AN ANALYTIC MODEL FOR UPPER ATMOSPHERE DENSITIES BASED UPON JACCHIA'S 1970 MODELS

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Abstract. An analytic model is presented which gives upper atmospheric densities as a function of the exospheric temperature and the altitude. The densities produced are identical to those produced by Jacchia's 1970 models (1970) for altitudes between 90 and 125 km and closely approximate Jacchia's values for altitudes greater than 125 km.

1. Introduction

In 1964, Jacchia published a set of models of the upper atmosphere. These models assumed constant boundary conditions at 120 km and were designed to produce atmospheric densities consistent with existent satellite drag data. Actually, both temperature and density experience considerable variation at 120 km. In spite of this obvious shortcoming Jacchia's 1964 model remained one of the better models available until the present time.

In order to determine the density at a given altitude for a given exospheric temperature, Jacchia's procedure required the numerical integration of the diffusion equation for each atmospheric constituent or the interpolation in two variables of extensively tabulated data. In 1965, Walker presented a minor modification to Jacchia's expression for the temperature profile. This modification made the diffusion differential equations exact differentials which, when integrated, provided an analytic representation for the density of the upper atmosphere. Many computer programs in use today throughout the country which require atmospheric density subroutines include as an option Walker's analytic approximation to Jacchia's 1964 model.

Recently, Jacchia (1970) published a new set of models, which assume constant boundary conditions at 90 km. All available atmospheric observations do indicate that atmospheric conditions have minimum variation at 90 km. Instead of one expression for the temperature profile, Jacchia employs two-one valid for altitudes between 90 and 125 km and a second valid above 125 km. It is apparent that the temperature equation valid between 90 and 125 km can be determined explicitly instead of being defined implicitly as it is by Jacchia. The resulting temperature expression can be substituted into the barometric differential equation, as mixing is assumed to be predominant from 90 to 105 km, and integrated by partial fractions yielding an analytic expression for the atmospheric density valid for altitudes between 90 and 105 km. Using the resulting terminal conditions at 105 km as initial conditions, the temperature expression can be substituted into the diffusion differential equation, as diffusive equilibrium is assumed above 105 km, and integrated by partial fractions yielding an analytic expression for the atmospheric density valid for altitudes between 105 and 125 km. The author has been able to modify Jacchia's temperature equation which is valid above 125 km in a manner similar to the way in which Walker modified Jacchia's 1964 temperature equation. That is, to modify the temperature equation in such a manner that upon substitution into the diffusion differential equation an exact differential results. Integration using the initial conditions obtained for 125 km then results in an analytic expression for the atmospheric density which is valid for altitudes greater than 125 km.

In this report an analytical model of the upper atmosphere, as outlined above, is developed which provides atmospheric density as a function of the exospheric temperature and the altitude. This model produces densities which are identical to those produced by Jacchia's 1970 model for altitudes between 90 and 125 km and which closely approximate Jacchia's values for altitudes between 125 and 1000 km. The model is developed in sufficient mathematical detail to permit the reader to produce his own computer routines based upon the text. The primary general assumption made is that the exospheric temperature, T_∞, is calculated using the equations presented in Jacchia (1970).

2. Temperature Profile for Altitudes between 90 and 125 km

Jacchia (1970, p. 9) defines the temperature profile for altitudes, Z, between 90 and 125 km by the fourth-degree polynomial:

$$T(Z) = T_x + \sum_{n=1}^{4} c_n (Z - Z_x)^n \tag{1}$$

where T_x, the temperature at the inflection point, $Z_x = 125$ km, is defined as the following function of the exospheric temperature, T_∞:

$$T_x = a + bT_\infty + c \exp(dT_\infty), \tag{2}$$

where $a = 444.3807$, $b = 0.02385$, $c = -392.8292$, and $d = -0.0021357$. The coefficients c_i are to be determined by satisfying the following conditions:

(i) For $Z = Z_0 = 90$ km

$$T(Z_0) = T_0 = 183\,K \tag{3}$$

and

$$(\mathrm{d}T/\mathrm{d}Z)_{Z=Z_0} = 0. \tag{3$'$}$$

(ii) For $Z = Z_x = 125$ km

$$(\mathrm{d}T/\mathrm{d}Z)_{Z=Z_x} = 1.90 \frac{(T_x - T_0)}{(Z_x - Z_0)} \tag{4}$$

and

$$(\mathrm{d}^2T/\mathrm{d}Z^2)_{Z=Z_x} = 0. \tag{4$'$}$$

Solving for c_i it is seen that $c_1 = 1.90\ (T_x - T_0)/(Z_x - Z_0)$, $c_2 = 0$, $c_3 = -1.70\ (T_x - T_0)/(Z_x - Z_0)^3$, and $c_4 = -0.80\ (T_x - T_0)/(Z_x - Z_0)^4$.

Hence

$$T(Z) = T_x + F \sum_{n=0}^{4} C_n Z^n \tag{5}$$

where $F = (T_x - T_0)/35^4$

$C_0 = -\ 89284375.0$

$C_1 = \ \ 3542400.0 \text{ km}^{-1}$

$C_2 = -\ \ 52687.5 \text{ km}^{-2}$

$C_3 = \ \ 340.5 \text{ km}^{-3}$

$C_4 = -\ \ 0.8 \text{ km}^{-4}$.

3. Molecular Mass Profile for Altitudes between 90 and 125 km

From 90 to 105 km Jacchia (1970, p. 4) uses the following empirical formula for the mean molecular mass, M:

$$M(Z) = \sum_{n=0}^{6} a_n (Z - 100)^n \tag{6}$$

where

$a_0 = \ \ 28.15204$

$a_1 = -\ 8.5586 \times 10^{-2} \text{ km}^{-1}$

$a_2 = \ \ 1.2840 \times 10^{-4} \text{ km}^{-2}$

$a_3 = -\ 1.0056 \times 10^{-5} \text{ km}^{-3}$

$a_4 = -\ 1.0210 \times 10^{-5} \text{ km}^{-4}$

$a_5 = \ \ 1.5044 \times 10^{-6} \text{ km}^{-5}$

$a_6 = \ \ 9.9826 \times 10^{-8} \text{ km}^{-6}$.

Equation (6) may be rewritten as:

$$M(Z) = \sum_{n=0}^{6} A_n Z^n \tag{7}$$

where

$A_0 = \ \ 83809.05064$

$A_1 = -\ 5196.932946 \text{ km}^{-1}$

$A_2 = \ \ 134.0855452 \text{ km}^{-2}$

$A_3 = -\ 1.842006056 \text{ km}^{-3}$

$A_4 = \ \ 1.421149 \times 10^{-2} \text{ km}^{-4}$

$A_5 = -\ 5.83912 \times 10^{-5} \text{ km}^{-5}$

$A_6 = \ \ 9.9826 \times 10^{-8} \text{ km}^{-6}$.

4. Atmospheric Density for Altitudes between 90 and 105 km

From 90 to 105 km mixing is assumed to prevail and the density, ϱ, is assumed to satisfy the barometric differential equation:

$$\mathrm{d}(\ln \varrho) = \mathrm{d}(\ln (M/T)) - Mg\,\mathrm{d}Z/RT\,, \tag{8}$$

where g is the acceleration due to gravity and $R=8.31432$ Joules/K-mole is the universal gas constant. The initial conditions assumed at $Z_0=90$ km are

$$\varrho_0 = \varrho\,(Z_0) = 3.46 \times 10^{-9}\ \mathrm{gm/cm^3}$$

and (8′)

$$T_0 = T\,(Z_0) = 183K\,.$$

The acceleration due to gravity at an altitude Z may be adequately approximated by the expression:

$$g = g_0 R_a^2/(Z + R_a)^2 \tag{9}$$

where $g_0=9.80665$ m/s^2 and $R_a=6356.766$ km. Substituting (9) into (8) and integrating results in:

$$\varrho\,(Z) = \varrho\,(Z_0)\,\frac{M\,(Z)\,T\,(Z_0)}{M\,(Z_0)\,T\,(Z)}\exp\left(k\int\limits_{Z_0}^{Z}\frac{f M\,(Z)\,\mathrm{d}Z}{(Z+R_a)^2\,P\,(Z)}\right) \tag{10}$$

where $k= -g_0/R\,(T_x - T_0)$; $f= 35^4\,R_a^2/C_4$; and $C_4\,P(Z) = \sum_{n=0}^{4} c_n^* Z^n$ $c_0^* = 35^4\,[1 + T_0/(T_x - T_0)] + C_0$ and $c_i^* = C_i$ for $1 \le i \le 4$.

At least for $300\,\mathrm{K} \leqslant T_x \leqslant 500\,\mathrm{K}$ ($600\,\mathrm{K} \leqslant T_\infty \leqslant 2000\,\mathrm{K}$) the polynominal $P(Z)$ has two real, positive, and unequal roots, r_1 and r_2, and two complex (non-real) roots, $r_3=x+iy$ and $r_4=x-iy$ ($y>0$). As T_x varies from 300 K to 500 K, r_1 varies from 167.77 to 164.42, r_2 from 57.34 to 65.29, x from 100.26 to 97.96, and y from 32.40 to 22.92.

Thus

$$I = \int\limits_{Z_0}^{Z}\frac{f M\,(Z)\,\mathrm{d}Z}{(Z+R_a)^2\,P\,(Z)} = f A_6\,(Z - Z_0) + \\ + \int\limits_{Z_0}^{Z}\frac{S\,(Z)\,\mathrm{d}Z}{(Z+R_a)^2\,(Z-r_1)\,(Z-r_2)\,(Z^2 - 2xZ + x^2 + y^2)}, \tag{11}$$

where

$$S\,(Z) = \sum_{n=0}^{5} B_n Z^n,$$

$$B_0 = \left(A_0 - \frac{R_a^2 A_6}{C_4}\,c_0^*\right) f$$
$$= (A_0 + 5.04227040425412\,c_0^*)\,f\,,$$

$$B_1 = \left(A_1 - (2c_0^* + R_a C_1)\,\frac{R_a A_6}{C_4}\right) f$$
$$= (17856541.74708379 + 0.00158642630679\,c_0^*)\,f\,,$$

$$B_2 = \left(A_2 - (2C_1 + R_aC_2)\frac{R_aA_6}{C_4} - \frac{A_6}{C_4}c_0^*\right)f$$
$$= \left(-259910.779829766 - \frac{A_6}{C_4}c_0^*\right)f,$$
$$B_3 = \left(A_3 - (C_1 + 2R_aC_2 + R_a^2C_3)\frac{A_6}{C_4}\right)f$$
$$= -1.236944975267146 \times 10^{17},$$
$$B_4 = \left(A_4 - (C_2 + 2R_aC_3 + R_a^2C_4)\frac{A_6}{C_4}\right)f$$
$$= 2.642300254604438 \times 10^{14}, \quad \text{and}$$
$$B_5 = \left(A_5 - (C_3 + 2R_aC_4)\frac{A_6}{C_4}\right)f$$
$$= 9.740305355778338 \times 10^{10}.$$

The integrand appearing in Equation (11) may be written as the following sum of partial fractions:

$$\frac{p_1}{Z + R_a} + \frac{p_2}{Z - r_1} + \frac{p_3}{Z - r_2} + \frac{2p_4(Z - x)}{Z^2 - 2xZ + x^2 + y^2} + $$
$$+ \frac{p_5}{(Z + R_a)^2} + \frac{p_6}{Z^2 - 2xZ + x^2 + y^2} \tag{12}$$

where

$$p_2 = S(r_1)/U(r_1),$$
$$p_3 = S(r_2)/-U(r_2),$$
$$p_5 = S(-R_a)/V(-R_a),$$
$$U(v) = (v + R_a)^2 (v^2 - 2xv + x^2 + y^2)(r_1 - r_2),$$
$$V(v) = (v^2 - 2xv + x^2 + y^2)(v - r_1)(v - r_2),$$
$$p_4 = (B_0 - r_1r_2R_a^2(B_4 + (2x + r_1 + r_2 - R_a)B_5) -$$
$$- r_1r_2R_a(x^2 + y^2)B_5 + r_1r_2(R_a^2 - (x_2 + y_2))p_5 +$$
$$+ W(r_1)p_2 + W(r_2)p_3)/X,$$
$$W(v) = r_1r_2R_a^2(v + R_a) + (x^2 + y^2)R_a(R_av + r_1r_2),$$
$$X = -2r_1r_2R_a(R_a^2 + 2xR_a + x^2 + y^2),$$
$$p_6 = B_4 + (2x + r_1 + r_2 - R_a)B_5 - p_5 - 2(x + R_a)p_4 -$$
$$- (r_2 + R_a)p_3 - (r_1 + R_a)p_2$$

and

$$p_1 = B_5 - 2p_4 - p_3 - p_2.$$

Upon integration we have for $90 < Z \leqslant 105$:

$$\varrho(Z) = \varrho(Z_0)\frac{M(Z)\,T(Z_0)}{M(Z_0)\,T(Z)}F_1^k \exp(kF_2), \tag{13}$$

where

$$F_1 = \left(\frac{Z + R_a}{Z_0 + R_a}\right)^{p_1}\left(\frac{Z - r_1}{Z_0 - r_1}\right)^{p_2}\left(\frac{Z - r_2}{Z_0 - r_2}\right)^{p_3}\left(\frac{Z^2 - 2xZ + x^2 + y^2}{Z_0^2 - 2xZ_0 + x^2 + y^2}\right)^{p_4}$$

and

$$F_2 = (Z - Z_0)\left(fA_6 + \frac{p_5}{(Z + R_a)(Z_0 + R_a)}\right) + \\ + \frac{p_6}{y}\operatorname{Arctan}\left(\frac{y(Z - Z_0)}{y^2 + (Z - x)(Z_0 - x)}\right).$$

5. Atmospheric Density for Altitudes above 105 km

The constituents of the atmosphere employed in determining atmospheric densities for altitudes above 105 km are nitrogen (N_2), argon (Ar), helium (He), oxygen (O_2 and O), and hydrogen (H). (Hydrogen is not included in the computation of the density for altitudes less than 500 kilometers.) Table I summarizes the pertinent data for these constituents. The total number of particles per cm^3 at an altitude of

TABLE I

		Fraction by volume at sea-level	Molecular mass (grams/mole)	Thermal diffusion coefficient
i	Constituent	Q_i	M_i	α_i
1	N_2	0.78110	28.0134	0.0
2	Ar	0.00934	39.948	0.0
3	He	0.00001289	4.0026	−0.38
4	O_2	0.20955	31.9988	0.0
5	O	0.0	15.9994	0.0
6	H	0.0	1.00797	0.0

105 km divided by Avogadro's number is calculated from

$$D = \varrho(105)/M(105) \tag{14}$$

where $\varrho(105)$ and $M(105)$ are computed from Equations (13) and (7) respectively. The number of particles of each constituent per cm^3 at 105 km divided by Avogadro's number, $d_i(105)$, is computed as follows:

For $i=1, 2, 3$ (N_2, Ar, He)

$$d_i(105) = Q_i\,\varrho(105)/M_s, \tag{15}$$

where $M_s = 28.960$ is the mean molecular mass at sea-level.

For $i=4$ (O_2)

$$d_i(105) = 2D\left(1 - M(105)/M_s\right). \tag{16}$$

And for $i=5$ (O)

$$d_i(105) = D\left((1 + Q_4)\,M(105)/M_s - 1\right). \tag{17}$$

Above 105 km all of the constituents under consideration except hydrogen are assumed to be in diffusive equilibrium. (Hydrogen is assumed to be in diffusive equilibrium above 500 km.) Thus the number of particles of each constituent per cm^3 at altitudes greater than 105 km divided by Avogadro's number, $d_i(Z)$, is computed by integrating the diffusion differential equation:

$$\frac{dd_i}{d_i} = \frac{-M_i g_0 R_a^2 \, \mathrm{d}Z}{RT(Z+R_a)^2} - \frac{\mathrm{d}T}{T}(1+\alpha_i). \tag{18}$$

6. Atmospheric Density for Altitudes between 105 and 125 km

For altitudes between 105 and 125 km the temperature profile, $T(Z)$, is given by Equation (5). Substituting (5) into (18) and integrating, the results for $i=1, 2, 3, 4, 5$ are:

$$d_i(Z) = d_i(105)\left(\frac{T(105)}{T(Z)}\right)^{1+\alpha_i} \exp\left(M_i kf \int_{105}^{Z} \frac{\mathrm{d}Z}{(Z+R_a)^2 \, P(Z)}\right) \tag{19}$$

where k, f, and $P(Z)$ are as defined previously. The integrand of Equation (19) may be written as a sum of partial fractions. The fractions are identical to those of Equation (12); however, the coefficients of the sum are now q_i instead of p_i ($i=1, \ldots, 6$). The q_i's are defined as follows:

$$\begin{aligned}
& q_2 = 1/U(r_1), \\
& q_3 = 1/-U(r_2), \\
& q_5 = 1/V(-R_a), \\
& q_4 = (1 + r_1 r_2 (R_a^2 - (x^2 + y^2))\, q_5 + W(r_1)\, q_2 + W(r_2)\, q_3)/X, \\
& q_6 = -q_5 - 2(x+R_a)\, q_4 - (r_2 + R_a)\, q_3 - (r_1 + R_a)\, q_2, \\
\text{and} \quad & q_1 = -2q_4 - q_3 - q_2.
\end{aligned}$$

(r_1, r_2, x, y, U, V, W, and X are the same as defined previously.)

The integrand of (19) may now be integrated yielding for $i=1, 2, 3, 4, 5$:

$$d_i(Z) = d_i(105)\left(\frac{T(105)}{T(Z)}\right)^{1+\alpha_i} F_3^{M_i kf} \exp(M_i kf F_4) \tag{20}$$

where

$$F_3 = \left(\frac{Z+R_a}{R_a+105}\right)^{q_1}\left(\frac{Z-r_1}{105-r_1}\right)^{q_2}\left(\frac{Z-r_2}{105-r_2}\right)^{q_3} \times \left(\frac{Z^2 - 2xZ + x^2 + y^2}{(105)^2 - 210x + x^2 + y^2}\right)^{q_4}$$

and

$$F_4 = \frac{q_5(Z-105)}{(Z+R_a)(R_a+105)} + \frac{q_6}{y}\,\mathrm{Arctan}\left(\frac{y(Z-105)}{y^2 + (Z-x)(105-x)}\right).$$

The density for $105 < Z \leqslant 125$ is then computed from

$$\varrho(Z) = \sum_{i=1}^{5} M_i d_i(Z). \tag{21}$$

7. Atmospheric Density for Altitudes above 125 km

For altitudes greater than 125 km Jacchia (1970, p. 10) defines the temperature profile by the equation

$$T(Z) = T_x + \frac{2}{\pi}(T_\infty - T_x) \times \\ \times \operatorname{Arctan}\left(0.95\pi\left(\frac{T_x - T_0}{T_\infty - T_x}\right)\left(\frac{Z - Z_x}{Z_x - Z_0}\right)(1 + B(Z - Z_x)^n)\right) \tag{22}$$

where $B = 4.5 \times 10^{-6}$ and $n = 2.5$. This expression provides continuity of the temperature profile and the derivative of the temperature profile at Z_x. However, the substitution of this expression into Equation (18) does not produce an exact differential equation. According to Jacchia (1970, p. 11): "The inverse tangent was selected among several suitable asymptotic functions ... The presence of the corrective term $[1 + B(Z - Z_x)^n]$ frees the temperature profile from strict dependence on the selected type of asymptotic function." Consequently, instead of using Equation (22) for the temperature profile, the following equation is employed:

$$T(Z) = T_\infty - (T_\infty - T_x)\exp\left[-\left(\frac{T_x - T_0}{T_\infty - T_x}\right)\left(\frac{Z - Z_x}{Z_x - Z_0}\right)\left(\frac{l}{R_a + Z}\right)\right]. \tag{23}$$

This equation produces a temperature profile which is continuous at Z_x regardless of the choice of l. The derivative of the temperature profile is also continuous at Z_x if l is chosen to be $1.9\ (R_a + Z_x) = 12315.3554$ km. Differentiating (23) it is seen that:

$$\mathrm{d}Z = \left(\frac{T_\infty - T_x}{T_x - T_0}\right)\left(\frac{Z_x - Z_0}{R_a + Z_x}\right)(R_a + Z)^2 \frac{\mathrm{d}T}{l(T_\infty - T)}. \tag{24}$$

Substituting (23) and (24) into (18) and integrating from T_x to T (equivalently from Z_x to Z) the result for $i = 1, 2, 3, 4, 5$ is:

$$d_i(Z) = d_i(125)\left(\frac{T_x}{T}\right)^{1 + \alpha_i + \gamma_i}\left(\frac{T_\infty - T}{T_\infty - T_x}\right)^{\gamma_i} \tag{25}$$

where

$$\gamma_i = \frac{M_i g_0 R_a^2}{R l T_\infty}\left(\frac{T_\infty - T_x}{T_x - T_0}\right)\left(\frac{Z_x - Z_0}{R_a + Z_x}\right) \tag{25'}$$

and $d_i(125)$ is obtained from Equation (20). The density for $125 < Z < 500$ is then computed from Equation (21).

The number of particles of hydrogen per cubic centimeter at 500 km, $n_H(500)$, is computed from

$$\log_{10} n_H(500) = 73.13 - (39.4 - 5.5 \log_{10} T_\infty) \log_{10} T_\infty, \tag{26}$$

and $d_6(500)$ is equal to $n_H(500)$ divided by Avogadro's number. Substituting (23) and (24) into (18) and integrating from $T(500)$ to $T(Z)$ (equivalently from $Z=500$ to Z) results in:

$$d_6(Z) = d_6(500)\left(\frac{T(500)}{T(Z)}\right)^{1+\alpha_6+\gamma_6}\left(\frac{T_\infty - T(Z)}{T_\infty - T(500)}\right)^{\gamma_6} \tag{27}$$

where γ_6 is as defined in (25′). Thus the density for $Z>500$ is computed from

$$\varrho(Z) = \sum_{i=1}^{6} M_i d_i(Z). \tag{28}$$

8. Discussion

The analytic atmospheric density model presented here has been computer programmed by the author. The model provides densities which are identical to Jacchia's 1970 model for altitudes between 90 and 125 km. Choosing $l=12315.3554$ km provides continuity of dT/dZ at $Z=Z_x$; however, this choice of l does not produce the best possible agreement between the temperature profiles (22) and (23) nor the densities produced by the analytic model and Jacchia's 1970 model for altitudes greater than 125 km and for all exospheric temperatures in the range of 600 K to 2000 K due to the effect of the corrective factor in Equation (22). The best agreement in density values for altitudes greater than 125 km, the goal of this study, can be obtained by treating l as a variable. Thus, in essence, l becomes the corrective factor in Equation (23).

The following philosophy was adopted by the author in determining l:

(i) l is to be a function of only the exospheric temperature.

(ii) l is to be chosen so that the maximum of the absolute values of the percentage difference in the density produced by the analytic model and Jacchia's 1970 model is a minimum for altitudes between 125 and 1000 km and for exospheric temperatures of 600 K, 1300 K, and 2000 K.

The choice of l which satisfy condition (ii) and the associated maximum absolute value of the percentage difference in density are summarized in Table II.

TABLE II

T_∞ (K)	l	Max abs (% difference in density)
600	11825	3.89
1300	13515	1.08
2000	14515	4.86

Condition (i) is satisfied by using Lagrange interpolation. For altitudes ranging from 125 to 1000 km and for exospheric temperatures ranging from 600 K to 2000 K, this manner of choosing l produces temperature profiles which differ from Jacchia's temperature profiles by less than 4% and densities which differ from Jacchia's 1970 densities by less than 4.86%.

The reader who wishes to implement the analytic atmospheric density model presented here should develop his own criteria for choosing l based upon his own requirements. That is, if one intends to use the analytic model to produce densities to be used in the prediction of the orbit of satellites whose perigee is 150 km and whose apogee is 500 km, then one might wish to choose l so that the maximum of the absolute value of the percentage difference in the densities produced by the analytic model and Jacchia's 1970 model is a minimum for altitudes ranging from 150 to 500 km and for exospheric temperature between 1000 K and 1600 K.

References

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Postscript. From a letter written by Jacchia to the referee I have learned that new evidence presented at the 1970 COSPAR meetings in Leningrad has prompted Jacchia to alter his 1970 models. The mathematical structure of the new models – not yet published – remains identical to that of the 1970 models. The alteration of the 1970 models is accomplished by changing the values of some of the parameters – most notable of which are the decrease in the height of the homopause from 105 to 100 km and the decrease in the ratio of the number of molecules of monatomic oxygen (O) to the number of molecules of diatonic oxygen (O_2) at the homopause boundary. Consequently the form of the equations presented in this article will remain valid for Jacchia's new models; however, some values of the parameters must be changed in order to obtain agreement in the density values.