## Heavy to light baryon weak form factors in the lightcone constituent quark model

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**Abstract.** Heavy to light baryon weak form factors are investigated in a lightcone constituent quark model. In a SU(4) symmetry broken scheme, both charged and neutral weak current-induced form factors are calculated at the  $q^2 = 0$  point including the leading relativistic effects in the spin composition of baryons. The corresponding semileptonic decays are described by assuming dipole dependence of form factors on  $q^2$ .

### I Introduction

With increasing data on heavy baryon decays being available at ARGUS, CERN, CLEO and Fermilab, the study of heavy baryon decays is becoming more topical. The past several years have seen discoveries of many new decay modes of charmed baryons, including new and high statistics measurements of  $\Lambda_c$  decays [1]. In particular, the measurements of semileptonic decay and even form factor ratios for the process  $\Lambda_c \to \Lambda e^+ \nu$  have been carried out [1, 2] recently. These new data now make it possible to carry out a serious theoretical investigation of heavy baryon decays.

Weak decays involve an interplay between weak and strong interactions. The weak form factors of decay amplitudes directly parameterize the nonperturbative dynamics of heavy baryon decays and play an important role in analyzing exclusive decays, and observing CP violations. During the past few years, several attempts have been made to study heavy baryon decay weak form factors, in particular those for charmed baryons, by using the nonrelativistic quark model [3], MIT bag model [3] an heavy quark effective theory (HQET) [4–7]. In the present work, we study heavy baryon weak form factors using the constituent quark model in the lightcone formalism, which was first formulated by Berestetski and Terent'ev [8] and has been applied to various hadronic processes [9, 10]. In the case of mesons, the lightcone quark model has been used to study mesonic weak form factors by a number of authors [11-17]. Recently, the investigation has been extended to light baryons [18, 19] where it was found that the lightcone quark model successfully explained the magnetic moments and weak decays of light baryons.

Weak from factors are determined by wavefunctions of initial and final baryons. In principle, baryon wavefunctions can be obtained by solving the bound state equation [20-22]. Unfortunately, due to the nonperturbative property of QCD at long-distance, at present, one can not determine the wavefunctions from first principle. In this paper, the phenomenological wavefunctions, which have proven to be successful in explaining electroweak properties of light baryons [18,19] and mesons [23], have been used. By fitting experiment data for semileptonic decay  $\Lambda_c \rightarrow \Lambda e^+ \nu$ , a consistent picture can be found in a SU(4) symmetry broken scheme of heavy baryon spinflavor wavefunctions. An advantage of our analysis is that the constituent quark masses determined by baryons are consistent with those determined by mesons. In addition, we employ Melosh transformation [24,25] to construct baryon states of definite spins, and use an approximation of retaining only the leading relativistic effects in the spin composition.

This paper is organized as follows: Following the Introduction, Sect. II introduces the lightcone formalism and baryon wavefunctions. Weak form factors are calculated and semileptonic decays of charmed baryons are analyzed in Sect. III. The final part, Sect. IV, contains a summary and discussion.

### II Lightcone formalism and baryon wavefunctions

In this section, we introduce some necessary notations and definitions, in particular baryon wavefunctions on the lightcone, which will be used to calculate weak form factors in the next section.

In the lightcone formalism [20,21,26], the physical hadron state is defined at the lightcone "time"  $\tau = t + z$ . A baryon bound state with total lightcone momentum  $(P^+, P_\perp)$ , which is assumed to be consisted of three constituent quarks  $q_1, q_2, q_3$  in the lightcone constituent quark model [8], can be written as

$$|B;P^+,P_\perp>=\tag{1}$$

$$\begin{split} \sum_{\lambda_i} &\int \frac{[dx][d^2k_{\perp}]}{\sqrt{x_1 x_2 x_3}} |q_1 q_2 q_3 : x_i p^+, x_i P_{\perp} + k_{\perp i}, \lambda_i > \\ &\times \psi(x_i, k_{\perp i}, \lambda_i), \end{split}$$

where the sum is over helicities, and  $x_i = k_i^+/P^+$  is the lightcone momentum fraction carried by the i-th quark  $q_i$ ,  $\lambda_i$  and  $k_{\perp i}$  are respectively the helicity and the transverse momenta relative to the momentum of the bound state, and

$$[dx] = \delta(1 - \sum_{i=1}^{3} x_i) \prod_{i=1}^{3} dx_i,$$
  
$$[d^2k_{\perp}] = 16\pi^3 \delta^{(2)} (\sum_{i=1}^{3} k_{\perp i}) \prod_{i=1}^{3} \frac{d^2k_{\perp i}}{16\pi^3}.$$
 (2)

The baryon wavefunction  $\psi(x_i,k_{\perp i},\lambda_i)$  satisfies the normalization condition

$$\int [dx] [d^2 k_\perp] |\psi(x_i, k_{\perp i}, \lambda_i)|^2 = 1$$
(3)

and the invariant mass operator (without interaction terms) is given by

$$M_0^2 = \frac{\mathbf{k}_{\perp 1}^2 + m_1^2}{x_1} + \frac{\mathbf{k}_{\perp 2}^2 + m_2^2}{x_2} + \frac{\mathbf{k}_{\perp 3}^2 + m_3^2}{x_3} \qquad (4)$$

where  $m_1$ ,  $m_2$ ,  $m_3$  are masses of the constituent quarks.

In order to construct a hadron state of definite spin, the longitudinal components  $k_{3i}$  are defined [25] such that the relative momenta  $\mathbf{k}_i = (\mathbf{k}_{\perp}, \mathbf{k}_3)_i$  and the spin satisfy vector commutation relations. Thus with

$$k_{3i} = \frac{1}{2} [M_0 x_i - \frac{m_i^2 + \mathbf{k}_{\perp i}^2}{M_0 x_i}], \qquad (5)$$

the total spin operator  $\mathbf{J}$  can be expressed as a sum of orbital and spin contributions,

$$\mathbf{J} = \sum_{i} \left( \mathbf{y}_{i} \times \mathbf{k}_{i} + \mathbf{j}_{i} \right), \tag{6}$$

where  $\mathbf{y}_i$  are coordinate operators of quarks, and the operators  $\mathbf{j}_i$  are related to the quark spin  $\mathbf{s}_i$  by a Melosh rotation [24, 25, 27]

$$\mathbf{j}_i = R_M(x_i, k_{\perp i}, m_i, M_0) \mathbf{s}_i. \tag{7}$$

The matrix representation of Melosh rotation is given by [25]

$$<\lambda'|R_M(x,k_{\perp},m,M)|\lambda> = \left[\frac{m+xM-i\sigma\cdot(\mathbf{n}\times\mathbf{k}_{\perp})}{\sqrt{(m+xM)^2+\mathbf{k}_{\perp}^2}}\right]_{\lambda'\lambda}$$
(8)

with the vector  $\mathbf{n} = (0, 0, 1)$ .

In terms of the eigenvalues of the longitudinal components of operators  $\mathbf{j}_i$ , the spin-flavor wavefunctions of charmed baryons with spin  $\frac{1}{2}^+$  can be written as in the nonrelativistic constituent quark model [28]

$$\begin{aligned} |\Lambda_c^+(\uparrow)\rangle &= \frac{1}{\sqrt{2}} (udc - duc)\chi_A, \qquad |\Sigma_c^{++}(\uparrow)\rangle = uuc\chi_S, \\ |\Sigma_c^+(\uparrow)\rangle &= \frac{1}{\sqrt{2}} (udc + duc)\chi_S, \qquad |\Sigma_c^0(\uparrow)\rangle = ddc\chi_S, \\ |\Xi_c^+(\uparrow)\rangle &= \frac{1}{\sqrt{2}} (usc - suc)\chi_A, \\ |\Xi_c^{+\prime}(\uparrow)\rangle &= \frac{1}{\sqrt{2}} (usc + suc)\chi_S, \qquad (9) \\ |\Xi_c^0(\uparrow)\rangle &= \frac{1}{\sqrt{2}} (dsc - sdc)\chi_A, \\ |\Xi_c^{0\prime}(\uparrow)\rangle &= \frac{1}{\sqrt{2}} (dsc + sdc)\chi_S, \\ |\Omega_c^0(\uparrow)\rangle &= ssc\chi_S; \end{aligned}$$

$$\chi_A = \frac{1}{\sqrt{2}} [|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle],$$
  
$$\chi_S = \frac{1}{\sqrt{6}} [2|\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle], \qquad (10)$$

where eigenstates  $|\uparrow\rangle$ ,  $|\downarrow\rangle$  of the longitudinal components of  $\mathbf{j}_i$  are related to the corresponding quark helicity eigenstates  $|\uparrow\rangle_F$ ,  $|\downarrow\rangle_F$  by the Melosh rotation  $R_M(x_i, k_{\perp i}, m_i, M_0),$ 

$$\uparrow (\downarrow) >= R_M(x_i, k_{\perp i}, m_i, M_0) |\uparrow (\downarrow) >_F.$$
(11)

It should be emphasized that the above spin-flavor wavefunctions define our SU(4) symmetry broken scheme. In SU(4) symmetry, the corresponding spin-favor wavefunctions include the extra terms, which are obtained from (9) by permutations of the quarks in the first and second positions with the charm quark [3].

The baryon wavefunction is a product of the spin-flavor wavefunction and the momentum wavefunction. In this paper, the momentum wavefunctions of baryons are assumed to be a simple function of the invariant mass  $M_0$ of the form,

$$\phi(x_i, k_{\perp i}) = Aexp[-M_0^2/2\beta^2]$$
(12)

where  $\beta$  is a scale parameter and the amplitude A is determined by the normalization condition (3). It should be emphasized that the above harmonic oscillator wavefunctions can be obtained directly by solving the equaltime B-S equation with a harmonic oscillator potential in the rest frame and applying the Brodsky-Huang-Lepage (BHL) prescription [22], which suggests that the valence Fock-state wavefunction depends only on the off-shell energy variable and the equal-time wavefunction in the rest frame is related to the wavefunction in the infinite momentum frame by equating the energy propagators in the two frames.

In the next section, we apply these wavefunctions to investigate weak form factors and semileptonic decays of charmed baryons.

# III Weak form factors and semileptonic decays of charmed baryons

By Lorentz invariance, the general decomposition of hadronic matrix element of weak currents is given by

$$< B_{f}(P_{f}, S_{f})|V_{\mu}|B_{i}(P_{i}, S_{i}) >$$

$$= \bar{u}_{f} \left[ f_{1}(q^{2})\gamma_{\mu} + if_{2}(q^{2})\sigma_{\mu\nu}q^{\nu}/m_{I} + f_{3}(q^{2})q_{\mu}/m_{I} \right] u_{i},$$

$$< B_{f}(P_{f}, S_{f})|A_{\mu}|B_{i}(P_{i}, S_{i}) >$$

$$= \bar{u}_{f} \left[ g_{1}(q^{2})\gamma_{\mu} + ig_{2}(q^{2})\sigma_{\mu\nu}q^{\nu}/m_{I} + g_{3}(q^{2})q_{\mu}/m_{I} \right] \gamma_{5}u_{i},$$
(13)

where  $q = P_i - P_f$  and  $m_i$  is mass of the initial baryon. All form factors  $f_i$  and  $g_i$  are real functions of  $q^2$  by T invariance of strong interaction. Usually, the dependence of these form factors on  $q^2$  are assumed to be either monopole or dipole. In this paper, we assume a dipole behavior for them, which is preferred since it is close to the calculated baryonic Isgur-Wise function. Thus,

$$f_i(q^2) = \frac{f_i(0)}{(1 - q^2/m_V^2)^2}, \, g_i(q^2) = \frac{g_i(0)}{(1 - q^2/m_A^2)^2} \quad (15)$$

where  $m_V$  and  $m_A$  are pole masses of the vector and axial vector meson with the same quantum number as the current under consideration, respectively.

In order to calculate these form factors, we choose a frame with  $q^+ = 0$  and put, without loss of generality,  $q_{\perp} = (q_x, q_y) = (q_x, 0)$  for convenience. By expressing the currents  $V_{\mu}$  and  $A_{\mu}$  in terms of creation and annihilation operators, it is not difficult to obtain

$$f_{1} = \frac{1}{2\sqrt{P_{i}^{+}P_{f}^{+}}} < B_{f}(P_{f},\uparrow)|V^{+}|B_{i}(P_{i},\uparrow)>,$$
  
$$g_{1} = \frac{1}{2\sqrt{P_{i}^{+}P_{f}^{+}}} < B_{f}(P_{f},\uparrow)|A^{+}|B_{i}(P_{i},\uparrow)>; \quad (16)$$

$$f_2 = \frac{m_I}{2\sqrt{P_i^+ P_f^+ q_\perp}} < B_f(P_f, \downarrow) | V^+ | B_i(P_i, \uparrow) >,$$

$$m_I$$

$$m_I = P_i(P_i, \downarrow) | A^+ | P_i(P_i, \uparrow) >, \quad (17)$$

$$g_2 = \frac{m_I}{2\sqrt{P_i^+ P_f^+ q_\perp}} < B_f(P_f,\downarrow) | A^+ | B_i(P_i,\uparrow) > \quad (17)$$

$$f'_{3} = \frac{m_{I}}{2q_{\perp}} < B_{f}(P_{f},\uparrow) | V_{x} | B_{i}(P_{i},\uparrow) >,$$
  
$$f_{3} = \frac{m_{I}}{m_{I} + m_{f}} [f'_{3} + f_{1}], \qquad (18)$$

$$g'_{3} = \frac{m_{I}}{2q_{\perp}} < B_{f}(P_{f},\uparrow)|A_{x}|B_{i}(P_{i},\uparrow)>,$$
  

$$g_{3} = -\frac{m_{I}}{m_{I}-m_{f}}[g'_{3}+g_{1}].$$
(19)

Calculation of form factors requires evaluation of the spin matrix elements. The eigenstates of longitudial components of operators  $\mathbf{j}_i$  can be expanded in terms of the quark helicity states as

$$|\uparrow\rangle^{(i)} = \frac{m_i + x_i M_0}{\sqrt{(m_i + x_i M_0)^2 + k_{\perp i}^2}} |\uparrow\rangle_F^{(i)} - \frac{k_{Ri}}{\sqrt{(m_i + x_i M_0)^2 + k_{\perp i}^2}} |\downarrow\rangle_F^{(i)}, \quad (20)$$

$$|\downarrow\rangle^{(i)} = \frac{k_{Li}}{\sqrt{(m_i + x_i M_0)^2 + k_{\perp i}^2}} |\uparrow\rangle_F^{(i)} + \frac{m_i + x_i M_0}{\sqrt{(m_i + x_i M_0)^2 + k_{\perp i}^2}} |\downarrow\rangle_F^{(i)}$$
(21)

where  $k_{L(R)i} = k_{xi} \mp k_{yi}$ . Hereafter, for convenience, we call  $|\uparrow\rangle^{(i)}$ ,  $|\uparrow\rangle^{(i)}$  and  $|\uparrow\rangle^{(i)}_F$ ,  $|\downarrow\rangle^{(i)}_F$  quark spin states and helicity states, respectively. Obviously, in the nonrelativistic limit  $k_{\perp i} = 0$ , one has  $|\uparrow >^{(i)} = |\uparrow >^{(i)}_{F}, |\downarrow >^{(i)} =$  $|\downarrow>_F^{(i)}$ . At the same time, one gets  $f_2 = g_2 = 0$  duo to the helicity conservation. However, recent measurement of form factors for  $\Lambda_c \to \Lambda e^+ \nu$  from CLEO found [2] that the form factor ratio  $R = -0.25 \pm 0.14 \pm 0.08$  in the framework of HQET. In our definition of form factors, the above measured ratio gives  $f_2/f_1 = g_2/g_1 =$  $-0.28 \pm 0.21$ , which hints that relativistic effects in the spin composition are necessary, i.e., there are necessary Melosh rotations connecting quark spin states and helicity states on the lightcone. On this point, one finds that heavy baryons and heavy mesons are quite different; in weak decays of heavy meson, no direct experimental evidence to support such a rotation has been found. This explains why BSW model [11] met with such great success. Since in the hadronic bound state, quarks interact with each other by exchanges of soft gluons, quark momentum fluctuates around a very low value, in particular, the transverse movementa of quarks on the lightcone are highly suppressed. One naturally expects that the main contribution comes from low transverse momenta of quarks. Therefore, it is very reasonable to consider the leading relativistic effects in the spin composition, i.e., expand the Melosh rotation matrix in the quark transverse momentum

$$|\uparrow\rangle^{(i)} = |\uparrow\rangle_{F}^{(i)} - \frac{k_{Ri}}{m_i + x_i M_0}|\downarrow\rangle_{F}^{(i)} + \dots, \qquad (22)$$

$$|\downarrow\rangle^{(i)} = \frac{k_{Li}}{m_i + x_i M_0} |\uparrow\rangle_F^{(i)} + |\downarrow\rangle_F^{(i)} + \dots, \qquad (23)$$

and keep only the terms up to  $O(k_{\perp})^1$  in the above expansion, and also in the expansions of all spin matrix elements. Under this approximation, the normalizations of quark spin states are preserved automatically,  $\langle \uparrow | \uparrow \rangle^{(i)} = \langle \downarrow | \downarrow \rangle^{(i)} = 1$ ; and for the individual spectator transition, the spin conserving and spin flip matrix elements are respectively of  $O(k_{\perp})^0$  and  $O(k_{\perp})^1$ . One can, similarly, also count the powers of the individual spin matrix elements between the decaying and the produced quarks in  $k_{\perp}$  under the action of a given current. Therefore, under

Table 1. Coefficients in form factors for weak currents induced charmed to light baryon transitions (in our notation, the second spectator always refers to the heavier one, and values shown in the table for all coefficients have been multiplied by a factor of  $\sqrt{3}$ )

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Transition	$(a_f, a_g)$	$(a_1, a_2, a_3)$	$(b_1, b_2, b_3)$	$(c_1, c_2, c_3)$	$(d_1, d_2, d_3)$
$\Lambda_c^+  ightarrow n$	$\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}$	$\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}$	$\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}$	$\sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}$	$\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}$
$\Sigma_c^{++} \to p$	$1, -\frac{1}{3}$	$\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$	$-\frac{1}{6}, 0, 0$	$1, -\frac{2}{3}, -\frac{2}{3}$	$-\frac{1}{3}, 0, 0$
$\Sigma_c^+  ightarrow n$	$\frac{1}{2\sqrt{2}}, -\frac{1}{6\sqrt{2}}$	$-\frac{1}{6\sqrt{2}}, \frac{1}{6\sqrt{2}}, -\frac{1}{6\sqrt{2}}$	$-\frac{1}{6\sqrt{2}}, -\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}$	$\frac{1}{2\sqrt{2}}, \frac{1}{6\sqrt{2}}, -\frac{3}{6\sqrt{2}}$	$-\frac{1}{6\sqrt{2}}, -\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}$
$\Xi_c^{\prime +} \to \Lambda$	$\frac{\sqrt{3}}{2}, -\frac{1}{2\sqrt{3}}$	$-\frac{1}{2\sqrt{3}}, \frac{\sqrt{3}}{2}, \frac{1}{2\sqrt{3}}$	$-\frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}$	$\frac{\sqrt{3}}{2}, -\frac{1}{2\sqrt{3}}, \frac{\sqrt{3}}{2}$	$-\frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}$
$\Xi_c^+ \to \Lambda$	$\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}$ , $\frac{1}{2}$ , $-\frac{1}{2}$	$\frac{1}{2}$ , $\frac{1}{2}$ , $-\frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}$
$\Xi_c^{*+} \to \Sigma^0$	$-\frac{1}{2}, \frac{1}{6}$	$\frac{1}{6}, \frac{1}{6}, -\frac{3}{6}$	$\frac{1}{6}, -\frac{1}{2}, \frac{1}{2}$	$\frac{3}{6}, -\frac{1}{2}, -\frac{1}{6}$	$\frac{1}{6}, -\frac{1}{2}, \frac{1}{2}$
$ \begin{array}{c} \Pi_{a} \Pi_{a} \Pi_{b} \Pi_{c} \Pi_{c$	$\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}, -\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}$	$\frac{\sqrt{3}}{2}, -\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}$	$\frac{\sqrt{3}}{2}, -\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}$	$\frac{\sqrt{3}}{2}, -\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}$
$\Xi_c^{**} \to \Sigma^-$	$-\frac{1}{\sqrt{2}}, \frac{1}{3\sqrt{2}}$	$\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, -\frac{3}{3\sqrt{2}}$	$\frac{1}{3\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}, \frac{3}{3\sqrt{2}}, -\frac{2}{3\sqrt{2}}$	$\frac{1}{3\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$
$\Xi_c^0 \to \Sigma_c^-$	$\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}$	$\sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$	$\sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$	$\sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$	$\sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$
$\Omega_c^0 \to \Xi^-$	$-1$ , $\frac{1}{3}$	$\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}$	$\frac{1}{6}, 0, 0$	$-1$ , $\frac{2}{3}$ , $\frac{2}{3}$	$\frac{1}{3}$ , 0, 0
$\begin{array}{c} A_c^+ \to A \\ \Sigma^{++} & \Sigma^+ \end{array}$	1, 1	1, 0, 0	1, 0, 0	1,0,0	1, 0, 0
$\Sigma_c^+ \to \Sigma^+$ $\Sigma^+ \to \Sigma^0$	$1, -\frac{1}{3}$ $1, -\frac{1}{3}$	$-\frac{1}{3}$ , $\frac{1}{3}$ , $\frac{1}{3}$ , $\frac{1}{3}$	$-\frac{1}{6}, 0, 0$	$1, -\frac{1}{3}, -\frac{1}{3}$ $1, -\frac{2}{3}, -\frac{2}{3}$	$-\frac{1}{3}, 0, 0$
$\Sigma_c^0 \to \Sigma^-$	$1, -\frac{1}{3}$	$-\frac{3}{1}$ , $\frac{3}{2}$ , $\frac{3}{2}$	$-\frac{1}{6}, 0, 0$	$1, -\frac{2}{3}, -\frac{2}{3}$	$-\frac{1}{3}, 0, 0$
$\begin{array}{c} \Xi_c^{\circ} \to \Sigma \\ \Omega_c^0 \to \Xi^- \\ \Lambda_c^+ \to \Lambda \\ \Sigma_c^{++} \to \Sigma^+ \\ \Sigma_c^{\circ} \to \Sigma^- \\ \Xi_c^{\prime+} \to \Xi^0 \\ \Sigma_c^{\circ} \to \Sigma^- \\ \Xi_c^{\prime+} \to \Xi^0 \\ \Xi_c^{\prime0} \to \Xi^- \\ \Xi_c^{0} \to \Xi^- \end{array}$	$\frac{1}{\sqrt{2}}, -\frac{3}{\sqrt{2}}$	$-\frac{1}{2}$ , $\frac{3}{2}$ , $\frac{3}{2}$ , $\frac{3}{2}$ , $\frac{1}{2}$	$-\frac{1}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{5}{\sqrt{2}}$	$-\frac{1}{2\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$
$\Xi_{c}^{+} \rightarrow \Xi^{0}$	$\sqrt{\frac{3}{2}}$ , $\sqrt{\frac{3}{2}}$	$\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{2}}, \frac{1}{2}, -\frac{1}{2}$	$\sqrt{\frac{3}{2}}$ , $\frac{1}{\sqrt{2}}$ , $-\frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$
$\Xi_{c}^{\prime 0} \rightarrow \Xi^{-}$	$\frac{1}{1}$ , $-\frac{1}{1}$	$-\frac{1}{1}$ , $\frac{5}{5}$ , $-\frac{1}{1}$		$\frac{\sqrt{2}}{1}$ , $\frac{1}{\sqrt{6}}$ , $\frac{\sqrt{6}}{\sqrt{6}}$	
$\Xi_c^0 \to \Xi^-$	$\sqrt{\frac{2}{3}}$ $\sqrt{\frac{3}{3}}$	$3\sqrt{2}$ , $3\sqrt{2}$ , $3\sqrt{2}$	$\sqrt{\frac{3}{2}}$ $\sqrt{\frac{1}{2}}$ $\sqrt{\frac{1}{2}}$	$\sqrt{\frac{2}{3}}$ $\sqrt{\frac{3}{2}}$ $\sqrt{\frac{3}{2}}$ $\sqrt{\frac{3}{2}}$	$\sqrt{\frac{3}{2}}$ $\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{2}}$
$\frac{1}{2}$	$\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$	$\sqrt{\frac{2}{3}}, \sqrt{6}, \sqrt{6}$	$\sqrt{\frac{2}{3}}, \sqrt{6}, \sqrt{6}$	$\sqrt{\frac{2}{3}}, \sqrt{6}, \sqrt{6}$	$\sqrt{\frac{2}{3}}, \sqrt{6}, \sqrt{6}$
$egin{aligned} \Lambda_c^+ & ightarrow p \ \Sigma_c^+ & ightarrow p \end{aligned}$	$\sqrt{\frac{1}{2}}$ , $\sqrt{\frac{1}{2}}$	$\sqrt{\frac{2}{2}}$ , $\sqrt{\frac{6}{5}}$ , $\sqrt{\frac{6}{5}}$	$\sqrt{\frac{1}{2}}$ , $\sqrt{\frac{1}{6}}$ , $-\sqrt{\frac{1}{6}}$	$\sqrt{\frac{1}{2}}$ , $\sqrt{6}$ , $-\sqrt{6}$	$\sqrt{\frac{1}{2}}$ , $\sqrt{\frac{1}{6}}$ , $-\sqrt{\frac{1}{6}}$
$\Sigma_c^0 \rightarrow n$	$\sqrt{2}$ ' $3\sqrt{2}$	$3\sqrt{2}$ , $3\sqrt{2}$ , $3\sqrt{2}$	$3\sqrt{2}, \sqrt{2}, \sqrt{2}, \sqrt{2}$	$\sqrt{2}$ , $3\sqrt{2}$ , $3\sqrt{2}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{c} \Sigma_c^0 \to n \\ \Xi_c^{\prime +} \to \Sigma^+ \end{array}$	$-\frac{1}{2}, \frac{3}{1}$	$\underline{1}^{3}, \underline{1}^{3}, \underline{3}^{3}, \underline{5}^{3}$	$\frac{1}{6}, \frac{1}{6}, \frac{1}{6}$	$-\frac{1}{5}$ , $\frac{5}{5}$ , $\frac{1}{-1}$	$\frac{1}{1}$ , $-\frac{1}{5}$ , $\frac{1}{5}$
$\frac{-c}{\Xi_c^+} \to \Sigma^+$	$\sqrt{\frac{2}{3}}$ , $\sqrt{\frac{3}{2}}$	$3\sqrt{2}$ , $3\sqrt{2}$ , $3\sqrt{2}$	$\frac{3\sqrt{2}}{\sqrt{3}}$ , $\frac{\sqrt{2}}{1}$ , $\frac{\sqrt{2}}{1}$	$\sqrt{2}$ , $3\sqrt{2}$ , $3\sqrt{2}$	$\frac{3\sqrt{2}}{\sqrt{3}}$ , $\frac{\sqrt{2}}{1}$ , $\frac{\sqrt{2}}{1}$
$\Xi_c^{\prime 0}$ A	$\bigvee_{2}^{2}, \bigvee_{2}^{2}$	$egin{array}{cccccccccccccccccccccccccccccccccccc$	$egin{array}{cccccccccccccccccccccccccccccccccccc$	$egin{array}{cccccccccccccccccccccccccccccccccccc$	$\bigvee 2 \ , \ \sqrt{6} \ , \ \sqrt{6} \ 1 \ 1 \ 1$
$ \begin{array}{c} \square_c \rightarrow \Lambda \\ \square^0 \rightarrow \Lambda \end{array} $	$-\frac{1}{2}$ , $\frac{1}{2\sqrt{3}}$	$\frac{1}{2\sqrt{3}}$ , $\frac{1}{2\sqrt{3}}$ , $\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$ , $\frac{1}{2\sqrt{3}}$ , $\frac{1}{2\sqrt{3}}$	$-\frac{1}{2}$ , $\frac{1}{2\sqrt{3}}$ , $-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$ , $\frac{1}{2\sqrt{3}}$ , $-\frac{1}{2\sqrt{3}}$
$\Xi_c \to \Lambda$ $\Xi'^0 \to \Sigma^0$	$-\frac{1}{2}, -\frac{1}{2}$	$\begin{array}{c} \overline{2} \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ $	$\frac{1}{2}$ , $\frac{1}{2}$ , $\frac{1}{2}$	$-\frac{1}{2}$ , $-\frac{1}{2}$ , $\frac{1}{2}$	$\frac{1}{2}$ , $\frac{1}{2}$ , $\frac{1}{2}$
$\Xi_c^0 \rightarrow \Sigma^0$	$\frac{\sqrt{3}^{2}}{\sqrt{3}}, \frac{\sqrt{3}}{\sqrt{3}}$	$\frac{6}{\sqrt{3}}, \frac{6}{1}, \frac{6}{1}$	$\frac{\sqrt{3}}{\sqrt{3}}$ , $\frac{1}{\sqrt{3}}$ , $\frac{1}{\sqrt{3}}$ , $\frac{1}{\sqrt{3}}$ , $\frac{1}{\sqrt{3}}$	$\frac{\sqrt{3}}{\sqrt{3}}$ , $\frac{1}{\sqrt{3}}$ , $\frac{1}{\sqrt{3}}$ , $\frac{1}{\sqrt{3}}$	$\frac{6}{\sqrt{3}}$ , $\frac{1}{1}$ , $\frac{1}{1}$
$\begin{array}{c} \Xi_c^{\prime 0} \to \Lambda \\ \Xi_c^0 \to \Lambda \\ \Xi_c^{\prime 0} \to \Sigma^0 \\ \Xi_c^0 \to \Sigma^0 \\ \Omega_c^0 \to \Xi^0 \end{array}$	$\frac{2}{-1}, \frac{1}{2}$	$\begin{array}{c} \left(\begin{array}{c} a_{1}, a_{2}, a_{3} \right) \\ \hline \sqrt{\frac{3}{2}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \\ & \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \\ -\frac{1}{6\sqrt{2}}, \frac{5}{6\sqrt{2}}, -\frac{1}{6\sqrt{2}} \\ & -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}} \\ & \frac{1}{2\sqrt{3}}, \frac{1}{2}, -\frac{1}{2} \\ & \frac{1}{6}, \frac{1}{6}, -\frac{5}{6} \\ \hline \frac{\sqrt{3}}{2}, -\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}} \\ & \frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, -\frac{5}{3\sqrt{2}} \\ & \sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \\ & \frac{1}{3}, -\frac{2}{3}, -\frac{2}{3} \\ & 1, 0, 0 \\ & -\frac{1}{3}, \frac{2}{3}, \frac{2}{3} \\ & -\frac{1}{3\sqrt{2}}, \frac{5}{3\sqrt{2}}, -\frac{1}{3\sqrt{2}} \\ & \sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \\ & -\frac{1}{3\sqrt{2}}, \frac{5}{3\sqrt{2}}, -\frac{1}{3\sqrt{2}} \\ & \sqrt{\frac{3}{2}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \\ & -\frac{1}{3\sqrt{2}}, \frac{5}{3\sqrt{2}}, -\frac{3}{3\sqrt{2}} \\ & \sqrt{\frac{3}{2}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \\ & \sqrt{\frac{3}{2}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \\ & \sqrt{\frac{3}{2}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \\ & \sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \\ & \frac{1}{3\sqrt{2}}, -\frac{5}{3\sqrt{2}}, -\frac{3}{3\sqrt{2}} \\ & \sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \\ & \frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}}, -\frac{2}{3} \\ & \frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}} \\ & -\frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}} \\ & \frac{1}{3}, -\frac{2}{3}, -\frac{2}{3} \\ & \frac{1}{3}, -\frac{2}{3}, -\frac{2}{3} \\ & \frac{1}{3}, -\frac{2}{3}, -\frac{2}{3} \\ \end{array}$	$\begin{array}{c} (b_1, b_2, b_3) \\ \hline \sqrt{\frac{3}{2}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \\ -\frac{1}{6}, 0, 0 \\ -\frac{1}{6\sqrt{2}}, -\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}} \\ \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \\ \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \\ \frac{1}{3}, -\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}} \\ \frac{1}{3\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \\ \frac{1}{6}, 0, 0 \\ -\frac{1}{6}, 0, 0 \\ -\frac{1}{6}, 0, 0 \\ -\frac{1}{6}, 0, 0 \\ -\frac{1}{6}, 0, 0 \\ -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{2}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \\ \sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}}, -\frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}} \\ -\frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}} \\ -\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}} \\ \frac{1}{6}, -\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}} \\ \frac{1}{6}, 0, 0 \end{array}$	$\begin{array}{c} (c_1, c_2, c_3) \\ \hline & \sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \\ & 1, -\frac{3}{3}, -\frac{2}{3} \\ \frac{1}{2\sqrt{2}}, \frac{1}{6\sqrt{2}}, -\frac{5}{6\sqrt{2}} \\ & \frac{3}{2}, -\frac{1}{2\sqrt{3}}, \frac{\sqrt{3}}{2} \\ & \frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{3}}, -\frac{1}{2} \\ & \frac{5}{6}, -\frac{1}{2}, -\frac{1}{6} \\ \hline & \frac{3}{2}, -\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}} \\ & -\frac{1}{\sqrt{2}}, \frac{3}{3\sqrt{2}}, -\frac{2}{3\sqrt{2}} \\ & \sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \\ & -1, \frac{2}{3}, \frac{2}{3} \\ & 1, 0, 0 \\ 1, -\frac{2}{3}, -\frac{2}{3} \\ & 1, -\frac{2}{3}, -\frac{2}{3} \\ 1, -\frac{2}{3}, -\frac{2}{3} \\ & 1, -\frac{2}{3}, -\frac{2}{3} \\ & 1, -\frac{2}{3}, -\frac{2}{3} \\ & \frac{1}{\sqrt{2}}, \frac{1}{3\sqrt{2}}, -\frac{3\sqrt{2}}{3\sqrt{2}} \\ & \sqrt{\frac{3}{2}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \\ & \frac{\sqrt{3}}{2}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \\ & \sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \\ & -\frac{\sqrt{3}}{2}, \frac{1}{2\sqrt{3}}, -\frac{\sqrt{3}}{2} \\ & -\frac{1}{2}, \frac{5}{3\sqrt{2}}, -\frac{1}{3\sqrt{2}} \\ & -\frac{1}{2}, \frac{5}{3\sqrt{2}}, -\frac{1}{2\sqrt{3}} \\ & -\frac{1}{2}, \frac{5}{3\sqrt{2}}, -\frac{1}{2\sqrt{3}} \\ & -\frac{1}{2}, \frac{5}{3\sqrt{2}}, \frac{1}{2\sqrt{3}} \\ & -\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}} \\ & -\frac{1}{2}, \frac{2}{3}, \frac{1}{2\sqrt{3}} \\ & -\frac{1}{2\sqrt{3}}, \frac{1}{2$	$\begin{array}{c} (d_1, d_2, d_3) \\ \hline \sqrt{\frac{3}{2}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{3}}, 0, 0 \\ -\frac{1}{6\sqrt{2}}, -\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}} \\ \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \\ \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \\ \frac{1}{3\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \\ \frac{\sqrt{3}}{2}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \\ \frac{1}{3}, 0, 0 \\ -\frac{1}{3}, 0, 0 \\ -\frac{1}{3}, 0, 0 \\ -\frac{1}{3\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{2}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{2}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \\ \sqrt{\frac{3}{2}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \\ \sqrt{\frac{3}{2}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \\ \sqrt{\frac{3}{2}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \\ \sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{2}}, -\frac{1}{2\sqrt{3}}, \frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}}, \frac{1}{\sqrt{2}\sqrt{3}} \\ -\frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}}, \frac{1}{\sqrt{2}\sqrt{3}} \\ \frac{1}{3}, 0, 0 \end{array}$

this approximation, for form factors  $f_1$  and  $g_1$ , which correspond to spin conserving transitions at the hadron level, only those subprocesses in which all individual quark spins are conserved are considered while processes involving simultaneous spin flip of the two quarks are ignored. For form factors  $f_2$  and  $g_2$ , which correspond to the spin flip transition at the hadron level, only those processes in which one quark spin is flipped are considered while all processes with simultaneous three quark spin flips are ignored. Similarly one can find the corresponding transition picture of the form factors  $f_3$  and  $g_3$  at the quark level.

Under this approximation, the complex spin matrix element calculations get greatly simplified. Form factors at  $q^2 = 0$  are given by

$$f_1 = a_f \cdot \int [dx] [d^2 k_\perp] \phi_i \phi_f, \qquad (24)$$

$$g_1 = a_g \cdot \int [dx] [d^2 k_\perp] \phi_i \phi_f, \qquad (25)$$

$$f_2 = -\int [dx][d^2k_\perp]\phi_i\phi_f[a_1\lambda_1 + a_2\lambda_2 + a_3\lambda_3], \quad (26)$$

$$g_2 = -\int [dx] [d^2k_{\perp}] \phi_i \phi_f [b_1 \hat{\lambda}_1 + b_2 \lambda_2 + b_3 \lambda_3], \quad (27)$$

$$f_{3}' = \int \frac{[dx][d^{2}k_{\perp}]}{x_{1}} \phi_{i} \phi_{f} \Big\{ a_{f} \Big[ \frac{2k_{x1}^{2}}{\beta_{f}^{2}x_{1}} - 1 \Big] \\ + (m_{q} - m_{Q}) \Big[ c_{1}\lambda_{1} + c_{2}\lambda_{2} + c_{3}\lambda_{3} \Big] \Big\}, \quad (28)$$

$$g'_{3} = \int \frac{[dx][d^{2}k_{\perp}]}{x_{1}} \phi_{i} \phi_{f} \Big\{ a_{g} \Big[ \frac{2k_{x_{1}}^{2}}{\beta_{f}^{2}x_{1}} - 1 \Big] \\ + (m_{q} + m_{Q}) \Big[ d_{1}\hat{\lambda}_{1} + d_{2}\lambda_{2} + d_{3}\lambda_{3} \Big] \Big\}, \quad (29)$$

where

$$\lambda_1 = \frac{1 - x_1}{m_q + x_1 M_f} - \frac{k_{x_1}^2}{M_f (m_q + x_1 M_f)^2}$$

$$-\frac{k_{x1}^2[m_Q - m_q + x_1(M_I - M_f)]}{\beta_f^2 x_1(m_q + x_1M_f)(m_Q + x_1M_I)},$$
 (30)

$$\lambda_{2} = -\frac{x_{2}}{m_{2} + x_{2}M_{f}} - \frac{x_{2}k_{1x}k_{2x}}{x_{1}M_{f}(m_{2} + x_{2}M_{f})^{2}} - \frac{x_{2}(M_{I} - M_{f})k_{x1}k_{x2}}{\beta_{f}^{2}x_{1}(m_{2} + x_{2}M_{f})(m_{2} + x_{2}M_{I})},$$
(31)

$$\lambda_{3} = -\frac{x_{3}}{m_{3} + x_{3}M_{f}} - \frac{x_{3}k_{x1}k_{x3}}{x_{1}M_{f}(m_{3} + x_{3}M_{f})^{2}} - \frac{x_{3}(M_{I} - M_{f})k_{x1}k_{x3}}{\beta_{f}^{2}x_{1}(m_{3} + x_{3}M_{f})(m_{3} + x_{3}M_{I})}, \qquad (32)$$

$$\hat{\lambda}_{1} = \frac{1 - x_{1}}{m_{q} + x_{1}M_{f}} - \frac{k_{x1}^{2}}{M_{f}(m_{q} + x_{1}M_{f})^{2}} - \frac{k_{x1}^{2}[m_{Q} + m_{q} + x_{1}(M_{I} + M_{f})]}{\beta_{f}^{2}x_{1}(m_{q} + x_{1}M_{f})(m_{Q} + x_{1}M_{I})}, \quad (33)$$

and

$$\begin{split} \phi_i &= A_i exp[-M_I^2/2\beta_i^2],\\ M_I^2 &= \frac{\mathbf{k}_{\perp 1}^2 + m_Q^2}{x_1} + \frac{\mathbf{k}_{\perp 2}^2 + m_2^2}{x_2} + \frac{\mathbf{k}_{\perp 3}^2 + m_3^2}{x_3} \quad (34) \end{split}$$

$$\phi_f = A_f exp[-M_f^2/2\beta_f^2],$$
  
$$M_f^2 = \frac{\mathbf{k}_{\perp 1}^2 + m_q^2}{x_1} + \frac{\mathbf{k}_{\perp 2}^2 + m_2^2}{x_2} + \frac{\mathbf{k}_{\perp 3}^2 + m_3^2}{x_3} \quad (35)$$

with  $m_Q$ ,  $m_q$  being the masses of the decaying and the produced quarks respectively, and  $m_2$ ,  $m_3$  the masses of the spectators. In the above expressions, constants  $a_f$  and  $a_q$  have values as in the nonrelativistic quark model as explained earlier. The spin transition terms  $\lambda_i (i = 1, 2, 3)$ and  $\hat{\lambda}_1$  are the basic ones, all spin matrix elements can be reduced or related to them. They have a direct physical meaning: the terms  $\lambda_1$  and  $\hat{\lambda}_1$  just correspond to the spin transition only between the decaying quark and the produced quark while spectators remain unaffected under the action of the "plus" component of vector and axial vector currents, respectively. The terms  $\lambda_2$ ,  $\lambda_3$  respectively, are related to the spin transitions of the second spectator quark and of the third spectator quark with other quarks remaining unaffected under the action of "plus" component of vector current. In Table 1, we show all calculated coefficients  $a_f$ ,  $a_g$ ,  $a_i$ ,  $b_i$ ,  $c_i$  and  $d_i$  for weak current induced charmed to light baryon transitions.

With these quantities, numerical calculation of form factors can be carried out directly. All integrals can finally be reduced to ones in two or three dimensions. Before doing the numerical calculation, we have to determine the parameters in baryon wavefunctions, the scale parameters  $\beta_i$ ,  $\beta_f$ , and masses of the constituent quarks  $m_u$ ,  $m_d$ ,  $m_s$ and  $m_c$ . For light baryons, by fitting the magnetic moments and semileptonic decays, it was found [18,19] that

 Table 2. Form factors for weak currents induced charmed to light baryon transitions

Transition	£	£	£	~	<i>a</i> -	0-
$\frac{11111111011}{\Lambda_c^+ \to n}$	$\frac{f_1}{0.34}$	$f_2$ -0.24		$\frac{g_1}{0.34}$	$\frac{g_2}{-0.04}$	$\frac{g_3}{-0.35}$
$\begin{array}{c} \Lambda_c \to n \\ \Sigma_c^{++} \to p \end{array}$	$0.34 \\ 0.28$	-0.24 0.35	-0.30	-0.09	-0.04 0.006	-0.00
$\begin{array}{ccc} \varSigma_c & \rightarrow p \\ \Sigma_c^+ & \rightarrow n \end{array}$	$0.20 \\ 0.10$	$0.35 \\ 0.12$	-0.11	-0.03	0.000	0.03
$\begin{array}{c} \Sigma_c \to n\\ \Xi_c^{\prime +} \to \Lambda \end{array}$	0.10 0.24	0.12 0.29	-0.26	-0.03	0.004 0.02	0.00
$\begin{array}{c} \Xi_c^- \to \Lambda \\ \Xi_c^+ \to \Lambda \end{array}$	$0.24 \\ 0.14$	-0.12	-0.20		-0.02	-0.18
$\Xi_{c}^{\prime +} \to \Sigma^{0}$	-0.14		0.14	0.14 0.05	-0.002	-0.04
$\Xi_c^+ \to \Sigma^0$	-0.14 0.24	-0.10	-0.09	$0.03 \\ 0.24$	-0.04	-0.29
$\begin{array}{c} \Xi_c^{\prime} \to \mathcal{L} \\ \Xi_c^{\prime 0} \to \mathcal{L}^- \end{array}$	-0.24		0.20	$0.24 \\ 0.07$	-0.004	-0.29
$\begin{array}{ccc} \Xi_{c}^{\circ} \to \Sigma \\ \Xi_{c}^{0} \to \Sigma^{-} \end{array}$	0.20	-0.25	-0.13	0.07 0.34	-0.05	-0.41
			0.13	$0.34 \\ 0.09$	-0.009	-0.41
$\Omega_c^0 \to \Xi^-$	-0.27					
$\begin{array}{c} \Lambda_c^+ \to \Lambda \\ \Sigma_c^{++} \to \Sigma^+ \end{array}$	0.35	-0.22	-0.08	0.35	-0.03	-0.32
	0.35	0.42	-0.29	-0.12	0.006	0.11
$\Sigma_c^+ \to \Sigma^0$	0.35	0.42	-0.29	-0.12	0.006	0.11
$\Sigma_c^0 \to \Sigma^-$	0.35	0.42	-0.29	-0.12	0.006	0.11
$\Xi_c^{\prime+} \to \Xi^0$	0.24	0.27	-0.20	-0.08	0.02	0.12
$\Xi_c^+ \to \Xi^0$	0.41	-0.30	-0.11	0.41	-0.05	-0.47
$\Xi_c^{\prime 0} \rightarrow \Xi^-$	0.24	0.27	-0.20	-0.08		0.12
$\Xi_c^0 \to \Xi^-$	0.41	-0.30	-0.11	0.41	-0.05	-0.47
$\Lambda_c^+ \to p$	0.34	-0.24	-0.11	0.34	-0.04	-0.35
$\Sigma_c^+ \to p$	-0.20	-0.25	0.21	0.07	0.29	-0.07
$\varSigma_c^0  ightarrow n$	-0.28	-0.35	0.30	0.09	-0.006	-0.09
$\Xi_c^{\prime +} \to \Sigma_c^+$	-0.20	-0.25	0.20	0.07	-0.003	-0.05
$\Xi_c^+ \to \Sigma^+$	0.34	-0.26	-0.13	0.34	-0.05	-0.41
$\Xi_c^{\prime 0} \to \Lambda$	-0.24	-0.29	0.26	0.08	-0.02	-0.10
$arepsilon_{m{c}}^{0}  ightarrow A$	-0.14	0.12	0.06	-0.14	0.03	0.18
$\Xi_c^{\prime 0} \rightarrow \Sigma^0$	-0.14	-0.18	0.14	0.05	-0.002	-0.04
$\Xi_c^0 \to \Sigma^0$	0.24	-0.19	-0.09	0.24	-0.04	-0.29
$\Omega^{\rightarrow}\Xi^{0}$	-0.27	-0.33	0.29	0.09	-0.009	-0.12

Table 3. Semileptonic decay rates of charmed baryons into light ones (in units of  $10^{10}s^{-1}$ )

Process	Decay Width			
$\Lambda_c^+ \to n e^+ \nu_e$	0.81			
$\Sigma_c^{++} \to p e^+ \nu_e$	0.32			
$\Sigma_c^+  ightarrow ne^+  u_e$	0.039			
$\Xi_c^{\prime +} \to \Lambda e^+ \nu_e$	0.19			
$\Xi_c^+ \to \Lambda e^+ \nu_e$	0.14			
$\Xi_c^{\prime +}  ightarrow \Sigma^0 e^+ \nu_e$	0.049			
$\Xi_c^+ \to \Sigma^0 e^+ \nu_e$	0.31			
$\Xi_c^{\prime 0} \to \Sigma^- e^+ \nu_e$	0.10			
$\Xi_c^0 \to \Sigma^- e^+ \nu_e$	0.63			
$\Omega_c^0 \to \Xi^- e^+ \nu_e$	0.20			
$\Lambda_c^+ \to \Lambda e^+ \nu_e$	7.0			
$\Sigma_c^{++} \to \Sigma^+ e^+ \nu_e$	3.3			
$\Sigma_c^+ \to \Sigma^0 e^+ \nu_e$	3.3			
$\Sigma_c^0 \to \Sigma^- e^+ \nu_e$	3.2			
$\Xi_c^{\prime +} \to \Xi^0 e^+ \nu_e$	2.3			
$\Xi_c^+ \to \Xi^0 e^+ \nu_e$	9.7			
$\Xi_c^{\prime 0}  ightarrow \Xi^- e^+  u_e$	1.5			
$\Xi_c^0 \to \Xi^- e^+ \nu_e$	9.7			

Table 4. Predictions of different models for some semileptonic processes (decay width  $\Gamma$  is in units of  $10^{10}s^{-1}$ ; NRQM refers to the nonrelativistic quark model, and MBM to the MIT bag model)

Transition	$f_1$	$f_2$	fз	$g_1$	$g_2$	$g_3$	Г	
$\Lambda_c^+ \to \Lambda e^+ \nu_e$	0.35	-0.22	-0.08	0.35	-0.03	-0.32	7.0	this work
	0.35	-0.09	0.25	0.61	0.04	-0.10	18.0	NRQM [3]
	0.46	-0.19	0.00	0.50	0.05	-0.44	14.8	MBM [3]
	0.36	-0.17	-0.17	0.47	-0.22	-0.22	9.8	[4]
	0.29	-0.14	-0.03	0.38	-0.03	-0.19	7.1	[5]
$\Xi_c^0 \to \Xi^- e^+ \nu_e$	0.41	-0.30	-0.11	0.41	-0.05	-0.47	9.7	this work
	0.48	-0.08	0.26	0.76	0.04	-0.12	28.8	NRQM [3]
	0.59	-0.31	-0.03	0.63	0.05	-0.74	23.9	MBM [3]
	0.40	-0.23	-0.23	0.50	-0.30	-0.30	8.5	[4]
	0.31	-0.19	-0.04	0.39	-0.06	-0.24	7.4	[5]

experimental data could be explained well with the following parameters:  $\beta_N = 0.56 GeV$ ,  $\beta_A = \beta_{\Sigma} = 0.60 GeV$ , and  $\beta_{\Xi} = 0.62 GeV$ ; light quark masses  $m_u = m_d =$  $0.27 GeV, m_s = 0.40 GeV$ , which are in good agreement with light quark masses determined by electroweak properties of light mesons in [23]. Inspired by the result that the consituent quark masses are the same for both mesons and baryons as suggested by the above investigations, we have chosen to fix the charm quark consituent mass at the value  $m_c = 1.45 GeV$ , which is detrmined by the electroweak properties of charmed mesons [23] with the same ansatz for meson wavefunctions. In addition, we assume that scale parameters for all charmed baryons under consideration are also the same. By fitting the central value [1, 5] of experimental semileptonic decay rate for  $\Lambda_c \to \Lambda e^+ \nu$ , we find  $\beta_i = 0.80 GeV$ . Obviously, this is a consistent scale parameter for charmed bayonic wavefunctions. However, it should be pointed out that in the SU(4) symmetry scheme, no reasonable scale parameter can be found. Indeed, the favor suppression factor in the SU(4) symmetry broken scheme, which was first noticed in [4, 29] and particularly emphasized by [5,28], is important for finding the reasonable parameters.

In Table 2, all weak current-induced heavy to light baryon transition form factors are calculated at  $q^2 = 0$ . The corresponding semileptonic decays widths are given in Table 3. In numerical calculations, we use  $V_{cd} = 0.222$ ,  $V_{cs} = 0.9745$ , and the pole masses  $m_A = 2.536$  GeV,  $m_V = 2.11$  GeV for  $c \rightarrow s$  transitions and  $m_A = 2.423$ GeV,  $m_V = 2.01$  GeV for  $c \rightarrow d$  transitions [1]. The charmed baryon masses are respectively  $m_{A_c} = 2.2851$ GeV,  $m_{\Sigma_c^{++}} = 2.4531$  GeV,  $m_{\Sigma_c^+} = 2.4538$  GeV, $m_{\Sigma_c^0} =$ 2.4524 GeV and  $m_{\Xi_c^+} = 2.4651$  GeV,  $m_{\Xi_c^0} = 2.4703$  GeV ,  $m_{\Omega_c} = 2.704 GeV$  [1] and  $m_{\Xi_c'} = 2.563$  GeV [30].

Now we make a comparison with other model calculations [3-5]. As in [4,5], our calculation is also in a SU(4) symmetry broken scheme where wavefunctions defined in (9) and (10) are used while in [3], the SU(4) symmetric heavy baryon spin-flavor wavefunctions are employed. This introduces a flavor suppression factor of  $1/\sqrt{3}$  in our calculations. As a result, our semileptonic decay ratios are much smaller than those in [3] for most of the channels but quite close to those of [4,5] with the nonrelativistic quark model in HQET. In Table 4, we list the results for those processes common to [3-5]. As seen from Table 4, all results are close to each other except those of [3]. Also, different models predict different ratios of form factors,  $f_2/f_1$ ,  $f_3/f_1$ ,  $g_2/g_1$  and  $g_3/g_1$ . Therefore, experimental information on the ratios of form factors will be very useful in testing different models and refining the model calculations. Here we want to emphasize that for the ratios  $f_2/f_1$ ,  $g_2/g_1$ in  $\Lambda_c \to \Lambda e^+ \nu$ , all calculations shown in Table 4 disagree with the result  $f_2/f_1 = g_2/g_1 = -0.28 \pm 0.21$  predicted by the recently experiment data [2]. This implies that there are indeed large  $1/m_c$  corrections to these ratios. In addition, while in [3-5] all form factors are calculated at the zero-recoil point, and form factors at the  $q^2 = 0$  point are obtained by an additional extrapolation assumption, our form factors are calculated directly at the  $q^2 = 0$  point. Also in our calculation, baryon momentum wavefunctions are flavor dependent while in nonrelativistic quark model calculations of [3–5] they are assumed to be flavor independent. Since at present no experimental data for other processes are available, we hope that future experiments will test our calculations directly.

### IV Summary and discussion

Heavy to light baryonic weak form factors are investigated in the lightcone constituent quark model. With Melosh rotation to contruct the spin state of baryons, all weak form factors are directly calculated at the  $q^2 = 0$  point. Assuming a dipole dependence of form factors on  $q^2$ , the corresponding semileptonic decays are also predicted. It should be emphasized that all our calculations are in a SU(4) symmetry broken scheme for spin-flavor wavefunctions of baryons as defined by (9) and (10).

The experimental measurement of form factor ratios of process  $\Lambda_c \to \Lambda e^+ \nu$  lends support to the notion that there are relativistic effects in the spin composition of baryon on the lightcone. In our calculations, an approximation that only the leading relativistic effects in the spin composition

of baryon need be considered has been used and proven to be very useful in simplifying the numerical calculation. We would like to emphasize that in the frame with  $q^+ = 0$ , the contribution from instantaneous interaction vanishes for the 'plus' component. However, for other components of the current, this is not true. Naturally, one would expect that our calculated form factors  $f_3$  and  $g_3$  could be less reliable than  $f_1, f_2, g_1$  and  $g_2$ . It is conceivable that experiments can test the present results soon and provide more information about the higher order contributions thus enabling us to refine the model.

In our lightcone quark model, as in studying properties of light baryons and mesons, the momentum wavefunctions of baryons are assumed to be a simple function of the invariant mass square  $M_0^2$ , i.e., a harmonic-typed wavefunction. By fitting experimental data on semileponic decay  $\Lambda_c \to \Lambda e^+ \nu$ , we determine the scale parameter, which is assumed to be universal for all charmed baryonic wavefunctions. It is expected that as more data are accumulated, the present simple picture will be improved.

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#### Notes added:

After this paper was submitted for publication, we became aware of new data [31]. Reference [31] lists,  $Br(\Lambda_c \rightarrow$  $Al^+\nu_l$  = (2.3 ± 0.2)%, which leads to  $\Gamma(A_c \rightarrow Al^+\nu_l)$  = (11.2±2.5)×10<sup>10</sup>s<sup>-1</sup>. Obviously, it is different from value used in this paper and [5],  $\Gamma(\Lambda_c \rightarrow \Lambda l^+ \nu_l) = (7.0 \pm$  $2.5) \times 10^{10} s^{-1}$ , which comes from  $Br(\Lambda_c \to \Lambda e^+ \nu_e) =$  $(1.4\pm0.5)\%$  by assuming that the process  $\Lambda_c \to \Lambda e^+ \nu_e$  saturates the inclusive mode  $\Lambda_c \to \Lambda e^+ X$  with [1]  $Br(\Lambda_c \to$  $Ae^+X$  = (1.4±0.5)%. In order to accomodate this new result, by changing  $\beta_i$  in our model from  $0.7 GeV \sim 1.0 GeV$ . one finds that in the SU(4) symmetry scheme,  $\beta_i = 0.70 \text{ GeV}$ , A.N.Kamai(Fienum, New York, 1909), 100 22. S.J.Brodsky, T.Huang and G.P.Lepage, Particle and  $\begin{array}{l} \Gamma(\Lambda_c \to \Lambda l^+ \nu_l) = 18.4 \times 10^{10} s^{-1}; \ \beta_i = 0.90 GeV, \ \Gamma(\Lambda_c \to \Lambda l^+ \nu_l) = 22.2 \times 10^{10} s^{-1}; \ \beta_i = 1.0 GeV, \\ \Gamma(\Lambda_c \to \Lambda l^+ \nu_l) = 21.6 \times 10^{10} s^{-1}. \end{array}$ suppressed factor is still needed. However, new data in [31] seems to be in conflict with their listing  $Br(\Lambda_c \rightarrow$  $Ae^{+}X) = (1.6 \pm 0.6)\%$  and  $Br(A_{c} \to A\mu^{+}X) = (1.5 \pm 0.6)\%$ 0.9)%, that is, the branching ratios of the inclusive channels are smaller than that of the corresponding exclusive mode  $\Lambda_c \to \Lambda l^+ \nu_l!$  For this reason in this paper, we have continued to use the old data  $\Gamma(\Lambda_c \rightarrow \Lambda l^+ \nu_l) =$  $(7.0 \pm 2.5) \times 10^{10} s^{-1}$  in our analysis.

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