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An Axiomatization of the Disjunctive Permission Value for Games with a Permission Structure[1]

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Abstract: Players that participate in a *cooperative game with transferable utilities* are assumed to be part of a *permission structure* being a hierarchical organization in which there are players that need permission from other players before they can cooperate. Thus a permission structure limits the possibilities of coalition formation.

Various assumptions can be made about how a permission structure affects the cooperation possibilities. In this paper we consider the *disjunctive approach* in which it is assumed that each player needs permission from at least one of his predecessors before he can act. We provide an axiomatic characterization of the *disjunctive permission value* being the *Shapley value* of a modified game in which we take account of the limited cooperation possibilities.

1 Introduction

A situation in which a finite set of players N can generate certain payoffs by cooperation can be described by a *cooperative game with transferable utilities* (or simple a TU-game), being a pair (N, v) where $v: 2^N \to \mathbb{R}$ is a *characteristic function* such that $v(\emptyset) = 0$. Since in this paper we take the player set N to be fixed we represent a TU-game by its characteristic function. We denote the collection of all TU-games on N by $\mathcal{G}^N$.

In a TU-game the players only differ with respect to their contributions to the payoffs that coalitions can obtain by cooperation. Besides that the players are assumed to be socially identical in the sense that every player can cooperate with every other player. Models have been developed in which there are social asymmetries between players in a TU-game. In, e.g., Aumann and Drèze (1974), Owen (1977), and Winter (1989), it is assumed that the players are part of a *coalition structure* which is a partition of the players into disjoint sets. These sets can be seen as social groups such that for a particular player it is easier to cooperate with players in his own group than to cooperate with players in other groups.

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Another example of models in which the players are socially different can be found in, e.g., Myerson (1977), Kalai, Postlewaite, and Roberts (1978), Owen (1986), Borm, Owen, and Tijs (1992), and van den Nouweland (1993). In these models an undirected graph describes limited communication possibilities between the players. The edges of such a graph represent binary communication links. Whether players can cooperate or not then depends on their position in the communication graph.

This paper is based on the models as developed in Gilles, Owen, and van den Brink (1992), van den Brink and Gilles (1996), and Gilles and Owen (1994). For a survey of these models we refer to van den Brink (1994). A related model can be found in Faigle and Kern (1993). In these models it is assumed that players that participate in a TU-game are part of a hierarchical organization in which there are players that need permission from other players before they are allowed to cooperate within a coalition. Thus the possibilities of coalition formation are determined by the positions of the players in this so-called *permission structure.* Various assumptions can be made about how a permission structure affects the cooperation possibilities in a TU-game. In this paper we take the *disjunctive approach* as considered in Gilles and Owen (1994). In this approach it is assumed that a player needs permission from at least one of his predecessors before he is allowed to cooperate with other players.

An *allocation rule* for games with a permission structure is a function that assigns to every game with a permission structure a distribution of the payoffs that can be obtained by cooperation. The main result of this paper is an axiomatic characterization of a particular allocation rule that is based on the disjunctive approach, namely the *disjunctive permission value.* The crucial axiom in this axiomatization is *fairness* which states that deleting a permission relation between two players has the same effect on the payoffs of both players. This axiom is closely related to fairness as stated in Myerson (1977) for games with limited communication possibilities. For these games fairness means that deleting a communication relation between two players has the same effect on both their payoffs.

In Gilles, Owen, and van den Brink (1992) an alternative approach to games with a permission structure is considered, namely the *conjunctive approach.* In this approach it is assumed that each player needs permission from *all* his predecessors in the permission structure before he is allowed to cooperate. In van den Brink and Gilles (1996) an axiomatization of the *conjunctive permission value* is given. This is an allocation rule that is based on this conjunctive approach. This value does not satisfy fairness.

In Section 2 we briefly discuss the disjunctive and conjunctive approach to games with a permission structure. Given a game with a permission structure corresponding modified games are derived in which we take account of the limited possibilities of coalition formation based on the disjunctive and conjunctive approaches. The disjunctive and conjunctive permission values are then defined as the *Shapley values* (Shapley (1953)) of the corresponding modified games.

In Section 3 we first show an important difference between the disjunctive and conjunctive approaches. In the disjunctive approach deleting a relation in a permission structure results in less possibilities of cooperation, while deleting a relation leads to more cooperation possibilities in the conjunctive approach. We then show that the disjunctive permission value satisfies fairness, i.e., the deletion of a relation between two players changes their disjunctive permission value by the same amount. This is not the case for the conjunctive permission value.

Finally, in Section 4 we give an axiomatization of the disjunctive permission value for games with a permission structure that uses fairness.

2 Games with a Permission Structure

We assume that players who participate in a TU-game are part of a hierarchical organization in which there are players that need permission from certain other players before they are allowed to cooperate. For a finite set of players N such a hierarchical organization is represented by a mapping $S: N \to 2^N$ which is called a *permission structure* on N. The players in $S(i)$ are called the *successors* of player $i \in N$ in the permission structure S. The players in $S^{-1}(i) := \{j \in N \mid i \in S(j)\}$ are called the *predecessors* of i in S. By $\hat{S}$ we denote the *transitive closure* of the permission structure S, i.e., $j \in \hat{S}(i)$ if and only if there exists a sequence of players $(h_1, \ldots, h_t)$ such that $h_1 = i$, $h_{k+1} \in S(h_k)$ for all $1 \leq k \leq t-1$ and $h_t = j$. The players in $\hat{S}(i)$ are called the *subordinates* of i in S, and the players in $\hat{S}^{-1}(i) := \{j \in N \mid i \in \hat{S}(j)\}$ are called the *superiors* of i in S.

In this paper we restrict our attention to a special class of permission structures that are also considered in Gilles and Owen (1994).

Definition 2.1: A permission structure S on N is *hierarchical* if the following two conditions are satisfied

(i) S is *acyclic*, i.e., for every $i \in N$ it holds that $i \notin \hat{S}(i)$;
(ii) S is *quasi-strongly connected*, i.e., there exists an $i \in N$ such that $\hat{S}(i) = N \backslash \{i\}$.

We denote the collection of all hierarchical permission structures on N by $\mathscr{S}_H^N$. These hierarchical permission structures are important for economic applications as discussed in van den Brink and Gilles (1994). In that paper it is also shown that in a hierarchical permission structure there exists a unique player i_0 such that $\hat{S}(i_0) = N \backslash \{i_0\}$. Moreover, for this player it holds that $S^{-1}(i_0) = \emptyset$. We call this player the *topman* in the permission structure.

A triple (N, v, S) with $v \in \mathscr{G}^N$ and $S \in \mathscr{S}_H^N$ is called a *game with a hierarchical permission structure*. As in Gilles and Owen (1994) we assume that each player needs permission from at least one of his predecessors before he is allowed to cooperate with other players. Consequently, a coalition can cooperate only if every player in the coalition, except the topman i_0, has a predecessor who also belongs to the coalition. (Note that this implies that the unique topman i_0 belongs to the coalition.) Thus, the formable coalitions are the ones in the set

$$\Psi_S := \left\{ E \subset N \;\middle|\; \begin{array}{l} \text{for every } i \in E \text{ there is a sequence of players } (h_1, \ldots, h_t) \\ \text{such that } h_1 = i_0,\ h_{k+1} \in S(h_k) \text{ for all } 1 \leq k \leq t-1, \\ \text{and } h_t = i \end{array} \right\}. \tag{1}$$

The coalitions in Ψ_S are called the *disjunctive autonomous* coalitions in S.

Definition 2.2: The *disjunctive sovereign part* of $E \subset N$ in $S \in \mathscr{S}_H^N$ is the coalition given by

$$\sigma(E) = \cup \{F \in \Psi_S \mid F \subset E\}.$$

The disjunctive sovereign part of $E \subset N$ is the largest formable subset of E. It consists of those players in E that can be reached by a 'permission path' starting at the topman such that all players on this path belong to coalition E. Using this concept we can transform the game $v \in \mathscr{G}^N$ into a modified game in which we take account of the limited possibilities of cooperation as determined by the permission structure S.

Definition 2.3: Let $v \in \mathscr{G}^N$ and $S \in \mathscr{S}_H^N$. The *disjunctive restriction* of v on S is the game $\mathscr{D}_S(v) \in \mathscr{G}^N$ given by

$$\mathscr{D}_S(v)(E) := v(\sigma(E)) \text{ for all } E \subset N.$$

An *allocation rule* for games with a permission structure is a function that assigns to every game with a permission structure (N, v, S) a distribution of the payoffs that can be obtained by cooperation according to v taking into account the limited cooperation possibilities determined by S. In this paper we discuss the *disjunctive permission value* $\psi : \mathscr{G}^N \times \mathscr{S}_H^N \to \mathbb{R}^N$ which is given by

$$\psi(v, S) := Sh(\mathscr{D}_S(v)) \text{ for all } v \in \mathscr{G}^N \text{ and } S \in \mathscr{S}_H^N,$$

where $Sh : \mathscr{G}^N \to \mathbb{R}$ denotes the Shapley value, i.e.,

$$Sh_i(v) = \sum_{E \ni i} \frac{\Delta_v(E)}{\#E}, \text{ for all } i \in N \text{ and } v \in \mathscr{G}^N, \tag{2}$$

with *dividends* given by $\Delta_v(E) := \sum_{F \subset E} (-1)^{\#E - \#F} v(F)$ for all $E \subset N$ (see Harsanyi (1959)).

An alternative allocation rule is the *conjunctive permission value* which is based on the *conjunctive approach* as developed in Gilles, Owen, and van den Brink

(1992). In this approach it is assumed that each player needs permission from *all* his superiors in the permission structure before he is allowed to cooperate. This implies that a coalition E is formable only if for every player $i \in E$ it holds that all superiors of i are part of the coalition. The set of formable coalitions in this approach thus is given by

$$\Phi_S := \{E \subset N \mid \text{for every } i \in E \text{ it holds that } \hat{S}^{-1}(i) \subset E\}. \tag{3}$$

The coalitions in the set Φ_S are called the *conjunctive autonomous* coalitions in S. Similarly as in the disjunctive approach the largest autonomous subset of E, $\sigma^c(E) = \cup\{F \in \Phi_S \mid F \subset E\}$, is refered to as the *conjunctive sovereign part* of E in S. It consists of all players in E whose superiors are all part of E. Given a game with a permission structure (N, v, S) the *conjunctive restriction* of v on S is the game $\mathscr{R}_S(v)$ given by $\mathscr{R}_S(v)(E) := v(\sigma^c(E))$ for all $E \subset N$. The *conjunctive permission value* for games with a hierarchical permission structure $\varphi: \mathscr{G}^N \times \mathscr{S}_H^N \to \mathbb{R}^N$ then is given by $\varphi(v, S) := Sh(\mathscr{R}_S(v))$ for all $v \in \mathscr{G}^N$ and $S \in \mathscr{S}_H^N$.

Example 2.4: Let $v \in \mathscr{G}^N$ and $S \in \mathscr{S}_H^N$ on $N = \{1, 2, 3, 4\}$ be given by

$$v(E) = \begin{cases} 1 & \text{if } E \ni 4 \\ 0 & \text{else} \end{cases}$$

and

$$S(1) = \{2, 3\},\ S(2) = S(3) = \{4\},\ S(4) = \emptyset.$$

The disjunctive and conjunctive restrictions of v on S, respectively, are given by

$$\mathscr{D}_S(v)(E) = \begin{cases} 1 & \text{if } E \in \{\{1,2,4\},\ \{1,3,4\},\ \{1,2,3,4\}\} \\ 0 & \text{else} \end{cases}$$

and

$$\mathscr{R}_S(v)(E) = \begin{cases} 1 & \text{if } E = \{1,2,3,4\} \\ 0 & \text{else} \end{cases}$$

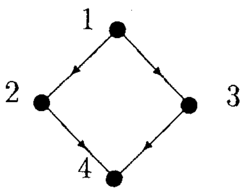

Fig. 1. Permission structure S of example 2.4

The disjunctive and conjunctive permission values are given by $\psi(v,S)=(\frac{5}{12},\frac{1}{12},\frac{1}{12},\frac{5}{12})$ and $\varphi(v,S)=(\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4})$.

3 A Fairness Axiom

In this section we discuss a particular axiom that plays an important role in the axiomatization of the disjunctive permission value that is presented in the next section. Suppose that $h \in N$ and $j \in S(h)$ are such that the permission structure that results after the deletion of the permission relation between players h and j is hierarchical. The axiom states that if we delete the relation between players h and j, then their disjunctive permission values decrease (or increase) by the same amount. Moreover, if player i dominates player h 'completely' in the sense that all permission paths from the unique topman i_0 to player h contain player i, then also player i's disjunctive permission value changes by that same amount. Given a permission structure $S \in \mathcal{S}_H^N$ and two players $h, j \in N$ such that $j \in S(h)$ we define the permission structure $S_{-(h,j)}$ by

$$S_{-(h,j)}(i)=\begin{cases} S(i)\backslash\{j\} & \text{if } i=h \\ S(i) & \text{else.} \end{cases}$$

Note that in order for $S_{-(h,j)}$ to be a hierarchical permission structure it must hold that $\#S^{-1}(j) \geq 2$. (If this is not the case then $S_{-(h,j)}^{-1}(j)=\emptyset$, and thus $j \notin \hat{S}_{-(h,j)}(i_0)$.) Before analyzing how the deletion of the relation between two players affect their disjunctive permission values we state a proposition which points out an important difference between the conjunctive and disjunctive approaches to games with a permission structure. If we delete a relation in a hierarchical permission structure (such that the permission structure stays hierarchical) then this leads to less autonomous coalitions in the disjunctive approach, while it leads to more autonomous coalitions in the conjunctive approach.

Proposition 3.1: For every $S \in \mathcal{S}_H^N$ and $h, j \in N$ such that $j \in S(h)$ and $\#S^{-1}(j) \geq 2$ it holds that $\Psi_{S_{-(h,j)}} \subset \Psi_S$ and $\Phi_{S_{-(h,j)}} \supset \Phi_S$.

Proof: Let $S \in \mathcal{S}_H^N$ and $h, j \in N$ be such that $j \in S(h)$ and $\#S^{-1}(j) \geqslant 2$.

(i) Suppose that $E \in \Psi_{S_{-(h,j)}}$. Since $S_{-(h,j)}(i) \subset S(i)$ for all $i \in N$ it follows with (1) that $E \in \Psi_S$.

(ii) Suppose that $E \in \Phi_S$. Since $S_{-(h,j)}^{-1}(i) \subset S^{-1}(i)$ it follows with (3) that $\hat{S}_{-(h,j)}^{-1}(i) \subset \hat{S}^{-1}(i) \subset E$ for all $i \in E$. Thus $E \in \Phi_{S_{-(h,j)}}$. □

Next we present a lemma which states that a disjunctive autonomous coalition E that does not contain player h and his successor $j \in S(h)$ is still disjunctive autonomous after the deletion of the relation between h and j.

Lemma 3.2: For every $S \in \mathscr{S}_H^N$ and $h, j \in N$ such that $j \in S(h)$ and $\#S^{-1}(j) \geq 2$ it holds that

$$E \in \Psi_S \text{ and } E \not\supset \{h,j\} \text{ implies that } E \in \Psi_{S_{-(h,j)}}.$$

Proof: Let $S \in \mathscr{S}_H^N$ and $h, j \in N$ be such that $j \in S(h)$ and $\#S^{-1}(j) \geq 2$.

Further, let $E \in \Psi_S$ and $E \not\supset \{h, j\}$.

If $E \not\ni j$ then it follows with (1), $E \in \Psi_S$ and the fact that $S^{-1}_{-(h,j)}(i) = S^{-1}(i)$ for all $i \in N \backslash \{j\}$ that $E \in \Psi_{S_{-(h,j)}}$.

If $E \ni j$ then by assumption $E \not\ni h$. Since $E \in \Psi_S$ it holds that $(S^{-1}(j) \backslash \{h\}) \cap E \neq \varnothing$. But then $S^{-1}_{-(h,j)}(j) \cap E \neq \varnothing$. Since $S^{-1}_{-(h,j)}(i) = S^{-1}(i)$ for all $i \in E \backslash \{j\}$ it then follows with (1) that $E \in \Psi_{S_{-(h,j)}}$. □

Now we are able to state the main result of this section which says that deleting the relation between two players h and $j \in S(h)$ (with $\#S^{-1}(j) \geq 2$) changes the disjunctive permission values of players h and j by the same amount. Moreover, also the disjunctive permission values of all players i that 'completely' dominate player h in the sense that all permission paths from the topman i_0 to player h contain player i, change by this same amount.

Theorem 3.3: For every $v \in \mathscr{G}^N$, $S \in \mathscr{S}_H^N$ and $h, j \in N$ such that $j \in S(h)$ and $\#S^{-1}(j) \geq 2$ it holds that

$$\psi_i(v, S) - \psi_i(v, S_{-(h,j)}) = \psi_j(v, S) - \psi_j(v, S_{-(h,j)}) \text{ for all } i \in \{h\} \cup \bar{S}^{-1}(h),$$

where $\bar{S}^{-1}(h) := \{i \in \hat{S}^{-1}(h) \mid E \in \Psi_S \text{ and } E \ni h \text{ implies that } E \ni i\}$.

Proof: Let $w_T = c_T u_T$, where u_T is the *unanimity game* of coalition $T \subset N$, and $c_T \in \mathbb{R}$ is some constant, i.e.,

$$w_T(E) = \begin{cases} c_T & \text{if } E \supset T \\ 0 & \text{else.} \end{cases}$$

Further, let $S \in \mathscr{S}_H^N$ and $h, j \in N$ be such that $j \in S(h)$ and $\#S^{-1}(j) \geq 2$.

From the definition of the Shapley value (equation (2)), and the fact that $\Delta_{\mathscr{D}_S(w_T)}(E) = 0$ for all $E \in 2^N \backslash \Psi_S$ (this follows from a more general result that is stated in Derks and Peters (1993)) it follows with Proposition 3.1 that for every $i \in N$ it holds that

$$\psi_i(w_T, S) = \sum_{\substack{E \in \Psi_S \\ E \ni i}} \frac{\Delta_{\mathscr{D}_S(w_T)}(E)}{\#E} = \sum_{\substack{E \in \Psi_{S_{-(h,j)}} \\ E \ni i}} \left(\frac{\Delta_{\mathscr{D}_S(w_T)}(E)}{\#E} \right) + \sum_{\substack{E \in \Psi_S \backslash \Psi_{S_{-(h,j)}} \\ E \ni i}} \left(\frac{\Delta_{\mathscr{D}_S(w_T)}(E)}{\#E} \right)$$

Next we establish the following facts:

(i) If $E \not\supset \{h, j\}$ then clearly $F \not\supset \{h, j\}$ for all $F \subset E$. But this implies that the disjunctive sovereign part of $F \subset E$ in permission structure S is the same as the disjunctive sovereign part of F in permission structure $S_{-(h,j)}$. Thus $\mathscr{D}_S(w_T)(F) = \mathscr{D}_{S_{-(h,j)}}(w_T)(F)$ for all $F \subset E$. For the dividends it then holds that $\Delta_{\mathscr{D}_S(w_T)}(E) = \Delta_{\mathscr{D}_{S_{-(h,j)}}(w_T)}(E)$ for all $E \not\supset \{h, j\}$.

(ii) Lemma 3.2 is equivalent to saying that

$E \in \Psi_S \setminus \Psi_{S_{-(h,j)}}$ implies that $E \supset \{h, j\}$.

Thus it follows with this lemma that

$$\sum_{\substack{E \in \Psi_S \setminus \Psi_{S_{-(h,j)}} \\ E \ni h}} \left(\frac{\Delta_{\mathscr{D}_S(w_T)}(E)}{\#E} \right) = \sum_{\substack{E \in \Psi_S \setminus \Psi_{S_{-(h,j)}} \\ E \ni j}} \left(\frac{\Delta_{\mathscr{D}_S(w_T)}(E)}{\#E} \right).$$

From this we can derive that

$$\begin{aligned}
\psi_h(w_T, S) - \psi_j(w_T, S) &= \sum_{\substack{E \in \Psi_{S_{-(h,j)}} \\ E \ni h}} \left(\frac{\Delta_{\mathscr{D}_S(w_T)}(E)}{\#E} \right) - \sum_{\substack{E \in \Psi_{S_{-(h,j)}} \\ E \ni j}} \left(\frac{\Delta_{\mathscr{D}_S(w_T)}(E)}{\#E} \right) \\
&= \sum_{\substack{E \in \Psi_{S_{-(h,j)}} \\ E \ni h, E \not\ni j}} \left(\frac{\Delta_{\mathscr{D}_S(w_T)}(E)}{\#E} \right) - \sum_{\substack{E \in \Psi_{S_{-(h,j)}} \\ E \not\ni h, E \ni j}} \left(\frac{\Delta_{\mathscr{D}_S(w_T)}(E)}{\#E} \right) \\
&= \sum_{\substack{E \in \Psi_{S_{-(h,j)}} \\ E \ni h, E \not\ni j}} \left(\frac{\Delta_{\mathscr{D}_{S_{-(h,j)}}(w_T)}(E)}{\#E} \right) - \sum_{\substack{E \in \Psi_{S_{-(h,j)}} \\ E \not\ni h, E \ni j}} \left(\frac{\Delta_{\mathscr{D}_{S_{-(h,j)}}(w_T)}(E)}{\#E} \right) \\
&= \sum_{\substack{E \in \Psi_{S_{-(h,j)}} \\ E \ni h}} \left(\frac{\Delta_{\mathscr{D}_{S_{-(h,j)}}(w_T)}(E)}{\#E} \right) - \sum_{\substack{E \in \Psi_{S_{-(h,j)}} \\ E \ni j}} \left(\frac{\Delta_{\mathscr{D}_{S_{-(h,j)}}(w_T)}(E)}{\#E} \right) \\
&= \psi_h(w_T, S_{-(h,j)}) - \psi_j(w_T, S_{-(h,j)}) \qquad (4)
\end{aligned}$$

Further, we can derive the following facts:

(iii) By definition of Ψ_S it holds that $E \in \Psi_S$ and $E \ni h$ imply that $E \supset \bar{S}^{-1}(h)$.

(iv) If $E \not\ni h$ then $E \in \Psi_S$ if and only if $E \in \Psi_{S_{-(h,j)}}$.

(v) From fact (i) stated above it follows that $\Delta_{\mathscr{D}_S(w_T)}(E) = \Delta_{\mathscr{D}_{S_{-(h,j)}}(w_T)}(E)$ for all $E \not\ni h$.

From this it follows that for every $i \in \bar{S}^{-1}(h)$ it holds that

$$\psi_i(w_T, S) - \psi_j(w_T, S) = \sum_{\substack{E \in \Psi_S \\ E \ni i, E \not\ni h}} \left(\frac{\Delta_{\mathscr{D}_S(w_T)}(E)}{\#E} \right) + \sum_{\substack{E \in \Psi_S \\ E \ni i, E \ni h}} \left(\frac{\Delta_{\mathscr{D}_S(w_T)}(E)}{\#E} \right)$$

$$- \sum_{E \in \Psi_S} \left(\frac{\Delta_{\mathscr{D}_S(w_T)}(E)}{\#E} \right)$$

$$= \sum_{\substack{E \in \Psi_{S_{-(h,j)}} \\ E \ni i, E \not\ni h}} \left(\frac{\Delta_{\mathscr{D}_{S_{-(h,j)}}(w_T)}(E)}{\# E} \right) + \sum_{\substack{E \in \Psi_S \\ E \ni h}} \left(\frac{\Delta_{\mathscr{D}_S(w_T)}(E)}{\# E} \right)$$

$$- \sum_{\substack{E \in \Psi_S \\ E \ni j}} \left(\frac{\Delta_{\mathscr{D}_S(w_T)}(E)}{\# E} \right).$$

Together with facts (i) and (ii) stated above we then can derive that for every $i \in \bar{S}^{-1}(h)$ it holds that

$$\psi_i(w_T, S) - \psi_j(w_T, S) = \sum_{\substack{E \in \Psi_{S_{-(h,j)}} \\ E \ni i, E \not\ni h}} \left(\frac{\Delta_{\mathscr{D}_{S_{-(h,j)}}(w_T)}(E)}{\# E} \right) + \sum_{\substack{E \in \Psi_{S_{-(h,j)}} \\ E \ni h}} \left(\frac{\Delta_{\mathscr{D}_{S_{-(h,j)}}(w_T)}(E)}{\# E} \right)$$

$$- \sum_{\substack{E \in \Psi_{S_{-(h,j)}} \\ E \ni j}} \left(\frac{\Delta_{\mathscr{D}_{S_{-(h,j)}}(w_T)}(E)}{\# E} \right)$$

$$= \psi_i(w_T, S_{-(h,j)}) - \psi_j(w_T, S_{-(h,j)}).$$

With (4) we can conclude that

$$\psi_i(w_T, S) - \psi_i(w_T, S_{-(h,j)}) = \psi_j(w_T, S) - \psi_j(w_T, S_{-(h,j)}) \quad \text{for all } i \in \{h\} \cup \bar{S}^{-1}(h).$$

For arbitrary games $v \in \mathscr{G}^N$ with a hierarchical permission structure $S \in \mathscr{S}_H^N$ it holds that

$$\mathscr{D}_S(v)(E) = v(\sigma(E)) = \sum_{T \subset N} \Delta_v(T) u_T(\sigma(E)) = \sum_{T \subset N} \Delta_v(T) \mathscr{D}_S(u_T)(E) \text{ for all } E \subset N.$$

Similarly $\mathscr{D}_{S_{-(h,j)}}(v)(E) = \sum_{T \subset N} \Delta_v(T) \mathscr{D}_{S_{-(h,j)}}(u_T)(E)$ for all $E \subset N$.

The theorem them follows directly from additivity of the Shapley value. □

An allocation rule that satisfies the condition stated in Theorem 3.3 is said to be *fair*. This concept of fairness is closely related to the fairness concept that is introduced in Myerson (1977) for games in which the possibilities of cooperation are restricted because of limited communication possibilities between the players. As the following example shows the conjunctive permission value is not fair.

Example 3.4: Consider the game with hierarchical permission structure of Example 2.4. Let S' be the permission structure that is obtained by deleting the relation between players 3 and 4.

Then

$$\mathscr{D}_{S'}(E) = \mathscr{R}_{S'}(E) = \begin{cases} 1 & \text{if } E \supset \{1,2,4\} \\ 0 & \text{else,} \end{cases}$$

and thus $\psi(v, S') = \varphi(v, S') = (\frac{1}{3}, \frac{1}{3}, 0, \frac{1}{3})$.

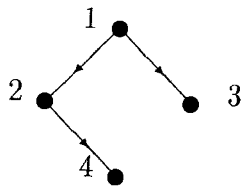

Fig. 2. Permission structure S' of example 3.4

Comparing this with the values for (N, v, S) in Example 2.4 yields

$$\psi_3(v,S)-\psi_3(v,S')=\frac{1}{12}-0=\frac{1}{12}=\frac{5}{12}-\frac{1}{3}=\psi_4(v,S)-\psi_4(v,S')$$

and

$$\varphi_3(v,S)-\varphi_3(v,S')=\frac{1}{4}-0=\frac{1}{4}\neq-\frac{1}{12}=\frac{1}{4}-\frac{1}{3}=\varphi_4(v,S)-\varphi_4(v,S').$$

Note that the conjunctive permission values of players 3 and 4 change in opposite directions.

4 An Axiomatization of the Disjunctive Permission Value

In this section we present six axioms on an allocation rule $f: \mathscr{G}^N \times \mathscr{S}_H^N \to \mathbb{R}^N$ that uniquely determine the disjunctive permission value for games with a hierarchical permission structure. Five of these axioms are also satisfied by the conjunctive permission value[3]. The sixth axiom is fairness.

The first two axioms are generalizations of *efficiency* and *additivity* of the Shapley value.

Axiom 4.1 (Efficiency): For every $v \in \mathscr{G}^N$ and $S \in \mathscr{S}_H^N$ it holds that

$$\sum_{i \in N} f_i(v,S) = v(N).$$

Axiom 4.2 (Additivity): For every $v, w \in \mathscr{G}^N$ and $S \in \mathscr{S}_H^N$ it holds that

$$f(v+w,S) = f(v,S) + f(w,S),$$

where $(v+w) \in \mathscr{G}^N$ is defined by $(v+w)(E) := v(E) + w(E)$ for all $E \subset N$.

[3] The first four axioms are already stated in van den Brink and Gilles (1996). The fifth axiom is a weaker version of the corresponding axiom in that paper.

If the players are not part of a permission structure then the zero player axiom of the Shapley value states that if player $i \in N$ is a *zero player*, i.e., $v(E) = v(E \setminus \{i\})$ for all $E \supset N$, then i gets a payoff equal to zero. However, if the players are part of a permission structure then, although player i is a zero player in game v, it might be that there are non-zero players that need his permission. In that case it seems reasonable that player i gets a non-zero payoff. However, if all subordinates of the zero player i are also zero players then again it seems reasonable that player i gets a zero payoff. Such a player i is called *inessential* in (N, v, S).

Axiom 4.3 (Inessential Player Property): For every $v \in \mathcal{G}^N$, $S \in \mathcal{S}_H^N$, and $i \in N$ such that every player $j \in \hat{S}(i) \cup \{i\}$ is a zero player in v, it holds that $f_i(v, S) = 0$.

The next two axioms are stated for the class of monotone TU-games. A TU-game v is *monotone* if for all $E \subset F \subset N$ it holds that $v(E) \leq v(F)$. The class of all monotone TU-games on N is denoted by $\mathcal{G}_M^N$. If player i is necessary for any coalition to obtain any positive payoff in a monotone game then i can always guarantee that the other players earn nothing by refusing any cooperation. In that case it seems reasonable that the *necessary* player i gets at least as much as any other player.

Axiom 4.4 (Necessary Player Property): For every $v \in \mathcal{G}_M^N$, $S \in \mathcal{S}_H^N$, and $i \in N$ such that $v(E) = 0$ for every $E \subset N \setminus \{i\}$ it holds that

$$f_i(v, S) \geq f_j(v, S) \text{ for all } j \in N.$$

As shown in van den Brink and Gilles (1996) the conjunctive permission value satisfies *structural monotonicity* which states that a player in a monotone game with a permission structure gets at least as much as any of his subordinates. The disjunctive permission value does not satisfy this axiom. The next axiom is a weaker version of structural monotonicity. It says that if player i dominates player j 'completely' in the sense that all permission paths from the topman i_0 to player j contain player i, then i gets at least as much as j if the game is monotone.

Axiom 4.5 (Weak Structural Monotonicity): For every $v \in \mathcal{G}_M^N$, $S \in \mathcal{S}_H^N$ and $i \in N$ it holds that

$$f_i(v, S) \geq f_j(v, S) \text{ for all } j \in \bar{S}(i),$$

where

$$\bar{S}(i) = \{j \in N \mid i \in \bar{S}^{-1}(j)\} = \{j \in \hat{S}(i) \mid E \in \Psi_S \text{ and } E \ni j \text{ implies that } E \ni i\}.$$

As said, the final axiom is fairness as discussed in the previous section.

Axiom 4.6 (Fairness): For every $v \in \mathscr{G}^N$, $S \in \mathscr{S}^N_H$, and $h, j \in N$ such that $j \in S(h)$ and $\#S^{-1}(j) \geq 2$ it holds that

$$f_i(v, S) - f_i(v, S_{-(h,j)}) = f_j(v, S) - f_j(v, S_{-(h,j)}) \text{ for all } i \in \{h\} \cup \bar{S}^{-1}(h).$$

These six axioms uniquely determine the disjunctive permission value for games with a hierarchical permission structure. Before proving this result we present the following lemma.

Lemma 4.7: Let $S \in \mathscr{S}^N_H$ and $w_T = c_T u_T$ where u_T is the unanimity game of $T \subset N$, and $c_T \geq 0$ is some non-negative constant.

(i) If $f; \mathscr{G}^N \times \mathscr{S}^N_H \to \mathbb{R}^N$ satisfies the inessential player property then $f_i(w_T, S) = 0$ for all $i \in N \backslash \alpha(T)$ where $\alpha(T) := T \cup \hat{S}^{-1}(T)$.
(ii) If $f: \mathscr{G}^N \times \mathscr{S}^N_H \to \mathbb{R}^N$ satisfies the necessary player property and weak structural monotonicity then there exists a constant $c \geq 0$ such that $f_i(w_T, S) = c$ for all $i \in \beta(T)$ where $\beta(T) := \{i \in \alpha(T) \mid T \cap (\{i\} \cup \bar{S}(i)) \neq \varnothing\}$.

Proof: Let $S \in \mathscr{S}^N_H$ and $w_T = c_T u_T$ with $c_T \geq 0$.

(i) If $i \in N \backslash \alpha(T)$ then $i \notin T$ and $\hat{S}(i) \cap T = \varnothing$. Thus i is inessential in (N, w_T, S). The inessential player property then implies that $f_i(w_T, S) = 0$.
(ii) If $i \in T$ then i is a necessary player in the monotone game w_T. From the necessary player property it then follows that there exists a constant $c \geq 0$ such that

$$f_i(w_T, S) = c \quad \text{for all } i \in T$$
$$f_i(w_T, S) \leq c \quad \text{for all } i \in N \backslash T$$

If $i \in \beta(T) \backslash T$ then $\bar{S}(i) \cap T \neq \varnothing$. Weak structural monotonicity then implies that also $f_i(w_T, S) = c$ for all $i \in \beta(T) \backslash T$. □

Next we state the main result of this paper.

Theorem 4.8: An allocation rule $f: \mathscr{G}^N \times \mathscr{S}^N_H \to \mathbb{R}^N$ is equal to the disjunctive permission value ψ if and only if it satisfies efficiency, additivity, the inessential player property, the necessary player property, weak structural monotonicity, and fairness.

Proof: In the previous section we already showed that ψ satisfies fairness.

Efficiency of ψ directly follows from efficiency of the Shapley value and the fact that $\sigma(N) = N$ for every $S \in \mathscr{S}^N_H$.

Additivity of ψ directly follows from additivity of the Shapley value and the fact that $\mathscr{D}_S(v) + \mathscr{D}_S(w) = \mathscr{D}_S(v + w)$ for all $v, w \in \mathscr{G}^N$ and $S \in \mathscr{S}^N_H$.

For every $v \in \mathscr{G}^N$ and $S \in \mathscr{S}_H^N$ it holds that an inessential player in (N, v, S) is a zero player in $\mathscr{D}_S(v)$. The zero player property of the Shapley value then implies that ψ satisfies the inessential player property.

In Gilles and Owen (1994) it is shown that for every $v \in \mathscr{G}_M^N$ and $S \in \mathscr{S}_H^N$ it holds that $\mathscr{D}_S(v) \in \mathscr{G}_M^N$. As is known the Shapley value can be written as

$$Sh_i(v) = \sum_{E \ni i} p(E) \cdot (v(E) - v(E \setminus \{i\})), \text{ for all } i \in N, \tag{5}$$

where $p(E) := \frac{(\#N - \#E)!(\#E - 1)!}{(\#N)!}$. Let $i \in N$ be a necessary player in the monotone game v. Then i is a necessary player in the monotone game $\mathscr{D}_S(v)$. From this we can derive that

(i) $\mathscr{D}_S(v)(E) - \mathscr{D}_S(v)(E \setminus \{i\}) = \mathscr{D}_S(v)(E) \geq \mathscr{D}_S(v)(E) - \mathscr{D}_S(v)(E \setminus \{j\})$ for all $j \in N$ and $E \subset N$;
(ii) $\mathscr{D}_S(v)(E) - \mathscr{D}_S(v)(E \setminus \{i\}) \geq 0$ for all $E \ni i$;
(iii) $\mathscr{D}_S(v)(E) - \mathscr{D}_S(v)(E \setminus \{j\}) = 0$ for all $j \in N$ and $E \not\ni i$.

With (5) it then follows that

$$\begin{aligned}
\psi_i(v, S) &= Sh_i(\mathscr{D}_S(v)) \\
&= \sum_{\substack{E \ni i \\ E \ni j}} p(E) \cdot (\mathscr{D}_S(v)(E) - \mathscr{D}_S(v)(E \setminus \{i\})) + \sum_{\substack{E \ni i \\ E \not\ni j}} p(E) \cdot (\mathscr{D}_S(v)(E) - \mathscr{D}_S(v)(E \setminus \{i\})) \\
&\geq \sum_{\substack{E \ni i \\ E \ni j}} p(E) \cdot (\mathscr{D}_S(v)(E) - \mathscr{D}_S(v)(E \setminus \{j\})) + \sum_{\substack{E \not\ni i \\ E \ni j}} p(E) \cdot (\mathscr{D}_S(v)(E) - \mathscr{D}_S(v)(E \setminus \{j\})) \\
&= Sh_j(\mathscr{D}_S(v)) = \psi_j(v, S) \text{ for every } j \in N.
\end{aligned}$$

Thus ψ satisfies the necessary player property.

Let $v \in \mathscr{G}_M^N$, $S \in \mathscr{S}_H^N$, and $i \in N$. From monotonicity of $\mathscr{D}_S(v)$ it then follows that

(i) $\mathscr{D}_S(v)(E) - \mathscr{D}_S(v)(E \setminus \{i\}) \geq \mathscr{D}_S(v)(E) - \mathscr{D}_S(v)(E \setminus \{j\})$ since $\sigma(E \setminus \{i\}) \subset \sigma(E \setminus \{j\})$ for all $j \in \bar{S}(i)$ and $E \subset N$;
(ii) $\mathscr{D}_S(v)(E) - \mathscr{D}_S(v)(E \setminus \{i\}) \geq 0$ for all $E \ni i$;
(iii) $\mathscr{D}_S(v)(E) - \mathscr{D}_S(v)(E \setminus \{j\}) = 0$ for all $j \in \tilde{S}(i)$ and $E \not\ni i$.

With this and (5) it then can be shown that ψ satisfies weak structural monotonicity in a similar way as is shown that ψ satisfies the necessary player property.

We thus conclude that ψ satisfies the six axioms.

Now suppose that $f: \mathscr{G}^N \times \mathscr{S}_H^N \to \mathbb{R}^N$ satisfies the six axioms.

Consider the hierarchical permission structure $S \in \mathscr{S}_H^N$ and the monotone game $w_T = c_T u_T$ where u_T is the unanimity game of $T \subset N$, and $c_T \geq 0$ is some non-negative constant.

Note that for every hierarchical structure $S \in \mathcal{S}_H^N$ it holds that $\sum_{i \in N} \#S(i) \geq \#N - 1$.

If $\sum_{i \in N} \#S(i) = \#N - 1$ then $\#S^{-1}(i) = 1$ for all $i \in N \backslash \{i_0\}$, and thus $\bar{S}(i) = \hat{S}(i)$ for all $i \in N$. In that case $T \cap (\{i\} \cup \bar{S}(i)) \neq \varnothing$ for all $i \in \alpha(T)$. Thus $\beta(T) = \alpha(T)$, where $\alpha(T)$ and $\beta(T)$ are as defined in Lemma 4.7. With that lemma it then follows that there exists a constant $c \geq 0$ such that

$$f_i(w_T, S) = \begin{cases} c & \text{if } i \in \alpha(T) \\ 0 & \text{else.} \end{cases}$$

Efficiency then implies that $c = (c_T / \#\alpha(T))$, and thus $f(w_T, S) = \psi(w_T, S)$.

Proceeding by induction we assume that $f(w_T, S') = \psi(w_T, S')$ for all $S' \in \mathcal{S}_H^N$ with $\sum_{i \in N} \#S'(i) < \sum_{i \in N} \#S(i)$.

Next we recursively define the sets L_k, $k \in \{0\} \cup \mathbb{N}$, by

$$L_0 := \varnothing,$$

and

$$L_k := \left\{ i \in N \backslash \bigcup_{t=1}^{k-1} L_t \,\middle|\, S(i) \subset \bigcup_{t=1}^{k=1} L_t \right\}, \text{ for all } k \in \mathbb{N}.$$

In van den Brink and Gilles (1994) it is shown that for hierarchical permission structures there exists an $M < \infty$ such that the sets $L_1, \ldots, L_M$ form a partition of N consisting of non-empty sets only.

Let $c^* \geq 0$ be such that $f_i(w_T, S) = c^*$ for all $i \in \beta(T)$. (The existence of such a constant c^* follows from Lemma 4.7.)

Next we describe a procedure which determines the values $f_i(w_T, S)$ as functions of the constant c^* for all $i \in N$.

Step 1: For every $i \in L_1$ one of the following two conditions is satisfied:

(i) If $i \in N \backslash \alpha(T)$ then $f_i(w_T, S) = 0$ by Lemma 4.7.
(ii) If $i \in \alpha(T)$ then $i \in T$ since $S(i) = \varnothing$. Thus $f_i(w_T, S) = c^*$.

Let $k = 2$.

Step 2: If $L_k = \varnothing$ then STOP.
Else, for every $i \in L_k$ one of the following three conditions is satisfied:

(i) If $i \in N \backslash \alpha(T)$ then $f_i(w_T, S) = 0$ by Lemma 4.7.
(ii) If $i \in \beta(T)$ then $f_i(w_T, S) = c^*$.
(iii) If $i \in \alpha(T) \backslash \beta(T)$ then by definition of $\alpha(T)$ and $\beta(T)$ there exists an $h \in \{i\} \cup \bar{S}(i)$ and a $j \in S(h)$ such that $\#S^{-1}(j) \geq 2$. Fairness then implies that

$$f_i(w_T, S) - f_i(w_T, S_{-(h,j)}) = f_j(w_T, S) - f_j(w_T, S_{-(h,j)}).$$

Using the induction hypothesis we can write

$$f_i(w_T, S) = f_j(w_T, S) + \psi_i(w_T, S_{-(h,j)}) - \psi_j(w_T, S_{-(h,j)}). \tag{6}$$

Since $j \in \hat{S}(i)$ implies that $j \in L_l$ with $l < k$ we already determined $f_j(w_T, S)$ as a function of c^*, and thus with (6) we have determined $f_i(w_T, S)$ as a function of c^*.

Step 3: Let $k = k + 1$. Go to Step 2.

Since there exists an $M < \infty$ such that the sets $L_1, \ldots, L_M$ form a partition of N consisting of non-empty sets only the procedure described above determines the values $f_i(w_T, S)$ as a function of c^* for all $i \in N$. Efficiency then uniquely determines the value c^*. Since the disjunctive permission value satisfies the axioms it then must hold that $f(w_T, S) = \psi(w_T, S)$.

Above we showed that $f(w_T, S) = \psi(w_T, S)$ for all games $w_T = c_T u_T$ with $c_T \geq 0$ and $S \in \mathcal{S}_H^N$. Suppose that $w_T = c_T u_T$ with $c_T \leq 0$, and let $v_0 \in \mathcal{G}^N$ denote the *null game*, i.e., $v_0(E) = 0$ for all $E \subset N$. From the inessential player property it follows that $f_i(v_0, S) = 0$ for all $i \in N$ and $S \in \mathcal{S}_H^N$. Since $-w_T = -c_T u_T$ with $-c_T \geq 0$ and $\mathcal{D}_S(w_T) + \mathcal{D}_S(-w_T) = \mathcal{D}_S(v_0)$ for all $T \subset N$ it follows from additivity of f and of the Shapley value that $f(w_T, S) = -f(-w_T, S) = -\psi(-w_T, S) = -Sh(\mathcal{D}_S(-w_T)) = -Sh(-\mathcal{D}_S(w_T)) = Sh(\mathcal{D}_S(w_T)) = \psi(w_T, S)$ for all $S \in \mathcal{S}_H^N$. Since every game $v \in \mathcal{G}^N$ can be expressed as a linear combination of unanimity games it then follows with additivity that $f(v, S) = \psi(v, S)$ for all $v \in \mathcal{G}^N$ and $S \in \mathcal{S}_H^N$. □

We conclude this paper by illustrating the independence of the axioms stated in Theorem 4.8.

Example 4.9: We illustrate the independence of the axioms in Theorem 4.8 by presenting six alternative allocation rules.

1. The conjunctive permission value $\varphi: \mathcal{G}^N \times \mathcal{S}_H^N \to \mathbb{R}^N$ which is discussed at the end of Section 2 and is axiomatized in van den Brink and Gilles (1996) satisfies all axioms of Theorem 4.8 except fairness.
2. Let the allocation rule $f^1: \mathcal{G}^N \times \mathcal{S}_H^N \to \mathbb{R}^N$ be given by

   $$f^1(v, S) = Sh(v) \text{ for all } v \in \mathcal{G}^N \text{ and } S \in \mathcal{S}_H^N.$$

   This allocation rule satisfies all axioms of Theorem 4.8 except weak structural monotonicity.
3. Let the allocation rule $f^2: \mathcal{G}^N \times \mathcal{S}_H^N \to \mathbb{R}^N$ be given by

   $$f_i^2(v, S) = \begin{cases} v(N) & \text{if } S^{-1}(i) = \emptyset \\ 0 & \text{else} \end{cases}$$

   for every $i \in N$, $v \in \mathcal{G}^N$, and $S \in \mathcal{S}_H^N$.

This allocation rule satisfies all axioms of Theorem 4.8 except the necessary player property.

4. Let the allocation rule $f^3: \mathscr{G}^N \times \mathscr{S}_H^N \to \mathbb{R}^N$ be given by

$$f_i^3(v, S) = \frac{v(N)}{\#N} \text{ for all } i \in N,\ v \in \mathscr{G}^N, \text{ and } S \in \mathscr{S}_H^N.$$

This allocation rule satisfies all axioms of Theorem 4.8 except the inessential player property.

5. Let $g_T \in \mathscr{G}^N$ be given by $g_T(E) = \begin{cases} 1 & \text{if } E \cap T \neq \emptyset \\ 0 & \text{else.} \end{cases}$

Now let the allocation rule $f^4: \mathscr{G}^N \times \mathscr{S}_H^N \to \mathbb{R}^N$ be given by

$$f^4(v, S) = \begin{cases} f^2(v, S) & \text{if } v = g_T,\ \#T \geq 2 \\ \psi(v, S) & \text{else} \end{cases}$$

for every $v \in \mathscr{G}^N$ and $S \in \mathscr{S}_H^N$.

This allocation rule satisfies all axioms of Theorem 4.8 except additivity.

6. Let the allocation rule $f^5: \mathscr{G}^N \times \mathscr{S}_H^N \to \mathbb{R}^N$ be given by

$$f_i^5(v, S) = 0 \text{ for all } i \in N,\ v \in \mathscr{G}^N \text{ and } S \in \mathscr{S}_H^N.$$

This allocation rule satisfies all axioms of Theorem 4.8 except efficiency.

Thus, all six axioms are necessary in order to uniquely determine the disjunctive permission value for games with a hierarchical permission structure.

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