of the grand coalition N that induces the ratio equilibrium (x^*, p^*, z^*) of the economy belongs to the core of the game $[N, w^\gamma]$.

Proof: Suppose, contrary to the assertion, that the strategy inducing the above state (x^*, p^*, z^*) is not in the core of the game $[N, w^\gamma]$. Then there exists a coalition $S \subset N, S \neq N$, and a Nash equilibrium with respect to S, $[(\tilde{x}_1, \tilde{p}_1), \ldots, (\tilde{x}_n, \tilde{p}_n)]$, such that

$$\tilde{x}_i + v_i(\tilde{z}) > x_i^* + v_i(z^*) \text{ for all } i \in S, \tag{17}$$

where $\tilde{z} = -\Sigma_{i\in N}\tilde{p}_i$.

Define a new Pareto efficient state of the economy $(\hat{x}_1, \ldots, \hat{x}_n; p_1^*, \ldots, p_n^*; z^*)$ as follows:

$$\hat{x}_i = g_i(\tilde{p}_i) - \frac{\pi_i^*}{\pi_N^*}\left(\sum_{j\in N} g_j(\tilde{p}_j) - \sum_{j\in N} g_j(p_j^*)\right), i = 1, \ldots, n.$$

We claim that inequality (17) implies that

$$\sum_{i\in S} \hat{x}_i + \sum_{i\in S} v_i(z^*) > \sum_{i\in S} \tilde{x}_i + \sum_{i\in S} v_i(\tilde{z}) \tag{18}$$

and

$$\sum_{i\in N\backslash S} \hat{x}_i > \sum_{i\in N\backslash S} x_i^*. \tag{19}$$

As these two inequalities together with (17) clearly contradict the Pareto efficiency of $(x_1^*, \ldots, x_n^*; p_1^*, \ldots, p_n^*; z^*)$, our theorem is proved if we establish them.

We first prove (18). By the definition of the new allocation,

$$\sum_{i\in S} \hat{x}_i = \sum_{i\in S} g_i(\tilde{p}_i) - \left(\sum_{i\in S} \pi_i^*/\pi_N^*\right)\left(\sum_{j\in N} g_j(\tilde{p}_j) - \sum_{j\in N} g_j(p_j^*)\right)$$
$$\geq \sum_{i\in S} \tilde{x}_i - \left(\sum_{i\in S} \pi_i^*/\pi_N^*\right)\left(\pi_N^*\left(\sum_{j\in N} \tilde{p}_j - \sum_{j\in N} p_j^*\right)\right),$$

using the concavity of the functions g_i, as well as the Pareto efficiency condition (3); but the last expression is equal to:

$$\sum_{i\in S} \tilde{x}_i - \left(\sum_{i\in S} \pi_i^*/\pi_N^*\right)\pi_N^*(z^* - \tilde{z}).$$

Thus,

$$\sum_{i\in S} \hat{x}_i + \sum_{i\in S} \pi_i^* z^* \geq \sum_{i\in S} \tilde{x}_i + \sum_{i\in S} \pi_i^* \tilde{z},$$

that is

$$\sum_{i\in S} \hat{x}_i + \sum_{i\in S} v_i(z^*) \geq \sum_{i\in S} \tilde{x}_i + \sum_{i\in S} v_i(\tilde{z}) + \left(\sum_{i\in S} v_i(z^*) - \sum_{i\in S} v_i(\tilde{z}) - \sum_{i\in S} \pi_i^*(z^* - \tilde{z})\right)$$
$$\geq \sum_{i\in S} \tilde{x}_i + \sum_{i\in S} v_i(\tilde{z}),$$

(*i.e.* (18)), since from concavity $v_i(z^*) - v_i(\tilde{z}) \geq \pi_i^*(z^* - \tilde{z})$ for all i.

Next we prove inequality (19). By definition,

$$\sum_{i\in N\setminus S} \hat{x}_i = \sum_{i\in N\setminus S} g_i(\tilde{p}_i) - \left(\sum_{i\in N\setminus S} \pi_i^*/\pi_N^*\right)\left(\sum_{j\in N} g_j(\tilde{p}_j) - \sum_{j\in N} g_j(p_j^*)\right)$$
$$= \sum_{i\in N\setminus S} g_i(\bar{p}_i) - \left(\sum_{i\in N\setminus S} \pi_i^*/\pi_N^*\right)\left(\sum_{j\in N} g_j(\bar{p}_j) - \sum_{j\in N} g_j(p_j^*)\right)$$
$$+ \left(\sum_{i\in N\setminus S} g_i(\tilde{p}_i) - \sum_{i\in N\setminus S} g_i(\bar{p}_i)\right)$$
$$+ \left(\sum_{i\in N\setminus S} \pi_i^*/\pi_N^*\right)\left(\sum_{j\in N} g_j(\bar{p}_j) - \sum_{j\in N} g_j(\tilde{p}_j)\right)$$

$$= \sum_{i \in N \setminus S} x_i^* + \left(\sum_{i \in N \setminus S} g_i(\bar{p}_i) - \sum_{i \in N \setminus S} g_i(\bar{p}_i) \right)$$
$$+ \left(\sum_{i \in N \setminus S} \pi_i^* / \pi_N^* \right) \left(\sum_{j \in N} g_j(\bar{p}_j) - \sum_{j \in N} g_j(\tilde{p}_j) \right)$$
$$= \sum_{i \in N \setminus S} x_i^* + \left[\left(\sum_{i \in N \setminus S} g_i(\tilde{p}_i) - \sum_{i \in N \setminus S} g_i(\bar{p}_i) \right) \right.$$
$$\left. - \left(\sum_{i \in N \setminus S} \pi_i^* / \pi_N^* \right) \left(\sum_{i \in N \setminus S} g_i(\tilde{p}_i) - \sum_{i \in N \setminus S} g_i(\bar{p}_i) \right) \right]$$
$$- \left(\sum_{i \in N \setminus S} \pi_i^* / \pi_N^* \right) \left(\sum_{i \in S} g_i(\tilde{p}_i) - \sum_{i \in S} g_i(\bar{p}_i) \right).$$

As Propositions 4 and 5 imply that $\tilde{p}_i \geq \bar{p}_i$ for all $i \in N \setminus S$ and $\tilde{p}_i \leq \bar{p}_i$ for all $i \in S$, we have $\Sigma_{i \in N \setminus S} \hat{x}_i > \Sigma_{i \in N \setminus} x_i^*$, *i.e.* (19). □

We had referred earlier to an international environmental externalities literature that concludes negatively on the issue of full cooperation. These authors (MÄLER 1989, BARRETT 1990, CARRARO and SINISCALCO 1993) all establish their claim under the alternative assumption of identical players–thus, identical countries. It is therefore of interest that the opposite result can be obtained here under the same assumption, as we now show.

Assumption 5: For all $i, j \in N, u_i = u_j$, i.e., the utility[13] functions of the countries are identical.

Proposition 6: Proposition 5 holds under Assumption 5 instead of Assumption 1″.

Proof: When the payoff functions are identical, it follows from the characterizations of the Nash equilibrium and of the partial agreement equilibrium with respect to S that for all $i, j \in S$, $\gamma_i(\bar{p}_i) = \gamma_j(\bar{p}_j)$ and $\gamma_i(\tilde{p}_i) = \gamma_j(\tilde{p}_j)$. Since $\Sigma_{i \in S} \tilde{p}_i \leq \Sigma_{i \in S} \bar{p}_i$ by Proposition 4, it follows that $\gamma_i(\tilde{p}_i) \geq \gamma_i(\bar{p}_i)$ for all $i \in S$, that is, $\tilde{p}_i \leq \bar{p}_i$ for all $i \in S$. □

Corollary: Theorem 2 holds under Assumption 5 instead of Assumption 1″.

[13] and not the production functions, as assumed in earlier versions of this paper.

Proof: Identical to the proof of Theorem 2. □

7 Concluding Remarks

For the cooperative game associated with an economy with detrimental environmental externalities, we have exhibited a strategy in the γ-core, that is, Pareto optimal joint actions of the all-players set N such that no coalition $S \subset N$ can do better for its members. The essence of our contribution lies (i) in identifying this fully cooperative strategy and (ii) in showing that to deter deviating behavior of any $S \subset N$ against it – that is, to deter free riding by any S, the breaking up of the players not in S into singletons acting rationally is sufficient.

We interpret our result as an argument supporting the view that, on logical grounds, full cooperation and efficiency can prevail in economies of this type, in spite of the fact that neither a strong Nash equilibrium nor a Pareto efficient coalition-proof Nash equilibrium can be shown to exist for them.

The proposed allocation has an equilibrium interpretation, nevertheless: it is analogous to that of ratio and therefore of Lindahl equilibrium in economies with public goods. For such economies, it is known that the ratio equilibrium belongs to the core (actually, the α-core). Since the γ-core is a smaller set of allocations, our result establishes an additional property of the ratio equilibrium.

Note also that the constructive nature of our result has the virtue of allowing one to compute the γ-core allocation in applied problems, as is done, for instance, in GERMAIN, TOINT and TULKENS 1996 for the international acid rains model of MÄLER's 1989.

Finally, while the restrictive assumptions made have permitted to introduce and illustrate the basic concepts in a more transparent way, extending our analysis to a wider class of utility functions and other information structures are obvious directions for further research.

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