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Game Theory

Cooperation in an Asymmetric Volunteer's Dilemma Game

Theory and Experimental Evidence[1]

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Abstract: The symmetric Volunteer's dilemma game (VOD) models a situation in which each of N actors faces the decision of either producing a step-level collective good (action "C") or freeriding ("D"). One player's cooperative action suffices for producing the collective good. Unilateral cooperation yields a payoff U for D-players and $U-K$ for the cooperative player(s). However, if all actors decide for "freeriding", each player's payoff is zero ($U>K>0$). In this article, an essential modification is discussed. In an asymmetric VOD, the interest in the collective good and/or the production costs (i.e. U or K) may vary between actors. The generalized asymmetric VOD is similar to market entry games. Alternative hypotheses about the behaviour of subjects are derived from a game-theoretical analysis. They are investigated in an experimental setting. The application of the mixed Nash-equilibrium concept yields a rather counter-intuitive prediction which apparently contradicts the empirical data. The predictions of the Harsanyi-Selten-theory and Schelling's "focal point theory" are in better accordance with the data. However, they do not account for the "diffusion-of-responsibility-effect" also observable in the context of an asymmetric VOD game.

I Introduction

In a N-person non-cooperative matrix game called "volunteer's dilemma" (Diekmann 1985), each actor has the choice between a favorite alternative D with payoff U and a less favorite alternative C with payoff $U-K$ ($U>K>0$). However, while C-players, irrespective of other actors' choices, obtain the maximin payoff $U-K$, D-players receive payoff U only if there is at least one other player choosing C. Otherwise D-players' payoff is zero.

The game described a collective good problem with a step production function. Cooperative actors pay K units of utility for the production of the collective good, while cooperators as well as defectors gain U. However, if all actors decide on freeriding, the worst payoff is obtained.

There are several applications of a volunteer's dilemma in economics, sociology, political science, and biology. Examples are the market entry decision of two

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firms (Sherman and Willett 1967), voting behavior (Brennan and Lomasky 1984), the sanctioning dilemma of N actors facing a norm violation, the investment decision of two or more privileged actors in a "privileged group" (Olson 1965), bystander intervention in emergencies as analyzed by Darley and Latané (1968) in social psychology, and "vigilance games" in biology.

Obviously, there are N asymmetric equilibria in pure strategies, which are usually not attainable without coordination. In addition, a symmetric mixed Nash-equilibrium exists with the probability of defection $q^* = \sqrt[N-1]{K/U}$ (Diekmann 1985). Since only one mixed Nash-equilibrium with symmetric payoff-vector exists, theories of equilibrium selection, which require that solutions are invariable concerning the renumbering of players, will clearly select the mixed strategy equilibrium (e.g. Harsanyi and Selten 1988). This solution implies that the probability of freeriding will increase with group-size N, which corresponds very well to the "diffusion of responsibility"-effect observed by Darley and Latané in situations of helping behaviour. Experimenal evidence confirms the negative correlation of group-size and freeriding in experiments on helping behaviour (Darley and Latané 1968) as well as experimental gaming (Diekmann 1986) although the mixed equilibrium solution systematically underestimates the percentage of cooperation.

Volunteer's dilemma assumes a symmetric decision situation for all players. An interesting question arises if this assumption is discarded. What are the consequences for the game theoretic solution if there is, for instance, a "strong" player with the ability to produce the collective good at a lower cost than his or her coplayers? In this article, an asymmetric volunteer's dilemma game will be explored allowing for an unequal distribution of costs K_i and interests U_i among $i = 1, 2, \ldots, N$ actors. It will be shown that the mixed Nash-equilibrium yields highly counterintuitive results unlikely to be observed in actual behaviour.

II Game Theoretical Analysis

Consider the binary decision N-Person matrix game with actor i's ($i = 1, 2, \ldots, N$) decision alternatives C_i and D_i respectively. With $U_i > K_i > 0$ the payoff structure is as follows:

(i) Strategy C_i yields $U_i - K_i$,

$$\text{(ii) while, for } D_i \begin{cases} U_i & \text{is obtained if there is at least one other actor choosing } C, \\ 0 & \text{otherwise.} \end{cases}$$

Obviously, the game has N efficient and strict equilibria with exactly one "volunteer" and $N-1$ "freeriders". Moreover, an additional equilibrium point in mixed strategies may exist.

If D_i is chosen with probability q_i, actor i's expected utility E_i is

$$E_i = q_i U_i \Big(1 - \prod_{j \neq i} q_j\Big) + (1-q_i)(U_i - K_i). \tag{1}$$

Partially differentiating with respect to q_i yields

$$\frac{\partial E_i}{\partial q_i} = -U_i \prod_{j \neq i} q_j + K_i. \tag{2}$$

The following system of N equations results if the derivatives are set equal to zero:

$$\prod_{j \neq i}^{N} q_j = \frac{K_i}{U_i} \quad i = 1, 2, \ldots, N \tag{3}$$

The solution of (3) is

$$q_i^* = \frac{U_i}{K_i} \left(\prod_{j=1}^{N} \frac{K_j}{U_j} \right)^{\left(\frac{1}{N-1}\right)} \tag{4}$$

This is a (weak) mixed Nash-equilibrium if $0 < q_i^* < 1$ for $i = 1, 2, \ldots, N$. For the special case $N = 2$ it follows from (4) that a mixed equilibrium does always exist. Also, for $U_i = U$ and $K_i = K$ there is always a solution under the restriction $0 < q_i^* < 1$, which is the mixed equilibrium of the symmetric game: $q^* = \sqrt[N-1]{K/U}$ (Diekmann 1985).

Substitution of q_i in (1) by q_i^* yields the payoff-vector of the mixed equilibrium strategy. Payoffs are $U_i - K_i$, which is identical to the payoff of the pure maximin-strategy[2].

This can be seen more easily by substitution of the product term in equation (1) by formula (3). It also becomes apparent that the expected value in the equilibirum E_i^* does not depend on q_i, i.e. the mixed Nash-equilibrium is weak.

$1 - q_i^*$ is the probability that actor i will decide on cooperation under the mixed equilibrium strategy. From (3) and (4), we obtain the probability that the collective good will be produced:

$$P = 1 - \prod_{i=1}^{N} q_i^* = 1 - \left(\prod_{i=1}^{N} \frac{K_i}{U_i} \right)^{\left(\frac{1}{N-1}\right)} \tag{5}$$

$P(N)$ is not necessarily a decreasing function of N. In a symmetric game, however, the likelihood of collective good production decreases with group-size.

Now, consider solution (4). The Nash-equilibrium strategy implies that actor i's defection probability will increase with decreasing production costs K_i or increasing interest in the collective good U_i. This is a very paradoxical result which hardly will be in line with observed behaviour of individual decision makers[3]. An explanation in

[2] Hence, the mixed equilibrium strategy is "unprofitable". For 2×2-games this is always true (Holler 1990). However, it does not necessarily hold true in N-person games.

[3] See, also, Wittman (1985) for a critique of the counter-intuitive implications of mixed equilibrium strategies in 2×2 games.

formal terms is that the mixed equilibrium strategy yields the maximin payoff, which is higher for "strong" players with either greater interest U_i or lower costs K_i. In order to achieve at least the maximin payoff, a "stronger" actor's defection probability has to be greater than the defection probability of co-players with a lower maximin payoff.

Referring to empirical behavior, however, the paradox does not vanish. Imagine, for example, a price cartel with one firm violating the price level agreed upon by the member firms of the cartel. If identification and individual sanctioning of that firm is possible, a sanctioning dilemma arises. Assume that all firms have the same interest in the collective good U_i (conservation of the cartel price), that sanctioning is costly (K_i), and that one firm k has the lowest sanctioning costs K_k (Figure 1). Then the mixed equilibrium solution implies that firm k has the lowest probability of sanctioning the norm-violator despite its highest sanctioning power in the group. As another example, consider three bystanders observing a victim in danger of being drowned in a lake. If only one of the observers is able to swim ($K_k < K_i$), it is not the swimmer but the non-swimmers who are expected to jump in the water and save the victim[4].

The opposite predicted by Schelling's (1960) concept of a "prominent solution" is probably much more realistic. As mentioned above, there are N pareto-efficient equilibria. The asymmetric game, however, contains a clue pointing to the single

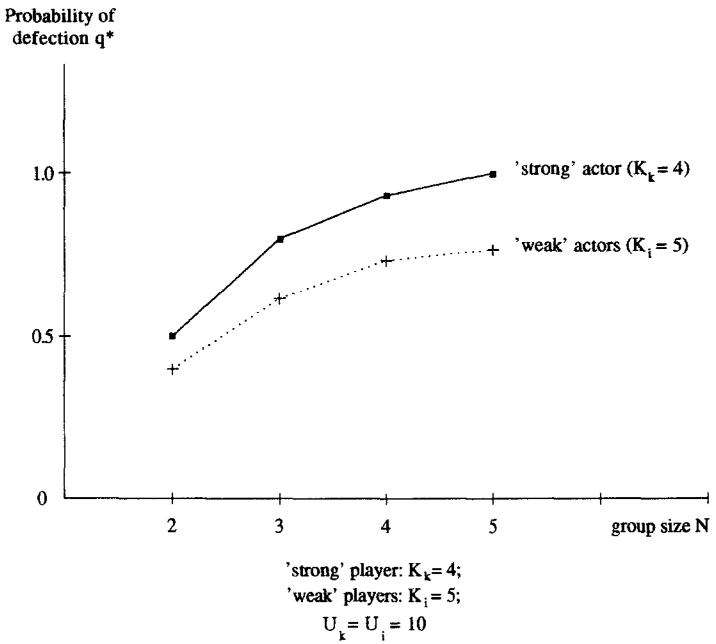

Fig. 1. Mixed equilibrium strategy for asymmetric volunteer's dilemma with one "strong" player and $N-1$ "weak" players

[4] See Weesie (1993) for an analysis of this example and the problem of "rational timing" in a "volunteer's timing dilemma".

"eligible" equilibrium. It is the strongest player who is expected to take action, thereby yielding a pareto-efficient equilibrium.

In the special case of an asymmetric volunteer's dilemma with one "strong" and $N-1$ "weak" players investigated in the experiment described below, the conclusion following from "prominence theory" can be drawn from the Harsanyi-Selten theory of equilibrium selection. Consider first the subclass of games with constant interest ($U_i = U =$ constant; $i = 1, 2, \ldots, N$) and strictly increasing costs, i.e. $K_1 < K_2 < \ldots < K_N$; $O < K_i < U_i$; $i = 1, 2, \ldots, N$. This subset of asymmetric volunteer's dilemma games belongs to the class of "regular market entry games" discussed in Selten and Güth (1982)[5]. By application of the Harsanyi-Selten theory, Selten and Güth derive the theorem that "the solution of the game is that equilibrium point where the firms with the lowest entry costs enter the market", whereby "market entry" corresponds to the "cooperative" choice (C) in the asymmetric volunteer's dilemma game. It follows from the theorem that the strict equilibrium point is selected as the solution of the game where the player with the lowest costs (the "strong" player) acts cooperatively, while the "weaker" co-players defect. However, due to the assumption of the strict order of costs, the theorem does not cover the situation of one strong and $N-1$ co-players with equal degree of weakness and $N>2$. For the strong player/weak co-players asymmetric volunteer's dilemma (i.e. $K_1 < K_2 = K_3 = \ldots = K_N$; $N>2$), the theorem has to be generalized. It can be shown that the Harsanyi-Selten theory selects the equilibrium point where player 1 employs the cooperative strategy ("market entry") and players, 2, 3, ..., N "defect"[6]. This is

[5] Another requirement is: $K_i + K_j \neq K_k + K_l$ for $i, j, k, l = 1, \ldots, N$ pairwise different. An asymmetric volunteer's dilemma subject to these restrictions satisfies assumptions (1) to (6) in Selten and Güth (1982). It is, moreover, a "regular market entry game" because dominated strategies are excluded by the defining properties of a volunteer's dilemma game. Note that "market entry" corresponds here to "cooperation". In other market entry games, which under certain conditions are structurally equivalent with volunteer's dilemma, "market entry" is the "defective" choice. For example, this is the case in the game of Sherman and Willet (1967), which is a symmetric volunteer's dilemma for $N=2$.

[6] I owe a sketch of the proof for this assertion to Reinhard Selten (personal communication):
"The primitive formations are those generated by the strict equilibrium points. These equilibrium points form the first candidate set. The restricted games for the risk dominance comparisons between any two elements of this set are 2×2-games. If player 1 is stronger than the players 2, ..., N, then the equilibrium point where player 1 cooperates risk dominates all other candidates and therefore emerges as the solution (there are no payoff dominance relationships among the candidates). Assume that player 1 is weaker than players 2, ..., N. Then the equilibrium point where player 1 cooperates is risk dominated by all other candidates. For reasons of symmetry these other candidates do no risk dominate each other. Therefore the equilibrium point where player 1 cooperates is eliminated. The second candidate set is formed by the other strict equilibrium points. A substitution step becomes necessary (see Harsanyi and Selten 1988, p. 228). The application of the tracing procedure to the centroid of the second candidate set must yield a symmetric equilibrium point. This can be either the strict equilibrium point where player 1 cooperates or the mixed equilibrium point. Obviously, in the first case the assertion holds. In the second case, the third candidate set consists of the mixed equilibrium point and the strict equilibrium point where player 1 cooperates. It can be seen without difficulty that this strict equilibrium point risk dominates the mixed equilibrium point. Obviously the mixed equilibrium point does not payoff dominate this strict equilibrium point. It follows that, in this case too, the strict equilibrium point where player 1 cooperates is the solution of the game."

the single strict and efficient equilibrium point in pure strategies with symmetric payoffs for players in a symmetric position.

In the strong-player-weak-co-players asymmetric situation, the mixed Nash-equilibrium predicts a higher probability of defection for the strong player who receives a payoff bonus compared to his weak co-players. The opposite conclusion follows from Schelling's "prominent" solution and the Harsanyi-Selten theory. While the former theory values "strength", the latter theories imply the "strength of weakness". By these theories, defective choices of weak players result in higher payoffs than the strong player's gain if players meet the mutual expectation of tacit coordination. Strong players are, so to speak, exploited by their weak co-players. However, if strong actors do not comply with their role as "rational hero", all free-riding actors might loose. The interesting question arises: How do real subjects behave in an asymmetric volunteer's dilemma? In the following, the opposing predictions are confronted in an "experimentum crucis".

III Experimental Test

An experimental test was arranged in order to find out which of the opposing predictions from, on the one hand, mixed Nash-equilibrium and, on the other hand, Schelling's theory of tacit coordination as well as the Harsanyi-Selten theory would better match the actual behaviour of decision makers. The experimental factor of main interest is the "weak player/strong co-players" versus "strong player/weak co-players" condition. In addition, the degree of weakness and strength and the group-size ($N=2$ versus $N=5$) were varied (see Table 1).

328 students of various disciplines at the University of Mannheim, West Germany, participated in the experiment. Subjects were randomly distributed over ten experimental groups. They were asked to fill out a questionnaire containing the volunteer's dilemma and three versions of the prisoner's dilemma[7]. Volunteer's dilemma was presented first in this sequence of games. Thus, sequence effects probably did not influence the subjects' decision behaviour in volunteer's dilemma. The game was presented in matrix form, with subjects' payoff and co-players' payoffs displayed in separate matrices (Table 1). In addition, the game was described verbally and subjects were asked to give their own choice, the expected choice of co-players, and resulting payoffs. Subjects had plenty of time to think over the decision problems carefully and were motivated by relatively high monetary gains. It was announced

[7] Participants were recruited in the university cafeteria during lunch time. Students who agreed to participate were asked to come to a separate room in the cafeteria building where the experiment was arranged. In order to avoid sequence effects, we did not choose a design where the different versions of the dilemma game were presented to the same subjects. Rather, we decided for the more "expensive" design of a random assignment to the various experimental conditions. Hence, each participant was confronted with one decision problem of volunteer's dilemma type. Subjects did not know their co-players and were, in fact, not matched to co-players.

that achieved points would be converted to money according to an exchange rate of 0.10 DM per point. The monetary payoff for a defective choice matched by co-player(s)' cooperative choice, consequently, amounts to DM 10.—, approximately US$ 6 (in fact, participants received an amount of 5.— to 15.— DM, depending on the experimental conditions to which they were randomly assigned).

Results are based on those subjects who correctly answered the question of expected payoffs, given the supposed choice of co-player(s). Hence, only the subset of 301 subjects who passed this "test of understanding" of the game structure was included in the analysis[8].

For the 2×2-games, all three experimental tests cleary contradict the mixed Nash-equilibrium hypothesis. While this hypothesis predicts that increasing strength of the co-player will decrease the probability of player's defection, experimental data show a significant trend in the opposite direction (Figure 2a). Moreover, player's strength is inversely related to defection proportions (Figure 2b). Again, the differences are highly significant and contradict the mixed Nash-equilibrium implications. Finally, there is an extreme and highly significant difference in defection rates in the third test-situation (Figure 2c), which directly contradicts the mixed Nash-equilibrium hypothesis.

Of course, we do not know whether actors really have employed mixed strategies. Besides various possible interpretations of the meaning of mixed strategies on the individual level (e.g. Harsanyi and Selten, 1980, 14 pp.), the experimental test allows for potential falsification on the group level. Whether or not subjects employ mixed strategies on the individual level, aggregate results on the group level clearly falsify the mixed Nash-equilibrium hypothesis. The implication of this hypothesis is that proportions of defection should vary in accordance with formula (4), which apparently is not true.

On the other hand, the experimental data are much more in accordance with Schelling's "prominence theory" and the Harsanyi-Selten theory. Although the behaviour of subjects does not coincide with the prediction of the strict deterministic version of the Harsanyi-Selten theory of equilibrium selection, the data at least approximate the theoretical expectations. The more extreme the payoff-differences between weak and strong actors, the better the theory is approximated.

In the symmetric conditions A and H (Table 1), the prediction of the Harsanyi-Selten theory is identical to the mixed Nash-equilibrium. In both conditions, however, the empirical defection proportions are overestimated by the theory. These findings are perfectly in accordance with three earlier experiments of the symmetric volunteer's dilemma (e.g. Diekmann 1986).

For group-size 5, subjects' behaviour shows a much less clear pattern. In the case of co-player's increasing strength, the sign of the significant difference is in the

[8] 27 subjects or 8.2% did not pass the test for the volunteer's dilemma. Not surprisingly, there are more errors in the asymmetric game compared to the symmetric versions. In the latter conditions (A and H in Table 1), no inconsistencies were detected at all. With this exception, no systematic variations of "failure rates" over experimental conditions were found. The distribution of excluded cases is as follows: A 0, B 0, C 4, D 3, E 3, F 5, G 5, H 0, I 5, J 2.

Table 1. Experimental design

Experimental condition	Group size	Type of game:	Cost benefit structure*	Subject's Matrix**	Matrix of co-player(s)	Predicted probability of defection by mixed Nash-equilibrium	Proportion of "defection" in experiment
A	2	symmetric:	K = 50, U = 100	50 50 100 0	50 50 100 0	0.50	0.39
B	2	weak player: strong co-player:	K = 50, U = 100 K = 40, U = 100	50 50 100 0	60 60 100 0	0.40	0.55
C	2	weak player: extremely strong co-player:	K = 50, U = 100 K = 10, U = 100	50 50 100 0	90 90 100 0	0.10	0.81
D	2	strong player: weak co-player:	K = 40, U = 100 K = 50, U = 100	60 60 100 0	50 50 100 0	0.50	0.33
E	2	extremely strong player: weak co-player:	K = 10, U = 100 K = 50, U = 100	90 90 100 0	50 50 100 0	0.50	0.05
F	2	very weak player: very strong co-player:	K = 80, U = 100 K = 20, U = 100	20 20 100 0	80 80 100 0	0.20	0.93
G	2	very strong player: very weak co-player:	K = 20, U = 100 K = 80, U = 100	80 80 100 0	20 20 100 0	0.80	0.16
H	5	symmetric:	K = 50, U = 100	50 50 50 50 50 0 100 100 100 100	50 50 50 50 50 0 100 100 100 100	0.84	0.72
I	5	weak player, three co-players: one strong co-player:	K = 50, U = 100 K = 40, U = 100	50 50 50 50 50 0 100 100 100 100	60 60 60 60 60 0 100 100 100 100	0.80	0.44
J	5	strong player: four weak co-players:	K = 40, U = 100 K = 50, U = 100	60 60 60 60 60 0 100 100 100 100	50 50 50 50 50 0 100 100 100 100	0.99	0.70

* First line refers to subject's costs and benefits.
** Subject's matrix of co-player(s) as displayed to subject in the questionnaire. Upper row = cooperative choice, lower row = defective choice, left column = other player's cooperative choice, right column = other player's defective choice. For group size 5, columns from left to right = number of other players choosing the cooperative alternative. In condition I, subjects were instructed that player's decision matrix was also presented to three of the four co-players.

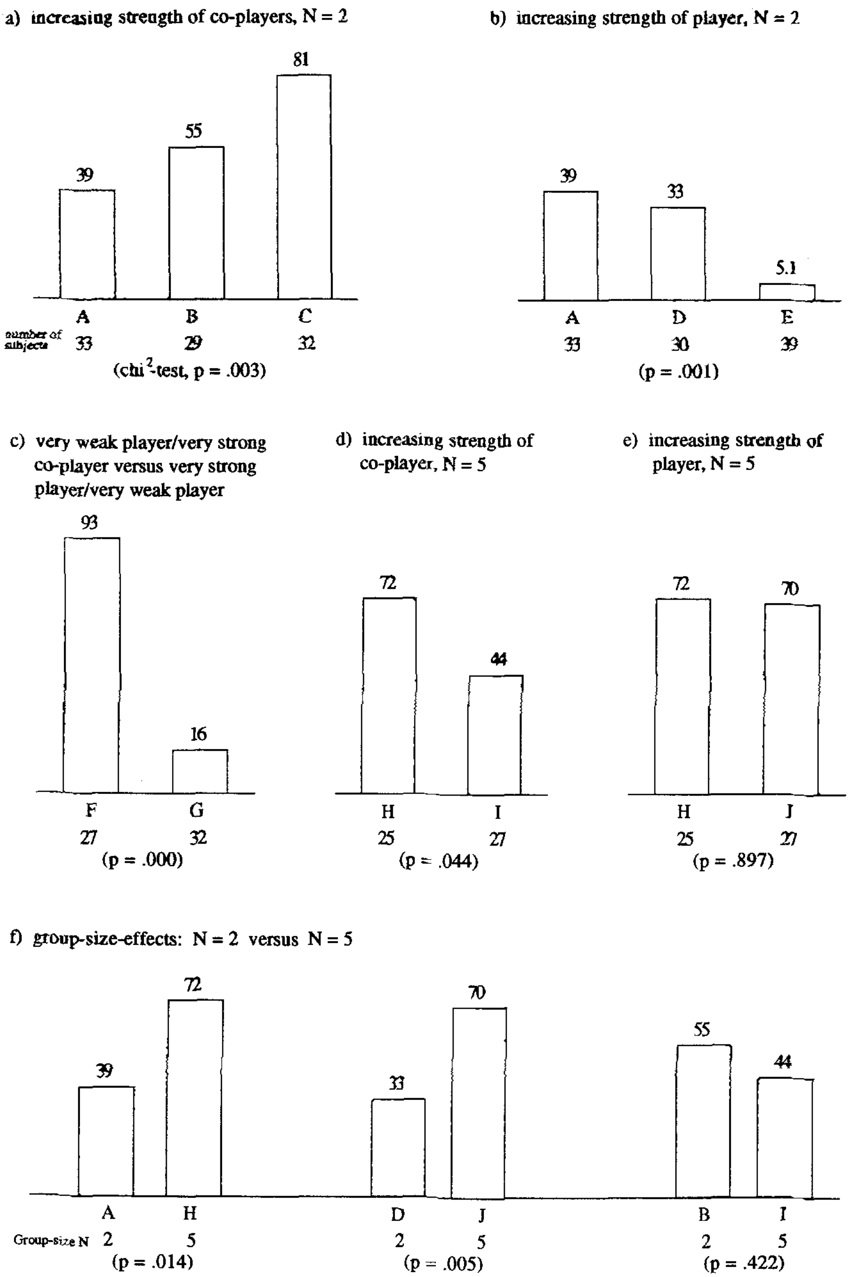

Fig. 2. % defection by experimental condition

direction of the mixed equilibrium hypothesis, although the degree of defective behaviour in condition I is much less than predicted by formula (4) (Figure 2d). On the other hand, no significant effect of player's increasing strength could be detected (Figure 2e). It may be the case that the extraordinarily low level of defection in group I is an "outlier-effect"[9]. In three earlier experiments with (symmetric) volunteer's dilemma, such a low level of defection was never observed. Whether this might be the case can only be answered by a replication of the experiment.

Group-size-effects as expected by the mixed Nash-equilibrium hypothesis could be observed in two out of three test situations (Figure 2g). This "diffusion of responsibility" mechanism is well supported by a great variety of experiments (e.g. Darley and Latané 1968, Diekmann 1986). Again, the difference is reversed for conditions B and I because of the extraordinary low proportion of defectors in the latter group.

IV Conclusions

Volunteer's dilemma is a game with some interesting properties paradigmatic for a variety of social situations. In the symmetric game, theories of equilibrium selection yield the solution of the mixed Nash-equilibrium which is, however, weak and non-efficient. The solution implies a decline of cooperation with increasing group-size. This mechanism is well known in social psychology as the effect of "diffusion of responsibility", which is confirmed by a large bulk of experimental data.

In the generalized, asymmetric version of the game the strict equilibria are also candidates for equilibrium selection. The mixed equilibrium solution, on the other hand, yields the counter-intuitive result that the "strongest" actor capable of producing the collective good on lowest costs has the highest probability of freeriding. In the special case of one "strong" and $N-1$ "weak" actors investigated in this article with experimental data, the Harsanyi-Selten theory as well as Schelling's "prominence theory" predict the opposite result (i.e., the strongest player will choose the cooperative strategy). The experimental results are ambiguous for group-size $N=5$. In the experimental conditions with 2×2-games the data clearly support the latter theories if these are interpreted in a probabilistic sense. In other words, the findings show that the higher the payoff-difference between strong and weak players, the more likely the strong player and the less likely actors in the role of weak players will opt for cooperation. Note however, that the Harsanyi-Selten theory underestimates the level of cooperation in the symmetric game and does not imply the "diffusion-of-responsibility" effect in the asymmetric game. The reason is that the selection of a strict equilibrium point in the asymmetric game is independent of group-size. In con-

[9] However, no "outlier effect" was observed in condition I for the decision behaviour in three prisoner's dilemma situations. Also, it is unlikely that presentation effects may have caused the ambiguous results for group-size $N=5$. Under both conditions ($N=2$ and $N=5$) the game was described verbally and presented in matrix form as depicted in Table 1.

trast to rationality theory, this psychological effect seems to be present in asymmetric dilemmas as well.

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