$f(0, w) = 0$ and $\partial_z f(0, w) = 0$, that transforms $v = F(z, \bar{z}, u)$ into $v = F(z, \bar{z}, u)$ such that

$$\tilde{F}_{p0} = 0, \ \tilde{F}^1_{p1} = 0.$$

Proof. The proof of this lemma is a repetition of the proof of [1, Lemma 3.3], applied to the first coordinate, i.e., to the equation

$$v^1 = F^1(z, \bar{z}, u).$$

The transformations, preserving the point $\xi = (0, 0)$, $\gamma = \{z = 0, v = 0\}$, the parameter u, and the form of Eq. (15), have the following form: $z \to P(w)z$, $w \to w$, where P is a matrix that depends holomorphically on w. The condition

$$e(u) \to \left(\frac{\partial}{\partial z^1}, \ldots, \frac{\partial}{\partial z^n}\right)$$

fixes P uniquely. The theorem is proved.

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FINITE-GAP SOLUTIONS OF BOUNDARY VALUE PROBLEMS FOR INTEGRABLE EQUATIONS

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Introduction. Recently, there has been a noticeable increase in the interest shown in boundary value problems for nonlinear equations integrable by the inverse problem method (IPM) [1]. This is connected, first of all, with the discovery [2] of nontrivial types of "integrable" boundary conditions, which lend themselves to the development of an analytical technique based on the introduction of an "ingredient" [3, 4] and which permit construction of an IMP analog for problems on a semiaxis and on an itnerval.

The present study extends the investigations initiated in [4] connected with the construction of algebraic-geometric solutions of boundary value problems. As our basic model we consider the XXZ-equation of Landau–Lifshits, which describes dynamics of the magnetization vector S(x, t) in uniaxial ferromagnetics with an anisotropic "light plane" [5]:

$$S_t = [S, S_{xx} + IS], \quad |S| = 1, \qquad (1)$$
$$S = (S_1, S_2, S_3) \in \mathbf{R}^3, \quad I = \operatorname{diag}(0, 0, -16\varepsilon^2), \quad \varepsilon > 0.$$

The integrable boundary condition [2] at point x = 0 has the form

$$(2i\alpha_0 S_- - [S, S_x]_-)|_{x=0} = 0, \quad \alpha_0 = \text{const} \in \mathbf{R}. \qquad (2)$$

By definition, $S_{\underset{(-)}{+}} = S_1 \underset{(-)}{+} iS_2$. Each of the terms in Eq. (2) has a physical meaning; therefore, the boundary condition (2) can prove to be useful in describing boundary effects in the one-dimensional ferromagnetics (1).

Our basic goal consists in constructing algebraic-geometric solutions of Eq. (1), which satisfy condition (2) at x = 0 and also the condition

$$(2i\alpha_1 S_- - [S, S_x]_-)|_{x=1-0} = 0, \qquad (3)$$

at $x = 1$, $\alpha_1 = \text{const} \in \mathbf{R}$.

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We remark that a problem similarly formulated was solved in [4] for the nonlinear Schrödinger (NS_-) equation using the "ingredient" technique, the use of which has led to rather nontrivial calculations. In particular, the results obtained in [4] reestablished anew (and modernized) the algebraic-geometric construction of finite-gap solutions over the whole axis.

In the present paper we employ a different approach, one based on a further reduction of the Baker–Akhiezer Ψ-function for "ordinary" finite-gap solutions over the whole axis, which substantially simplifies calculations, particularly in the case where finite-gap solutions of the equation in question are known, along with the corresponding Ψ-function. Specifically, for Eq. (1), instead of considering a problem with an "ingredient," we pose the problem of singling-out from the known [6, 7] smooth finite-gap solutions over the whole axis a special class of solutions preserving $\forall t \in \mathbf{R}$ the boundary conditions (2) and (3).

As we show below, this problem admits an effective solution. We remark that the results of our study agree with those which can be obtained using the "ingredient" ideology.

1. Boundary Value Problem for the Landau–Lifshits Equation. 1.1. Equation (1) may be represented (see [5]) in the form of the linear system compatibility condition

$$\begin{cases} \Psi_x = U\Psi, \ \Psi_t = V\Psi, \\ U = -i\sum_{j=1}^{3} S_j w_j \sigma_j, \\ V = 2i\sum_{j=1}^{3} S_j w_1 w_2 w_3 w_j^{-1}\sigma_j - i\sum_{j=1}^{3} [S, S_x]\, w_j \sigma_j, \end{cases}$$

where $w_1 = w_2 = \sqrt{\lambda^2 - \varepsilon^2}$, $w_3 = \lambda$; σ_j are Pauli matrices; $\Psi = (\psi_1, \psi_2)^T$.

From the form of the U–V pairs it is readily seen that to satisfy boundary condition (2) it is sufficient to require that the following equation be identically satisfied with respect to t (see Remark 4 of [4]):

$$\psi_1(x = 0,\ t,\ \lambda = i\alpha_0) = 0 \qquad \forall t. \tag{4}$$

Actually, this condition implies the vanishing of an anti-diagonal element of the matrix V ($x = 0$, t, $\lambda = i\alpha_0$), which is equivalent to boundary condition (2).

This remark can serve as the basis for the derivation of integrable boundary conditions for other nonlinear equations (see [2]). The main item here is that with integrable boundary conditions we can reduce the problem on a semiaxis to a problem on the whole axis under special limitations on the initial data. The realization of this problem makes use of various technical methods depending on the class of functions in which a solution is sought. In the general case reliance on an effective use of remark (4) is difficult. Much more adequate is the "ingredient" approach [2] or its interpretation in the spirit of the Bäcklund transformation (see the Conclusion). However, in the finite-gap class, where an explicit representation for the Baker–Akhiezer Ψ-function is known [6, 7], it is natural to use formula (4) to single-out solutions of the boundary value problem.

1.2. We proceed to the details. To avoid excessive detail, we consider only one component, namely, the simplest component of finite-gap solutions corresponding to the case [6] in which 1) the spectral curve Γ contains the points $\pm\varepsilon$ as branch points; 2) all the other branch points λ_i are real; 3) the dynamics is bounded on the simplest real torus: $\Delta = 0$ in formula (11), which corresponds (see [8]) to boundedness for all $x, t \in \mathbf{R}$ of the Baker–Akhiezer function.

Let us recall the necessary facts from the finite-gap theory of Eq. (1). The spectral curve Γ is given by the equation

$$z^2(\lambda) = \prod_{i=1}^{2g+1} (\lambda - \lambda_i)(\lambda^2 - \varepsilon^2), \qquad \lambda_i \in \mathbf{R}. \tag{5}$$

Point $P \in \Gamma$ is specified by the pair $P = (\lambda, z)$; $\infty^{\pm}$ are two infinitely distant points on Γ; the upper sign "+" indicates membership in the upper branch Γ^+.

On a canonical basis of cycles a_i, b_i, $i = 1, \ldots, g$, we define, in the standard way, a vector ω of normalized ($\int_{a_i} \omega_j = \delta_{ij}$) holomorphic differentials, an Abelian mapping $A(P) = \int_{\infty^+}^{P} \omega$, the Riemann B-matrix, and the theta-function of curve: $\Theta(x \mid B) = \sum_{m \in \mathbf{Z}^g} \exp\left(\pi i\left(\langle Bm, m\rangle + 2\langle x, m\rangle\right)\right)$.

We define normalized ($\oint_{a_i} d\,\Omega_j = 0,\ j = 1, 2$) Abelian integrals $\Omega_1(P)$ and $\Omega_2(P)$, having in a neighborhood of the points $\infty^{\pm}$ the behavior

$$\Omega_k(P) \sim \mp (k \cdot \lambda^k(P) + O(1)), \quad P \to \infty\pm, \quad k = 1, 2.$$

Expressions for the finite-gap solutions have the form

$$\begin{aligned} &S_- = 2WB'/R,\ S_+ = 2CD/R,\ S_3 = (WD - B'C)/R, \\ &W = \Theta(\Omega + D),\ B' = \Theta(\Omega + D - r), \\ &C = \Theta(\Omega + D + n),\quad D = -\Theta(\Omega + D - r + n), \\ &R = WD - B'C, \quad n = \left(\tfrac{1}{2}, \tfrac{1}{2}, \ldots, \tfrac{1}{2}\right). \end{aligned} \tag{6}$$

Here $\Omega = (V^1x - V^2t)/2\pi$, V^k is a vector of b-periods of the integral $\Omega_k(P)$, $k = 1, 2$; $r = \int_{\infty-}^{\infty+} \omega = \int_\gamma \omega$, where contour γ does not intersect the a-cycles on curve Γ (Fig. 1). For vector S to be real, it is necessary and sufficient that phase vector $D \in \mathbf{C}^g$ be such that

$$2\,\mathrm{Re}\,D = -(r + n + \Delta) = -\Delta, \tag{7}$$

where $\Delta \in \mathbf{Z}^g/2\mathbf{Z}^g$ is an arbitrary g-dimensional vector whose components can take on the values 0 and 1. We consider the simplest case $\Delta = (0, 0, \ldots, 0)$. We require the following formula for the ψ_1-component of the Baker–Akhiezer function, which has the form

$$\psi_1 = f(x, t)\frac{\Theta(A(P) + \Omega + D)}{\Theta(A(P) + D)}\exp(i(\Omega_1 x + \Omega_2 t)), \tag{8}$$

where f(x, t) is a non-vanishing function.

1.3. We proceed to consider the boundary value problems (2), (3). For satisfaction of requirement (4) it is sufficient to have vanishing of the theta-function:

$$\begin{aligned} &\Theta\left(\left(A(P_0) - \frac{V^2 t}{2\pi} + D\right) \middle| B\right) = 0 \qquad \forall t, \\ &\lambda(P_0) = i\alpha_0. \end{aligned} \tag{9}$$

We shall determine the restrictions that need to be placed on curve Γ and the phase vector D in order that identity (9) be satisfied. The results obtained in [4] furnish the main consideration. Namely, in [4] it was shown that for a solution of a boundary value problem for NS_- using the "ingradient" technique it is necessary that curve Γ admit symmetry of the type τ: $\lambda \to -\lambda$ and be of odd genus $g > 1$: $g = 2k + 1$, $k = 0, 1, \ldots$.

We require that these conditions be satisfied in our case also. Let the sheets of Γ be permuted by the involution τ: $\lambda \to -\lambda$. It is easy to see that $0^{\pm}$ and $\infty^{\pm}$ are fixed points of the involution τ. We select a basis of cycles on Γ so that under the action of τ it is transformed in the following way (see Fig. 1)

$$\begin{aligned} \tau a_0 &= -a_0, & \tau a_j &= -a_{j+k}, \\ \tau b_0 &= -b_0, & \tau b_j &= -b_{j+k}, \end{aligned} \qquad j = 1, \ldots, k. \tag{10}$$

We note now that by virtue of the symmetry of curve Γ and the basis of cycles (10) the theta-function $\Theta(x \mid B)$ reduces, according to a theorem of Fay [9], to combinations of theta-functions of dimensionality k and k + 1:

$$\Theta\left(\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} \middle| B\right) = \sum_{\delta \in \frac{1}{2}\mathbf{Z}^g/2\mathbf{Z}^g} \Theta[(\delta, 0), 0]\left(\begin{pmatrix} z_1 + z_2 \\ z_0 \end{pmatrix} \middle| 2\Pi\right)\Theta[\delta, 0]((z_1 - z_2) \mid 2T). \tag{11}$$

Here $z_1, z_2 \in \mathbf{C}^k$, $z = (z_1, z_0, z_2)^{\mathrm{T}} \in \mathbf{C}^g$, and $\Theta[\alpha, \beta](z)$ is a theta-function with characteristics α, β; T is a matrix, immaterial for our purposes, of dimensions k × k. The matrices $\Pi \in \mathrm{Mat}(k+1, k+1)$ and $B \in \mathrm{Mat}(g, g)$ have the form

$$\Pi = \begin{pmatrix} \Pi_{ij} & \Pi_{i0} \\ \Pi_{0j} & \Pi_{00} \end{pmatrix} = \begin{pmatrix} 2\int_{b_j} \omega_i & \int_{b_0} \omega_i \\ \int_{b_j} \omega_0 & \frac{1}{2}\int_{b_0} \omega_0 \end{pmatrix}, \quad B = \begin{pmatrix} \frac{\Pi_{ij} + T_{ij}}{2} & \Pi_{j0} & \frac{\Pi_{ij} - T_{ij}}{2} \\ \Pi_{0j} & 2\Pi_{00} & \Pi_{0j} \\ \frac{\Pi_{ij} - T_{ij}}{2} & \Pi_{j0} & \frac{\Pi_{ij} + T_{ij}}{2} \end{pmatrix}. \tag{12}$$

Formulas (11) and (12) show that only the theta-function of dimension k on the left side of Eq. (9) depends on the variable t.

Actually, by virtue of relations (10) we have the identities

$$d\Omega_1(\tau P) = -d\Omega_1(P),\quad d\Omega_2(\tau P) = d\Omega_2(P),\qquad V_j^1 = V_{j+k}^1,\ V_0^2 = 0,\ V_j^2 = -V_{j+k}^2, \tag{13}$$

from which it follows that in the representation (11) the dynamics with respect to x and with respect to t "separates" into theta-functions of dimension k + 1 and k, respectively.

We assume, by definition, each vector $A \in \mathbf{C}^g$; $\hat{A}_i \equiv A_i + A_{i+k}$, $i = 1, \ldots, k$, $g = 0, 1, \ldots, 2k$. To satisfy condition (9) it is sufficient to require that all theta-constants of dimension k + 1 in formula (11) be zero, which leads to the conditions

$$\begin{cases} A_0(P_0) + D_0 = \frac{1}{2} + \left(\frac{BN}{2}\right)_0 + M_0, \\ \hat{A}_i(P_0) + \hat{D}_i = (BN)_i + M_i, \qquad i = 1, \ldots, k, \end{cases}\qquad N, M \in \mathbf{Z}^g/2\mathbf{Z}^g;\ N_0 = 1,\ N_i = N_{i+k},\ M_i = M_{i+k}. \tag{14}$$

Indeed, under these conditions these theta-constants have the form

$$\Theta_\delta = \Theta[(\delta, 0), 0]\left(\begin{matrix} (BN)_i \\ \frac{1}{2} + \left(\frac{BN}{2}\right)_0 \end{matrix}\,\middle|\, 2\Pi\right),$$

from which it follows, upon taking relations (12) into account, that $\Theta_\delta = 0$.

The requirement of compatibility of the condition of reality (7) for $\Delta = 0$ and the boundary conditions (14) on vector D have the form

$$\begin{cases} \operatorname{Re} A_0(P_0) = 1/2, \\ \operatorname{Re} \hat{A}_i(P_0) = 0 \end{cases} \quad (\mathrm{mod}\ 1),$$

which will be automatically satisfied providing that

$$\operatorname{Re} \lambda(P_0) = 0,\ P_0 \in \Gamma^-. \tag{15}$$

Thus we have the following theorem.

<u>THEOREM.</u> Let curve Γ, of odd genus $g > 1$, be specified by Eq. (5) and admit the involution $\tau: \lambda \to -\lambda$. On vector D we impose restrictions (7) with $\Delta = 0$ and conditions (14), where point P_0 is chosen in accordance with provision (15). Formulas (6) then define finite-gap solutions of the boundary value problem on semiaxis $\mathbf{R}_+$ with coupling constant $\alpha_0 = (-i)\cdot\lambda(P_0)$.

We consider only the boundary value problem (2), (3) on the interval [0, 1]. Using the above described approach, we see that to satisfy the boundary condition at point x = 1 we must have

$$\psi_1(x = 1, t, \lambda = i\alpha_1) = 0 \qquad \forall t, \\ \Updownarrow \\ \Theta\left(\left(A(P_1) + \frac{V^1}{2\pi} - \frac{V^2 t}{2\pi} + D\right)\middle| B\right) = 0, \tag{16}$$

where $\lambda(P_1) = i\alpha_1$. Analysis of condition (16) is carried out in exactly the same way as was done at x = 0.

If we require joint satisfaction of conditions (14) and (16), we then arrive at constraints on curve Γ:

$$\begin{cases} A_0(P_1) - A_0(P_0) + \frac{V_0^1}{2\pi} = \left(\frac{BM}{2}\right)_0, \\ \hat{A}_i(P_1) - \hat{A}_i(P_1) + \frac{\hat{V}_i^1}{2\pi} = (BM)_i, \end{cases}\qquad M \in \mathbf{Z}^g/2\mathbf{Z}^g,\ M_0 = 0,\ M_i = M_{i+k},\ \operatorname{Re} \lambda(P_1) = 0. \tag{17}$$

These constraints are similar to those obtained in [4] for the NS_- model. (In this connection, the following change should be made to correct a misprint in [4]: change the sign before the vector in formula (28): $b \to -b$.)

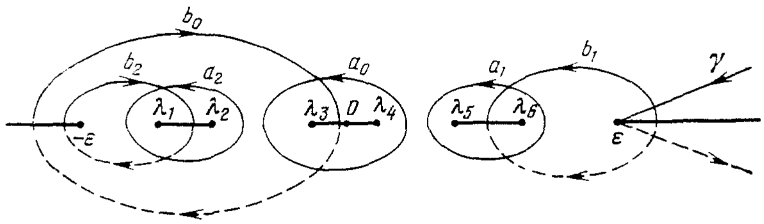

Fig. 1. Curve Γ. g = 3. Landau–Lifshits equation.

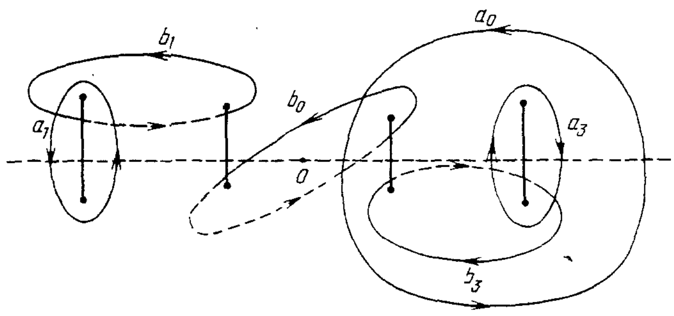

Fig. 2. Curve Γ. g = 3. Nonlinear Schrödinger equation.

Thus, if in addition to the conditions of the theorem we require satisfaction of the constraints (17), the formulas (6) then yield the solution of boundary value problem (2), (3) for the Landau–Lifshits equation (1).

2. Nonlinear Schrödinger Equation (NS+). It is obvious that the approach described above can be applied for singling-out integrable boundary conditions in the class of finite-gap solutions of other integrable equations. We remark that in the case in which finite-gap solutions are constructed over a Riemann surface with complex branch points, there arises the additional technical problem of compatibility of real requirements of type (7) and boundary requirements of type (14).

We illustrate these remarks by way of the NS_+ model:

$$ip_t + p_{xx} + 8\,|\,p\,|^2 p = 0, \tag{18}$$

on the semiaxis $x \geqslant 0$ with the following boundary condition [2] at x = 0:

$$p_x + 2\alpha_0 p\,|_{x=0},\ \alpha_0 = \mathrm{const} \in \mathbf{R}. \tag{19}$$

Formulas for the finite-gap solutions of model (18) and for the Baker–Akhiezer Ψ-function are well known [10] and can be easily reproduced in the framework of the scheme from [4] if in the corresponding formulas no account is taken of the "ingradient" effect. As is well known [10], the reality constraint leads to the fact that the branch points λ_i of the spectral curve Γ must have a nonzero imaginary part. For simplicity (see Remark 1) we consider the case when

$$\mathrm{Re}\,\lambda_i \neq 0,\ \mathrm{Im}\,\lambda_i \neq 0.$$

By analogy with [4], we select a curve Γ of odd genus g = 2k + 1, k = 0, 1,..., admitting two non-interchangeable branches of symmetry: an involution $\tilde{\tau}$: $\lambda \to -\lambda$ and a complex conjugate anti-involution π: $\lambda \to \bar{\lambda}$.

We select a basis of cycles on Γ, transforming in the following way:

$$\begin{aligned} &\tilde{\tau} a_0 = -a_0, \quad \tilde{\tau} a_i = -a_{i+k}, \\ &\tilde{\tau} b_0 = -b_0, \quad \tilde{\tau} b_i = -b_{i+k}, \qquad i = 1, \ldots, k, \\ &\pi a_0 = -a_0, \quad \pi a_j = -a_j, \qquad j = 1, \ldots, 2k, \\ &\pi b_0 = b_0 + \textstyle\sum_j^g m_j a_j, \quad \pi b_j = b_j + \sum_s^g n_s a_s, \end{aligned} \tag{20}$$

where m_j, n_j are integral coefficients. Curve Γ, along with the basis of cycles, is shown in Fig. 2 for the case g = 3. It is not hard to see that for the differentials $d\Omega_k(P)$, k = 1, 2 we have the identities

$$d\bar{\Omega}_k(\pi P) = d\Omega_k(P),$$
$$d\Omega_1(\tilde{\tau}P) = d\Omega_1(P), \quad d\Omega_2(\tilde{\tau}P) = d\Omega_2(P),$$

from which follows the reality of the winding vectors V^k:

$$\bar{V}^k = V^k, \quad k = 1, 2,$$

and, in addition, fulfillment of the symmetries (13).

In the indicated basis of cycles the "realness" requirement induces the following constraint on the phase vector D (cf. with [4]):

$$\operatorname{Im} D = 0, \tag{21}$$

and a "boundary" requirement of the type (4) again leads, as is readily verified from explicit formulas for the Ψ-function, to constraints of the form (14).

Thus we arrive at the compatibility conditions (21) and (14):

$$\begin{cases} 2\operatorname{Im} A(P_0) = \operatorname{Im}(BN)_0, \\ \operatorname{Im}\hat{A}(P_0) = \operatorname{Im}(BN)_i, \quad i = 1, \ldots, k, \end{cases} \tag{22}$$

where the vector $N \in \mathbf{Z}^g/2\mathbf{Z}^g$ has the same properties as in condition (14). By virtue of positive definitenes of the matrix Im B the right side of relations (22) is nonzero. On the other hand, since the coupling constant $\alpha_0 \in \mathbf{R}$, it follows that

$$\operatorname{Re}\lambda(P_0) = 0.$$

We note now that if the point $P_0 = P_0^+$ is chosen on the upper branch Γ^+, the condition (22) is then not satisfied. Actually, in this case the path of integration ℓ in the formula $A(P_0^+) = \int_{\infty^+}^{P_0^+}\omega = \int_l \omega$ can be chosen so that $\pi\ell = \tilde{\tau}\ell$, which leads, taking into account the identies

$$\omega_0(\tilde{\tau}P) = -\omega_0(P), \quad \hat{\omega}_i(\tilde{\tau}P) = -\hat{\omega}_i(P),$$
$$\bar{\omega}_0(\pi P) = -\omega_0(P), \quad \hat{\omega}_i(\pi P) = -\hat{\omega}_i(P), \quad i = 1, 2, \ldots, k, \tag{23}$$

to realness of the vector $A(P_0^+)$.

Conversely, in case point $P_0 = P_0^-$ is chosen on the lower branch Γ^-, then on the left side of relations (22) we have the vector $\operatorname{Im} R' = -\operatorname{Im}(2r_0, \hat{r}_i)^{\mathrm{T}}$, $i = 1, \ldots, k$; $R' \in \mathbf{C}^{k+1}$, $r = \int_\gamma \omega$. The contour of integration in this case can be selected so that $\gamma - \tau\gamma = b_0$. Taking relations (23) into account, it is readily seen that $2\operatorname{Im} R = \operatorname{Im}\int_{\gamma-\tilde{\tau}\gamma}\omega = \operatorname{Im} BN$, where $N = -(1, 0, \ldots, 0) \in \mathbf{R}^g$.

Thus, choosing the special vector $N = -(1, 0, \ldots, 0)$, we can satisfy requirement (22) and we can construct finite-gap solutions of boundary value problem (19) for model (18) of the NS_+. We remark that to solve the problem on the interval [0, 1] we again need to require satisfaction of the additional constraints (17).

Remark 1. If we consider the more general case of a spectral curve Γ with purely imaginary branch points $\tilde{\lambda}_j$, $j = 1, \ldots, 2m$, $\operatorname{Re}\tilde{\lambda}_j = 0$, we must then expect additional constraints on the coupling constants α_0, α_1, resulting from compatibility conditions of type (22). By analogy with the soliton case [2], we can assume that these constraints will have the form $|\alpha_0|, |\alpha_1| \leqslant \min|\tilde{\lambda}_j|$, $j = 1, \ldots, 2m$.

Conclusion. In the present paper, on the basis of two fundamental models, we have shown how it is possible to single-out finite-gap solutions of integrable boundary value problems, staying within the scope of the traditional IPM technique (and not involving the "ingredient" technique).

A similar idea can be employed to solve a Cauchy problem on the semiaxis $x \geqslant 0$ with an integrable boundary condition at $x = 0$ and a condition of rapid decrease as $x \to +\infty$. Constraints on scattering data for a problem on the whole axis, which make it possible to preserve during evolution over time t a boundary condition at point $x = 0$, have a simple form and contain in explicit form the symmetry $\lambda \to -\lambda$. The corresponding formulations were first obtained by V. O. Tarasov. We note also an observation of I. T. Khabibullin, which connects the deviation of the boundary condition with the possibility of introducing the symmetric

$x \to -x$ reduction in the Bäcklund transformation, permitted by the initial equation.

In all the schemes enumerated it is possible to reduce a problem on a semiaxis to a problem on the whole axis for integrable equations like the Landau–Lifshits equation, the nonlinear Schrödinger equation, the sine–Gordon equation, and similar types, possessing the explicit symmetry $x \to -x$.

A problem which remains open is that of singling-out integrable boundary conditions for equations which do not possess a similar property, for example, for the Korteweg–de Vries equation.

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UNIFORM RESOLVENT CONVERGENCE OF LINEAR OPERATORS UNDER PERTURBATIONS

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UDC 517.983.28

In this paper, sufficient conditions for resolvent convergence of perturbed operators to a non-perturbed one are obtained. The problem of determining this convergence is closely connected with the investigation of the stability of eigenvalues under perturbations [1] and the behavior of solutions of singularly perturbed problems [2]. Sufficient conditions obtained here are different from those studied earlier for self-adjoint [1, 3, 4] and non-self-adjoint [4, 5] operators. They allow us to consider a new class of perturbations, mainly for non-self-adjoint operators. Also, for certain already investigated perturbations of self-adjoint operators, these conditions simplify verification of uniform resolvent convergence.

Let H be a separable Hilbert space with the scalar product $(\cdot, \cdot)$ and the norm $\|\cdot\|$. For any linear operator T in H, denote by D(T) its domain of definition, by P(T) the resolvent set of the operator T and by O(T) the remainder of the spectrum (i.e., the set of all $\lambda \in \mathbf{C}$ such that the set of values of the operator $T - \lambda E$ is not dense in H). If the T operator can be closed, we denote its closure by T.

We shall consider a family of operators of the form $T(\varepsilon) = T_0 \pm \varepsilon T_1$, $\varepsilon > 0$, where T_0, T_1 are linear operators in H, $D(T(\varepsilon)) = D(T_0) \cap D(T_1)$ for $\varepsilon > 0$ and $\bar{D} = H$.

We state the basic result.

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