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The Older the More Valuable:
Divergence Between Utility and Dollar Values of Life as One Ages

By

Yew-Kwang Ng, Melbourne, Victoria, Australia*

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While the utility value of life may decrease monotonically with age, the dollar value may increase dramatically until a fairly old age (by ten-fold to age 60 for one plausible set of parameters). Crucial for this result is a high enough real rate of interest (e.g. 4–5 %) which makes accumulation desirable, leading to a lower marginal utility of money when one gets older, explaining the divergence. This divergence raises perplexing questions as to which value of life should be used and whether the old should be taxed and the young subsidized.

1. Introduction: Why Is Ross Parish Buying a Bigger Car upon His Sixtieth Birthday?

Upon his sixtieth birthday, my colleague, Ross Parish, told me that he was planning to buy a bigger and hence safer car. I said that, for some reasons, older people are more safety-conscious, while rationally it should be the young whose lives are more valuable to be so. Parish then remarked that he did not believe in valuing lives of different ages by counting the number of years saved as done by some practitioners, adding that the life of a person aged 60 is not less valuable than that of one aged 20 due to the accumulated wisdom. I replied that the life of a person aged 40 should be more valuable than a person aged 75 (I dared not say 60) and that while the method of counting years saved

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may not be ideal, it is an improvement over counting one life as one with no reference to age. I also had in mind the idea that life-cycle variation in productivity is not relevant since payment in accordance with marginal productivity should account for that. I did not say this as I was sure both Parish and I believed that each of us earned far less than his real marginal contributions.

At the time, while I was aware of the existence of the pioneering works of Schelling (1968), Mishan (1971), and Jones-Lee (1976) on the willingness-to-pay (for a reduction in the risk to life) approach to the valuation of life, I was unaware of the explicit treatment of age in the important papers by Arthur (1981) and Shepard and Zeckhauser (1982). Had I known of these results, I would have made photocopies of them (all showing decreasing values of life with age[1]) for Parish to prove the correctness of my intuition. Instead, I attempted to show that the paradox of valuing one's life more as one gets older is based on some kind of ignorance or irrationalities. (For a biological explanation of this paradox, see Ng, forthcoming.) The intuition was that, in the absence of ignorance, irrationalities, capital market imperfection, etc., a young person of low income should know that he would gain experience and earn much higher incomes in the future and hence should still place a higher dollar value on his life when young, borrowing to finance for this if necessary.

After ridding my homemade model of its mistakes, I was astonished to obtain the result, for the very first set of plausible parameters selected, that the dollar value of life increases dramatically until a fairly old age (60's for a model with a lifespan of 80). Parish's intuition is correct after all! Crucial for the result is a high enough real rate of interest (5 % is used in Table 1 below). If this rate exceeds the rate of time or uncertainty discount plus the rate of depreciation in capacity to derive utility, it induces an expected utility maximizer to accumulate wealth when young and consume more when old. The marginal utility of a dollar when young is much higher as it can be compounded longer. Thus, despite a decreasing value of life in utility terms, the dollar value of life may increase dramatically as one ages, since the marginal utility of a dollar may decrease faster. If the real rate of interest is low, the result does not hold. However, the discount rate that has to be exceeded by the real rate of interest is only the pure uncertainty discount on future *utility*, which should be quite small for rational individuals

[1] Shepard and Zeckhauser (1982, figure 7) have a result of increasing values until age 30–40 but this is for the Robinson Crusoe case with no capital market.

(see note 5 below). The required rate of interest may thus well apply in many cases.

The divergence in the utility and dollar values of life thus discovered raises perplexing welfare-theoretic and practical policy issues as to which value of life should be used in policy decision affecting risks to life and whether the old should be taxed and the young subsidized. This has important implications for practical policy decisions, especially in health care and accident prevention. For example, such practices as having an age cut-off for certain expensive treatments (e.g. renal dialysis in Britain) are presumably based on the declining (with age) utility values of life. With the dollar value, the prescription may be quite different. Ignoring the dollar value implies forgoing Pareto improvements (assuming no "procedural preferences" against efficiency calculation, on which see Ng, 1988; see also the argument in Ng, 1984, that a dollar should be treated as a dollar whomsoever it goes). To put it differently, saving the life of an older person may involve less gain in utility terms, "but will be paid with less valuable resources" (using a referee's phrase).

2. The Economist Measure of the Value of Life

The economist approach to the valuation of life is based on the (maximum) willingness to pay to avoid a certain risk of death or the minimum amount of compensation required for a marginal increase in the probability of death. The (dollar) value of life is obtained by multiplying the compensating variation (CV) or equivalent variation (EV) for a marginal change (one way or the other) in the probability of death by the inverse of this probability change.[2] For example, if an individual is willing to pay a maximum of \$ x (e.g. \$ 100) to avoid a probability of death equal to y (e.g. 0.01 %), his life is valuated at \$ x/y (e.g. \$ 100/0.0001 = \$ 1,000,000). The value of life thus measured may exceed the life-time income of the individual (Bergstrom, 1982).

For simplicity, it is assumed that there are no externalities (values of a life to others), no irrational aversion to death, that individuals are expected utility maximizers, and that the agony of death as such is negligible in comparison to the value of life itself (see Jones-Lee et al., 1985, on different modes of death and injuries). For a person

[2] For a marginal change, $CV = EV$. In practice, we cannot operate with truly infinitesimal changes. When CV and EV differ significantly, the measure of marginal dollar equivalent may be used instead (Ng, 1979/1983, Appendix 4A).

with very small value of life who strongly dislikes being killed in a fire, his willingness to pay to avoid fire hazards may reflect more his aversion to fire than his value of life.

The estimation of CV can be done either by: (1) actually asking the individual directly, (2) inferring from his revealed preference (e.g. his choices between different airlines), or (3) calculating from his utility function of income, including the point of zero utility.[3] If he is a rational expected utility maximizer, and if there are no significant mistakes made either by the individual or the investigator, the three measures obtained should be approximately equal to each other. However, significant mistakes can easily be made by the individual in reporting his willingness to pay especially in the present issues involving life and death. On the other hand, data are usually inadequate for a good estimate based on revealed preference. Thus, the method based on the utility function is properly a better estimate. Since there is a sizeable literature (on which see Seidl, 1988) on the utility functions of income, results obtained there can help in the estimation of the value of life.

The economist measure of the value of life outlined above may not be universally acceptable, especially by non-economists. Readers with doubts are referred to Ng (1989).

3. The Value of Life for People of Different Ages

Since our objective is to be exploratory and indicative rather than realistic and definitive, we lose little by adopting a very simplified model, thus gaining illustrative clarity. Ignoring uncertainty and the associated pure time discount, an individual expects to live with certainty until age T and to drop dead then.[4] He also knows his (exogenous) life-time

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[3] For surveys of various studies and comparison of alternative measures, see Jones-Lee (1976); Jones-Lee (Ed.) (1982); Jones-Lee et al. (1985); Berger, Blomquist, Kenkel, and Tolley (1986); and Harrington and Portney (1987). See also Ippolito and Ippolito (1984) on the use of a change in the information about the risk to life to estimate the value of life.

[4] A probability of survival function (of age) may be introduced as done in Shepard and Zeckhauser (1982). However, strictly speaking, the individual should then continuously update this survival function as he continues to survive since the probability that one will survive to age x is different given the different ages that one has already survived to. Nevertheless, mathematical simplicity necessitates abstracting away this complication. Since our analysis is more illustrative than immediately applicable, we lose little by taking the even simpler case of certainty and hence do away with the logical inelegance of assuming that individuals do not update their survival function. Typically,

income with certainty and can lend and borrow at the same market (real) rate of interest i. Thus, he maximizes at any age a, with respect to the control variable $c(t)$,

$$\int_a^T u\{c(t)\}\, e^{-rt}\, \mathrm{d}t \equiv V(a)\ , \tag{1}$$

where $V(a)$ = value of remaining life in utility at age a, u = utility, c = consumption, t = time, and r is the rate of depreciation (with age) in the capacity to derive utility, *not* the rate of discount as commonly used.[5] As usual, $u' > 0$, and $u'' < 0$.

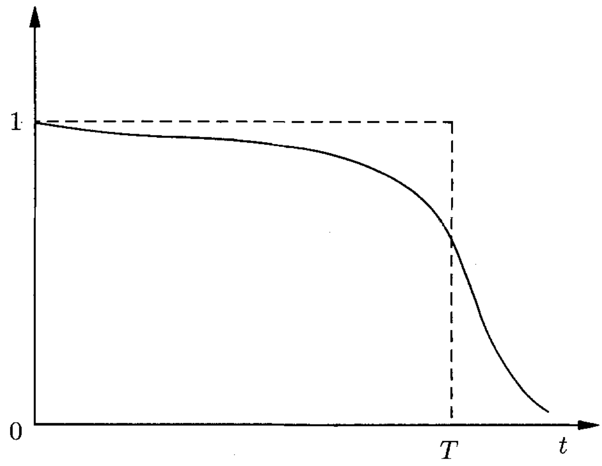

Fig. 1

the probability of survival at birth looks something like the curve in Figure 1. Our approximation involves replacing it by T. With an appropriate choice of T, the approximation is not misleading.

[5] Shepard and Zeckhauser (1982) use a discount rate equal to the interest rate and hence obtain the simple solution that the optimal trajectory of consumption is constant through time. While discounting future *consumption* at the market rate of interest may be reasonable, discounting future *utility* at this rate is questionable, especially since the probability of survival has already been accounted for. Assuming expected utility maximization, we adopt no further discount on future utility. If desired, a discount rate can be added with only notational and computational complications. Our r may be taken as the rate of depreciation (taken as zero by other analysts) plus the pure rate of time discount. Alternatively, a higher rate of time discount can be approximated by a smaller T in our model.

Given non-satiation and no motive for leaving bequests, we write the budget constraint at age a as

$$\int_a^T c(t)\, e^{-i(t-a)}\, \mathrm{d}t = Y\, e^{ia} - \int_0^a \bar{c}(t)\, e^{-i(t-a)}\, \mathrm{d}t \equiv Y(a)\ , \qquad (2)$$

where i is the market rate of interest, Y is the life-time earnings (plus gifts and inheritance received, if any) of the individual valued at age zero, and $Y(a)$ is the life-time income left (i.e. unconsumed) at age a valued also at age a, and a bar over c indicates its historical value.

The maximization of (1) subject to (2) gives[6]

$$u'\{c^*(t)\} = e^{(r-i)(t-a)}\, u'\{c^*(a)\}\ , \qquad (3)$$

which states that, from any age a, the individual's optimal choice involves arranging his future consumption path such that the marginal utility of consumption (after accounting for the depreciating ability to enjoy consumption) decreases at the rate of interest i. Given diminishing marginal utility ($u'' < 0$), consumption increases (decreases) with time if i is larger (smaller) than r. Intuitively, if the rate of interest is larger than the rate of depreciation in the ability to enjoy consumption, (from a position of equal consumption through time) it is better to save now and consume more in the future until this advantage is offset by the diminishing marginal utility of consumption.

Proposition 1: Consumption increases (decreases) with age if the rate of interest is larger (smaller) than the rate of depreciation in the capacity to enjoy consumption.

In a model allowing for the uncertainty of continued survival or for individual impatience, allowance must also be made for a discount factor. In this case, r may be taken as the rate of depreciation plus the rate of uncertainty discount. (Note that this uncertainty discount on future utility is much lower than the discount on future consumption which may equal the rate of interest, see note 5.)

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[6] We may view this as a calculus of variation problem or an optimal control problem in which case we may use $\dot{Y}(t) = iY(t) - c(t)$ which may be obtained by differentiating $Y(a)$ in (2) and replacing a by t; we also have the terminal condition $Y(T) \geq 0$. In either case, the same first-order condition (3) can be obtained.

Equations 2 and 3 determine the optimal consumption path through to age T from any age a (including $a = 0$), given Y, i, r, and T. To obtain explicit solutions, we need to have a specific utility function. Consider

$$u\{c(t)\} = \alpha\{c(t)\}^{\epsilon} - k \ , \tag{4}$$

where α, ϵ, and k are positive constants. For the special case of $k = 0$ assumed by most analysts, this utility function involves constant elasticity ($= \epsilon$) with respect to consumption. However, ignoring the positive constant k implies that utility is positive no matter how small (positive) consumption is. Common sense suggests that one needs a certain minimum level of consumption to avoid negative utility and to ensure survival itself. This problem does not arise for most economic issues (e.g. consumer demand) where the utility function may be taken to be unique up to any positive monotonic transformation; the addition of any constant would have no effect at all. However, for the present problem of the value of life, whether one enjoys positive or negative utility is important. The neglect of this by most economic analysts is probably related to the fallacy of misplaced abstraction (Ng, 1979/1983, Sect. 1.4).

For the case of (4), we may solve, from (2) and (3), at any age a, for the optimal consumption path from age a to T, for $\epsilon i - r \neq 0$,[7]

$$c(t) = (\epsilon i - r) \cdot Y(a) \cdot e^{\frac{(i-r)(t-a)}{1-\epsilon}} \cdot (1-\epsilon)^{-1} \cdot \left\{ e^{\frac{(\epsilon i - r)(T-a)}{1-\epsilon}} - 1 \right\}^{-1} . \tag{5}$$

Substitute this optimal solution for $c(t)$ into (1) and integrate, yielding,

$$V(a) = \frac{\alpha \cdot (1-\epsilon)^{1-\epsilon} \cdot \{Y(a)\}^{\epsilon} \cdot \left\{ e^{\frac{(\epsilon i - r)T - \epsilon a(i-r)}{1-\epsilon}} - e^{-ra} \right\}}{(e\, i - r)^{1-\epsilon} \cdot \left\{ e^{(\epsilon i - r)(T-a)/(1-\epsilon)} - 1 \right\}^{\epsilon}} + $$
$$+ (e^{-rT} - e^{-ra})k/r \ . \tag{6}$$

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[7] For the case $\epsilon i - r = 0$, we have instead of (5),

$$c(t) = (T-a)^{-1} \cdot Y(a) \cdot e^{(i-r)(t-a)/(1-\epsilon)} \ . \tag{5a}$$

Equations (6)–(9) should also be changed correspondingly.

Differentiating (6) with respect to $Y(a)$, we have,

$$\frac{\partial V(a)}{\partial Y(a)} = \frac{\epsilon \cdot \alpha \cdot (1-\epsilon)^{1-\epsilon} \cdot \{Y(a)\}^{\epsilon-1} \cdot \{\cdot\}}{(\epsilon i - r)^{1-\epsilon} \cdot \{\cdot\cdot\}} \,. \tag{7}$$

Divide (6) by (7), yielding,

$$D(a) \equiv \frac{V(a)}{\partial V(a)/\partial Y(a)} = \frac{Y(a)}{\epsilon} + \frac{(e^{-rT} - e^{-ra})k/r}{\partial V(a)/\partial Y(a)} \,, \tag{8}$$

where $D(a)$ is the dollar value of remaining life at age a. (This is similar to the dollar value of an ordinary good, e.g. the dollar value of an apple is its utility value divided by the marginal utility of a dollar.) This is a valid measure of the individual's own valuation of his life based on his CV of a marginal risk to life at the respective age, assuming expected utility maximization and with the utility of death normalized at zero. (This is compelling since the agony of death has been abstracted away; see Sect. 1.) Thus, from the current assumed situation of perfect certainty, suppose the minimum amount to persuade the individual to accept a small probability of death β is M dollars. In utility terms, the gain of M dollars, for a small M, is approximately equal to M times the marginal utility of income. We thus have $V(a) \simeq (1-\beta)\{V(a) + M\,\partial V(a)/\partial Y(a)\}$ which gives

$$M \simeq \frac{\beta V(a)}{(1-\beta)\,\partial V(a)/\partial Y(a)} \,.$$

Since $D(a) = M/\beta$, we have

$$D(a) \simeq \frac{V(a)}{(1-\beta)\,\partial V(a)/\partial Y(a)} \simeq \frac{V(a)}{\partial V(a)/\partial Y(a)}$$

for a small β. The approximation is exact for an epsilon β.

Putting $a = 0$ in (5) to calculate the values of $\bar{c}(t)$ for t from 0 to a and substituting the resulting solution into the second equation in (2), we have, after integration and simplification,

$$Y(a) = Y\left\{e^{\frac{(\epsilon i - r)T}{1-\epsilon} + ia} - e^{(i-r)a/(1-\epsilon)}\right\} \Big/ \left\{e^{\frac{(\epsilon i - r)T}{1-\epsilon}} - 1\right\} \,. \tag{9}$$

Obviously, $Y(0) = Y$, and $Y(T) = 0$ as required. The solution of $Y(a)$ in (9) can be substituted into (5) to (8) to give the final solutions for $c(t)$ (for t from a to T), $V(a)$, $\partial V(a)/\partial Y(a)$, and $D(a)$, respectively. If precise values of ϵ, α, i, r, T, k, and Y are known, exact numerical values for these variables can be calculated for all ages from zero to T. This is done for a plausible set of these parameter values for selected ages, as reported in Table 1. The parametric values used in Table 1 are the first set selected, partly based on observation (e.g. i) and common sense (e.g. ϵ which must lie between zero and one; Arthur uses values for ϵ from 0.4 to one; the use of a higher ϵ is favourable to our case).[8] The value of α is a scale factor; if α and k are changed proportionately, there will be no effect on the values of $D(a)$. The value of $k/\alpha = 32$ means that an individual needs to consume about \$ 1,000 per period (year) to avoid negative utility.

Table 1

$\epsilon = 1/2$, $i = 1/20$, $r = 1/80$, $T = 80$, $Y = \$ 200{,}000$, $k = 3200$, $\alpha = 100$

Age a	Current Value of Life-time Unconsumed Earnings $Y(a)$ (in \$ 1,000)	Utility Value of Life $V(a)$ (in 1,000 utils)	Marginal Utility of Life-time Income $\partial V(a)/\partial Y(a)$ (in util per dollar)	Dollar Value of Life $D(a)$ (in \$ 1,000)
0	200	553	1.787	309
20	488	537	0.658	817
40	1,080	462	0.242	1,908
50	1,487	393	0.147	2,682
60	1,828	299	0.089	3,355
70	1,694	170	0.054	3,156
79	297	19	0.034	559

4. The Divergence in the Utility and the Dollar Values

Rather unexpectedly, the dollar value of life increases rather sharply with age, by more than ten times from age zero to age 60, as reported

[8] At the time of writing, the nominal interest rates for Australia/US are about 18 % / 10 % and inflation rates about 7 % / 4 %. I use a lower real rate closer to the historical average. It is true that the after-tax return is considerably lower. However, there are many forms of accumulation (such as in low-tax superannuation and properties) that yield much higher returns. It may still be thought that a 5 % real rate is much too high in historical terms. One may then view it as being adopted to make the result more dramatic.

in Table 1. It falls slightly in the sixties and sharply in the seventies. However, the utility value of life decreases with age almost throughout the whole range. The profile of the utility values of life is consistent with the common sense view. But that of the dollar values of life appears counter-intuitive but can be explained intuitively. As stated in Proposition 1, if the rate of interest exceeds the rate of depreciation in the capacity to enjoy consumption, optimal consumption increases with age. This has the following effects. First, the marginal utility of consumption decreases with age. Second, the total utility may increase with age. Third, a dollar obtained at a younger age is more valuable (than one at an older age) since it can be invested (at compound interest) longer to yield more utility in the future. At $i = 0.05$, a dollar at age zero becomes more than \$ 30 at age 70. Thus, it may pay to invest and consume later in life despite capacity depreciation and diminishing marginal utility of consumption. The first and third factors make the marginal utility of income $\partial V(a)/\partial Y(a)$ decrease quickly with age and the second factor makes the utility value of life decrease slowly with age at younger ages. All these make the dollar value of life possibly increase at a fast rate up until a fairly old age when the utility value of life itself starts to decrease at a fast rate.

Proposition 2: Despite a decreasing (with age) utility value of life, the dollar value of life may increase sharply until a fairly old age. Such a contrast is more likely to prevail if the rate of interest is substantially higher than the rate of depreciation in the capacity to enjoy consumption, ceteris paribus.

To my knowledge, no other analysts have obtained such a drastically contrasting result. For example, the figures for the dollar value of life obtained by Arthur (1981, p. 63) for $\epsilon = 0.6$ are (in \$ 1,000) 668, 664, 619, 520, 399, 265, 139, 54, 31, respectively at age 0, 10, 20, ..., 80. The explanation of the difference is that other analysts have not allowed adequately for the possibility of a markedly decreasing (over age) value of $\partial V(a)/\partial Y(a)$. For example, Shepard and Zeckhauser (1982) discount future utility at the rate of interest (but do not allow for possible depreciation in capacity) and hence obtain the result that the optimal consumption path is a constant level of consumption.

The crucial role of the excess of the rate of interest over the rate of capacity depreciation can be seen by contrasting the case when this excess vanishes. Table 2 is computed taking everything the same as in Table 1 except that $i = 1/80$. This change (from $i = 1/20$ to $i = 1/80$) alone changes the picture completely, giving now a "traditional result" as reported in Table 2 where both the utility and the dollar values of life decrease monotonically with age.[9]

Table 2

Age a	Current Value of Life-time Unconsumed Earnings $Y(a)$ (in \$ 1,000)	Utility Value of Life $V(a)$ (in 1,000 utils)	Marginal Utility of Life-time Income $\partial V(a)/\partial Y(a)$ (in util per dollar)	Dollar Value of Life $D(a)$ (in \$ 1,000)
0	200	156	0.795	196
20	167	102	0.619	164
40	124	59	0.482	122
50	99	41	0.426	97
60	70	26	0.376	69
70	37	12	0.331	37
79	4	1	0.296	4

From (12), if $k = 0$, $D(a) = Y(a)/\epsilon$, the dollar value of life equals the current value of unconsumed life-time income multiplied by $1/\epsilon$. This result is consistent with those obtained by other analysts (e.g. Arthur, 1981, and Shepard and Zeckhauser, 1982) who have $k = 0$.

5. Perplexing Policy Implications

The divergence between the utility and dollar values of life raises interesting and perplexing welfare theoretic and policy issues. Should we use the utility or the dollar value of life in choices involving risks to life, especially those with respect to people of different ages? Suppose that one out of every hundred thousand individuals will be struck by an accident that involves instant death. Should we (i.e. the society) prefer less deaths falling upon the young or the old? With the question posed this way, most people (myself included) would prefer a smaller number of young people dying (and of course unavoidably more older people dying since the total number of deaths is being held constant).

[9] In the computation, note the $\epsilon i - r$ is negative when $i = r$ since $\epsilon < 1$. We proceed by rewriting the $(\epsilon i - r)^{1-\epsilon}$ in (6) as $-r \cdot (-r)^{\epsilon} \cdot (1 - \epsilon)^{1-\epsilon}$ where $(1 - \epsilon)^{1-\epsilon}$ cancel with the same term in the numerator, and $(-r)^{\epsilon}$ is combined with $\left\{e^{(\epsilon i - r)(T-a)/(1-\epsilon)} - 1\right\}^{\epsilon}$ to turn the whole term positive.

This is consistent with the use of the utility value of life. However, the young may be willing to pay no more than \$ 10 each to remove the one in 100,000 risk of death, while those in their sixties may be willing to pay more than \$ 30 each. Suppose that the risk can be removed at a cost of \$ 20 per head, and that this can be done selectively. It is then obviously a Pareto improvement (in comparison to no removal) to remove the risk for the old but not for the young, since the old can pay the \$ 20 per head cost and still be better off but the young will be worse off after such a payment. Should we then remove the risk only for the old? Or should we remove the risk for the young as well with the costs financed from the old, effectively involving a transfer from the old to the young?

Despite the fact that it was against my initial intuition, it is reassuring that a little economic analysis may explain an apparent (Parish) paradox which seems to be a fairly widespread fact. On the other hand, the perplexing questions raised by the divergence between the utility and the dollar values of life are rather disturbing. In the presence of a divergence in the marginal utilities of a dollar between the rich and the poor in the atemporal framework of normal welfare analysis, the maximization of a utilitarian or some other reasonable (quasi-concavity is sufficient but not necessary) social welfare function requires the transfer of purchasing power from the rich to the poor until the marginal gain is offset by the marginal costs (excess burden of taxation, policing, etc.)[10] Given this optimal transfer, a dollar can be treated as a dollar whomsoever it goes to (Ng, 1984). This transfer does not raise perplexing problems and is widely practised in the form of progressive income taxation and welfare expenditures. However, the divergence in the utility and the dollar values of life raises quite different issues. As discussed above, the divergence may be quite drastic even in a model of identical individuals each with the same endowment, earning abilities, and utility function, except that individuals of different ages exist at any one time. To transfer resources from the old to the young seems to be unfair (violating horizontal equity at least over the transitional stage); not to do so seems to forgo the possibility of increasing social welfare. The elderly have low marginal utility per dollar; a dollar transferred to the young would yield much higher utility. Also, when the policy of transfer has attained its steady state, everyone will be made better off. The policy of transferring from the old to the young is of course diametrically opposite to that of old-age pensions. The latter may be

[10] In the case of a strictly quasi-concave social welfare function, the marginal gain is the marginal utility weighted by welfare weights which takes account of the level of total utility.

justified by the presence of imprudent people. However, this may better justify forced superannuation than pensions. The possible desirability of transferring in favour of the young is still present due to the divergence between the utility and dollar value of life over ages. Perhaps this could be a relevant factor in our current concerns about the low rate of savings and the burden of supporting the increasing proportion of aged people.

Should we use the utility or the dollar values of life in guiding policy choice? I have been strongly in favour of treating a dollar as a dollar; this suggests using the dollar value of life. However, in the presence of the divergence between the utility and the dollar values of life (due to age but not to wealth differentials), the issue appears more complicated.

It is true that the problem of tranferring to the young can be viewed as one of socially optimal (forced) capital accumulation or growth involving intergenerational transfers and the associated conflict of interest.[11] Just as a dollar may be treated as a dollar once socially optimal transfers between the rich and the poor have been effected in the atemporal framework, the same is true for the intertemporal framework once socially optimal accumulation has been achieved. However, there is a

[11] However, it should be emphasized that our case for transfer is quite different from the transfer or national debt problem of achieving a golden-rule capital-labour ratio (see, e.g., Diamond, 1965; Stein, 1969; and Ihori, 1978). The latter purports to aim for Pareto optimality, focusing on steady-state comparisons. Our transfer problem is based on social optimality, "robing the old to benefit the young." The transfer is a dollar for a dollar. In the debt problem to achieve golden rule, the transfer is effected by a debt which is to be repaid *with interest*. With such repayment, a transfer is self-defeating in our framework. Also, the transfer to achieve a golden-rule is typically from the future (i.e. younger) into the present generation. The gain of the earlier generations being financed by an increasing national debt made possible by a positive population growth rate (which allows a "biological rate of interest" to match it). (In models allowing for the exogeneity of population growth and parental care about their children, the Pareto inefficiency of competitive equilibrium in the infinite economy is eliminated; see Nerlove, Razin, and Sadka, 1987, pp. 89–93; Willis, 1987.) Rather, our transfer problem is similar to the problem of the social optimal rate of savings (or capital accumulation). The problem arises because the rates of time preference of individuals (especially the aged) of limited lifespan may be much higher than the social rate. Our approach highlights the problem in terms of the possible huge differences in the marginal utilities of life-time income between people of different ages. The introduction of parental care as in Nerlove, Razin, and Sadka (1987) and Willis (1987) does not solve the problem here since their Pareto-efficient program is obtained by maximizing the utility of the present generation only.

significant difference. In the atemporal case, there is (at least ideally speaking) a government to represent the interest of the whole society (with apologies to the public choice school) which may thus maximize social welfare impartially. In the intertemporal case, future generations have no control on our present policies. Our choice on social accumulation may reflect our self-interest than an impartial trade-off between the present and the future. Thus, while a social optimal transfer may be achieved in the atemporal case, it cannot be achieved in the intertemporal case. Nevertheless, it may be said that, given whatever intertemporal choice we made that reflect whatever degree of bias we have, to be consistent we should then stick to this and hence proceed to use the dollar instead of the utility value of life.

If this last point is accepted, we may have to adopt policies that value the life of an old person many times more than that of a young person, diametrically opposite to the practice of counting the number of years saved used by many practitioners. Given that it is difficult to persuade policy makers to accept the economist approach of using the dollar value of life, economists may achieve partial success by collaborating with the extreme left-wing in insisting on "a life is a life" irrespective of age and wealth. What an irony!

Mathematical Appendix

For the case of (4), the maximization of (1) subject to (2) yields

$$c(t) = c(a)\, e^{(i-r)(t-a)/(1-\epsilon)} \; . \tag{M1}$$

Substitute $c(t)$ from (M1) into (2),

$$\int_a^T c(a)\, e^{(i\epsilon-r)(t-a)/(1-\epsilon)}\, dt = Y(a) \; . \tag{M2}$$

Upon integration, we have

$$\begin{aligned} Y(a) &= \left| \frac{1-\epsilon}{i\epsilon - r}\, c(a)\, e^{(i\epsilon-r)(t-a)/(1-\epsilon)} \right|_a^T \\ &= \frac{(1-\epsilon)\, c(a)}{i\epsilon - r} \left\{ e^{(i\epsilon-r)(T-a)/(1-\epsilon)} - 1 \right\} , \end{aligned} \tag{M3}$$

from which,

$$c(a) = \frac{(i\epsilon - r) \cdot Y(a)}{(1-\epsilon)\{e^{(i\epsilon - r)(T-a)/(1-\epsilon)} - 1\}} . \tag{M4}$$

Substitute $c(a)$ from (M4) into (M1),

$$c(t) = \frac{(i\epsilon - r) \cdot Y(a) \cdot e^{(i-r)(t-a)/(1-\epsilon)}}{(1-\epsilon)\{e^{(i\epsilon - r)(T-a)/(1-\epsilon)} - 1\}} . \tag{M5}$$

Substitute $c(t)$ from (M5) into (4) and the resulting expression for $u\{c(t)\}$ into (1), we have, after integration, (5) in the text.

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Address of author: Professor Yew-Kwang Ng, Monash University, Faculty of Economics, Commerce and Management, Clayton, Melbourne, Victoria 3168, Australia.