International Journal of Game Theory (1995) 24: 49–56

Game Theory

Γ-component Additive Games

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Abstract: According to Maschler, Peleg and Shapley (1972) the bargaining set of a *convex* game coincides with its core and the kernel consists of the nucleolus only. In this paper we prove the same properties for Γ-component additive games (= graph restricted games in the sense of Owen (1986)) if Γ is a tree. Furthermore, we give a description of the nucleolus of this type of games which makes it easier accessible for computation

Key words: cooperative game, bargaining set, kernel, nucleolus, graph-restricted games

1 Introduction

Cooperation often requires communication. Especially if the number of players is large, communication lays severe restrictions on the cooperation possibilities. In the tradition of Owen (1986), Myerson (1977) and Van den Nouweland (1993) we model the cooperation and the communication aspects separately. As usual the potential profits from cooperation are given by a TU-game (N, v) which we assume to be superadditive. Communication possibilities are modeled by an (undirected) graph $\Gamma = (N, E, \alpha)$ on the player set N. The potential profit of a coalition S can only be effected in as far as the player in S can communicate i.e., the actual value of a coalition S is

$$\sum_{T \in S/\Gamma} v(T)$$

where S/Γ is the set of *connected components of* S. So the introduction of a communication graph Γ defines a projection R_Γ from the cone of superadditive games SA^N onto a cone in the space of all games with player set N, G^N:

$$R_\Gamma(v)(S) := \sum_{T \in S/\Gamma} v(T).$$

In this paper the graph Γ will be a *tree* i.e., each pair of points (i, j) in Γ is connected by exactly one path. Under these conditions (namely, (N, v) is superadditive and Γ is a tree) we will prove the following results:

(a) The game $(N, R_\Gamma(v))$ is balanced.
(b) The bargaining set $\mathcal{M}(R_\Gamma(v))$ and the core $\mathcal{C}ore(R_\Gamma(v))$ coincidence.

0020–7276/95/1/49–56 \$ 2.50

(c) The kernel $\mathcal{K}(R_\Gamma(v))$ consists of the nucleolus $\mathcal{N}(R_\Gamma(v))$ only.
(d) The nucleolus of the game $R_\Gamma(v)$ is the unique point x satisfying the equalities $x(N) = v(N)$ and $\bar{s}_{ij}(x) = \bar{s}_{ji}(x)$ for all $(i, j) \in \Gamma$. The function $\bar{s}_{ij}$ is defined by

$$\bar{s}_{ij}(x) := \max\{v(S) - x(S) \mid i \in S \subset N \backslash j, S \text{ is connected in } \Gamma\}.$$

Comment: In Maschler, Peleg and Shapley (1972) it is proved that *convex games* have the properties (a), (b) and (c). In Muto, Potters and Tijs (1987, 1988[a], 1988[b]) *information market games, big boss games* and *clan games* have been shown to have these properties too. In fact, big boss games (and therefore also information market games) are games of type $R_\Gamma(v)$ where Γ is the 'spiter's web graph'. There are only edges between the 'big boss' and each of the other players). In Curiel et al. (1992) we considered what we called *σ-component additive games*. These games are exactly the zero-normalized superadditive games of the type $R_\Gamma(v)$ wherein Γ is the 'line graph' $\sigma(1)$–$\sigma(2)$–$\cdots$–$\sigma(n)$. Also for games of this type and in particular for *sequencing games* (cf. Curiel, Pederzoli and Tijs (1988)) the results of this paper hold.

For the convenience of the reader we will briefly repeat some of the definitions of the main concepts we need. This gives us also the opportunity to introduce our notation that may deviate at some points from the notation used in other papers in the field.

Notations from Graph Theory

An undirected graph Γ consists of a finite set of *nodes* N, a finite set of *edges* E and a map $\alpha : E \rightarrow \mathcal{P}_2(N)$. The set $\mathcal{P}_2(N)$ is the set of 2-point sets in N. If $\alpha(e) = \{i, j\}$ for some edge $e \in E$, we say that 'the edge e connects the nodes i and j'. We abbreviate the notation $\alpha(e) = \{i, j\}$ by $(i, j) \in \Gamma$.

A path from $i \in N$ to $j \in N, j \neq i$ is a sequence of *different* nodes $i = i_0, i_1, ..., i_p = j$ with $(i_{k-1}, i_k) \in \Gamma$ for $k = 1, ..., p$. An undirected graph is a *tree* if each pair of (different) points is connected by exactly one path.

If Γ is a tree, the choice of a *root* r, a node of Γ, introduces a partial order on the nodes of Γ:

$$i \preccurlyeq j \quad \text{iff the path from } r \text{ to } j \text{ contains the node } i.$$

Moreover we can introduce $\Gamma_{\succcurlyeq i}$ as the subgraph on the points $\{j \in N \mid i \preccurlyeq j\}$. The graph $\Gamma_{\succcurlyeq i}$ is also a tree for all $i \in N$ and $\Gamma_{\succcurlyeq r} = \Gamma$.

Finally, the set of *connect coalitions* in Γ is denoted by C_Γ and the set of components (= largest connected subcoalitions) of a coalition S is denoted by S/Γ.

Concepts from the Theory of TU-games

A cooperative game (N, v) is *superadditive* if $v(S) + v(T) \leq v(S \cup T)$ whenever $S \cap T = \emptyset$. The cone of superadditive games with player set N is denoted by SA^N. The game (N, v) is called *balanced* if for every nonegative solution $\{y_S\}_{S \subset N}$ of the vector equation $\sum_{S \subset N} y_S e_S = e_N$ the inequality $\sum_{S \subset N} y_S v(S) \leq v(N)$ holds. The vector e_S are, as usual, the characteristic vectors of coalitions S i.e., $E_{S,i} = 1$ if $i \in S$ and $e_{S,i} = 0$ if $\in \notin S$. The cone of balanced games (with player set N) is denoted by BA^N.

The Theorem of Shapley (1967) and Bondareva (1963) states that (N, v) is balanced if and only if the core of game

$$\mathcal{C}\mathrm{ore}(v) := \{x \in \mathbf{R}^N \mid x(S) \geq v(S) \text{ for all coalitions } S \subset N \text{ and } x(N) = v(N)\}$$

is nonempty. For vectors $x \in \mathbf{R}^N$ we write $x(S)$ as a shorthand notation for $\sum_{i \in S} x_i$. A game (N, v) is *zero-normalized* if $v(i) = 0$ for all $i \in N$ and the *zero-normalization* of a game (N, v) is the game (N, v_0) with $v_0(S) := v(S) - \sum_{i \in S} v(i)$. As all the concepts in this paper are invariant under or covariant with the addition of *additive games* we will only consider zero-normalized games. A game (N, v) is called *zero-monotonic* if $v_0(S) \leq v_0(T)$ whenever $S \subset T$ i.e., the zero-normalization of (N, v) is monotonic. Note that superadditive games are zero-monotonic.

The *imputation set* of a cooperative game (N, v) is the set

$$\mathcal{I}(v) := \{x \in \mathbf{R}^N \mid x(N) = v(N) \text{ and } x_i \geq v(i) \text{ for all } i \in N\}.$$

The elements of the *preimputation set* $\mathcal{I}^*(v)$ only satisfy the efficiency conditions $x(N) = v(N)$.

Solution Concepts

Here we will repeat the definitions and the most characteristic properties of the solutions from the Aumann-Maschler-Schmeidler complex. We start with the *bargaining set* (Aumann and Maschler (1964)). Suppose, (N, v) is a game with nonempty imputation set. If x is an imputation, an *objection of player $i \in N$ against player j (with respect to the imputation x)* is a coalition $S \subset N$ with $i \in S \subset N \setminus j$ and a vector $y \in \mathbf{R}^S$ such than $y_k > x_k$ for all players $k \in S$ and $y(S) = v(S)$. If $x \in \mathcal{I}(v)$ and the objection (S, y) of player i against player j are given, a *counter objection* is a coalition T with $j \in T \subset N \setminus i$ and a vector $z \in \mathbf{R}^T$ with $z(T) = v(T)$ and $z \geq (y_{|S \cap T}, x_{|T \setminus S})$. If an imputation x can counter every objection then x is, by definition, an element of the *bargaining set* $\mathcal{M}(v)$. As core elements don't allow any objection, the core is a subset of the bargaining set.

If x is an preimputation of a game (N, v), the *surplus* of player i against player j is defined by $s_{ij}(x) := \max_{S: i \in S \subset N \setminus j} (v(S) - x(S))$. The point x is an element of the *prekernel* $\mathcal{K}^*(v)$ if $s_{ij}(x) = s_{ji}(x)$ for all pairs (i, j), $i \neq j$. If x is an imputation and, for all pairs $i \neq j$, $s_{ij}(x) > s_{ji}(x)$ implies $x_j = v(j)$, then x is, by definition, an element of the *kernel* $\mathcal{K}(v)$ (see Davis and Maschler (1965) for more details). For zero-monotonic games (and therefore, in particular for superadditive games) the kernel and the prekernel are the same. Moreover, the kernel is a subset of the bargaining set.

To define the (pre)nucleolus of a game we need the following:

(a) The *excess map* $E : \mathcal{I}^*(v) \to R^{2^N \setminus \{\emptyset, N\}}$ defined by $E(x)_S := v(S) - x(S) =: \mathrm{exc}(S, x|v)$.
(b) The *coordinate ordering map* $\theta : \mathbf{R}^M \to \mathbf{R}^m$ wherein $M = 2^N \setminus \{\emptyset, N\}$ and $m = |M|$. By definition, $\theta(x) = y$ means that there is a bijective map π from M to $\{1, ..., m\}$ with $y_{\pi(S)} = x_S$ for all $S \subset N, S \neq \emptyset, N$ and $y_1 \geq y_2 \geq \cdots \geq y_m$.

(c) The *lexicographic order* $\preccurlyeq_{\text{lex}}$ on $\mathbf{R}^m$ is a complete transitive order defined by

$$x, y \in \mathbf{R}^m, \quad x \preccurlyeq_{\text{lex}} y \text{ iff } x = y \text{ or, for some } k \in \{1, 2, \dots, m\},$$
$$x_i = y_i \text{ for } i < k \text{ and } x_k < y_k.$$

The *prenucleolus* $\mathcal{N}^*(v)$ is defined as the set of points $x \in \mathcal{I}^*(v)$ where $\theta \circ E$ takes its lexicographic minimum in $\mathcal{I}^*(v)$ (Sobolev (1975)). The *nucleolus* $\mathcal{N}(v)$ is defined as the set of points in $\mathcal{I}(v)$ where $\theta \circ E$ takes its lexicographic mimimum in $\mathcal{I}(v)$ (Schmeidler (1969)). It is easy to prove that the (pre)nucleolus consists of one point that lies in the (pre)kernel. For zero-monotonic games the nucleolus and the prenucleolus are the same point in $\mathcal{I}(v)$. The following property characterizes the prenucleolus uniquely (Sobolev (1975), cf. also Kohlberg (1971)):

The prenucleolus is the unique point x in $\mathcal{I}^*(v)$ with the property that, for all $t \in \mathbf{R}$, the collection $\mathcal{B}_t(x) := \{S \subset N, S \neq \emptyset, N \mid \text{exc}(S, x) \geq t\}$ is balanced or empty.

2 Balancedness

Let N be a finite set of players and let $\Gamma = (N, E, \alpha)$ be a connected undirected graph on N. The graph Γ models the communication possibilities (restrictions) of players in a game (N, v). By lack of communication a coalition $S \subset N$ can only obtain the sum of the values of its connected components in the graph Γ. So we can introduce the *graph restricted game* $R_\Gamma(v)$ by

$$R_\Gamma(v)(S) := \sum_{T \in S/\Gamma} v(T)$$

where S/Γ is the set of connected components of S in Γ. The map R_Γ is a linear projection and $R_\Gamma(v_0) = R_\Gamma(v)_0$ (the map R_Γ commutes with zero-normalization). As R_Γ is a projection (i.e., $R_\Gamma \circ R_\Gamma = R_\Gamma$) the image of R_Γ consists of the games (N, v) with $R_\Gamma(v) = v$.

Proposition 1: (cf. Owen (1986) for a more general result)
If Γ is a tree, the image of R_Γ is the linear space in G^N generated by the games

$$\{u_T \mid T \in \mathcal{C}_\Gamma\}.$$

Proof: Every game (N, v) can be decomposed into a linear combination of unanimity games $\{u_T \mid T \subset N\}$.[1] If $v = \sum_{T \subset N} y_T u_T$, then $R_\Gamma(v) = \sum_{T \subset N} y_T R_\Gamma(u_T)$ (by linearity of R_Γ). Let $H(T)$ be the intersection of all *connected coalitions* containing T. As in a tree Γ the intersection of two (or any finite number of) connected coalitions is also connected, $H(T)$ is a connected coalition. It is clear that $R_\Gamma(u_T) = u_{\widehat{H(T)}}$. QED

[1] The unamimity game u_T is the simple game with $u_T(S) = 1$ iff $T \subset S$.

In Owen (1986) it is proved that for any graph Γ the game $R_\Gamma(u_T)$ is the monotonic simple game with $\mathcal{H}(T)$, the collection of minimal connected coalitions that contain T, as collection of minimal winning coalitions. Further, it is proved that $R_\Gamma(v)$ is superadditive if (N, v) is superadditive. A game (N, v) is called a *Γ-component additive game* if (N, v) is a superadditive zero-normalized game with $R_\Gamma(v) = v$. In case Γ is a tree it is known that $R_\Gamma(v)$ is even balanced.

Proposition 2: (Le Breton et al. (1992)) *Γ-component additive games are balanced if Γ is a tree*

The proof follows from the fact that the Bondareva/Shapley conditions has to be checked for connected coalitions only, that a balanced collection of *connected* coalitions is a union of partitions and the superadditivity of the game.

3 The Bargaining Set

The main result of this section will be that, for games of type $R_\Gamma(v)$ where (N, v) is superadditive and Γ is a tree, the bargaining set and the core coincide.

Let Γ be a tree on the player set N and let (N, v) be a *Γ-component additive game* i.e., (N, v) is superadditive, zero-normalized and $v(S) = \sum_{T \in S/\Gamma} v(T)$ for all coalitions $S \subset N$.

Let x be an element of $\mathcal{I}(v)/Core(v)$. The construction of a justified objection (an objection without counter objection) with respect to x will be the result of the following Lemma.

Lemma 3: *If x is an imputation of a Γ-component additive game and x is not in the core, there is a coalition S_0 and a vector $z \in \mathbf{R}_+^N$ with the following properties:*

(a) $z_j = 0$ *if* $j \notin S_0$.
(b) $z(S_0) = \mathrm{exc}(S_0, x)$.
(c) $z(S) \geq \mathrm{exc}(S, x)$, $S \in \mathcal{C}_\Gamma$.
(d) $\mathrm{exc}(T, x) < 0$ *if* $T \cap S_0 = \emptyset$.

Proof: The construction of $z \in \mathbf{R}_+^N$ and S_0 requires two steps.

The construction of z. First we construct the vector z and *a* coalition T_0 containing the nodes where $z_i > 0$. Choose any node r in Γ as root and start with $T_0 = \emptyset$. The coordinates z_i are defined inductively going from the extreme nodes of Γ to r. If k is an extreme node of Γ, we take $z_k = 0$. Further, we put the node k into T_0 iff $\mathrm{exc}(\{k\}, x) = 0$ (i.e., $x_k = 0$). The coordinate z_i can be defined as soon as the coordinates z_j with $j > i$ have been defined. If so, we put

$$\bar{z}_i := \max\{\mathrm{exc}(S, x) - z(S \setminus i) \mid S \in \mathcal{C}_\Gamma, i \in S \subset \Gamma_{\geqslant i}\}$$

and $z_i = \bar{z}_i \vee 0$. We extend T_0 with i iff $\bar{z}_i \geq 0$. Otherwise, T_0 doesn't change. Notice that $i \in T_0$ if $z_i > 0$ but also if $z_i = \bar{z}_i = 0$. For each point $i \in T_0$ there is a connected coalition S_i with $i \in S_i \subset \Gamma_{\succcurlyeq i}$ with $z(S_i) = \text{exc}(S_i, x)$. After finitely many steps we have defined the vector $z \geq 0$ completely. We have $z(S) \geq \text{exc}(S, x)$ for all $S \in \mathcal{C}_\Gamma$ and equality for at least one coalition S_i with $i \in S_i \subset \Gamma_{\succcurlyeq i}$ if $i \in T_0$.

The construction of S_0. The collection $\{S_i\}_{i \in T_0}$ may be a redundant covering of the set T_0 i.e., it is possible that a coalition S_j can be skipped without loosing the covering of T_0. In order to obtain an irredundant covering of T_0 we investigate whether the coalitions S_i are necessary to cover T_0. We start this scanning procedure with the coalitions S_i with the property that the path from r to i does not contain a point of $T_0, \neq i$. We give these coalitions a label $^*(S_i \rightarrow S_i^*)$. E.g., if r is in T_0, only the coalition S_r obtains immediately a label. Proceeding inductively, if a node $j \in S_i^*$ for some $i \prec j$ we skip the coalition S_j. If all nodes $i \in T_0$ with $i \prec j$ have been investigated and coalition S_j has not been skipped, we give the coalition S_j a label $(S_j \rightarrow S_j^*)$. After a while all remaining coalitions are labeled and we have a collection $\{S_i^*\}_{i \in T_1}, T_1 \subset T_0$, still covering the set T_0. We prove that the coalitions S_i^* are mutually disjoint. Suppose that $k \in S_i^* \cap S_j^*$. The path from r to k contains the nodes i and j. Let us assume w.l.o.g that $i \prec j$. Then the path from i to k contains the point j and as S_i^* is connected and $i, k \in S_i^*$ we have $j \in S_i^*$. Then the coalition S_j would have been skipped.

Then we take $S_0 := U_{i \in T_1} S_i^*$ and we have:

$$z(N) = z(S_0) = \sum_{i \in T_1} z(S_i^*) = \sum_{i \in T_0} \text{exc}(S_i^*, x) \leq \text{exc}(S_0, x) \leq z(S_0).$$

Hence, $z(S_0) = \text{exc}(S_0, x)$ and $z(S) \geq \text{exc}(S, x)$ for all $S \in \mathcal{C}_\Gamma$.

So, the pair (z, S_0) satisfies the conditions (a), (b) and (c). If T is a connected coalition with $T \cap S_0 = \emptyset$, we have $z(T) = 0 \geq \text{exc}(T, x)$. If $\text{exc}(T, x)$ would be zero and $i \in T$ is the point of T closest to the root r, the node i would be in $T_0 \subset S_0$ (see definition of z_i and T_0). Hence also condition (d) is satisfied. QED

After these preparations the proof of the main theorem is easy.

Theorem 4: *For Γ-component additive games (N, v) the bargaining set $\mathcal{M}(v)$ and the core $\mathcal{C}ore(v)$ coincide, if Γ is a tree.*

Proof: Let x be an imputation of (N, v) outside the core. Let $S_0 \subset N$ and $z \in \mathbf{R}_+^N$ be as in Lemma 3. Take $i \in S_0$ with $z_i > 0$ and $j \notin S_0$. Notice that $x(S) + z(S) \geq v(S)$ for all coalitions $S \subset N$ and that therefore, the vector z cannot be the zero-vector. This implies that $S_0 \neq N$ and $z(S_0) = z(N) > 0$. An objection (S_0, y) of player i against player j is defined as follows. For $k \in S_0$ and $k \notin i$, take the coordinate $y_k := x_k + z_k + s_0^{-1} z_i$ and the coordinate $y_i := x_i + s_0^{-1} z_i, s_0 = |S_0|$. Then (S_0, y) is an objection of player i against player j. Let T be a coalition with $j \in T \subset N \setminus i$. If $T \cap S_0 = \emptyset$, the excess $\text{exc}(T, x) < 0$ (see condition (d)) and if $T \cap S_0 \notin \emptyset$ we have $z(T \cap S_0) = z(T) \geq \text{exc}(T, x)$ and therefore, $y(T \cap S_0) + x(T \setminus S_0) > v(T)$. There is no counter objection for (S_0, y). The point x is not in the bargaining set. QED

4 The Kernel

In this section we prove the properties (c) and (d) of the introduction.

Theorem 5: *If Γ is a tree on the player set N and (N, v) is a Γ-component additive game, the kernel $\mathcal{K}(v)$ consists of the nucleolus $\mathcal{N}(v)$ only.*

Proof: Let $x \in \mathcal{K}(v)$. From Theorem 4 we know that $x \in \mathcal{C}\text{ore}(v)$. We prove that for each $t \in \mathbf{R}$, the collection $\mathcal{B}_t(x)$ is balanced or empty.

(a) If $S \in \mathcal{B}_t(x)$, then also $S/\Gamma \subset \mathcal{B}_t(x)$ as all excesses are negative or zero and (N, v) is Γ-component additive.
(b) If $S \in \mathcal{B}_t(x)$ is connected and $i \in S \subset N\backslash j$ with $(i, j) \in \Gamma$, there is also a connected coalition $T \in \mathcal{B}_t(x)$ with $j \in T \subset N \backslash i$. This is because $s_{ij}(x) \geq t$ implies that $s_{ji}(x) \geq t$ (here we use that $\mathcal{K}(v) = \mathcal{K}^*(v)$ and (a)).

From (a) and (b) we infer that each connected coalition $S \in \mathcal{B}_t(x)$ can be extended to a partition of N included in $\mathcal{B}_t(x)$. Then also each not-connected coalition $S \in \mathcal{B}_t(x)$ is element of a balanced collection in $\mathcal{B}_t(x)$. From Sobolev's theorem we find: $x = \mathcal{N}^*(v) = \mathcal{N}(v)$. QED

Remark: In the proof of the Theorem we only used the fact that $s_{ij}(x) = s_{ji}(x)$ for $(i, j) \in \Gamma$ and $x \in \mathcal{C}\text{ore}(v)$.

Further, for $x \in \mathcal{C}\text{ore}(v)$, we have

$$s_{ij}(x) = \bar{x}_{ij}(x) := \max\{v(S) - x(S) \mid S \in \mathcal{C}_\Gamma \text{and} i \in S \subset N\backslash j\}.$$

Corollary: *If Γ is a tree on N and (N, v) is a Γ-component additive game, we have*

$$x \in \mathcal{I}^*(v) \quad and \quad \bar{s}_{ij}(x) = \bar{s}_{ji}(x) forall (i, j) \in \Gamma \quad \Leftrightarrow \quad x = \mathcal{N}(v).$$

Proof: Suppose that a preimputation x satisfies the equalities $\bar{s}_{ij}(x) = \bar{s}_{ji}(x)$, $(i, j) \in \Gamma$ and $x \notin \mathcal{C}\text{ore}(v)$. Let $\mathcal{B}$ be the collection of all connected coalitions with positive excess. The collection $\mathcal{B}$ is not empty. So take $S \in \mathcal{B}$ with $\text{exc}(S, x)$ maximal.

If $(i, j) \in \Gamma$ with $i \in S \subset N\backslash j$, we have $\bar{s}_{ij}(x) > 0$ and therefore $\bar{s}_{ji}(x) > 0$. Hence there is a coalition $T \in \mathcal{B}$ with $j \in T \subset N \backslash i$. As S and T are connected in the tree Γ, the coalition $S \cap T = \emptyset$ and $S \cup T$ is connected. Then $S \cup T$ is clearly a connected coalition with a larger excess (at least $\text{exc}(S, x) + \text{exc}(T, x)$). So outside the core no preimputation x satisfies $\bar{s}_{ij}(x) = \bar{s}_{ji}(x)$ for all $(i, j) \in \Gamma$ and inside the core only the nucleolus does. QED

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Received October 1993
Revised version January 1994