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Comparison of Size Effect for Different Types of Strength Tests

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With 4 Figures

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Summary

Comparison of Size Effect for Different Types of Strength Tests. Different theories have been proposed to explain and predict size effect. Notable is Weibull's "Weakest link theory". In addition various theories have been founded on strain energy consumption at failure. The present paper suggests a theoretical approach, based on energy considerations and mode of failure, which holds for diverse types of mechanical testing. Maximum size effect is assumed to be associated with failure through development of a single fracture plane and a lack of size effect is associated with failure affecting a volume of material. The quantitative expression of the theoretical approach is based on the relationship between load at failure (P) and cross sectional area of the specimen (A): $P=KA^n$ where n expresses size effect and K is a constant.

The theoretically lowest n-value equals 0.75 expressing maximal size effect whereas the theoretically highest n-value is 1.00 expressing lack of size effect. n-values evaluated from published data for various mechanical tests indeed lie between these limits and appear to be related to type of test, material properties and specimen shape. Point load tests and Brazilian tests are generally associated with large size effects. This may be attributed to the test conditions which promote development of single fracture planes. On the other hand relatively small size effects are generally found for uniaxial compression tests. Failure in these tests characteristically occurs through multiple fracturing and crushing and therefore affects a volume of material rather than a single plane. Brittleness seems to be associated with large size effects and ductility with small size effects.

Introduction

The observation of size effect is at variance with the assumption that strength is a fundamental material property and that failure occurs whenever the strength, at a point within the rock body, is exceeded. Assuming elastic behaviour, stresses generated at any point within a rock body, may

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be expressed as a function of the general form:

$$\sigma = K \cdot \frac{P}{D^2}. \tag{1}$$

Where: σ — Stress induced at a point within the rock body;

P — External load applied to the rock body.

K — Factor depending on the shape of the rock body and type of test.

D — Characteristic dimension of the rock body.

If strength is a constant property, unaffected by size, load at failure must be proportional to the squared dimensions of the rock body regardless of type of test or specimen shape.

Though the existence of the above proportional relationship is occasionally reported, indicating a lack of size effect, deviations are commonly observed. Ordinairily strength is found to become smaller with increasing size for all common mechanical strength tests.

Weibull's "Weakest link theory", attributes failure to statistically distributed structural flaws, or inhomogeneities. The number of flaws and hence also the probability of failure increases with the volume of the rock-body. Weibull's theory led to an expression of the form of:

$$\frac{\log \sigma_1}{\log \sigma_2} = m \cdot \frac{\log V_2}{\log V_1}. \tag{2}$$

Where: σ_1, σ_2 — Strength of specimen 1 and 2.

V_1, V_2 — Volume of specimens.

m — Constant depending of material properties.

The expression implies that failure load is dependent on the specimen's volume and is a function of material properties. An argument against the "Weakest link theory", as a physical explanation for size effect, is that it is based on the assumption that local rupture will extend into total failure. This is at variance with evidence indicating that multiple failure preceeds total collapse (Hudson et al. (1971), Reichmuth (1968), Brady et al. (1973)). However Weibull's formula has been widely used and generally a good fit is found with experimental data.

Lundborg (1967) performed series of uniaxial compression tests and Brazilian tensile tests on a granite rock using specimens of equal shape but different size. A linear relationship with a high degree of correlation was established between $\log \sigma$ and $\log V$ yielding $m = 12$ for uniaxial compression tests and $m = 6$ for Brazilian tensile tests. The results clearly demonstrate that apart from material properties and volume also type of test has a strong influence on size effect.

Sunday (1974) performed point load test on disc shaped specimens, of varying thickness, prepared from three types of granite. For each of the rocktypes a linear relation of $\log P$ to $\log V$ was established (supporting Weibull's theory). The degree of correlation was however low. This may be attributed to variations in specimen shape (thickness to diameter Ratio).

Other investigators (Reichmuth (1968), Brook (1977) link load at failure with the cross sectional area of the rock specimen. The relation takes the form of:

$$P = C \cdot A^n. \tag{3}$$

Where: A — Characteristic cross section of the rock specimen between loading points.

n — Constant.

C — Constant.

Brook (1974) plotted point load test data published by Broch and Franklin (1972) and by Sundae (1974) for specimens of varied shape (discs, blocks, cores, cubes). For point load tests, load at failure was found to depend primarily on the cross sectional area of the specimen and to be influenced by the specimen's shape only to a minor degree. Brook developed a theoretical approach to size effect, based on energy considerations, and concluded that n-values should approximate 0.75. Test results analysed by Brook seem to agree with this theoretical constant. Deviations were attributed by Brook to different combinations of size and shape effects in different rocks.

Reichmuth (1968) also considered failure load to depend on minimum cross sectional area in point load tests and developed correction formulas to compensate for size effect. These formulas are based on a "shape factor" and a "relative brittleness index" which expresses ease of cracking and is related to brittleness and the contact geometry of loading points and rock specimens.

Wagner and Schümann (1971) studied size effect in stamp tests in which a flat ended cylindrical "stamp" is pressed against a flat surface till indentation of the rock surface is achieved. The study was performed with stamps of different diameter and different rock types. The formula used to express size effect is of the form of:

$$\sigma = Q \cdot a^{-\alpha}. \tag{4}$$

Where: Q — Constant.

a — Radius of stamp.

Based on theoretical considerations the authors claim that α would equal -0.5 for a perfectly brittle material and that it would equal 0.0 for ductile materials. Results appear to confirm the theoretical values.

An anomalous relationship between strength and size is reported by some authors for specimens of relatively small size. Dreyer (1972) found for small cubical specimens of rock salt that upto a "critical size" strength increased with size. For larger specimens strength was found to become constant. Dreyer attributed this phenomenon to the relatively small number of crystals constructing small specimens, providing little mutual restraint to deformation. Broch and Franklin (1972) found size effect in specimens of less than 25 mm to be "large and variable". A similar observation was made by Pratt (1972). Bieniawski (1968) performed uniaxial compression tests on cubical coal specimens ranging in size from 1.9 cm to 152 cm. Strength appeared to be constant for specimens smaller than 6.4 cm whereas for larger sizes a steady decrease in strength was apparent with size. The phenomenon was attributed by Bieniawski to the fact that under a certain size, specimens are no longer affected by discontinuities since the spacing between discontinuities exceeds specimen size.

Theoretical Background

An explanation of size effect may be based on energy considerations as applying to two extreme modes of failure.

a) Failure along a well defined plane within the rockbody expending all stored elastic strain energy through rupture along this plane (perfectly brittle behaviour).

b) Failure involving a volume of material (perfectly elastoplastic behaviour).

Considering failure mode a, the energy stored within a strained elastic body (U) may be expressed as:

$$U = \sigma \cdot \varepsilon \cdot V. \tag{5}$$

Where: ε — Strain.

V — Volume of body.

Expressing strain by stress and modulus of elasticity (E) we obtain:

$$U = K \cdot \frac{\sigma^2}{E} \cdot a^3. \tag{6}$$

Where: a — Linear dimension.

K — Constant expressing shape.

In case of failure along a single plane, it may be assumed that the released energy per unit area of failure plane (U_p) is constant and independent of size in equally shaped rock bodies:

$$U_p = \frac{U}{A} = K \cdot \frac{\sigma^2}{E} \cdot a. \tag{7}$$

Expressing stress by force (P) and area (A) we obtain.

$$P = C \cdot A^{0,75}. \tag{8}$$

Considering failure mode b, involving a volume of material in failure, the energy consumption per unit volume may be assumed to be constant and independent on the size of the rockbody. Therefore:

$$U_p = \frac{U}{V} = K \cdot \frac{\sigma^2}{E}. \tag{9}$$

Expressing stress by force and area we obtain:

$$P = C \cdot A^{1.00}. \tag{10}$$

The actual mode of failure of rock may be assumed to lie between the two extreme modes of failure. Size effect may therefore be expressed by an equation similar to that used by Reichmuth and Brook (Eq. 3).

$$P = C \cdot A^n.$$

Where n ranges between 0.75 and 1.0.

Analysis of Available Test Results

Results of studies of size effect, by various authors are summarized in Tables 1—5. These studies include uniaxial compression tests, triaxial compression tests, Brazilian tests. point load tests, direct shear tests and stamp bearing tests. To create a common basis for comparison of the published results, size effect is expressed by n-values (Eq. 3) in Tables 1—5.

Table 1. *Uniaxial Compression Tests*

Source	Rock type	n value	Correlation coef.	Dimensions in cm	Specimens tested	Remarks
Hudson, Brown and Fairhurst (1971)	Marble	0.837		5.0, 10.0	6	L/D = 3:1
	Marble	0.987		5.0, 10.0	6	2:1
	Marble	0.908		5.0, 10.0	6	1:1
	Marble	0.930		5.0, 10.0	6	1/2:1
	Marble	1.083		5.0, 10.0	6	1/3:1
Lundborg (1967)	Granite	0.875	0.998	1.9— 5.8	20	1:1
Skelly et al. (1977)	Coal	0.895		2.5— 30.5	155	1:1
Abu-Sayed et al. (1976)	Quartz	0.915		2.5— 14.7	42	$2^1/_2$:1
Mogi (1972)	Concrete	0.947		5.0— 80.0	8?	
Dreyer (1972)	Rock Salt	1.000		10.0— 23.0	17	cubes
Bieniawski (1968)	Coal	0.770	0.999	15.0—152.0	21	cubes
Townsend et al. (1977)	Coal	0.775		2.5— 17.8	20	cubes
Mogi (1962)	Marble	0.954		2.0— 10.0	5	cubes

Table 2. *Point Load Tests*

Source	Rock type	n value	Correlation coef.	Dimensions in cm	Specimens tested	Remarks
Broch and Franklin (1972)	Sandstone	0.795	0.999	1.25—7.6	80	16 cores for each diameter
	Quartz Dolerite	0.713	0.996	1.25—7.6	80	
	Granite	0.760	0.998	1.25—7.6	80	
	? (sample 216)	0.780	0.999	1.25—7.6	80	
Size correction chart ISRM (1971)		0.755—0.775				
Reichmuth (1968)	Limestone	0.780	0.999	1.3 —5.5	144	L/D=2
Brook (1974)	Granite	0.854		1.1 —4.3	31	7 cubes and other shapes

Table 3. *Brazilian Tensile Tests*

Source	Rock type	n value	Correlation coef.	Dimensions in cm	Specimens tested	Remarks
Lundborg (1967)	Granite	0.750	0.991	1.9—5.65	21	L/D=1
Sundae (1974)	Sandstone	0.773		2.5, 4.9	15	L/D=1/2
Brook (1974)	Sandstone	0.920		1.9—7.4	5	L/D=1/3
Habib et al. (1966)	Limestone	0.868		1.0—7.0	16	

Table 4. *Stamp Bearing Tests*

Source	Rock type	n value	Correlation coef.	Dimensions in cm	Specimens tested	Remarks
Wagner and Schümann (1971)	Quartzite	0.733	0.998	0.38—3.05	12	dimension-stamp diameter
	Shale	0.820	0.998	0.38—3.05	12	
	Norite	0.845	0.999	0.38—3.05	12	
	Marble	0.850	0.997	0.76—3.05	9	
	Sandstone	0.915	0.999	0.76—3.05	9	

Table 5. *Direct Shear Tests*

Source	Rock type	n value	Correlation coef.	Dimensions in cm^2	Specimens tested	Remarks
Il'nitskaya (1969)	Marble	0.840	0.998	7.6—89.0	29	
	Gabbro	0.921	0.997	8.6—90.0	32	
Pratt et al. (1974)	Quartz Diorite	0.809	0.999	252—5130	?	

Some authors (Bieniawksi (1968), Dreyer (1972), Townsend et al. (1977)) used Weibull's equation (Eq. 2) to express size effect by Weibull's m-value. Weibull's formula is mathematically equivalent to Eq. 3 for equally shaped specimens and the following conversion between n and m values could be made:

$$n = \frac{3}{2} \cdot m + 1. \tag{11}$$

Other authors (Wagner and Schümann (1971), Mogi (1962), Skelly et al. (1977)) used Eq. (4) to express size effect by the corresponding α values. Also this equation is mathematically equivalent to Eq. (3) for equally shaped specimens. The conversion between n and α takes the form:

$$n = \frac{2 - \alpha}{2}. \tag{12}$$

For those cases where the authors presented the original data on which their calculations were based, n-values could be directly calculated.

As mentioned before, strength results for relatively small specimens are often anomalous to the general trend of size effect commonly giving a relatively high strength. For this reason results for small specimens were ommited in the calculation of size effect in a number of cases. Calculated n-values for test results presented by Bieniawski (1968) for coal cubes were based on specimens of more than 6″ though results for smaller cubes were also given by Bieniawski. Likewise results for specimens with a diameter of 3/4″ were omitted in the calculation of n-values based on data presented by Hudson et al. (1971) for uniaxial compression tests on cylindrical specimens. A similar procedure was adopted with regard to some results of stamp test presented by Wagner and Schümann; n-values were calculated for specimens larger than 3.8 mm.

Discussion

According to the theoretical models, n-values expressing size effect, must range between 0.75 for maximal size effect (failure along a single plane) and 1.0 for no size effect (failure involving a volume of material). The range of n-values calculated from published results for diverse types of mechanical testing (Tables 1—5) indeed lies between these values with the exception of only two cases: ($n = 0.713$ in Table 2, $n = 0.733$ in Table 4).

n-Values appear to be related to the following factors:

a) Type of test.

b) Material properties.

c) Specimen shape.

Available data are however insufficient, at this stage, for an evaluation of the quantitative contribution of each of these factors and their interdependence.

a) A clear dependence on type of test was shown by Lundborg (1967) on basis of a comparison between size effect in Brazilian tensile tests and uniaxial compression tests performed on equally specimens from the same rock. Brazilian tensile test showed the maximal size effect ($n = 0.75$) whereas uniaxial compression tests showed a much smaller size effect ($n = 0.88$). Differences in size effect associated with different types of test are also demonstrated in the other data presented in the tables and especially those for point load tests and for uniaxial compression tests. Point load tests show the largest size effect with n-values frequently close to 0.75. Uniaxial compression tests, on the other hand, commonly show much smaller size effects. These differences may be attributed to different mechanisms of failure. In point load tests and in Brazilian tests, load application is through very small areas of contact, promoting the development of single uncomplicated fractures and a minimal amount of crushing. In uniaxial compression tests loads are applied through relatively large areas of contact. Failure characteristically occurs by means of series of fractures which parallel the line of load application and involves a considerable amount of crushing. This type of failure therefore affects a volume of material rather than a single plane. A situation theoretically associated with a minimal size effect.

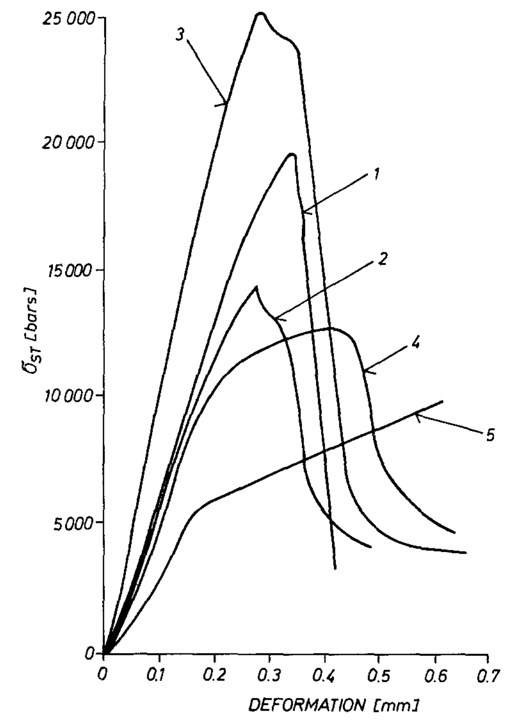

Fig. 1. Load-displacement curves for different rocks
(after Wagner and Schümann, 1971)
1 Quartzite; *2* quartzitic shale; *3* Norite; *4* Marble; *5* Sandstone

b) The dependence of size effect on material properties appears to vary for different types of test. Wagner and Schümann in stamp tests, found significant differences in size effect which could be related to the relative brittleness of the rocks tested; maximum size effect was obtained for brittle materials such as Quartzite and Norite whereas a much smaller size effect was obtained for a much ductile sandstone (see Figs. 1 and 2). Size effects

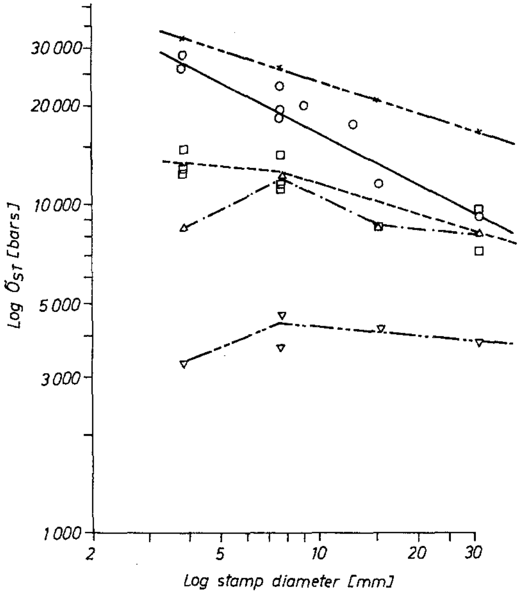

Fig. 2. Linearized representation of the relation between stamp-load bearing strength and stamp diameter for different rocks (after Wagner and Schümann, 1971)

○ Quartzite; ◇ quartzitic shale; × Norite; △ Marble; ▽ Sandstone

evaluated for uniaxial compression tests, presented by different authors for different rock types (Table 1) also seem to indicate a dependence on brittleness; smallest size effects appear to be associated with comparatively ductile materials such as rock salt and weak sandstone. On the other hand dependence on material properties seems to be minor in point load tests and Brazilian tests. This is best expressed by the small range of n-values evaluated for point load results presented by Broch and Franklin (1972) who conducted a systematic study of different rocks. Mode of failure appears to be the dominant factor controlling size effect in these tests.

Indirect evidence in support of a relationship between size effect and brittleness is provided by Habib and Vouille (1966) for triaxial compression tests (see Fig. 3). It was found that for large confining pressures size effect disappears. Since high confining pressures are also associated with ductile rather than brittle failure the results seem to support the above relationship. (Habib himself attributed the phenomenon to the closure of microcracks resulting in an increased mechanical homogeneity).

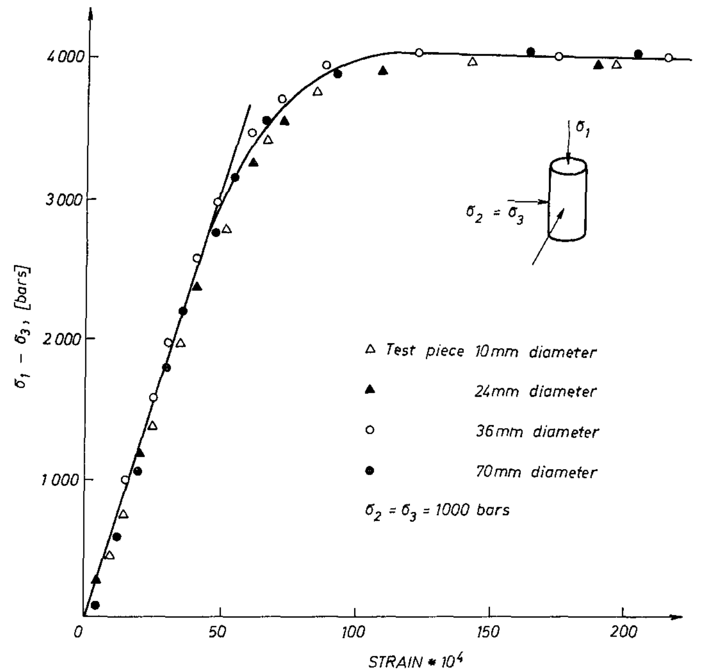

Fig. 3. Triaxial tests on equally shaped cylindrical limestone specimens of different size (after Habib and Vouille, 1966)

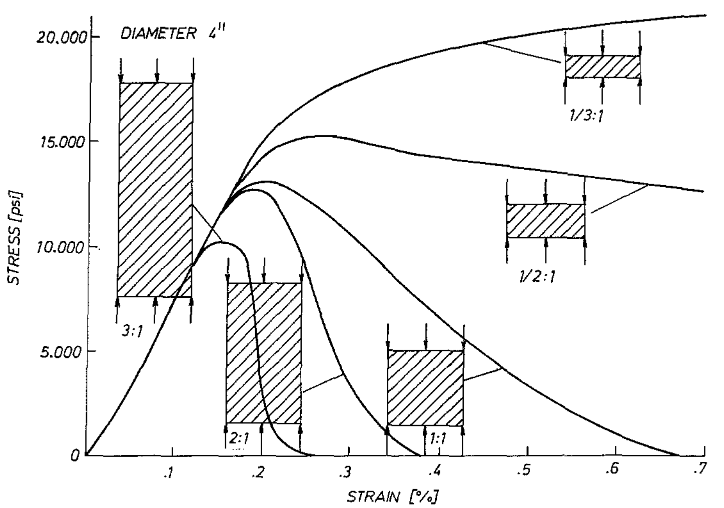

Fig. 4. The effect of slenderness on stress-strain curve for marble loaded in uniaxial compression (after Hudson et al., 1971)

The association of size effect with brittleness agrees with theoretical considerations. Brittle behaviour is quantitatively defined by the slope of the post failure stress strain curve which in turn is an expression of rate of energy release. Hence the more brittle the behaviour of a material, the larger its ability to release stored elastic strain energy abruptly along single fractures, a situation theoretically associated with maximum size effect.

c) The relation between shape and size effect is evident in results of uniaxial compression tests presented by Hudson et al. (1971) which show that increased size effect is associated with increased slenderness (length to diameter ratio, see Fig. 4). Data presented by Hudson et al. also indicate that increased slenderness is associated with an increasingly brittle post failure behaviour. The results therefore also correspond to the relation between size effect and brittleness as discussed above. An apparent lack of influence of specimen shape on size effect may be inferred from data presented by Brook for point load tests. Comparing results of tests performed on specimens of various shape (blocks, cylinders, irregular lumps) specimens strength was found to depend primiarily on the cross sectional area through the specimens between loading points and seemed unrelated to the shape of cross sectional area at least for the range of specimen shapes tested.

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