A NON-ROTATING ORIGIN ON THE INSTANTANEOUS EQUATOR: DEFINITION, PROPERTIES AND USE

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Abstract. The exact description of the Earth's rotation raises the problem of the choice of a reference point on the instantaneous equator both in space and in the Earth. We propose to use, as the reference point in space, a 'non-rotating origin' (Guinot 1979) such that its hour angle, reckoned from the origin of the longitudes (or 'non-rotating origin' in the Earth), represents strictly the sidereal rotation of the Earth. Such an origin on the instantaneous equator depends only on the motion of the pole of rotation; it is practically realizable from a chosen fixed reference and we give the formulae to obtain it in space and in the Earth. We show that the estimation of the sidereal rotation is not critically affected by the precision with which the trajectory of the pole is known. We therefore propose a definition of the Universal Time which will remain valid even if the adopted model for the precession and the nutation is revised. We show that the use of the non-rotating origin also simplifies the transformation of coordinates between the terrestrial and celestial reference systems. An additional simplification of this transformation would be obtained when using, in the precession and nutation matrixes, the development of the celestial coordinates of the pole as function of time in place of the various usual equatorial and ecliptic parameters. The use of the non-rotating origin instead of the equinox would thus have advantages for both conceptual and practical reasons.

Résumé. La description exacte de la rotation terrestre pose le problème du choix d'un point de référence sur l'équateur instantané, aussi bien dans l'espace que dans la Terre. Nous proposons d'utiliser, comme point de référence dans l'espace, une origine 'non-tournante' (Guinot 1979) définie de sorte que son angle horaire, compté depuis l'origine des longitudes (ou origine 'non-tournante' dans la Terre), représente strictement la rotation sidérale de la Terre. Une telle origine ne dépend què du mouvement du pôle de rotation; elle est réalisable à partir d'une référence fixe choisie et nous donnons le formulaire permettant de l'obtenir, dans l'espace et dans la Terre. Nous montrons que la détermination de la rotation sidérale n'est pas affectée d'une manière critique par la précision avec laquelle la trajectoire du pôle est connue. En conséquence, nous proposons une définition du Temps Universel qui reste valide même quand on procède à une révision du modèle de la précession et de la nutation. Nous montrons que l'usage de l'origine non-tournante facilite aussi la transformation de coordonnées entre les systèmes terrestre et céleste. Une simplification supplémentaire de cette fransformation peut être obtenue en remplaçant les paramètres classiques de précession-nutation, par les coordonnées du pôle céleste dans l'espace. L'utilisation de l'origine non-tournante de manière générale, à la place de l'équinoxe, aurait ainsi des avantages à la fois de nature conceptuelle et pratique.

1. Introduction

The new techniques for measuring the orientation of the Earth in space, satellite or lunar laser ranging and very long baseline interferometry (VLBI), have brought a

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large gain of accuracy over conventional techniques: the uncertainties have been reduced by a factor 1/10 to 1/100 in the last ten years.

These techniques also have the advantages of using terrestrial and celestial reference frames which have much better intrinsic qualities than the plumb-lines and stellar catalogues of the optical astrometry. Now the Terrestrial Reference System can be attached to the figure of the Earth and to its center of mass, and has direct geodetic and geophysical significance. The Celestial Reference System is accurately realized by the coordinates of remote radio-sources, with a negligible residual rotation.

The traditional practice in astrometry does not always follow these improvements especially for the definition of the angle which expresses the motion of the Earth around its rotation axis.

The aim of this paper is to give an exact and clear conceptual definition of this angle and of the Universal Time UT1. But a conceptual definition has no value if it cannot be implemented with sufficient accuracy: the errors of 'realization' are thoroughly discussed.

In this work, the concept of the 'non-rotating origin' introduced by Guinot (1979) is used. The definition and properties of this reference point are investigated as well as its practical use in fundamental astronomy in reduction of the observations and in transformations between the Terrestrial and Celestial Reference Systems.

2. The Concepts

In this section we will deal only with conceptual definitions and with simple mathematical properties which are their consequences. Of course, the choice of these definitions is inspired by the possibility of realizing them. We will also give some notations used throughout this paper.

2.1. The Celestial Reference System

The Celestial Reference System, CRS, previously designated as the 'non-rotating reference system' by Guinot (1979), is designated in this paper in accordance with the terminology of the MERIT/COTES working group. It is defined as having the direction of its reference axes fixed among the directions of the most remote bodies of the universe, as seen from the barycenter of the solar system.

We can represent the CRS by a celestial sphere of center O with a fundamental great circle of pole $\mathscr{C}_0$ and an origin Σ_0 on this circle (Figure 1).

We will use the spherical polar coordinates of a point P: $d = \mathscr{C}_0 P$, $\mathrm{E} = \widehat{\Sigma_0 \mathscr{C}_0 P}$. Alternatively, we will use trirectangular coordinates: OX towards $O\Sigma_0$, OZ towards $O\mathscr{C}_0$, and OY completing the direct triad.

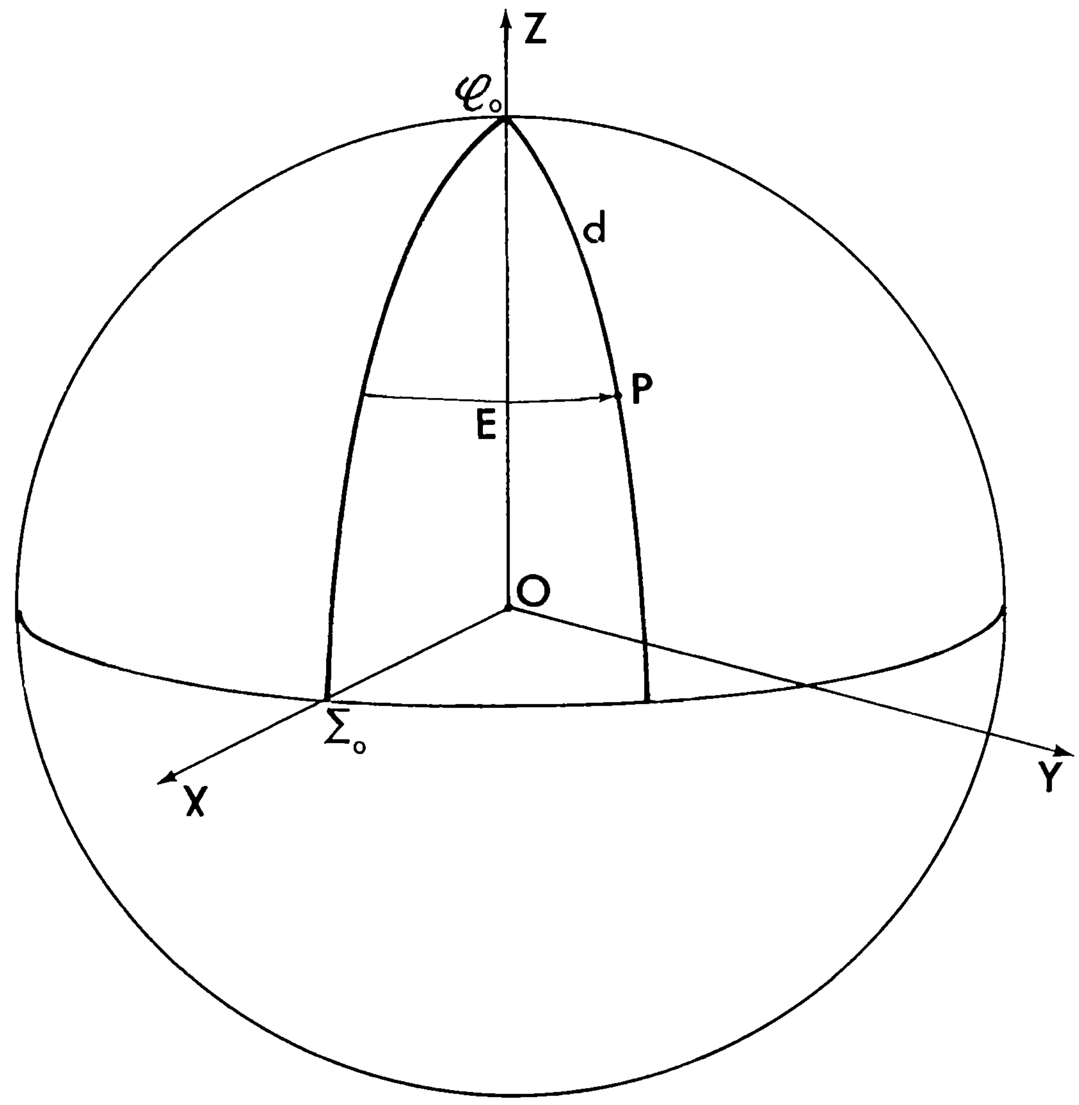

Fig. 1. The Celestial Reference System.

2.2. The Terrestrial Reference System

The Terrestrial Reference System, TRS, is defined by the condition that there is no net rotation or translation between the surface of the Earth and this reference system for an average over the whole surface of the solid Earth. This definition, although it is far from being satisfactory, is sufficient in the framework of the present study. The terrestrial directions can be represented on a celestial sphere of center O, with a fundamental great circle of pole R_0, and an origin (the longitude origin) Π_0 on this circle (Figure 2). We will use the polar coordinates of a point P: $g = R_0 P$, $F = \widehat{\Pi_0 R_0 P}$.

2.3. The Rotation of the Earth

Due to the diurnal rotation of the Earth, the orientation of the TRS with respect to the CRS is function of time. The intersection of the vector of instantaneous rotation of the Earth with the celestial sphere is the pole of instantaneous rotation P.

The rotation of the Earth can be described by the motions of P in the CRS and in the CTS and by the angular rotation of the CTS with respect to the CRS around OP.

The pole of instantaneous rotation is the one which logically agrees with the following theoretical developments. The consequences of the practical use of the Celestial Ephemeris Pole (CEP) instead of P will be considered in 5.2.

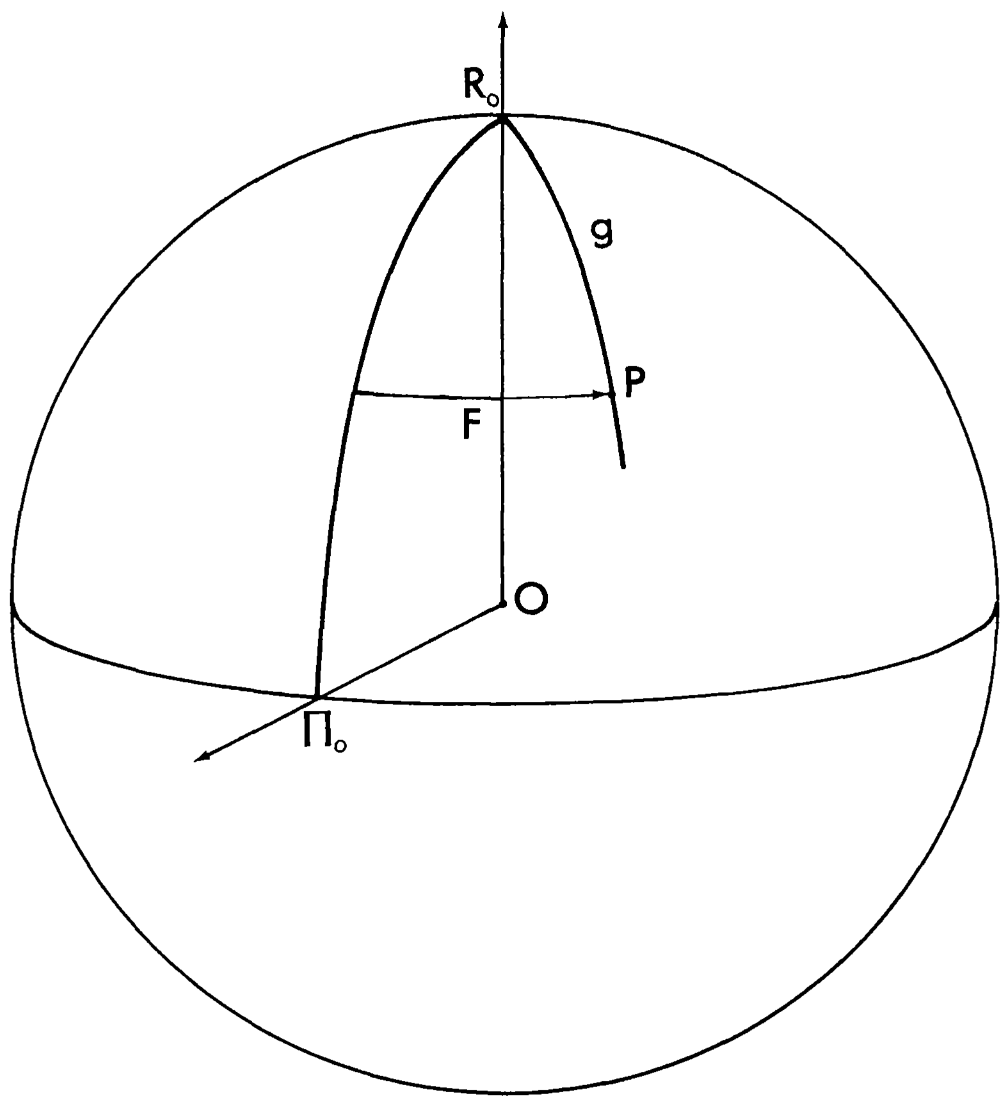

Fig. 2. The Terrestrial Reference System.

2.4. The Non-Rotating Origin

Let us consider a trirectangular coordinate system having its z-axis along OP and its x-axis along $O\,\sigma$, σ being an origin on the moving equator.

σ is kinematically defined in such a way that, as P moves in the CRS, $[Oxyz]$ has no component of instantaneous rotation with respect to the CRS around Oz (Guinot 1979, 1981). This definition is not sufficient, since the initial position of σ, at some epoch, is arbitrary. This matter will be considered in 2.5.

The point σ is designated as the non-rotating origin (NRO) on the moving instantaneous equator.

It is also useful to define a non-rotating origin ϖ in the TRS, in a similar manner, which will be the origin of the longitudes on the moving instantaneous equator of the Earth.

2.5. The Definition of the Quantity s

The position of σ at a date t depends, by definition, on the motion of P in the CRS.

Let us consider the positions P_0, P of the pole and σ_0, σ of the non-rotating origin respectively at epoch t_0 and date t. N_0 and N are the nodes of the equators of P_0 and P in the fundamental plane of $(\mathscr{C}_0, \Sigma_0)$ (Figure 3). To derive σ at the date t, it is necessary first to fix the origin σ_0 on the equator of epoch t_0 and then to take into account the whole history of the motion of P between t_0 and t.

Practically, σ would be obtained by giving the quantity $\Sigma_0 N$ and the quantity s as defined by:

$$s = (\sigma N - \Sigma_0 N) - (\sigma_0 N_0 - \Sigma_0 N_0), \tag{2.1}$$

$\sigma_0 N_0$ being arbitrarily chosen.

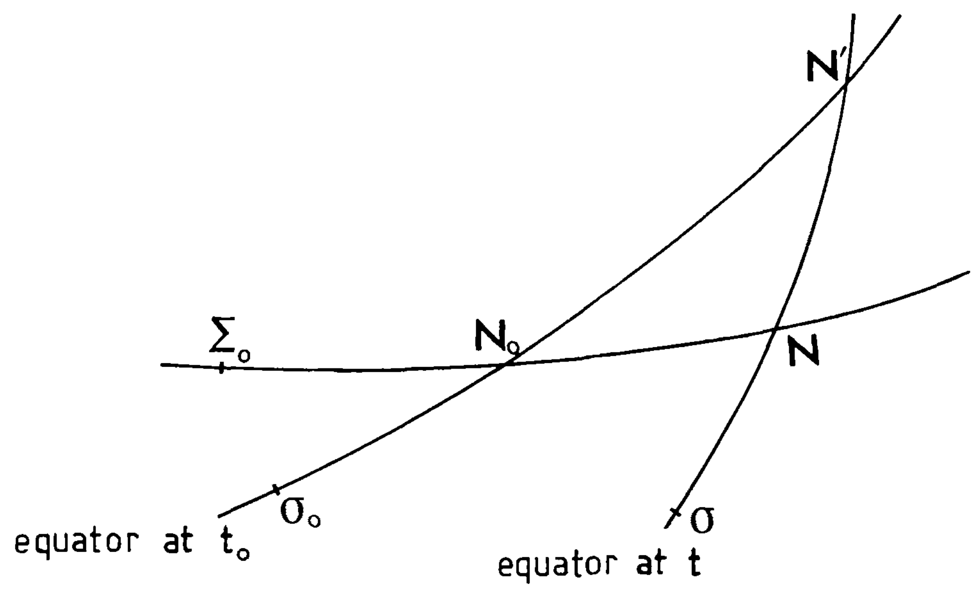

Fig. 3. Derivation of σ at a date t.

The expression of s can be obtained as follows by using the kinematical definition of the NRO.

If $\mathbf{n}$, $\mathbf{n}_0$ and $\mathbf{l}$ denote respectively the unit vectors along $\overrightarrow{OP}$, $\overrightarrow{O\mathscr{C}_0}$ and $\overrightarrow{ON}$ (figure 4), the instantaneous rotation vector between the $[Oxyz]$ system (as defined in 2.4) and the CRS due to the motion of P in the CRS is:

$$\mathbf{\Omega} = \dot{E}\mathbf{n}_0 - (\dot{E} + \dot{s})\mathbf{n} + \dot{d}\mathbf{l}.$$

with the polar coordinates E, d of P defined in 2.1 and the dot representing the time derivative.

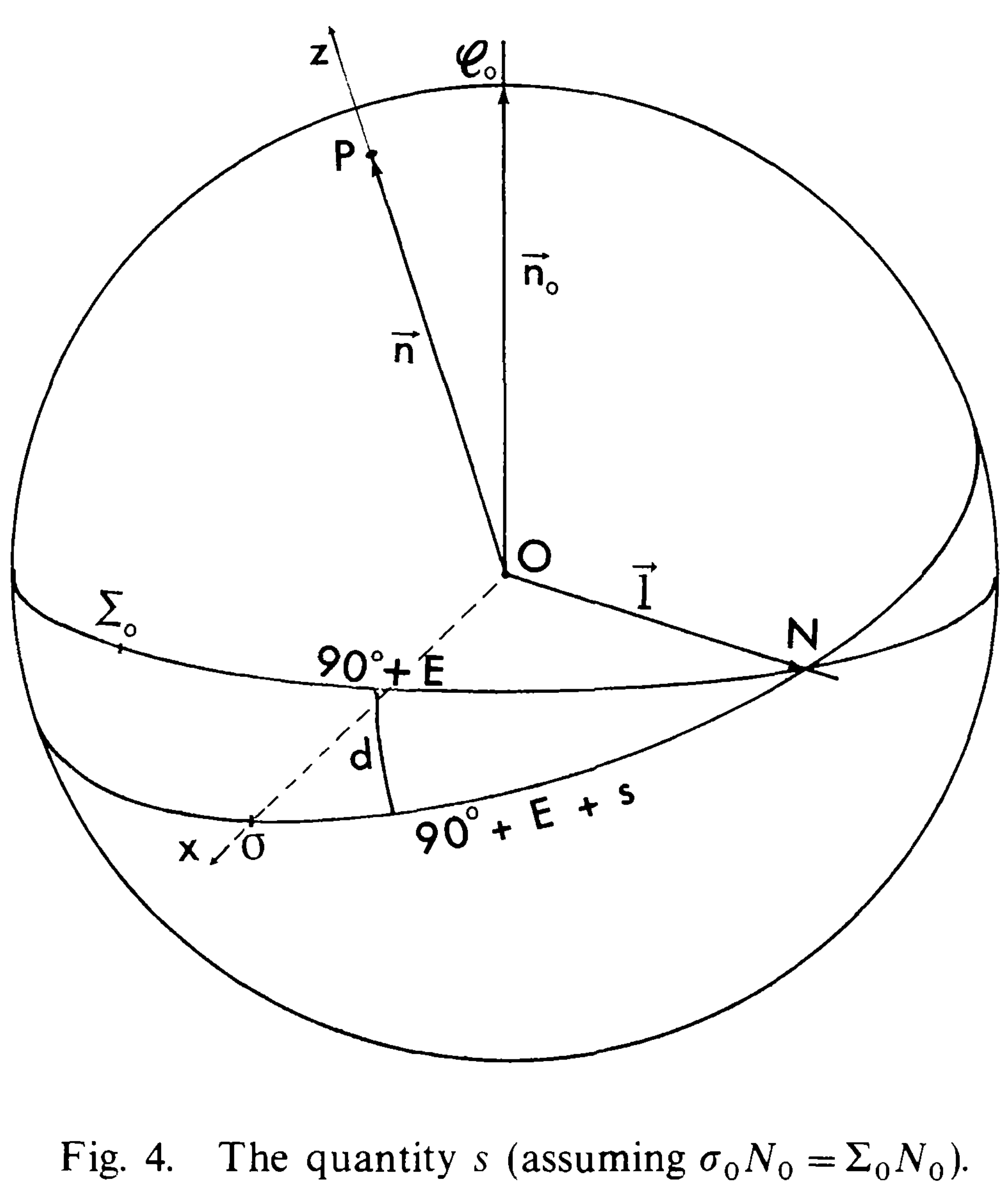

Fig. 4. The quantity s (assuming $\sigma_0 N_0 = \Sigma_0 N_0$).

The component of this rotation vector along Oz is:

$$\mathbf{\Omega}\cdot\mathbf{n}=\dot{E}\cos d-\dot{E}-\dot{s}=\dot{E}\ (\cos d-1)-\dot{s}.$$

The kinematical definition of σ gives: $\mathbf{\Omega}\cdot\mathbf{n}=0$ and then:

$$\dot{s}=\dot{E}\ (\cos d-1)$$

or

$$s=\int_{t_0}^{t}(\cos d-1)\ \dot{E}\ \mathrm{d}t-(\sigma_0N_0-\Sigma_0N_0). \tag{2.2}$$

Although s in (2.2) formally depends on the location of $\mathscr{C}_0$ with respect of P_0 and P, as long as σ_0 is known, σ is uniquely obtained whichever be the location of $\mathscr{C}_0$. This is obvious from the kinematical definition of the non-rotating origin, and it can be also shown trigonometrically. A consequence is that we can reckon d and E from P_0, designated as $\bar{d}$ and $\bar{E}$, so that we get:

$$\bar{s}=\sigma N'-\sigma_0N'=\int_{t_0}^{t}(\cos\bar{d}-1)\dot{\bar{E}}\,\mathrm{d}t,$$

N' being the node of the equator of P in the equator of P_0 (figure 3).

An alternate form of (2.2) is:

$$s=-\int_{t_0}^{t}\frac{(\mathbf{n}\wedge\dot{\mathbf{n}})\cdot\mathbf{n}_0}{1+\mathbf{n}\cdot\mathbf{n}_0}\mathrm{d}t-(\sigma_0N_0-\Sigma_0N_0), \tag{2.3}$$

or, in rectangular coordinates:

$$s=-\int_{t_0}^{t}\frac{(X\dot{Y}-Y\dot{X})}{1+Z}\,\mathrm{d}t-(\sigma_0N_0-\Sigma_0N_0). \tag{2.4}$$

The equations (2.2), (2.3) and (2.4) do not give any relation between Σ_0 and σ_0. The problem arises to obtain unambiguously σ at a date t from Σ_0. By convention we take:

$$\sigma_0N_0=\Sigma_0N_0. \tag{2.5}$$

Then, σ can be obtained from:

$$\sigma N-\Sigma_0N=\int_{t_0}^{t}(\cos d-1)\ \dot{E}\ \mathrm{d}t. \tag{2.6}$$

The concept of the non-rotating origin also applies to the TRS, using the formula corresponding to (2.3) and (2.4) for obtaining ϖ from Π_0 when P moves with respect to R_0. If M_0 and M are the nodes of the equators of P_0 and P in the equator of R_0 and ϖ_0 the position of ϖ at epoch t_0, we have (with the convention $\varpi_0 M_0 = \Pi_0 M_0$):

$$s' = \varpi M - \Pi_0 M = \int_{t_0}^{t} (\cos g - 1)\ \dot{F}\, dt. \tag{2.7}$$

2.6. Properties of the Quantity s

Some properties of the quantity s can be revealed by its preceding definition, either from its trigonometric expression (2.6) or from its vectorial expression (2.3).

The expression (2.6) shows that any circular motion of P in the CRS ($d = d_0$, $\dot{E} = e_0$, d_0 and e_0 being constants) generates a secular term in s, the coefficient of which is $(\cos d_0 - 1)\ e_0$.

In the case where the origin $\mathscr{C}_0$ is taken close to the pole P_0 at t_0, P_0P being usually small, a sufficient approximation for the vectorial expression (2.3) of s is:

$$s = -\frac{1}{2}\int_{t_0}^{t} (\mathbf{n} \wedge \dot{\mathbf{n}}) \cdot \mathbf{n}_0\, dt. \tag{2.8}$$

In the tangent plane at $\mathscr{C}_0$ to the celestial sphere (axes $\mathscr{C}_0\xi$, $\mathscr{C}_0\eta$, parallel to OX and OY),
with $\mathbf{P} \equiv \overrightarrow{\mathscr{C}_0 \mathrm{P}}$, $\dot{\mathbf{P}} \equiv \dot{\overrightarrow{\mathscr{C}_0 \mathrm{P}}}$ (figure 5),

$$(\mathbf{n} \wedge \dot{\mathbf{n}}) \cdot \mathbf{n}_0 = (\mathbf{P} \wedge \dot{\mathbf{P}}) \cdot \mathbf{n}_0,$$

and thus s can be written as:

$$s = -\frac{1}{2}\int_{t_0}^{t} (\mathbf{P} \wedge \dot{\mathbf{P}}) \cdot \mathbf{n}_0\, dt. \tag{2.9}$$

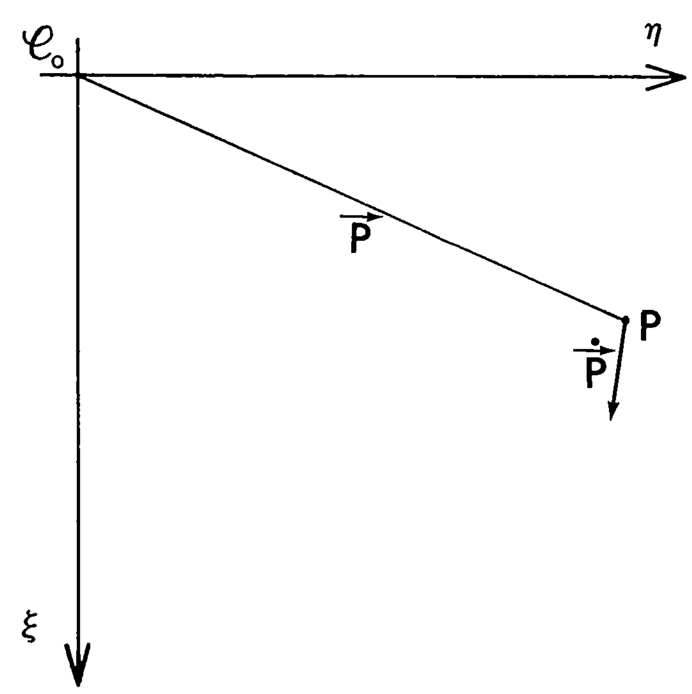

Fig. 5. The motion of P in the CRS.

Such an expression can be applied to the motion of P as a sum of $(n+1)$ components:

$$\mathbf{P}=\sum_{i=0}^{n}\mathbf{P}_i \tag{2.10}$$

$\mathbf{P}_i$ $(i=0,\ldots,n)$ representing the luni-solar precession and the n terms usually retained for the nutation, therefore:

$$s=-\frac{1}{2}\int_{t_0}^{t}\sum_{i=0}^{n}(\mathbf{P}_i\wedge\dot{\mathbf{P}}_i)\cdot\mathbf{n}_0\,\mathrm{d}t-\frac{1}{2}\int_{t_0}^{t}\sum_{i=0}^{n}\sum_{\substack{j=0\\ j\neq i}}^{n}(\mathbf{P}_i\wedge\dot{\mathbf{P}}_j)\cdot\mathbf{n}_0\,\mathrm{d}t. \tag{2.11}$$

It is interesting to note that to the sum of $(n+1)$ terms giving the effect of the precession and of each component of the nutation, taken independently of each other, are added the $n(n+1)$ cross terms.

This illustrates that it is not possible, when dealing with rigorous formulae, to separate simply the motion of the mean pole (precession) and the nutation. These complexities are not a specific drawback of the non-rotating origin: they affect the equinox as well.

2.7. The Stellar Angle and Instantaneous Ascension

Let θ be the hour angle $\widehat{\varpi 0 \sigma}$ of the non-rotating origin from the prime meridian $P\varpi$. This angle, reckoned positive westward, is designated (Guinot 1979) as the stellar angle. It expresses the sidereal rotation of the Earth and its time derivative gives directly the angular velocity of the Earth.

In the instantaneous equatorial system, one of the coordinates of a celestial body is the usual declination δ. The other coordinate is reckoned from σ, instead of the equinox with the same sign convention as the right ascension α. This new coordinate is designated as the 'instantaneous ascension' and denoted by A.

2.8. Relation between the Stellar Angle and the Sidereal Time

The Greenwich Apparent Sidereal Time (GST) is the hour angle of the true equinox from the prime meridian $P\varpi$. This angle is reckoned positive westward as is the angle θ. So the relation between GST and θ is:

$$\mathrm{GST}=\varpi\gamma=\theta+\alpha(\sigma). \tag{2.12}$$

$\alpha(\sigma)$ being the right ascension of the non-rotation origin σ.

The Greenwich Mean Sidereal Time (GMST) is defined as the Greenwich Apparent Sidereal Time corrected for the 'equation of the equinoxes'. This last term is used here, following Aoki and Kinoshita (1983), in a wider sense than generally accepted, for the complete periodic part of the right ascension (denoted $[\alpha(\gamma_m)]_p$) of the 'mean equinox', γ_m, referred to the true equinox and equator (Woolard &

Clemence 1966).

$$\text{So: GST} = \text{GMST} + [\alpha(\gamma_m)]_p \tag{2.13}$$

$$\text{Or: GST} = \text{GMST} + \alpha(\gamma_m) - [\alpha(\gamma_m)]_s = \text{GMST} + \alpha(\sigma) + A(\gamma_m) - [\alpha(\gamma_m)]_s$$

$[\alpha(\gamma_m)]_s$ being the secular part of the right ascension $\alpha(\gamma_m)$ and $A(\gamma_m)$ the instantaneous ascension (cf 2.7) of the 'mean' equinox γ_m.

$$\text{Thus: } \theta = \{\text{GMST} - [\alpha(\gamma_m)]_s\} + A(\gamma_m). \tag{2.14}$$

2.9. UT1

The definition of UT1, as recommended by IAU (Aoki *et al.* 1982) is not a conceptual definition but a conventional one (Xu *et al.* 1986), which gives a relationship between the mean sidereal time and UT1 to be used with the FK5-based astronomical reference system.

The definition does not correspond to a clear concept, as it refers both to the non-rotating origin and to the equinox and as the time argument is a mixture between UT1 and the Barycentric Dynamical Time TDB (see 6.1 thereafter).

A more convenient conceptual definition of UT1 has thus to be given which can be extended to possible non-rotating systems of reference in space in which the equinox could not be determined.

Following the definition given in 2.7, the stellar angle expresses the sidereal rotation of the Earth. UT1 should be defined (Guinot 1979) as an angle proportional to θ. The origin of UT1 is chosen so that 12h of UTI coincides approximately, at some epoch t_0, with the Greenwich mean noon and the coefficient of proportionality is chosen so that a day of UT1 in the long term remains approximately in phase with the mean solar day (this last condition cannot be strictly fulfilled on account of the precession).

With such a definition, UT1 strictly increases linearly with time if the rotation of the Earth is uniform.

3. Transformation from the CRS to the TRS

Following classical notations (Lambeck 1973, Kinoshita *et al.* 1979, Zhu and Mueller 1983), the transformation of the CRS to the TRS can be expressed as a product of rotation matrixes:

$$[\text{TRS}] = S(t)\cdot N(t)\cdot P(t)\cdot D(t_0)\,[\text{CRS}], \tag{3.1}$$

$D\ (t_0)$ being the matrix of rotation from the CRS to the system $[x_0\ y_o\ z_0]$ of the epoch t_0 and P being the matrix of rotation for precession, N for nutation and S for Earth rotation (including polar motion).

3.1. Classical Expression of the Transformation

In the classical representation (Mueller 1981), the precession matrix P expresses the transformation between the CRS and a 'mean equator and equinox frame' for the considered date t. It is given by:

$$P = R_3(-z_A)R_2(\theta_A)R_3(-\zeta_A), \tag{3.2}$$

using the usual equatorial parameters z_A, ζ_A, θ_A and their conventional representation (Lieske *et al.* 1977) for the date t.

The nutation matrix N expresses the transformation between the preceding mean celestial frame and the 'true equator and equinox frame' of the same date t.

It is given by:

$$N = R_1(-\varepsilon - \Delta\varepsilon)R_3(-\Delta\Psi)R_1(\varepsilon), \tag{3.3}$$

where ε is the mean obliquity of the ecliptic at the date t and $\Delta\varepsilon$, $\Delta\Psi$ respectively the nutation in obliquity and ecliptic longitude referred to the mean ecliptic of date, using the conventional representation (Wahr 1981, Seidelmann 1982) corresponding to the Celestial/Ephemeris Pole (CEP).

The Earth rotation matrix expresses the transformation from the preceding true frame to the TRS.

It is given by:

$$S = R_2(-x_p)R_1(-y_p)R_3(\mathrm{GST}), \tag{3.4}$$

where GST is the Apparent Sidereal Time, or rotational angle of the axis $O\varpi$ with respect to the first axis of the preceding 'true celestial frame', measured in the equator of the CEP; x_p, y_p are the coordinates of this pole with the usual convention (x_p towards $O\Pi_0$, y_p towards the longitude 90° W).

The Apparent Sidereal Time, GST, can be deduced from the conventional relationship between the Greenwich mean sidereal time, GMST, and UT1 at the date t (Aoki *et al.* 1982) and from the transformation (2.13) between the mean and the true sidereal time. The modified 'equation of the equinoxes' (Woolard 1953, Aoki and Kinoshita 1983) has to be used in this transformation in order to be consistent with the rotation matrixes (3.1) to (3.4).

3.2. Expression of the Transformation Using the Non-Rotating Origin

The use of the non-rotating origin σ, as conceptually defined in section 2, avoids the intermediate reference to the moving equinox of date and allows to clearly express the sidereal rotation of the Earth through the stellar angle θ.

The transformation matrix from the CRS to the TRS using this new reference σ, in place of the equinox γ, can be expressed as the product: $S'N'P'$, each of these matrixes being deduced from the corresponding one S, N or P by referring the involved parameters to σ.

The precession and nutation matrixes N', P', N, P are such that:

$$N'(t)P'(t) = R_3[\alpha(\sigma)]N(t)P(t),$$

so, the relation (2.12) between the stellar angle θ and the sidereal time GST gives the following relation between the transformations using the true equinox or the NRO:

$$R_3(\mathrm{GST})N(t)P(t) = R_3(\theta)N'(t)P'(t). \tag{3.5}$$

In fact as it has appeared in 2.6., the precession and nutation should not be separated: all the informations on these motions are contained in the displacement of the celestial pole in the CRS.

The transformation matrix to be used is thus:

$$[\mathrm{TRS}] = R_2(-x_p)R_1(-y_p)R_3(\theta)NP(t)\,[\mathrm{CRS}] \tag{3.6}$$

$$\text{with: } NP\;(t) = R_3(-s)\,R_3(-E)R_2(d)R_3(E). \tag{3.7}$$

The quantity s, appearing in the NP matrix, can be deduced from the motion of the celestial pole between the epoch t_0 and the date t. It can be taken into account by its development as function of time corresponding to the conventional representation of this motion (see Section 4).

The conventional series for the coordinates of the celestial pole in the CRS are thus the only necessary developments to be used in order to compute the transformation matrix NP as given by (3.7) at the date t. Their use (in place of the various usual ecliptic and equatorial parameters) together with the use of the NRO (in place of the moving equinox) would greatly simplify the practical transformation (3.1) of the CRS to the TRS.

4. Development of *s* as a Function of Time

This section is an application of the concepts of section 2, to the conventional representation of the celestial and terrestrial motions of the pole of rotation.

The full literal development of s, with the conventionally retained terms of the nutation would be very complex, but even at a much higher level of accuracy than actually needed, only a few terms have to be retained. These terms have small amplitudes so that their dependence on the model of the motion of the pole of rotation has nearly no measurable consequence, as it will be shown in 5.2.

4.1. Approximate Development of *s*

In order to evaluate the order of magnitude of the components of s, and to give an insight on its behaviour, we establish first an approximate development, using the properties of 2.6. In fact as we will see later, this development is correct up to the approximation of $10^{-3\prime\prime}$.

We will assume that the current position of the pole at t is P, reckoned from the position of the mean pole at the date $t_0 = 0$. The effects of an offset of the pole at $t_0 = 0$ are subsequently considered in (f).

The celestial displacement of P along the axes $P\xi$ (directed towards the equinox) and $P\eta$ (directed in the opposite direction of the ecliptic pole) is the sum of the precession represented here with a sufficient approximation by:

$$\begin{cases}\xi_0 = ct & (c = 2004''/c) \\ \eta_0 = -et^2 & (e = 22.41''/c^2),\end{cases} \tag{4.1}$$

and of n terms of nutation, each of these terms being:

$$\begin{cases}\xi_i = a_i & \sin(\omega_i t - \varphi_i) \\ \eta_i = b_i & \cos(\omega_i t - \varphi_i).\end{cases} \tag{4.2}$$

(We neglect the small change of orientation of the nutation axes when the mean pole moves).

Therefore, using (2.11), s can be written as the sum:

$$s = s_{0,0} + \sum_{i=1}^{n} (s_{0,i} + s_{i,0} + s_{i,i}) + \sum_{i=1}^{n} \sum_{j=i+1}^{n} (s_{i,j} + s_{j,i}), \tag{4.3}$$

$$\text{with:} \quad s_{i,j} = -\frac{1}{2}\int_0^t (\xi_i \dot{\eta}_j - \eta_i \dot{\xi}_j)\,\mathrm{d}t. \tag{4.4}$$

In the following, the units are the arc second and the julian century of dynamical time TDB. The evaluation uses the numerical coefficients in obliquity and ecliptic longitude tabulated in the 1980 IAU theory of nutation corrected for the pole of instantaneous rotation. The origin of t is therefore J2000.0, but the numerical development of s has little sensitivity to this origin, except for the constant term.

(a) PRECESSION

The component $s_{0,0}$ due to the precession can be written from (4.4) as:

$$s_{0,0} = \frac{1}{6}\,ect^3. \tag{4.5}$$

(b) NUTATION

Using (4.4), one finds easily that the component $s_{i,i} (i \neq 0)$ for each nutation is:

$$s_{i,i} = +\frac{1}{2}\omega_i a_i b_i t. \tag{4.6}$$

(c) CROSS TERMS PRECESSION × NUTATION

For each nutation, (4.4) leads to a sum of cross terms with precession such that:

$$\begin{aligned} s_{0,i} + s_{i,0} = {} & b_i c\left\{\frac{1}{\omega_i}\sin(\omega_i t - \varphi_i) - \frac{t}{2}\cos(\omega_i t - \varphi_i)\right\} \\ & - a_i e\left\{-\frac{2}{\omega_i^2}\sin(\omega_i t - \varphi_i) + \frac{2t}{\omega_i}\cos(\omega_i t - \varphi_i) + \frac{t^2}{2}\sin(\omega_i t - \varphi_i)\right. \\ & \left. + \left(\frac{b_i c}{\omega_i} + 2\frac{a_i e}{\omega_i^2}\right)\sin\varphi_i\right\} \end{aligned} \tag{4.7}$$

(d) CROSS TERMS NUTATION × NUTATION

For two nutations respectively represented by ξ_i, η_i and ξ_j, η_j, the cross terms are:

$$s_{i,j} + s_{j,i} = \frac{1}{4}(a_i b_j - a_j b_i)\frac{\omega_i - \omega_j}{\omega_i + \omega_j}\{\sin[(\omega_i + \omega_j)t - \varphi_{ij}] + \sin\varphi_{ij}\},$$

$$+\frac{1}{4}(a_i b_j + a_j b_i)\frac{\omega_i + \omega_j}{\omega_i - \omega_j}\{\sin[(\omega_i - \omega_j)t - \Psi_{ij}] + \sin\Psi_{ij}\}, \tag{4.8}$$

with: $\varphi_{ij} = \varphi_i + \varphi_j$, $\Psi_{ij} = \varphi_i - \varphi_j$.

Thus, the nutation cross terms are sine waves in s with angular velocities $\omega_i + \omega_j$ and $\omega_i - \omega_j$. All these terms are very small. Only three ones have an amplitude included between $5 \times 10^{-6}{}''$ and $5 \times 10^{-5}{}''$ and will be considered in the precise development of s (cf. 4.2).

(e) NUMERICAL DEVELOPMENT

Using the expressions given in (a), (b), (c), (d), the approximative development of s is up to $5 \times 10^{-4}{}''$ after a century:

$$s = 0.036'' t^3 + 0.004'' t - 0.003''(\sin\Omega - \sin\Omega_0)$$
$$-0.045'' t\cos\Omega - 0.003'' t\cos 2\odot, \tag{4.9}$$

Ω being the mean tropic longitude of the Moon's node, Ω_0 its value at t_0 and $\odot$ the mean tropic longitude of the Sun.

The diurnal celestial motion of P (sway) would have a negligible contribution, even in the precise development of s (see 4.2), and will not be considered.

(f) COMPLEMENTARY TERMS DUE TO AN OFFSET OF THE POLE AT $t_0 = 0$

The coordinates of P being:

$$\begin{cases} \xi = u + \xi_0 + \sum_{i-1}^{n} \xi_i \\ \eta = v + \eta_0 + \sum_{i=1}^{n} \eta_i, \end{cases} \tag{4.10}$$

the complementary terms of s are:

$$s = \frac{1}{2}\text{uet}^2 + \frac{1}{2}\text{vct} - \frac{1}{2}u\sum_{i=1}^{n} b_i\cos(\omega_i t - \varphi_i) + \frac{1}{2}v\sum_{i=1}^{n} a_i\sin(\omega_i t - \varphi_i). \tag{4.11}$$

With, for instance, $u = v = 0.01''$, the only non-negligible term is:

$$\frac{1}{2}\text{vct} = 5 \times 10^{-5}{}'' t. \tag{4.12}$$

(g) COMPLEMENTARY TERMS DUE TO THE 'POLAR MOTION' WITH RESPECT TO R_0

The same computations can be made for the motion of P with respect to the TRS.

We assume that this 'polar motion' is the sum of components:

$$\mathbf{p} = \mathbf{p}_s + \mathbf{p}_c + \mathbf{p}_a + \sum_{i=0}^{n} \mathbf{p}_i \tag{4.13}$$

$\mathbf{p}_s$ representing the secular drift of rate $0.3''/c$, $\mathbf{p}_c$ the Chandlerian wobble of amplitude lower than $0.5''$, $\mathbf{p}_a$ the annual wobble of amplitude $0.10''$, and $\Sigma_{i=0}^{n} \mathbf{p}_i$ the diurnal nutations (P being the pole of instantaneous rotation).

The only corresponding s' terms which may exceed $10^{-5''}$ after one century, are:

$$s'_{aa} = -0.00\,002''t, \; s'_{cc} \text{ such that } |s'_{cc}| < 0.00\,032''t, \tag{4.14}$$

for the non-predictable part, and:

$$s'(\text{diurnal nutation}) = 0.00\,008''t \tag{4.15}$$

$$(s'_{oo} = 0.00004''t, s'_{3131} = 0.00003''t, s'_{gg} = 0.00001''t).$$

4.2. Development of s Including all Terms up to $5 \times 10^{-6''}$ after One Century

The development of s is established with the assumption that $\mathscr{C}_0$ and Σ_0 coincide respectively with the mean pole and mean equinox of epoch J2000.0. The precession quantities are based on the IAU (1976) System of Astronomical Constants (Lieske *et al.* 1977), and we adopt the 1980 IAU theory of nutation (Seidelmann 1982).

The current theory of nutation gives the coordinates of the Celestial Ephemeris Pole (CEP). In principle, we should use here the coordinates of the pole of instantaneous rotation. Although there are some advantages in using the coordinates of the CEP, as it is shown in section 5, we have made the necessary corrections in order to give s computed for the pole of instantaneous rotation, in addition to its development for the CEP (only six terms are different at the considered level of accuracy).

The basic quantities $\sin d \cos E$, $\sin d \sin E$ and $\cos d$ can be written as:

$$\begin{cases} \sin d \sin E = -\sin\varepsilon_0 \cos(\omega_A + \Delta_1\varepsilon) + \cos\varepsilon_0 \sin(\omega_A + \Delta_1\varepsilon)\cos(\Psi_A + \Delta_1\Psi) \\ \sin d \cos E = \sin(\omega_A + \Delta_1\varepsilon)\sin(\Psi_A + \Delta_1\Psi) \\ \cos d = \cos\varepsilon_0 \cos(\omega_A + \Delta_1\varepsilon) + \sin\varepsilon_0 \sin(\omega_A + \Delta_1\varepsilon)\cos(\Psi_A + \Delta_1\Psi) \end{cases} \tag{4.16}$$

ε_0 being the obliquity of the ecliptic at epoch t_0, $\omega_A + \Delta_1\varepsilon$ the inclination of the instantaneous equator of date on the ecliptic of epoch t_0, $\Psi_A + \Delta_1\Psi$ the ecliptic longitude, along the ecliptic of epoch t_0, of the ascending node of the instantaneous equator in this fixed ecliptic.

The approximation $1 + \cos d = 2$, which can be shown to give an error on s lower than $5 \times 10^{-6''}$ after one century, has been made in the computation.

The expression of s, accurate up to $5 \times 10^{-6''}$ after one century, can then be written as (with t reckoned from J2000.0) and with the same notations as in formula (3.3):

$$s = \frac{\sin\varepsilon_0}{2}\int_0^t [(\omega_A - \varepsilon_0 + \Delta\varepsilon)(\dot{\Psi}_A + \Delta\dot{\Psi}) - (\Psi_A + \Delta\Psi)(\dot{\omega}_A + \Delta\dot{\varepsilon})]\, dt$$

$$+ \frac{\sin^2\varepsilon_0 \cos\varepsilon_0}{12}[(\Psi_A + \Delta\Psi)^3]_0^t. \tag{4.17}$$

The theoretical developments of the precession and nutation quantities in function of time as:

$$\omega_A = \varepsilon_0 + \varepsilon_2 t^2 + \cdots, \quad \Psi_A = \Psi_1 t + \Psi_2 t^2 + \cdots$$

$$\Delta\varepsilon = \sum_{i=1}^{n} b_i \cos(\omega_i t - \varphi_i), \quad \Delta\Psi \sin\varepsilon_0 = \sum_{i=1}^{n} a_i \sin(\omega_i t - \varphi_i)$$

yield then, a development of s similar to (4.3).

Table I gives the development of s as function of time including all the terms with an amplitude exceeding $5 \times 10^{-6\prime\prime}$ after one century. In this development, Ω is the mean tropic longitude of the Moon's node, ☾ and ☉ are respectively the mean tropic longitudes of the Moon and of the Sun, and p and p_s are respectively the mean tropic longitudes of the lunar perigee and of the perihelion.

The expression of s as given in Table I shows that the approximate development (4.9) gives in fact a complete development of this quantity up to $10^{-3\prime\prime}$ after one century. All the additional terms which appear in Table I, except three ones, would have appeared in the approximate development if performed up to $5 \times 10^{-6\prime\prime}$ after one century.

The development as given in Table I has to be used for s in the transformation matrix (3.7) in order that the angle θ expresses the specific rotation of the Earth along the moving equator with a $10^{-5\prime\prime}$ accuracy after one century. If an accuracy of only $10^{-3\prime\prime}$ after one century is needed, it is sufficient to use the expression (4.9).

5. Effects of the Errors on the Realization of the Reference Systems and on the Model of the Trajectory of the Pole of Rotation

5.1. Realization of the CRS and of the TRS

The realization of the ideal system $(\mathscr{C}_0, \Sigma_0)$ may have a residual rotation Γ in space (for instance due to the errors of the proper motions of a stellar reference frame). In this case, the instantaneous equatorial frame attached to the NRO would have a residual rotation around its polar axis, which is the projection of Γ on this axis. To this respect, the NRO has no advantage when compared to the equinox.

Let $(\mathscr{C}'_0, \Sigma'_0)$, $(\mathscr{C}''_0, \Sigma''_0)$ be two realizations of the CRS, P be the instantaneous pole of rotation at the date t and σ', σ'' the corresponding positions of the NRO in the

TABLE I
Development of s up to $5 \times 10^{-6\prime\prime}$

The development is computed for the coordinates of the Celestial Ephemeris Pole. When the coordinates of the pole of instantaneous rotation are used, some amplitudes differ slightly: they are given between parentheses.

Origin of the term	n° of nutation	Terms of s in $10^{-5\prime\prime}$, t in centuries			Period in days
precession			$+3629\,t^3$		
nutation	all	(+394)	$+\ 385\,t$		
precession × nutation	1	(−4472)	$-\ 265$	sin Ω (*)	6798.4
			$-4471\,t$	cos Ω	6798.4
			$-\ 37\,t^2$	sin Ω	6798.4
			$+\ 1\,t^2$	cos Ω	6798.4
	2		$+\ 1$	sin 2Ω	3399.2
			$+\ 43\,t$	cos 2Ω	3399.2
	9	(−277)	$-\ 279\,t$	cos 2☉	182.6
			$-\ 3\,t^2$	sin 2☉	182.6
	31	(−44)	$-\ 47\,t$	cos 2☾	13.7
	11		$-\ 11\,t$	cos (3☉ − p_s)	121.7
	33	(−9)	$-\ 10\,t$	cos (2☾ − Ω)	13.6
	34		$-\ 6\,t$	cos (3☾ − p)	9.1
	12		$+\ 5\,t$	cos (☉ + p_s)	365.2
	13		$+\ 3\,t$	cos (2☉ − Ω)	177.8
	36	(+2)	$+\ 3\,t$	cos (☾ + p)	27.1
	10		$-\ 3\,t$	cos (☉ − p_s)	365.3
	38		$+\ 2\,t$	cos (☾ − p + Ω)	27.7
	39		$-\ 2\,t$	cos (−☾ + p + Ω)	27.4
	41		$-\ 1\,t$	cos (3☾ − p − Ω)	9.1
	40		$-\ 1\,t$	cos (3☾ + p − 2☉)	9.6
	3		$+\ 1\,t$	cos (2p − Ω)	1305.5
	42		$-\ 1\,t$	cos (4☾ − 2☉)	7.1
	45		$-\ 1\,t$	cos (4☾ − 2p)	6.9
	44		$+\ 1\,t$	cos (☾ + 2☉ − p)	23.9
nutation × nutation	9,16 / 10,12		$+\ 1$	sin 2p_s	3.8×10^6
	1,9		$-\ 1$	sin (2☉ − Ω)	177.8
	1,2 / 31,33		$+\ 1$	sin Ω (*)	6798.4
+Constant	(−s at $t = 0$)				

(*) To be added to the other term of same argument.

equator of P. If there is no residual rotation in space between these two realizations of the CRS, the kinematical definition of the NRO shows that the time derivative $d(\sigma'\sigma'')/dt$ is equal to 0. $(\sigma'\sigma'')$ is thus constant whatever be the motion of P. As at $t = t_0$, σ' and σ'' are coincident with σ_0 on the equator of P_0, this constant is equal to 0. The position of σ on the equator of P is thus non-dependent on the choice of the CRS provided it has no residual rotation in space.

A similar conclusion also applies to the TRS: the position of $\bar{\omega}$ on the equator of P is non-dependent on the choice of the TRS provided it has no residual rotation with respect to the Earth.

5.2. Change of Model of the Pole Trajectory

5.2.1. Theory

Let us consider two models denoted 'a' and 'b'. At date t, P_a and P_b are the corresponding positions of the pole in the CRS, σ_a and σ_b are the NRO's on their equators (figure 6). The question arises to know whether the change-over of system Oxyz at date t, previously linked to σ_a, then to σ_b, introduces a finite rotation ρ of this system around its Oz axis:

$$\rho = \sigma_b \nu - \sigma_a \nu, \tag{5.1}$$

ν being the ascending node of the equator of P_a in the equator of P_b.

As:
$$\begin{cases} s_a = \sigma_a N_a - \Sigma_0 N_a \\ s_b = \sigma_b N_b - \Sigma_0 N_b, \end{cases} \tag{5.2}$$

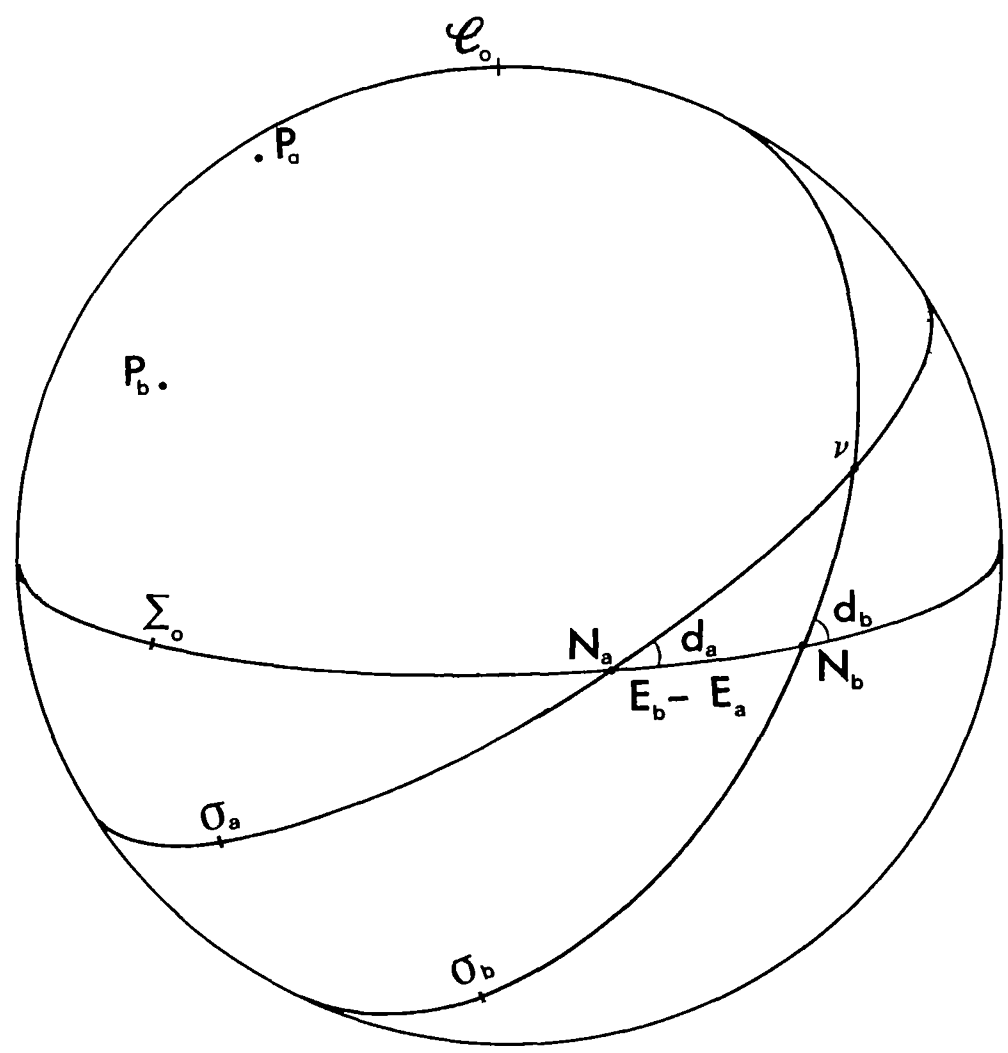

Fig. 6. Change in the model of the pole trajectory.

N_a and N_b being the nodes represented on figure 6, one finds easily:

$$\rho = (s_b - s_a) + (N_a N_b + N_b v - N_a v). \tag{5.3}$$

Using the trigonometrical expression (2.6) of s, we can write:

$$s_b - s_a = \int_{t_0}^{t} [\dot{E}_b(\cos d_b - 1) - \dot{E}_a(\cos d_a - 1)]\, dt.$$

In the spherical triangle N_aN_bv, the angles in N_a and N_b are respectively d_a and $180° - d_b$, N_aN_b is $E_b - E_a$, and we have the following trigonometric relation:

$$\mathrm{tg}\frac{1}{2}(N_bv - N_av) = -\mathrm{tg}\frac{1}{2}(E_b - E_a)\frac{\cos\frac{1}{2}(d_a + d_b)}{\cos\frac{1}{2}(d_b - d_a)}.$$

Starting from now, we will assume that P_a and P_b are sufficiently close to $\mathscr{C}_0$, so that terms in d^4 can be neglected. One finds:

$$N_a N_b + N_b v - N_a v = \frac{1}{2} d_a d_b \sin(E_b - E_a) = \frac{1}{2}(\mathbf{P}_a \wedge \mathbf{P}_b)\cdot \mathbf{n}_0$$

and:

$$\dot{\rho} = \frac{1}{2}[-\dot{E}_b d_b^2 + \dot{E}_a d_a^2 + (d_a\dot{d}_b + \dot{d}_a d_b)\sin(E_b - E_a) + d_a d_b(\dot{E}_b - \dot{E}_a)\cos(E_b - E_a)] \tag{5.4}$$

or equivalenty:

$$\rho = -\frac{1}{2}\int_{t_0}^{t} [\overrightarrow{P_aP_b} \wedge (\dot{\mathbf{P}}_a + \dot{\mathbf{P}}_b)]\cdot \mathbf{n}_0\, dt, \tag{5.5}$$

$\mathbf{n}_0$ being the unit vector toward $\mathscr{C}_0$, as previously.

We can also write: $\rho = (s_b - s_a) + \frac{1}{2}(\mathbf{P}_a \wedge \mathbf{P}_b)\cdot \mathbf{n}_0$, (5.6) which shows that for a change in the amplitude of a periodic motion, $\rho = s_b - s_a$ since $\mathbf{P}_a \wedge \mathbf{P}_b = 0$.

5.2.2. Application to the motion of the pole in the CRS

Henceforward, P_a will be the ideal position of the pole of instantaneous rotation, and P_b the position given by an erroneous model. ρ represents a spurious rotation which is reduced by the adoption of a new and better model and is, therefore, the maximum rotation introduced by a change of model for P_b at date t.

Let us assume that the model of the error $P_a\ P_b$ is:

$$\Delta\xi = u + kt + \sum_{i=1}^{n} \Delta a_i \sin(\omega_i t - \varphi_i)$$

$$\Delta\eta = v - k'ct^2 + \sum_{i=1}^{n} \Delta b_i \cos(\omega_i t - \varphi_i),$$

c being, as previously, the precession in declination, Δa_i, Δb_i, the errors on the nutation coefficients, k and k' small angular velocities, scaling the precession error.

The main terms of ρ are, for reasonable values of k and $k'(<0.1''/\text{century})$:

$$\rho = \text{vct} + \sum_{i=1}^{n} \omega_i \int_{t_0}^{t} [b_i \Delta a_i \sin^2(\omega_i t - \varphi_i) + a_i \Delta b_i \cos^2(\omega_i t - \varphi_i)]\, dt.$$

For instance with:

$u = v = 0.01''$,

$\Delta a_i = \Delta b_i = 0.01''$ (on the 5 larger terms),

the contributions on ρ, larger than $10^{-5''}$, are:

shift of the pole (vct):	$10 \times 10^{-5''}(t - t_0)$
term $n^\circ 1$ of nutation:	$2 \times 10^{-5''}(t - t_0)$
term $n^\circ 9$ of nutation:	$3 \times 10^{-5''}(t - t_0)$
term $n^\circ 31$ of nutation:	$8 \times 10^{-5''}(t - t_0)$

If $\dot{\mathbf{P}}_a$ and $\overrightarrow{P_a P_b}$ are colinear, as it would nearly occur in a readjustment of the general constant of precession, ρ is null, from (5, 5).

5.2.3. Application to the motion of the pole in the TRS

ρ' represents a spurious rotation due to a change in the adopted polar motion. The contribution of the secular drift on ρ' appears to be negligible and for the periodic terms, we have:

$$\rho' = s'_b - s'_a.$$

For instance, if the polar motion is ignored, $s'_b = 0$, and the contribution of its components have already been considered in 4.1 (g). The contribution of the Chandlerian motion may not be negligible. The effects of diurnal terms either due to an error of model for P_b, or due to the diurnal nutation are considered thereafter in 5.2.4 and 5.2.5.

5.2.4. Application to the determination of θ (and of UT1)

Let us represent the CRS and the TRS by two concentric spheres of unit radius. The relative orientation of the two spheres is given observationally by the location of terrestrial directions among celestial directions. The position of the rotation pole P in the CRS is, in general, given by a model. The corresponding position p in the TRS is therefore obtained.

For the evaluation of the stellar angle θ, the origins σ and ϖ on the moving equator are obtained from the computation of s and s' respectively from the motion of P in the CRS and of p in the TRS.

(a) Case where p_b corresponds strictly to P_b (diurnal motions of p_b taken into account):

Instead of the ideal stellar angle θ_a, derived from s_a and s'_a computed for the motions of P_a and p_a respectively in the CRS and the TRS, an erroneous value is computed from s_b and s'_b, and

$$\Delta\theta = \theta_b - \theta_a = \rho - \rho', \tag{5.7}$$

with

$$\rho' = -\frac{1}{2}\int_{t_0}^{t} [\overrightarrow{p_a p_b} \wedge (\dot{\mathbf{p}}_a + \dot{\mathbf{p}}_b)] \cdot \mathbf{n}_0 \, dt. \tag{5.8}$$

The effects of a small difference of the orientations of $\mathbf{n}_0$ and $\mathbf{n}'_0$ are neglected.

A property of the pole of instantaneous rotation is the equality of its velocity vector in the CRS and the TRS.

We have:

$$\overrightarrow{p_a p_b} = \overrightarrow{P_a P_b}, \ \dot{\mathbf{p}}_a = \dot{\mathbf{P}}_a, \ \dot{\mathbf{p}}_b = \dot{\mathbf{P}}_b - \Omega \wedge \overrightarrow{P_a P_b}$$

Ω being here the rotation vector of the TRS. Then

$$\Delta\theta = -\frac{1}{2}\int_{t_0}^{t} \Omega (P_a P_b)^2 \, dt. \tag{5.9}$$

$\Delta\theta$ may become important. For instance, with $P_a P_b = 0.1'' (t - t_0)$,

$$\Delta\theta = -0.002'' \, (t - t_0)^3.$$

(b) Case where the pole p'_b in the TRS is p_b freed from its diurnal motions.

This case is more realistic since the diurnal terms are usually not observed, and p'_b may be closer to the ideal p_a than p_b. One must evaluate separately ρ for the error $\overrightarrow{P_a P_b}$, and ρ', for $\overrightarrow{p_a p'_b}$. The equators of P_b and p'_b are different, but close enough so that (5.7) still helds. The error $\Delta\theta$ is strongly reduced when compared to the case (a).

5.2.5. Consequences of the adoption of the CEP instead of the instantaneous pole of rotation

As the concept of the NRO implies the use of the pole of instantaneous rotation, the use of the CEP can be represented as an error, with the main component given by:

$$\begin{cases} \xi_{CEP} - \xi_p = -0.0031'' \sin(\omega_9 t - \varphi_9) - 0.0071'' \sin(\omega_{31} t - \varphi_{31}) \\ \eta_{CEP} - \eta_p = -0.0087'' + 0.0029'' \cos(\omega_9 t - \varphi_9) + 0.0066'' \cos(\omega_{31} t - \varphi_{31}) \end{cases} \tag{5.10}$$

with:

$$\omega_9 = 1.26 \times 10^3 \quad \text{(semi-annual term)}$$
$$\omega_{31} = 1.68 \times 10^4 \quad \text{(semi-monthly term)}.$$

The spurious rotation due to the use of CEP are:

- in the CRS: $\rho = -0.00013''(t - t_0)$

(the quasi-periodic terms of $s_{CEP} - s_P$ being cancelled by the second term of ρ in (5.6)).

- in the TRS (assuming that the terms of the forced diurnal nutation have been used when computing s'),

$\rho' = -0.00\,008''\ (t - t_0)$,

- on θ, $\Delta\theta = -0.00005''\ (t - t_0)$.

5.2.6. PRACTICAL IMPLICATIONS, PROPOSALS.

(a) READJUSTMENT OF THE MODELED TRAJECTORY OF THE POLE IN THE CRS; EFFECTS ON THE UT1 DEFINITION.

As shown in the preceding sections, there is a good probability that, for $(t - t_0) <$ one century, it is sufficient to recompute s from the epoch with the new model, taking into account the convention given in 2.5 for the integration constant.

However, if the transition from the old to the new non-rotating origin brings a significant rotation around the polar axis (either due to a large $t - t_0$, or to an important shift of the pole model, or to an increased need of accuracy), at the date t_1 of change of model, it is always possible to impose:

$$\sigma_a \nu_1 = \sigma_b \nu_1,$$

where ν_1 is the node of the new equator in the old equator at t_1.

Using one or the other of these methods, it is, in particular, possible to keep a fixed relation between the stellar angle and UT1, without introducing a step of UT1 and its derivative, the duration of the day, at the instant of the changeover. In contrast, the continuity of UT1 imposes a change of the relation between the Greenwich mean Sideral time and UT1 when the motion of the pole in re-modelized.

(b) USE OF THE CELESTIAL EPHEMERIS POLE

As long as the CEP is conventionally adopted instead of the pole of instantaneous rotation, we suggest that s be computed from the motion of the CEP, in order to locate the NRO, σ, on the equator of the CEP. The spurious rotation thus introduced has a constant rate and is very slow (and can be taken into account when necessary, see Table II). Furthermore, the use of the CEP avoids to compute the effects on s' of the diurnal nutation in the TRS.

(c) MOTIONS OF THE POLE WITH RESPECT TO THE EARTH

These motions should be taken into account when computing s' and consequently θ and UTI. The corresponding sway in space has negligible effects.

TABLE II

Effects of the adoption of the CEP in place of the pole of instantaneous rotation P. The exact value of s is computed for P. The Table gives the error $\Delta s = s_{CEP} - s_P$, the rotations ρ and ρ' and the error $\Delta\theta$ on the stellar angle. It gives also the value of s' (with the exception of the terms of geophysical origin which are the same in both cases) to be used for the motion of the above poles in the TRS. All terms larger than $5 \times 10^{-6}{}''$ are retained and t is in centuries.

	Pole of instantaneous rotation	Celestial Ephemeris Pole
	unit: $1 \times 10^{-5}{}''$	unit: $1 \times 10^{-5}{}''$
Δs	0	$+1\,t\cos\Omega - 2\,t\cos 2\odot - 3\,t\cos 2☾$ $-1\,t\cos(2☾ - \Omega) + 1\,t\cos(☾ + p)$
s'	$+\ 8\,t$	0
ρ	0	$-13\,t$
ρ'	0	$-\ 8\,t$
$\Delta\theta$	0	$-\ 5\,t$

6. Relation between the Stellar Angle and UT1

We have proposed in 2.9 to define UT1 as being proportional to the stellar angle θ. The purpose of this section is to give a numerical relation between θ and UT1, which would lead as an implementation, in the case of the IAU 1976 system of astronomical constants and the FK5 system, to the conventional relation between the Greenwich mean sideral time GMST and UT1 (Aoki *et al.* 1982).

6.1. The Genesis of the Current GMST/UT1 Relation

This relation is based on the concept that UT1 should be proportional to the sidereal rotation of the Earth, and thus to the stellar angle, but some approximations have been introduced in its development.

One should have:

$$\theta = \theta_0 + kT_U, \tag{6.1}$$

where: $T_U = [\text{UT1} - (\text{UT1})_0]/36525$ (6.2)

including the number of days elapsed since $(\text{UT1})_0 = \text{JD}2451545.0$ UT1 (2000 January 1, 12h UT1).

In the relation (2.14) between θ and GMST, $A(\gamma_m)$ is obtained by:

$$A(\gamma_m) = -\int_0^t m_{\gamma_m}\,dt. \tag{6.3}$$

t being the dynamical time reckoned from the epoch J2000.0 = JED 2 451 545.0 TDB, and m_{γ_m} the component of the instantaneous rotation of the 'mean equator and equinox frame' (as defined in 3.1), with respect to the CRS, along the Oz axis. One can write, the nutation in declination being $\Delta\theta_A$:

$$m_{\gamma_m} = m \cos \Delta\theta_A,$$

and:

$$\theta = \text{GMST} - [\alpha(\gamma_m)]_s - \int_0^t m \cos \Delta\theta_A \, dt \tag{6.4}$$

where m is the speed of precession in right ascension. The secular term $[\alpha(\gamma_m)]_s$, given by Aoki and Kinoshita (1983) as $(\Delta q)_s$, is also the linear term of $-s$ due to the nutation evaluated in 4.2 (adopting the value for the CEP):

$$[\alpha(\gamma_m)]_s = -0.00385'' \, t = -0.000257 \text{ st} \equiv A_s t.$$

As, with a sufficient approximation:

$$\int_0^t m \cos \Delta\theta_A \, dt = \int_0^t m \, dt = At + Bt^2 + Ct^3, \tag{6.5}$$

the coefficients A, B, C being obtained from the numerical expression of m given by Lieske *et al.* (1977),

$$\text{GMST} = \theta_0 + kT_U + (A + A_s)t + Bt^2 + Ct^3. \tag{6.6}$$

However, it was decided to neglect the difference between t and T_U and to write:

$$\text{GMST} = \theta_0 + (k + A + A_s)T_U + BT_U^2 + CT_U^3. \tag{6.7}$$

(The departure between (6.6) and (6.7) should not exceed 1×10^{-5}s during the 21st century).

θ_0 and $k + A + A_s$ of (6.7) have been chosen in order to:

(1) be consistent with the origin of right ascensions (equinox) of the FK5,
(2) maintain the continuity of UT1 as determined by the observation of the FK4/FK5 stars, both in value and rate, at the epoch of change of catalogue and of IAU constants (in 1984 January 1st).

6.2. The θ/UT1 Relation

From the numerical expression of (6.5) and (6.7), the latter being derived from the conventional formula giving GMST and 0h UT1, one gets, with $t = T_U$,

$$\theta = 0.779\,057\,273\,264 + 1.002\,737\,811\,911\,354 \text{ (UT1} - 2000 \text{ January 1, 12h UT1)}, \tag{6.8}$$

UT1 being expressed in days and θ in revolutions, or equivalently:

$$\theta = 24\,110.548\,41\text{s} + 8639\,877.317\,38\text{s } T_U, \tag{6.9}$$

↑ at 0h UT1

where T_U is reckoned from JD = 2451 545.0 (in UT1) in Julian centuries of UT1, and has therefore the form $(n + 0.5)/36525$, n being an integer.

As explained in section 5, this expression can be kept, with no appreciable step of UT1 and of d (UT1)/dt, in case:

- of a change of model for precession and/or nutation,
- of a readjustment of the pole among the celestial objects realizing the CRS.

But, if the changeover from an old realization of the CRS to a new one involves a relative rotation rate between the two frames of reference, then the rate of d(UT1)/dt would change.

The full advantage of the NRO will be taken only when the frame of reference will be realized by positions of quasars (Guinot 1986).

7. Conclusions

The non-rotating origin on the instantaneous equator has been conceptually defined in order to give an exact and clear description of the Earth's rotation. Although this non-rotating origin cannot be directly observed, its motion in the Celestial Reference System (CRS) depends simply and solely on the motion of the rotation pole which is observable and its position can therefore be computed in the CRS at any date t.

The cross terms between the luni-solar precession and the nutation, which must be taken into account in a description of the Earth's rotation with an accuracy matching the precision of modern measurements, appear here in a clear way thanks to the concept of the non-rotating origin.

This concept applied to the Terrestrial Reference System (TRS) clarifies the definition of the origin of the longitudes in the moving equator and reveals some terms due to the Chandler motion which may not be negligible if a precision of 0.0001″ in the angle of rotation of the Earth is required.

The realization of the non-rotating origin depends on the model of the pole trajectory. However, we have shown that the possible errors in the recent model for this pole trajectory do not bring any appreciable spurious rotation around the polar axis of the instantaneous equatorial system linked to the non-rotating origin. It is therefore possible to give a simple and invariable definition of the Universal Time as an angle proportional to the angle which exactly expresses the motion of the Earth around its rotation axis.

The non-rotating origin, used jointly with the pole coordinates in the CRS (instead of the classical development of the precession and the nutation), avoids complexities in the transformation of coordinates between the CRS and the TRS.

The additional knowledge of the coordinates of the pole of the ecliptic in the CRS, with low accuracy, provides all which is needed for computing apparent positions.

Our conclusion is that the concept of the non-rotating origin:

(a) should be employed when dealing with the Earth's rotation and in particular in order to give a definition of the Universal Time which is well adapted to the new methods of observation,

(b) could in a general manner replace advantageously the equinox especially for the observations which are not sensitive to the orientation of the ecliptic.

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