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# Innovation and Imitation under Imperfect Patent Protection

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The paper develops a model in which the spillover of R&D is a consequence of a rational investment in imitation. The model incorporates the innovator's choice between patenting and secrecy as a protection device. The analysis demonstrates that an increase in patent breadth always discourages resorting to secrecy, whereas the influence of increased patent life is the opposite with large spillovers. An increase in patent life can also reduce innovative activity with large spillovers. Under endogenous imitation, short patents are socially optimal.

*Keywords:* patent policy, secrecy, spillovers.

*JEL classification:* O34, O31.

## 1 Introduction

The purpose of the patent institution is to encourage inventive effort and disclosure of research findings by conferring temporary monopoly power on the innovator. Since the seminal studies on optimal patent length and breadth by Nordhaus (1969, 1972), there has been extensive research on these issues. An excellent survey of the literature of optimal patent breadth–length mix is provided by Denicolò (1996). He demonstrates that seemingly contradictory results in different models, e.g., Gilbert and Shapiro (1990), Klemperer (1990), and Gallini (1992), are caused by the dissimilar impacts of patent breadth on social welfare and post-innovation profits in these models.

Even casual observations, however, confirm that imitation is pervasive in many industries in spite of patent protection. Since it may sometimes be beneficial for innovators to trust secrecy instead of acquiring a patent and disclosing all the details of the innovation to rivals, it is surprising that the theory of patents mainly concerns the decision to patent and knowledge spillovers as automatic and costless results of research activity. In this study I construct a simple model where spillovers are a consequence of a follower's investment in imitation. The model incorporates the essential insights from Gallini (1992) where patent breadth raises the imitation costs and the innovator can choose whether to patent the innovation or keep it secret.[1] She imposes the restriction that the imitators can invent around the patent at a fixed cost, whereas the unpatented innovation becomes freely available to everyone. Extending the analysis in Gallini (1992), I introduce rational imitation where the outcome is uncertain regardless of the innovator's decision to patent the innovation or to keep it secret.

It has been thought that an increase in patent length merely increases the length of the innovator's monopoly. Modeling imitation explicitly, however, demonstrates that the increase in patent length also accelerates investments in imitation, because the imitator can no longer afford just to "wait and see" until the patent expires. It turns out to be crucial from the innovator's point of view whether the expected spillover is "large" or "small." If imitation is likely to be successful, secrecy becomes attractive when patent life increases. The main finding in Gallini (1992), the social optimality of short patents, is confirmed, but the optimal patent does not need to be broad if the spillovers are high enough. Because a patent policy matters only if it reduces the rate of spillover, the prospects for the welfare-improving policy are rather restricted.

In the remainder of the paper, I first describe the model in Sect. 2, and in Sect. 3 I characterize the behavior of the imitator. The behavior of the innovator including the patent decision is analyzed in Sect. 4. Some implications for the patent policy can be found in Sect. 5, and the concluding remarks are in Sect. 6.

## 2 The Model

Consider a duopoly with an innovator and an imitator where the innovator initially invests in R&D to develop a new product or process. Success is followed by the decision to patent, and when patenting is preferred to secrecy, the protection is assumed to be valid for $L$ years. Acquiring a patent does not generally prevent imitation; the imitator has an opportunity to try to invent around the patent, that is, produce a noninfringing substitute. After the decision to patent has been made, the product is introduced onto the market and the imitator moves. The im-

1 There are also some links between this paper and a recent welfare analysis of imitation by Kanniainen and Stenbacka (1997).

itator can thus invest only after the uncertainty concerning the success of innovation is resolved. The roles of the innovator and the imitator are here taken as given in order to focus on subsequent interaction.[2] Without liquidity constraints nothing prevents the innovator or the imitator from investing again, but a more dynamic model would not be essential for the present argument. I thus presume that neither the innovator nor the imitator receive payoffs from the unsuccessful attempt.

Irrespective of the patent decision, an imitation can be obtained only through costly investment which involves uncertainty about success. These features of my model contrast with that of Gallini (1992), where the imitation of a patented innovation can be obtained with certainty at a fixed cost and the imitation of an unpatented innovation is costless. The production stage following the investment in imitation is reduced to fixed payoffs $\pi^{\mathrm{m}}$ and $\pi^{\mathrm{d}}$ ($\pi^{\mathrm{m}} \geq 2\pi^{\mathrm{d}} \geq 0$) for a monopoly and a duopoly, depending on the success of the imitator. It is assumed that cost functions for the innovator and the imitator, $C_R(\alpha) = \frac{1}{2}R\alpha^2$ and $C_D(\beta) = \frac{1}{2}D\beta^2$, are strictly convex. Parameters $R$ and $D$ reflect the exogenous efficiency of the existing innovation and imitation technology. It is assumed that they are large enough so that $\alpha \leq 1$ and $\beta \leq 1$ and, accordingly, $\alpha$ and $\beta$ can be regarded as the innovation and imitation success probabilities. For simplicity, I work directly with $\alpha$ and $\beta$ instead of treating investment levels as decision variables. An easy way to guarantee that innovation is more profitable than imitation is to assume that $D$ is sufficiently large or $\pi^{\mathrm{d}}$ is sufficiently small.

The subsequent position of the imitator depends on whether the innovator chooses to patent (p) the innovation or keep it secret (s). When the innovation is patented, the imitator can scrutinize the patent application to identify its technical details, but must be careful not to infringe the patent. The effect of patenting on imitation costs in contrast

2 To endogenize the order of moves, economists usually deviate from the standard models of simultaneous decisions by abandoning perfect information. For instance, in Normann (1997) one firm knows the state of the demand curve while the other is uninformed, and only sequential moves pass the equilibrium refinements applied. Kultti and Niinimäki (1998) show that simultaneous moves are never an equilibrium when neither firm knows the true state. Alternatively, the innovation here might be thought of as an application of some basic scientific breakthrough the obtaining of which indicates that the firms might have engaged in a stochastic pre-development game of Poisson type as in Denicolò (1996) or a real option game as in Lambrecht (1997). The winner of the pre-development game would then be a natural candidate for the innovator.

to secrecy is introduced by patent breadth as

$$w = \frac{D_p}{D_s} \iff D_p = D_s w \ . \tag{1}$$

If $w$ is finite but greater than one, the patent protection is imperfect but raises the imitation costs. Such a view is supported by the much-cited queries by Levin et al. (1987) and Mansfield et al. (1981). The increased cost of imitation could be viewed as arising from the risk of a court action as in Waterson (1990).

The approach here is clearly rather stylized in its modeling of patent breadth. The breadth affects the imitation costs for example, but not the product-market competition. However, Gallini (1992) suggests that the conclusions remain essentially unchanged even if patent breadth affects the product-market equilibrium by raising the production costs instead of the actual imitation costs.[3] Like Gallini (1992), I assume that imitation is costless after the patent expires.

The sequence of decisions is summarized in Fig. 1. As is usually done, I proceed in reverse order, beginning with the imitator's decisions.

## 3 The Effect of Patents on Imitation

If the innovator chooses to conceal the innovation, the problem of the imitator is

$$\max_{\beta_s} P_s(\beta_s) = \beta_s \pi^d - \tfrac{1}{2} D_s \beta_s^2 \ . \tag{2}$$

The maximization yields

$$\beta_s = \pi^d / D_s \ . \tag{3}$$

The parameter $\beta_s$ can be regarded as an endogenized rate of spill-

3 The restriction could also be justified by assuming that if the imitator is successful, the innovator receives $\pi^d - \varepsilon$ and the imitator $\pi^d + \varepsilon$ in which $\varepsilon$ is normally distributed with zero mean; the imitation may be an inferior substitute or an improvement to the original innovation. The innovator could also be unwilling to pay the patent-renewal fees when an imitation appears. As pointed out by Gallini (1992), several different innovations with the same purpose may be patentable and the imitation in this model is perhaps best thought of as a noninfringing substitute or a differentiated product with symmetric demands. See Denicolò (1996), for a detailed discussion of the modeling of patent breadth.

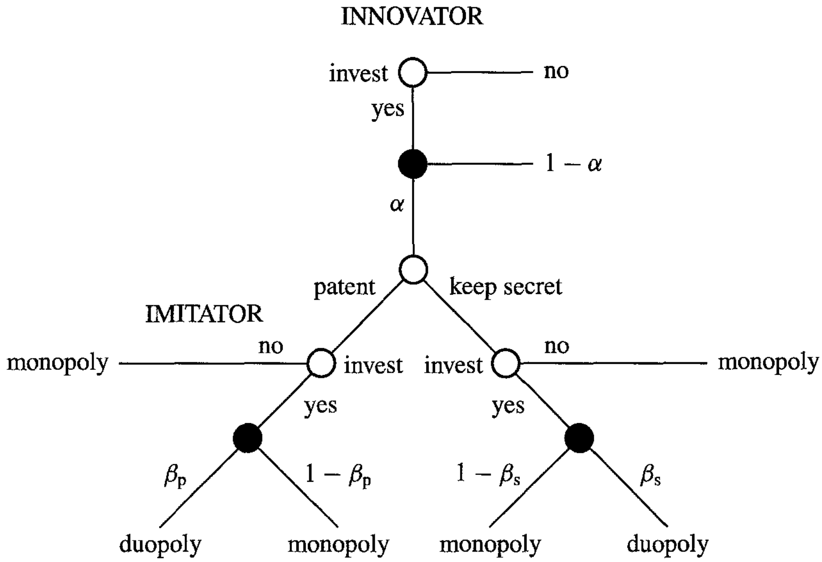


Fig. 1: The sequence of decisions

over because it is determined from the imitator's investment problem (2). Analogously, when the innovation is patented the problem is $\max_{\beta_p} P_p(\beta_p) = \beta_p \pi^d + (1-\beta_p)e^{-rL}\pi^d - \frac{1}{2}D_p\beta_p^2$. A successful imitator immediately gets the duopoly profits whereas with a failure, the imitator has to wait until the patent expires. The first-order condition can be written as

$$\beta_p = Z\pi^d / D_s \ , \tag{4}$$

where $Z = (1 - e^{-rL})/w$ captures the impact of patent system on R&D spillovers. With the patented innovations the probability of imitation clearly increases with patent length and decreases with patent breadth. A similar result with regard to patent length is in Gallini (1992), where free entry to "the markets of imitation" implies that there is a threshold length of patent, before which the probability of imitation is zero and after which it is unity.

## 4 The Effect of Patents on Patenting Behavior and Innovative Activity

One of the main arguments for the patent system is that it leads to public access to research information, but the same reason discourages the innovator from applying for the patent. As already noted in Penrose (1951), this argument for the patent system is indeed very weak, since patenting is useful for the innovator only if it efficiently retards the dissemination of research findings. This almost self-evident observation is easy to prove. The values of the patented and unpatented innovations are defined as

$$V^{\mathrm{p}} = (1 - \mathrm{e}^{-rL})[\pi^{\mathrm{m}} - (\pi^{\mathrm{m}} - \pi^{\mathrm{d}})\beta_{\mathrm{p}}] + \mathrm{e}^{-rL}\pi^{\mathrm{d}} \tag{5}$$

and

$$V^{\mathrm{s}} = \pi^{\mathrm{m}} - (\pi^{\mathrm{m}} - \pi^{\mathrm{d}})\beta_{\mathrm{s}} \ . \tag{6}$$

Because adding a fixed patenting cost would not bring additional insights, patenting is assumed to be costless. The innovation is thus patented if $V^{\mathrm{p}} \geq V^{\mathrm{s}}$, and direct calculation employing Eqs. (3)–(6) demonstrates that this is equivalent to

$$\Psi(L, w) \equiv \frac{(\pi^{\mathrm{d}}/D_{\mathrm{s}})[1 - (1 - \mathrm{e}^{-rL})Z]}{\mathrm{e}^{-rL}} \geq 1 \ . \tag{7}$$

The condition clearly shows that patenting may be optimal only if $Z < 1$, which in turn implies that $\beta_{\mathrm{p}} < \beta_{\mathrm{s}}$. An active patent system thus necessarily reduces the rate of the spillover.

To see how the properties of patents affect patenting behavior, it is useful to define $Z^* = D_{\mathrm{s}}/2\pi^{\mathrm{d}}$, which generates the spillover level of one half in (4), i.e., $\beta_{\mathrm{p}} = 1/2$.

*Proposition 1:* An increase in patent breadth always encourages patenting, whereas an increase in patent life encourages patenting only when $Z < Z^*$ and discourages it when $Z > Z^*$.

*Proof:* Because $V^{\mathrm{s}}$ is independent of the properties of the patents, it is sufficient to differentiate $V^{\mathrm{p}}$ from (5) with respect to $L$ and $w$. We immediately see from (4) and (5) that $\mathrm{d}V^{\mathrm{p}}/\mathrm{d}w > 0$. Differentiating $V^{\mathrm{p}}$ with respect to $L$ gives

$$\mathrm{d}V^{\mathrm{p}}/\mathrm{d}L = r\mathrm{e}^{-rL}(\pi^{\mathrm{m}} - \pi^{\mathrm{d}})(1 - 2Z\pi^{\mathrm{d}}/D_{\mathrm{s}}) \ . \tag{8}$$

Clearly, the sign of the term in the last parenthesis in (8) determines the sign of $\mathrm{d}V^{\mathrm{p}}/\mathrm{d}L$. □

It follows that an increase in patent life enhances the patent protection only with small spillovers ($\beta_p < 1/2$). With large spillovers ($\beta_p > 1/2$), an increase in patent life actually encourages protection by secrecy. As to the incentives to innovate, the same reasoning yields a new result.

*Corollary 1:* Provided that innovations are patented, an increase in patent breadth always encourages innovative activity, whereas an increase in patent life encourages innovative activity only if $Z < Z^*$ and discourages it if $Z > Z^*$.

*Proof:* The problem of the innovator is to choose $\alpha$ so as to maximize $P_R(\alpha) = \alpha \max(V^{\mathrm{p}}, V^{\mathrm{s}}) - \frac{1}{2}R\alpha^2$. The investments in innovation are thus given by

$$\alpha = \frac{\max(V^{\mathrm{p}}, V^{\mathrm{s}})}{R} \ . \tag{9}$$

When the innovation is patented, $\alpha = V^{\mathrm{p}}/R$. Proposition 1 shows that $\mathrm{d}V^{\mathrm{p}}/\mathrm{d}w > 0$ and that the sign of $\mathrm{d}V^{\mathrm{p}}/\mathrm{d}L$ is determined by the sign of $\{1 - 2Z\pi^{\mathrm{d}}/D_{\mathrm{s}}\}$. □

A long tradition in the literature beginning with Nordhaus (1969) declares that the innovative effort is everywhere an increasing function of patent length.[4] An increase in patent life here, however, may even dilute the incentives to innovate. While a longer patent increases the length of the monopoly period, it also increases the possibility that the innovation will be imitated, and the latter effect dominates for large spillovers ($\beta_p > 1/2$).

## 5 The Socially Optimal Patent Policy

The aim of this section is to briefly re-examine the much debated issue of the socially optimal design of the patent when the imitator's optimization and the innovator's patent decision account for the spillovers. Following the previous literature, patent length and breadth have been chosen so as to maximize the social utility from existing innovation,

4 The exceptions are Gallini (1992) and Cadot and Lippman (1995). In Gallini (1992) the innovative effort is a nondecreasing function of patent length, since extending patent life beyond some fixed time fails to provide additional protection. Cadot and Lippman (1995) in turn demonstrate by means of a model of repeated innovations that longer patents retard the incentive to introduce a new product generation.

constraining incentives to innovate to a predetermined level, i.e., the problem is

$$\max_{L,w} S_{\mathrm{p}} = (1-\mathrm{e}^{-rL})(W^{\mathrm{m}} + \beta_{\mathrm{p}}(W^{\mathrm{d}} - W^{\mathrm{m}})) + \mathrm{e}^{-rL} W^{\mathrm{d}} - \tfrac{1}{2} D_{\mathrm{p}} \beta_{\mathrm{p}}^2 \quad (10)$$

subject to

$$\alpha = \bar{\alpha} \ , \quad (11)$$

where $\alpha$ and $\beta_{\mathrm{p}}$ are determined by (9) and (4), and where $W^{\mathrm{m}} = cs^{\mathrm{m}} + \pi^{\mathrm{m}}$ and $W^{\mathrm{d}} = cs^{\mathrm{d}} + 2\pi^{\mathrm{d}}$ depict social welfare as the total of consumer surplus and industry profits under monopoly and duopoly. Because the patent policy matters only if innovations are patented, it is assumed that the desired reward for the innovator is high enough to guarantee patenting, i.e., $\bar{\alpha} \geq V^{\mathrm{s}}/R$.

The general theorem stated by Denicolò (1996) predicts that the optimal patent has maximum breadth and minimum length when both $S$ and $\alpha$ are convex in the breadth of the patent, and the opposite is true if both are concave. It is therefore useful to establish some properties of functions $S(w)$ and $\alpha(w)$.

*Lemma 1:* $\alpha'(w) > 0$, $\alpha''(w) < 0$, and $S'(w) \gtreqless 0 \wedge S'' \lesseqgtr 0 \iff 2(W^{\mathrm{d}} - W^{\mathrm{m}}) - \pi^{\mathrm{d}} \lesseqgtr 0$.

*Proof:* See Appendix 1.

The incentives to innovate are thus increasing and concave in the breadth of the patent, whereas post-innovation social welfare is generally decreasing and convex. Note that the general theorem in Denicolò (1996) is silent about the optimal policy when $S$ is convex but $\alpha$ is concave in patent breadth. As it stands, however, Lemma 1 leaves some prospects for counter-examples; as in Klemperer (1990), static social welfare may actually be increasing for some values of $w$. In what follows, I assume for brevity of the presentation that $S'(w) < 0$. Following Gallini (1992) I solve the optimal patent breadth–length mix by adjusting the length to achieve the desired incentives for the innovator.

*Proposition 2:* The socially optimal patent has the minimum length guaranteeing the incentive to innovate, and maximum breadth if $Z < Z^*$ but minimum breadth if $Z > Z^*$.

*Proof:* See Appendix 2.

This outcome is easy to explain. The presumption that the imitation is costless after the patent expires makes short patents an attractive way to spread new technologies. In the light of Corollary 1 the policy tools are substitutes for small spillovers with regard to innovation as in Nordhaus (1972). Shorter patents can be accomplished by increasing patent breadth correspondingly. However, if the rate of spillover is large enough, Corollary 1 implies that patent breadth and length are complements. Shorter patents increase the inventive effort by discouraging the imitation. As a result, patent scope can be narrowed to lessen imitation costs, with patent length simultaneously adjusted downwards to retain the incentive to innovate.

These findings only slightly revise the claims in Gallini (1992) and Denicolò (1996) that patents should always be broad if the imitation is costly. In short, Proposition 2 restates the conclusion in Gallini (1992) that short patents disseminate the new ideas at less cost than costly imitation. In assessing the observations here, however, it should be borne in mind that as in Gallini (1992) the inference that short patents are typically optimal fails to imply that patent life should be zero. The optimal patent length $L(w)$, determined by constraint (11), is positive, because when the length is zero, constraint (11) is satisfied only when $\bar{\alpha} = V^{\mathrm{s}}/R$ with $\beta_{\mathrm{s}} = 1$. Notice also that the reward that guarantees patenting can be so great that society as a whole would do better with secrecy. The circumstances in which patenting is both socially and privately preferable thus need to be specified.

Analogously to $S_{\mathrm{p}}$ in (10), static welfare when the innovations are kept secrect is $S_{\mathrm{s}} = W^{\mathrm{m}} + \beta_{\mathrm{s}}(W^{\mathrm{d}} - W^{\mathrm{m}}) - \frac{1}{2}D_{\mathrm{s}}\beta_{\mathrm{s}}^{2}$, where $\beta_{\mathrm{s}}$ is given by (3). Clearly, society prefers patenting to secrecy only if $S_{\mathrm{p}} \geq S_{\mathrm{s}}$. This turns out to be equivalent to $[2(W^{\mathrm{d}} - W^{\mathrm{m}}]/[2(W^{\mathrm{d}} - W^{\mathrm{m}}) - \pi^{\mathrm{d}}] \geq \Psi(L, w)$. By (7), the patent policy can make a difference only if $\Psi(L, w) \geq 1$. An active patent policy thus has a rationale only if the desired reward $\bar{\alpha}$ is such that $[2(W^{\mathrm{d}} - W^{\mathrm{m}})]/[2(W^{\mathrm{d}} - W^{\mathrm{m}}) - \pi^{\mathrm{d}}] \geq \Psi(L, w) \geq 1$ holds. In other words, if the competition at the production stage is fierce, the patent system can hardly increase welfare.

## 6 Conclusion

The diffusion of new technology depends on imitation as much as invention. Patents affect imitation in opposite directions; while they disclose information, they also protect the original innovation from direct copying. One should model the imitation explicitly in studying the impact of patent systems on innovative activity and social welfare, and allow secrecy instead of a patent as an instrument of protection. Constructing a simple model where the imitator's optimization behavior determines the level of spillover I find that an increase in the breadth of the patent always strengthens the incentive to choose the patent protection. In contrast, the influence of increased patent life may be the reverse for large spillovers. A long patent life destroys the imitator's waiting option: the imitator can no longer just "wait and see" but is obliged to attempt to invent around the patent. As a result, investments in innovation may decrease with patent length under large spillovers.

Concerning policy, the result in Gallini (1992) and Denicolò (1996) about the optimality of short patents when social welfare is convex in patent breadth continues to hold. Incorporating rational imitation into the analysis, however, shows that length and breadth are substitutes as policy tools only for small spillovers. For large spillovers, they are complements so that both of them can be cut. The innovator's option of keeping the innovation secret, however, can stringently constrain the scope of efficient patent policy.

## Appendix[5]

### *1 Proof of Lemma 1*

Noting that $\beta = Z\pi^{\mathrm{d}}/D_{\mathrm{s}}$ and that $Z_w = -Z/w < 0$, the first and second derivatives of $\alpha$ with respect to $w$ can be given as $\alpha_w = Z\beta \cdot (\pi^{\mathrm{m}} - \pi^{\mathrm{d}})/R > 0$, and $\alpha_{ww} = -2\beta Z(\pi^{\mathrm{m}} - \pi^{\mathrm{d}})/wR < 0$. Taking the partial derivative of $S$ from (10) with respect to $w$ yields

$$\begin{aligned} S_w &= Z_w \frac{\pi^{\mathrm{d}}}{D_{\mathrm{s}}}[(1 - \mathrm{e}^{-rL})(W^{\mathrm{d}} - W^{\mathrm{m}}) - D_{\mathrm{p}}\beta] - \tfrac{1}{2}\beta^2 D_{\mathrm{s}} \\ &= -\frac{Z}{w}\frac{\pi^{\mathrm{d}}}{D_{\mathrm{s}}}[(1 - \mathrm{e}^{-rL})(W^{\mathrm{d}} - W^{\mathrm{m}}) - D_{\mathrm{p}}\beta + \tfrac{1}{2}D_{\mathrm{s}}w\beta] \\ &= -\frac{\beta}{w}[(1 - \mathrm{e}^{-rL})(W^{\mathrm{d}} - W^{\mathrm{m}}) - \tfrac{1}{2}K_{\mathrm{p}}\beta] \\ \iff \quad S_w &= -\frac{Z\beta}{2}[2(W^{\mathrm{d}} - W^{\mathrm{m}}) - \pi^{\mathrm{d}}] \,. \qquad \text{(A.1)} \end{aligned}$$

Similarly, $S_{ww} = -\beta Z_w[2(W^{\mathrm{d}} - W^{\mathrm{m}}) - \pi^{\mathrm{d}}]$. The signs of the deriva-

5 In Appendix 1 and 2, $\beta = \beta_{\mathrm{p}}$ and $S = S_{\mathrm{p}}$, and the subscripts $w$ and $L$ denote the derivatives.

tives $S_w$ and $S_{ww}$ are clearly determined by the sign of the term $[2(W^{\mathrm{d}} - W^{\mathrm{m}}) - \pi^{\mathrm{d}}]$. □

## 2 *Proof of Proposition 2*

Let $L(w)$ be the patent length which maintains the innovation activity at the required level defined by (11). The social value of existing innovation as a function of $w$ is now $S(w, L(w))$. Differentiating (11) gives

$$\mathrm{d}L/\mathrm{d}w = -\alpha_w/\alpha_L \ . \tag{A.2}$$

Take the total differential of $S(w, L(w))$ with respect to $w$ to obtain

$$\mathrm{d}S/\mathrm{d}w = S_w + S_L(\mathrm{d}L/\mathrm{d}w) \ . \tag{A.3}$$

Substitution of (A.2) into (A.3) implies

$$\begin{aligned} \frac{\mathrm{d}S}{\mathrm{d}w} &= S_w - S_L\frac{\alpha_w}{\alpha_L} \\ &= \frac{wS_w\alpha_w}{L\alpha_L}\left(\frac{\alpha_L L}{\alpha_w w} - \frac{S_L L}{S_w w}\right) = -\frac{wS_w\alpha_w}{L\alpha_L}(\xi^S - \xi^\alpha) \ , \end{aligned} \tag{A.4}$$

where $\xi^S$ and $\xi^\alpha$ are shorthands for the respective elasticity ratios. Recall that $S_w < 0$ by assumption and Lemma 1 indicates that $\alpha_w > 0$. Therefore, $-S_w\alpha_w w/L > 0$, and (A.4) shows that the sign of $\mathrm{d}S/\mathrm{d}w$ is equal to the sign of $(\xi^S - \xi^\alpha)/\alpha_L$. Let us next prove that $\xi^S > \xi^\alpha$. Differentiating $S$ with respect to $L$ yields

$$\begin{aligned} S_L &= r\mathrm{e}^{-rL}[W^{\mathrm{m}} + \beta(W^{\mathrm{d}} - W^{\mathrm{m}})] + r\mathrm{e}^{-rL}Z\frac{\pi^{\mathrm{d}}}{D_{\mathrm{s}}}(W^{\mathrm{d}} - W^{\mathrm{m}}) \\ &\quad - r\mathrm{e}^{-rL}W^{\mathrm{d}} - D_{\mathrm{p}}\beta r\mathrm{e}^{-rL}\frac{\pi^{\mathrm{d}}}{D_{\mathrm{p}}} \\ \iff \quad S_L &= -r\mathrm{e}^{-rL}[(W^{\mathrm{d}} - W^{\mathrm{m}})(1 - 2\beta) + \beta\pi^{\mathrm{d}}] \ . \end{aligned} \tag{A.5}$$

By means of (8), (9), (A.1), and (A.5) the condition $\xi^S > \xi^\alpha$ can be rearranged to

$$\frac{-2Lre^{-rL}[(W^{\mathrm{d}} - W^{\mathrm{m}})(1-2\beta) + \beta\pi^{\mathrm{d}}]}{-wZ\beta[2(W^{\mathrm{d}} - W^{\mathrm{m}}) - \pi^{\mathrm{d}}]} > \frac{RLre^{-rL}(\pi^{\mathrm{m}} - \pi^{\mathrm{d}})(1-2\beta)}{RwZ(\pi^{\mathrm{m}} - \pi^{\mathrm{d}})} .$$

Simplify this to obtain $[2(W^{\mathrm{d}} - W^{\mathrm{m}})]/[2(W^{\mathrm{d}} - W^{\mathrm{m}}) - \pi^{\mathrm{d}}] > 1 - \beta$. The sign of $\mathrm{d}S/\mathrm{d}w$ in (A.4) is thus given by the sign of $\alpha_L$. By Corollary 1, $\alpha_L > 0$ when $Z < Z^*$ and $\alpha_L < 0$ when $Z > Z^*$. Corollary 1 also demonstrates that $\mathrm{d}L/\mathrm{d}w$ in (A.2) is negative when $Z < Z^*$ and positive when $Z > Z^*$. Hence, patent life is always adjusted downward. □

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## *References*

Cadot, O., and Lippman, S. A. (1995): "Barriers to Imitation and the Incentive to Innovate." INSEAD Working Paper no. 23, Institut Européen d'Administration des Affaires, Fontainebleau.

Denicolò, V. (1996): "Patent Races and Optimal Patent Breadth and Length." *Journal of Industrial Economics* 44: 249–266.

Gallini, N. (1992): "Patent Policy and Costly Imitation." *RAND Journal of Economics* 23: 52–63.

Gilbert, R., and Shapiro, C. (1990): "Optimal Patent Length and Breadth." *RAND Journal of Economics* 21: 106–112.

Kanniainen, V., and Stenbacka, R. (1997): "Towards a Theory of Endogenous Imitation with Implications for Technology Policy." Mimeo, University of Helsinki, Helsinki.

Klemperer, P. (1990): "How Broad Should the Scope of a Patent Be?" *RAND Journal of Economics* 21: 113–130.

Kultti, K., and Niinimäki, J. (1998): "Demand Uncertainty in a Cournot Duopoly." *Finnish Journal of Business Economics* 47: 24–32.

Lambrecht, B. (1997): "Strategic Sequential Investments and Sleeping Patents." Mimeo, University of Cambridge, Cambridge.

Levin, R., Klevorick, A., Nelson, R., and Winter, S. (1987): "Appropriating the

Returns from Industrial Research and Development." *Brookings Papers on Economic Activity* 3: 783–820.

Mansfield, E., Schwartz, M., and Wagner, S. (1981): "Imitation Costs and Patents: an Empirical Study." *Economic Journal* 91: 907–918.

Nordhaus, W. (1969): *Invention, Growth and Welfare*. Cambridge, MA: MIT Press.

—— (1972): "The Optimal Life of the Patent: Reply." *American Economic Review* 62: 428–431.

Normann, H.-T. (1997): "Endogenous Stackelberg Equilibria with Incomplete Information." *Journal of Economics / Zeitschrift für Nationalökonomie* 66: 177–187.

Penrose, E. T. (1951): *The Economics of the International Patent System*. Baltimore: The John Hopkins Press.

Waterson, M. (1990): "The Economics of Product Patents." *American Economic Review* 80: 860–869.

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