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Capital-Accumulation Games under Environmental Regulation and Duopolistic Competition

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In a differential game between two symmetric firms, provided with a clean and a dirty production activity, it is analyzed how investment and emissions are affected by environmental regulation. If both firms face the same environmental policy, a stricter policy reduces long-run investment in the dirty activity, while the impact on the clean activity is ambiguous. Both long-run emissions of each firm and total emissions decrease. This result does not necessarily hold if both firms face different policy instruments: Each firm's investment levels increase with a stricter environmental policy towards its rival, which causes more emissions by this firm.

Keywords: differential game, investment, pollution, emission taxes, tradeable emission permits, open-loop Nash equilibria, duopolistic competition.

JEL classification: C73, D92, L11, L13, Q25, Q28.

1 Introduction

Using environmental-policy instruments against polluting firms, such as taxing emissions, creating a market for tradeable emission permits, prescribing standards, or subsidizing investment in cleaner production activities or abatement, governments try to internalize externalities caused by pollution.

Empirical studies show that improvements in air quality are most commonly tried to achieve by taxing emissions of polluting firms, while a market for tradeable emission permits has only been implemented in a few countries. Table 1 provides an overview on instruments used by governments for air-quality improvement in OECD countries.[1]

1 A detailed overview on instruments for air-quality improvement, waste disposal, and water protection in Germany and other OECD countries can be found in Michaelis (1995).

Table 1. Instruments for air-quality improvement in OECD countries

Tax			Permit system		
CO_2	SO_2 and NO_x	CFC	NO_x	SO_2	CFC
Denmark Netherlands Finland Norway Sweden	France Portugal Sweden	Denmark Finland[a] Norway[a] USA[a]	USA Switzerland	USA	USA Canada Australia

[a] Input tax

Usually, investment behavior of a single profit-maximizing firm is influenced in such a way that long-run emissions are reduced if the government introduces an emission tax. Either investment in productive capital is reduced or abatement expenditures are increased.[2] But the possibility of factor substitution may imply that a higher tax rate does not necessarily lead to a reduction in emissions by a profit-maximizing firm: Xepapadeas (1992) argues that the level of long-run emissions only decreases with an increasing tax rate if the tax rate is sufficiently low that all productive inputs are reduced. Otherwise, a reduction in productive capital may lead to an increased usage of other factors of production and to increased emissions.

Since Buchanan (1969), it is well-known that market imperfections imply that imposing the Pigouvian tax may not be socially optimal. Subsequent studies suggest that strategic interactions on the product market have to be taken into consideration in formulating environmental policy.[3] However, in the recent literature dealing with the influence of environmental policy on the dynamic investment policy of polluting firms, market imperfections apart from monopoly are widely neglected.[4]

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2 See Kort (1994b, 1996), Hartl and Kort (1996b), and Xepapadeas (1992).

3 For the effect of taxes on emissions of oligopolistic firms, see, e.g., Katz and Rosen (1986), Dixit (1986), Dierickx et al. (1988), Simpson (1995), Okuguchi and Yamazaki (1994), Levin (1985), Okuguchi (1993), Dung (1993), Carraro and Soubeyran (1996), Germain (1989), Katsoulacos and Xepapadeas (1994), and Ebert (1992). The choice of the socially optimal tax rate within oligopoly is considered by Ebert (1991), Katsoulacos and Xepapadeas (1994), Simpson (1995), Okuguchi and Yamazaki (1994), Dierickx et al. (1988), and Requate (1993a, b). Note that an output tax can be considered as an emission tax if the emission–output ratios are fixed and abatement is absent.

4 See, e.g., Uimonen (1994), Kort (1994b, 1996), Hartl and Kort (1996b), who analyze the effect of market-based environmental policy, while Kopp and Smith (1980),

This paper extends a single-firm dynamic investment model of Kort (1996) to duopolistic competition, where two production activities instead of one productive and one abatement activity are considered. The influence of emission taxes and tradeable emission permits on optimal investment policies of both firms is analyzed, in particular if firms face different environmental-policy instruments. This may be possible if they are located in two different countries and engage in international trade.[5] Additionally, it is analyzed in which way a stricter environmental policy affects emissions.

The paper is organized as follows. In Sect. 2, the model is presented. Assuming open-loop investment strategies, necessary and sufficient conditions for Nash equilibria are derived in Sect. 3. In Sect. 4, the characteristics of the long-run equilibrium and its stability properties are discussed. Some related models are dealt with in Sect. 5. The main results are summarized in Sect. 6.

2 The Model

Two symmetric firms are considered, each of them provided with two linear production activities. Output Q^j of firm j, $j \in \{A, B\}$, is given by

$$Q^j := q_1 \cdot K_1^j + q_2 \cdot K_2^j \,, \tag{1}$$

where K_i^j denotes the capital stock associated with production activity i of firm j, $i \in \{1, 2\}$, and q_i is the corresponding production coefficient. Both firms produce an identical good causing emissions E^j with

$$E^j := e_1 \cdot K_1^j + e_2 \cdot K_2^j \,, \tag{2}$$

where e_i denotes the emission coefficient associated with activity i. Production activity 1 is less productive than activity 2, but also characterized by a lower emission–output ratio, i.e.,

$$q_1 < q_2 \,, \tag{3}$$

and

$$e_1/q_1 < e_2/q_2 \,. \tag{4}$$

Kort (1994a), and Hartl and Kort (1996a, b, 1997) consider different kinds of environmental standards. Exceptions are the contributions by Feenstra et al. (1996, 1997).

5 See Feenstra et al. (1996, 1997) who compare taxes and standards in an international-trade model of two firms with one production activity.

Capital stocks change over time due to capital depreciation and gross investment, which is assumed to be irreversible and bounded from above. The assumption of irreversible investment can be traced back to Arrow (1968). The capital stocks are interpreted as productive equipment, which is, according to Pindyck (1991), firm- or at least industry-specific and characterized by a low resale value. Therefore, the capital-accumulation equation is given by

$$\dot{K}_i^j := I_i^j - \alpha \cdot K_i^j \,, \tag{5}$$

where I_i^j denotes gross investment into capital stock i of firm j, with $0 \leq I_i^j \leq I_{\max}$, and α represents the capital depreciation rate. Assuming a linear inverse demand function and duopolistic competition between the two firms, the revenue R^j of firm j can be represented as follows:

$$R^j := -a \cdot (Q^j + Q^{\bar{j}}) \cdot Q^j + b \cdot Q^j \,, \tag{6}$$

where $b > 0$ represents the prohibitive price, b/a with $a > 0$ is the saturation point, and $\bar{j}$ is defined by

$$\bar{j} := \begin{cases} A & \text{if } j = B, \\ B & \text{otherwise.} \end{cases}$$

In the following, two different environmental-policy instruments are considered, namely emission taxes and tradeable emission permits. For the first instrument, it is assumed that the government imposes a tax on each unit of emissions, where the tax rate is constant. If, in contrast, tradeable emission permits are employed, the following situation is considered. At the beginning of the planning horizon each firm is provided with a certain number of emission permits.[6] This number of permits entitles the firm to emit a corresponding amount per unit of time. If the firm exceeds this amount, it has to purchase additional permits. Conversely, if the firm does not use its full number of permits, it can sell the rest to other firms. The permits are assumed to be of infinite validity.[7] For the sake of simplicity,

6 The government prescribes a level of total emissions, which is not allowed to be exceeded. This level is divided into small parts, where each part corresponds to one emission permit. These permits are issued to polluting firms (see Weimann, 1995, for further details). This process is assumed to be already finished.

7 According to Siebert (1992), permits may be defined on a temporary basis or without a time limit. While permits defined on a temporary basis give more flexibility

it is assumed that one emission permit corresponds to exactly one unit of emissions and that the price for one emission permit is given exogenously.[8]

The costs of firm j consist of production costs L^j, investment costs A^j, and costs due to environmental regulation C^j. The production costs are represented by the firm's labor costs, where, for simplicity, the labor intensities are equal for both activities. These costs are given by

$$L^j := w \cdot l \cdot (K_1^j + K_2^j) , \tag{7}$$

where w denotes the wage rate, and l is the labor intensity of both activities. Investment costs include costs of acquisition of productive capital as well as costs of installation. They are assumed to be additively separable, strictly increasing, and strictly convex:[9]

$$A^j := A(I_1^j) + A(I_2^j) , \tag{8}$$

with $A' > 0$ and $A'' > 0$. The costs due to environmental regulation can be represented by

$$C^j := \rho^j \cdot E^j + \bar{\rho}^j \cdot \dot{E}^j , \tag{9}$$

where

$$\rho^j := \begin{cases} \rho & \text{if taxes are imposed on the emissions of firm } j, \\ 0 & \text{otherwise,} \end{cases} \tag{10}$$

and

$$\bar{\rho}^j := \begin{cases} \bar{\rho} & \text{if the government employs tradeable emission permits towards firm } j, \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

to governments in the regulation of emissions, the resulting allocation may induce over-investment in abatement: Investment decisions of polluting firms would not only depend on permit price and marginal abatement costs, but also on the number of permits not available in the future (Cansier, 1993). Moreover, even with infinite validity of permits the government can achieve a reduction of emissions by letting the permits depreciate over time or by repurchasing permits (Cansier, 1993).

8 This can be made plausible by the assumption that the market for the emission permits is fully competitive: Both firms are the only suppliers of the produced good, but they are not the only demanders for emission permits if the pollutant is assumed to be not industry-specific.

9 The assumption of strict convexity is not unusual in the literature on optimal dynamic investment (see, e.g., Söderström, 1976, and several models in Van Hilten et al., 1993). Holt et al. (1960) even argue that costs of installation of productive capital can be appropriately approximated by linear-quadratic functions (see Gould, 1968, p. 49).

ρ denotes the emission tax rate and $\bar{\rho}$ denotes the price of an emission permit, which can be either sold or purchased.

It is assumed that each firm chooses its investment levels in both activities such that its present value of earnings is maximized, where the planning horizon is assumed to be infinite.

Summarizing the foregoing model assumptions, we get the following differential game:

$$\max_{I_1^j, I_2^j} \int_0^\infty \exp(-r \cdot t) \cdot \big[-a \cdot (Q^j + Q^{\bar{j}}) \cdot Q^j + (b \cdot q_1 - w \cdot l - \alpha \cdot (1 - \bar{\rho}^j \cdot e_1) - \rho^j \cdot e_1) \cdot K_1^j + (b \cdot q_2 - w \cdot l - \alpha \cdot (1 - \bar{\rho}^j \cdot e_2) - \rho^j \cdot e_2) \cdot K_2^j - A(I_1^j) - \bar{\rho}^j \cdot e_1 \cdot I_1^j - A(I_2^j) - \bar{\rho}^j \cdot e_2 \cdot I_2^j\big] \,\mathrm{d}t$$

subject to

$$\dot{K}_1^j = I_1^j - \alpha \cdot K_1^j \,, \tag{12}$$

$$\dot{K}_2^j = I_2^j - \alpha \cdot K_2^j \,, \tag{13}$$

$$0 \le I_1^j \le I_{\max} \quad \text{and} \quad 0 \le I_2^j \le I_{\max} \,, \tag{14}$$

where $0 < r < 1$ denotes the discount rate.

3 Open-Loop Nash Equilibria

In the following, it is assumed that each firm is unable to observe the changes in the capital stocks of its rival and that both firms are able to precommit to their investment paths. In such a situation, it is reasonable to analyze open-loop investment strategies, where both firms choose their investment levels simultaneously.

Generally, the assumption of open-loop investment strategies is somewhat crucial because it requires precommitment to investment paths by the firms (Reynolds, 1987). Moreover, open-loop investment strategies do not explicitly take strategic interaction between both firms into account: At each point of time investment depends only on the initial values of capital stocks and time. Therefore, each firm decides on its investment levels without taking into consideration changes in the capital stocks due to investment decisions of the rival. Strategic interactions are taken into account only implicitly, i.e., in the same way as in Cournot–Nash oligopoly, because all decisions are made at exactly one point of time, namely at the beginning of the planning horizon (Fudenberg and Tirole, 1991, p. 529). In contrast, investment will depend on the current values of the capital stocks and time if feedback strategies are assumed.

The dependence on initial values is the reason why Nash equilibria in open-loop investment strategies are generally not subgame perfect. However, Reynolds (1987) argues that each firm may not be able to observe the changes in the capital stocks of its competitor or that adjusting investment decisions at each point of time to current capital stocks may be very costly, which serves as an explanation for precommitment to investment paths. In such a case, the open-loop investment strategies can be interpreted as degenerate feedback strategies. Consequently, the corresponding Nash equilibria are degenerately subgame perfect. Given that both firms are able to precommit to their investment paths, open-loop investment strategies are an appropriate approach (Dockner and Takahashi, 1990, p. 249).

Defining now the Hamiltonian

$$\begin{aligned} H^j = & -a \cdot (Q^j + Q^{\bar{j}}) \cdot Q^j \\ & + (b \cdot q_1 - w \cdot l - \alpha \cdot (1 - \bar{\rho}^j \cdot e_1) - \rho^j \cdot e_1) \cdot K_1^j \\ & + (b \cdot q_2 - w \cdot l - \alpha \cdot (1 - \bar{\rho}^j \cdot e_2) - \rho^j \cdot e_2) \cdot K_2^j \\ & - A(I_1^j) - \bar{\rho}^j \cdot e_1 \cdot I_1^j - A(I_2^j) - \bar{\rho}^j \cdot e_2 \cdot I_2^j \\ & + \lambda_1^j \cdot (I_1^j - \alpha \cdot K_1^j) + \lambda_2^j \cdot (I_2^j - \alpha \cdot K_2^j) , \end{aligned} \tag{15}$$

we get the following necessary optimality conditions:

$$\left\{ \begin{array}{l} \lambda_i^j \geq A'(I_{\max}) + \bar{\rho}^j \cdot e_i \\ \lambda_i^j \leq A'(0) + \bar{\rho}^j \cdot e_i \\ \text{otherwise} \end{array} \right\} \Longrightarrow \left\{ \begin{array}{l} I_i^j = I_{\max} \\ I_i^j = 0 \\ I_i^j = (A')^{-1}(\lambda_i^j - \bar{\rho}^j \cdot e_i) \end{array} \right\} \tag{16}$$

and

$$\begin{aligned} \dot{\lambda}_i^j = & (r + \alpha) \cdot \lambda_i^j + 2 \cdot a \cdot q_i \cdot Q^j + a \cdot q_i \cdot Q^{\bar{j}} - b \cdot q_i \\ & + w \cdot l + \alpha \cdot (1 - \bar{\rho}^j \cdot e_i) + \rho^j \cdot e_i , \end{aligned} \tag{17}$$

$$\dot{K}_i^j = I_i^j - \alpha \cdot K_i^j \tag{18}$$

for $i \in \{1, 2\}$ and $j \in \{A, B\}$.

Due to the concavity of H^j, the necessary conditions will be also sufficient for optimality if the limiting transversality conditions

$$\lim_{t \to \infty} \exp(-r \cdot t) \cdot \lambda_i^j(t) \cdot (\tilde{K}_i^j(t) - K_i^j(t)) \geq 0 \qquad (19)$$

for all admissible $\tilde{K}_i^j$, $i \in \{1, 2\}$, $j \in \{A, B\}$, are met (see Seierstad and Sydsæter, 1987, p. 234).

The necessary conditions imply the following properties for each firm's investment behavior, which hold for both activities.

Proposition 1: The shadow price at which a firm begins to invest the maximum amount is higher if emission permits are employed than if emission taxes are imposed. The same holds for the shadow price at which a firm stops investment. If the shadow price is the same for both environmental-policy instruments and if there is an interior solution, investment will be lower if emission permits are employed than if emission taxes are imposed.

Proof: Since $\bar{\rho}^j$ is equal to zero if taxes are imposed, the assertions follow from (16). □

The shadow price λ_i^j represents firm j's marginal willingness to pay for an additional unit of its capital stock i. If it is higher than the marginal costs of an additional unit of investment in this capital stock, the firm will increase the capital stock. Otherwise, investment will be zero. The Hamiltonian can be interpreted as the utility function of firm j, where utility is derived from profit and from the changes in the capital stocks. If the shadow price λ_i^j is higher than marginal costs, utility can be raised by increasing the corresponding capital stock. Accordingly, the firm can increase utility by a reduction of the capital stock if the shadow price is lower than marginal costs. Hence, in equilibrium, the shadow price equals marginal costs. If emissions are taxed, marginal costs of an additional unit of investment consist only of marginal investment costs A', since taxes do not punish investment directly, but only the resulting amount of capital stock. In contrast, if tradeable emission permits are employed, marginal costs of an additional unit of investment are higher because each additional unit of capital stock causes additional emissions. Consequently, additional emission permits must be bought which yields additional marginal costs $\bar{\rho} \cdot e_i$. Figure 1 illustrates the result of Proposition 1.

Next, both production activities are compared under the different environmental policy instruments.

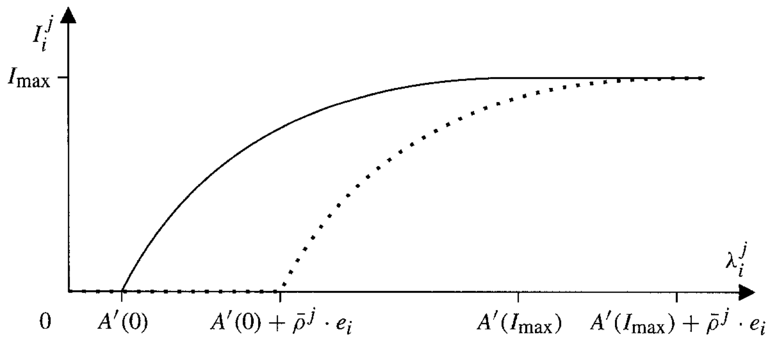

Fig. 1. Investment into capital stock i if taxes are imposed (solid line) and if tradeable emission permits are employed (dotted line)

If tradeable emission permits are employed, it holds for the investment behavior of both firms.

Proposition 2: The shadow price at which a firm begins to invest the maximum amount is lower for the clean activity. The same holds for the shadow price at which a firm stops investment. If the shadow prices for increasing the capital stocks are equal for both activities and if there is an interior solution, investment in the clean activity will be higher than investment in the dirty activity.

Proof: If tradeable emission permits are employed, then $\bar{\rho}^j = \bar{\rho}$. Furthermore, $q_1 < q_2$ together with $e_1/q_1 < e_2/q_2$ implies $e_1 < e_2$. Then the claims follow from (16). □

The marginal costs of an additional unit of investment, which have to be compared to the marginal willingness to pay for an increase in the capital stock, are higher for the capital stock with the higher emission coefficient. The reason is that an additional unit of this capital stock causes more additional emissions than an additional unit of the other capital stock, and the firm has to buy a higher amount of additional permits. This result is illustrated in Fig. 2a.

Investment behavior under emission taxes is described by Proposition 3.

Proposition 3: A firm begins maximum investment in both capital stocks at the same shadow price. It also stops investment in both capital stocks at

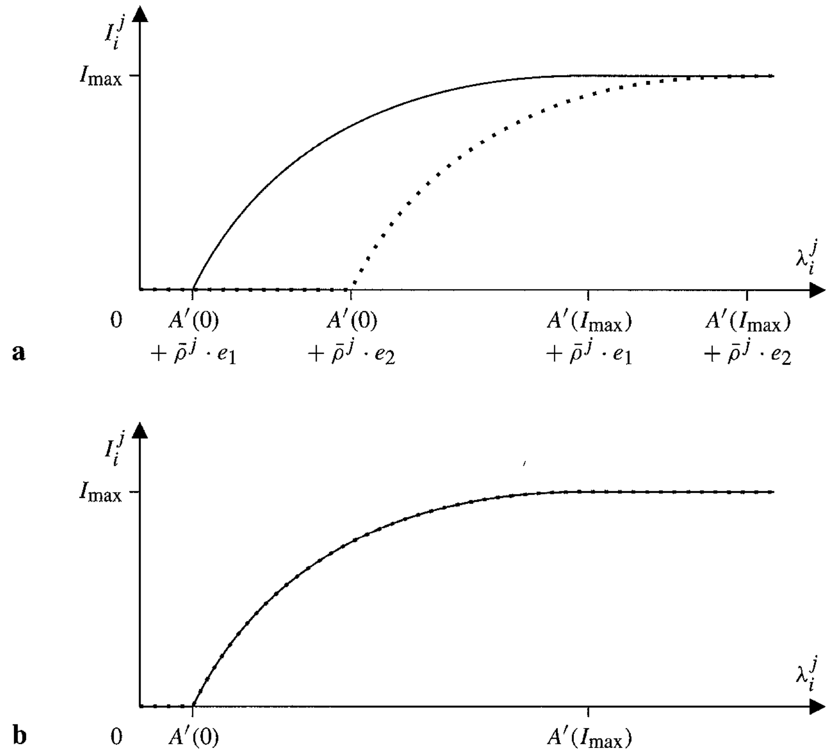

Fig. 2. Comparison of investment in the clean activity 1 (solid line) and the dirty activity 2 (dotted line) (a) if tradeable emission permits are employed or (b) if emission taxes are imposed

the same shadow price. If the shadow prices for increasing the capital stocks are equal for both activities and if there is an interior solution, investment in both capital stocks will be equal.

Proof: If emission taxes are imposed, it holds: $\bar{\rho}^j = 0$. The assertions then follow from (16). □

This result, illustrated in Fig. 2b, can be explained by the fact that the marginal costs of an additional unit of investment are equal for both activities if emission taxes are imposed.

4 Long-Run Equilibrium and Stability Properties

In the following, the analysis is confined to interior solutions. In order to derive predictions on long-run tendencies, the steady-state levels of investment are determined, and the stability properties of the equilibrium are established.

The steady state is characterized by the following equilibrium conditions:

$$K_i^j = I_i^j/\alpha \,, \tag{20}$$

$$\lambda_i^j = A'(I_i^j) + \bar{\rho}^j \cdot e_i \,, \tag{21}$$

and

$$\begin{aligned}\lambda_i^j \cdot (r+\alpha) = & -2 \cdot a \cdot q_i \cdot (q_1 \cdot K_1^j + q_2 \cdot K_2^j) \\ & - a \cdot q_i \cdot (q_1 \cdot K_1^{\bar{j}} + q_2 \cdot K_2^{\bar{j}}) \\ & + b \cdot q_i - w \cdot l - \alpha \cdot (1 - \bar{\rho}^j \cdot e_i) - \rho^j \cdot e_i \,.\end{aligned} \tag{22}$$

This leads to the following investment levels:

$$\begin{aligned}&(A'(I_1^j) + \bar{\rho}^j \cdot e_1) \cdot (r+\alpha) + w \cdot l + \alpha \cdot (1 - \bar{\rho}^j \cdot e_1) + \rho^j \cdot e_1 \\ &= -2 \cdot a \cdot q_1^2 \cdot \frac{I_1^j}{\alpha} - 2 \cdot a \cdot q_1 \cdot q_2 \cdot \frac{I_2^j}{\alpha} \\ &\quad - a \cdot q_1^2 \cdot \frac{I_1^{\bar{j}}}{\alpha} - a \cdot q_1 \cdot q_2 \cdot \frac{I_2^{\bar{j}}}{\alpha} + b \cdot q_1 \,,\end{aligned} \tag{23}$$

$$\begin{aligned}&(A'(I_2^j) + \bar{\rho}^j \cdot e_2) \cdot (r+\alpha) + w \cdot l + \alpha \cdot (1 - \bar{\rho}^j \cdot e_2) + \rho^j \cdot e_2 \\ &= -2 \cdot a \cdot q_1 \cdot q_2 \cdot \frac{I_1^j}{\alpha} - 2 \cdot a \cdot q_2^2 \cdot \frac{I_2^j}{\alpha} \\ &\quad - a \cdot q_1 \cdot q_2 \cdot \frac{I_1^{\bar{j}}}{\alpha} - a \cdot q_2^2 \cdot \frac{I_2^{\bar{j}}}{\alpha} + b \cdot q_2\end{aligned} \tag{24}$$

for $j \in \{A, B\}$.

Conditions (23) and (24) show that in the steady state marginal revenue of an additional unit of investment equals marginal costs for each of the capital stocks. The change in revenue is due to the change of output resulting from the increase in the capital stocks. The marginal costs of an additional unit of investment consist of marginal direct costs and marginal indirect costs, where the marginal indirect costs indicate the change of costs which is not directly caused by the additional unit of investment, but by the resulting change in the capital stock. They consist of marginal labor costs $w \cdot l$, marginal costs of capital depreciation α, and marginal costs of an additional unit of capital stock due to environmental regulation $\rho^j \cdot e_i - \alpha \cdot \bar{\rho}^j \cdot e_i$. The marginal direct costs are a direct consequence of the additional unit of investment. They are represented by the product of the sum of the marginal investment costs A' and the marginal costs due to environmental regulation $\bar{\rho}^j \cdot e_i$ and the sum of the depreciation rate and the discount rate. These marginal costs can be interpreted as marginal opportunity costs of production: If the firm does not invest an additional unit in productive capital but in financial assets, it will receive interest payments $(A' + \bar{\rho}^j \cdot e_i) \cdot r$ and save additional capital depreciation costs $(A' + \bar{\rho}^j \cdot e_i) \cdot \alpha$.

Inserting the maximum condition (21) into the system of state and costate equations, it can be shown that the Jacobian of the linearized system, evaluated at the steady state, possesses 8 real eigenvalues, where 4 are positive and 4 are negative (Stimming, 1996). This implies that there exists a 4-dimensional manifold in a neighborhood of the steady state such that any optimal path starting on this manifold converges to the steady state in a monotonic way (see Brock and Malliaris, 1989, p. 126).[10]

Proposition 4 deals with the impact of a stricter environmental policy on the long-run levels of investment and production.[11]

Proposition 4: 1. Tax/tax, permits/permits. If both firms face the same environmental-policy instrument, a stricter policy will reduce long-run investment in the dirty activity in both firms while the effect on the clean activity is ambiguous. Output always decreases.

2. Tax/permits. a. If the two firms face different environmental-policy instruments, a stricter policy towards one firm will reduce long-run investment in the dirty activity by the firm, while the effect on its clean activity is ambiguous. Its output always decreases. Long-run investment and production of the competitor is stimulated. Aggregate output decreases.

10 This behavior is often referred to as local saddlepoint stability. For a similar capital-accumulation game with only one capital stock global saddlepoint stability is shown by Fershtman and Muller (1984), where an extension of their result to a game with more than one capital stock was considered to be difficult. Dockner and Takahashi (1990) show the saddlepoint property for more general games, but they need the assumption of a discount rate equal to zero. In the current model, r is assumed to be positive in order to ensure convergence of the objective functions.

11 The results in Propositions 4–8 are derived by comparative statics. The proofs are available from the author upon request.

b. If the two firms face different environmental-policy instruments, a simultaneous marginal increase of the tax rate and the permit price will reduce long-run investment in the dirty activity by the firm facing the tax. The effect on long-run investment in the clean activity as well as on the competitor's investment levels is ambiguous. While the impact on output levels is ambiguous, aggregate production decreases.

This result can be explained as follows. Conditions (23) and (24) state that in the steady-state equilibrium, marginal revenue of an additional unit of investment equals marginal costs. If the environmental policy towards one firm becomes stricter, its marginal costs of an additional unit of investment rise. Therefore, marginal costs of an additional unit of production increase, where, due to (4), this increase is higher for the dirty activity than for the clean one [divide (23) by q_1 and (24) by q_2 and subtract (23) from (24)]. Consequently, the firm will primarily reduce dirty production, and investment in the corresponding capital stock will decrease.

A stricter environmental policy towards one firm leads to lower long-run investment in at least one activity by this firm. A lower level of supply yields a higher marginal profit of the rival. Since in a steady state marginal profit is equal to zero, long-run investment of the rival will increase.

If environmental policy towards both firms becomes stricter, there are two forces at work: First, a stricter policy towards one firm makes production less attractive for this firm (regulation effect); second, a reduced supply by a firm increases marginal revenue of the other firm which leads to increased attractivity of production (strategic effect). Since the discount rate is lower than one, the effect of a marginal increase of the tax rate is stronger than the effect of a marginal increase of the permit price. Therefore, the regulation effect is relatively stronger and the strategic effect relatively weaker towards the firm facing the tax, compared to the case where both firms face the same policy instrument. Towards the other firm, the strategic effect is relatively stronger while the regulation effect is relatively weaker.

Proposition 5 deals with the impact of a stricter environmental policy on the long-run levels of emissions.

Proposition 5: 1. Tax/tax, permits/permits. If both firms face the same environmental-policy instrument, a stricter policy reduces long-run emissions of both firms and the long-run level of aggregate emissions.

2. Tax/permits. a. If the two firms face different environmental-policy instruments, a stricter policy towards one firm will reduce long-run

emissions of the firm, but increase long-run emissions of the competitor. The effect on total emissions is ambiguous. If investment costs are linear-quadratic, total emissions decrease.

b. If the two firms face different environmental-policy instruments, a simultaneous marginal increase of the tax rate and the permit price has ambiguous effects on the long-run levels of emissions. If investment costs are linear-quadratic, long-run emissions by the firm facing the tax as well as total emissions decrease.

The result that a stricter environmental policy towards only one firm does not necessarily lead to lower total emissions in the long run is not surprising. A stricter policy towards one firm will lead to higher long-run investment and, consequently, to higher long-run emissions of the other firm. Hence, even if the emissions of one firm decrease, a stricter policy does not necessarily reduce total emissions in the long run. Furthermore, a stricter policy towards both firms, but with different instruments, also leads to ambiguous results concerning the impact on aggregate emissions.

These results can be compared to similar conclusions from the literature on tax incidence with market imperfections: Levin (1985) and Okuguchi (1993) show in a Cournot oligopoly where firms differ with respect to their cost functions that imposing individual tax rates towards the firms and increasing the tax rates does not necessarily reduce the sum of weighted output. If the individual weights are interpreted as firm-specific emission–output ratios, the sum of weighted output represents the level of aggregate emissions. If the tax rate is increased, marginal costs of production of the individual firms are increased differently: The increase in marginal costs is lower for the firms with the lower emission–output ratio. This may bring about that in these firms the strategic effect dominates the regulation effect, resulting in an increase of emissions. If one firm faces an emission tax and one firm faces tradeable emission permits, firms usually differ with respect to their cost functions. If the tax rate and the permit price are increased simultaneously, marginal costs of the two firms are increased differently. The regulation effect might be dominated by the strategic effect in one firm, which leads to an increase in emissions.

This conclusion as well as the result that employing a stricter policy towards only one firm does not necessarily reduce total emissions in the long run is important in connection with international environmental policy. If both firms are located in different countries, a unilaterally stricter environmental policy in one country may cause an increase in global emissions.

Furthermore, this may also hold if both countries employ a stricter policy, but with different instruments.

The phenomenon that some countries reduce emissions, but total emissions are reduced by a lower percentage or even increased, is known as leakage effect (see Barrett, 1994). Its quantification by the leakage rate, i.e., the increase in emissions by countries outside the group of emission-reducing countries divided by the decrease in emissions by the countries inside the group (compare Barrett, 1994), indicates that leakage rates may be negative as well as greater than one.[12] The last case mirrors that a unilateral reduction of emissions by a group of countries lead to an increase in global emissions.

The results emphasize the need for international policy coordination. Nevertheless, unilateral measures by some countries are explicitly allowed in international environmental agreements[13] and are common practice (compare Hoel, 1991; Pezzey, 1992). Furthermore, as international cooperation is characterized by a prisoner's dilemma type of situation, international agreements often fail. Without side payments, the size of stable coalitions is rather small (see Carraro and Siniscalco, 1993; Barrett, 1995a, b; Stähler, 1996), but the coalition can be expanded by transfer payments to countries outside by linking environmental policy to other economic policies (Carraro and Siniscalco, 1992). If cooperation between countries fails even in the two-country case, the cooperative pollution level might be asymptotically sustained if the countries' game is infinitely repeated in time.[14]

If parameters are such that a unilaterally stricter policy reduces global emissions, we get the following result comparing taxes and permits with respect to their ecological effectiveness:

Corollary 6: If ρ is (a) equal to, (b) greater than, or (c) less than $r \cdot \bar{\rho}$, employing tradeable emission permits is (a) as effective as, (b) less effective than, or (c) more effective than imposing an emission tax with respect to a reduction of global emissions.

12 Pezzey (1992), Winters (1992), and Ulph (1994) survey some of the results.

13 See parts of the Vienna Convention for the Protection of the Ozone Layer (1985), given in Hoog and Steinmetz (1993).

14 This has been shown by Dockner and van Long (1993) in a two-country model, where countries use Markov-perfect strategies.

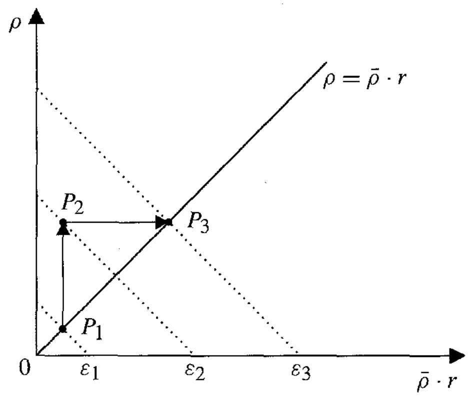

Fig. 3. Long-run level of total emissions depending on environmental policy at home and abroad

Proof: Steady states are equal for both instruments if $\rho = r \cdot \bar{\rho}$. The assertion then follows directly from Proposition 5. □

The result is illustrated in Fig. 3. Each of the curves ε_1, ε_2, and ε_3 represents environmental-policy combinations by the two countries which induce a certain level of total emissions, where $\varepsilon_1 > \varepsilon_2 > \varepsilon_3$. Assume that both firms face different policy instruments and that the emission-tax rate exceeds the product of the permit price and the discount rate such that P_2 together with the emission level ε_2 is realized. Total emissions are higher than they would be if both firms faced an emission tax, associated with the emission level ε_3. Total emissions are lower than they would be if both firms faced tradeable emission permits, implying total emissions equal to ε_1.

5 Related Models

One of the main results of the previous section was that a stricter environmental policy towards one firm does not necessarily reduce the long-run level of total emissions if both firms face different policy instruments. Furthermore, a stricter policy towards both firms, but with different instruments, also has an ambiguous impact on total emissions. The foregoing discussion of these results led to the conclusion that direct regulation effects may be dominated by indirect strategic effects. Consequently, the duopolistic structure of the model was considered to be responsible for these results. However, an analysis of related duopoly models shows that strategic effects are not the only driving forces.

Proposition 7: If the duopolists are provided with only one activity, a stricter environmental policy towards one firm will always reduce total emissions in the long run. A stricter policy towards both firms, but with different instruments, will also induce lower total emissions. The same results hold if the duopolists are provided with one productive and one cleaning activity.

In contrast to the original model, a reduction of the long-run level of total emissions as a consequence of a stricter environmental policy can generally be derived. Therefore, it might be expected that the second production activity is responsible for the ambiguity results in the original model, because it is the only element which has been omitted from the original. Such an expectation is reinforced by the result of Xepapadeas (1992), according to which the possibility of factor substitution for production brings about that a stricter environmental policy towards a monopolistic firm does not necessarily reduce its emissions. Furthermore, it is in line with a similar result of Hoel (1991), who shows in a static model with two countries that a unilateral reduction of emissions by one country always leads to lower total emissions.[15]

However, analyzing a monopoly model with two production activities derived from the original model by omitting the second firm, we obtain that even with possible factor substitution a stricter environmental policy always decreases the monopolist's long-run emissions.

Proposition 8: In the monopoly case with two productive capital stocks a stricter environmental policy always leads to a lower long-run emission level.

In contrast to the original model and similar to the duopoly model with one activity, the long-run level of total emissions is generally reduced.

15 Note that if the firms are provided with only one production activity, the corresponding conditions for long-run investment can be transformed into reaction functions of emissions of the two firms, which exhibit a negative slope, greater than -1. This is exactly the structure of Hoel's model, if not countries but firms are the players. If the duopolists are provided with two production activities, it is not possible to derive such similarities to Hoel's model.

Therefore, we conclude that it is a combined effect of the second production activity and the duopolistic structure which let us obtain the result of the original model.

6 Conclusion

In a differential game of capital accumulation between two symmetric firms, it has been analyzed how investment decisions and emissions are affected by environmental regulation. Two kinds of environmental-policy instruments were considered, namely emission taxes and tradeable emission permits.

It has been shown that in the long run a stricter policy towards one firm reduces investment in the dirty production activity and therefore dirty production in this firm. The impact on the clean activity is ambiguous. A stricter policy towards only one firm stimulates long-run investment of its rival, which may lead to a higher long-run level of aggregate emissions. Furthermore, a stricter environmental regulation towards both firms, but with different instruments, also has an ambiguous effect on total emissions. Therefore, air-quality regulation without international coordination of both environmental policy and policy instruments may not only counteract air-quality improvement, but it could be rather advantageous to mitigate environmental policy.

All results are based on the restrictive assumption that firms use open-loop investment strategies. If instead feedback investment strategies are assumed, strategic interaction is explicitly taken into account. However, getting analytical results seems to be almost impossible, although Reynolds (1987) derived analytical results in a similar, but lower-dimensional capital-accumulation game. While numerical analysis can be conducted, the results generally depend on the specification of the parameters.

It would be an interesting modification of the model if the assumption of a perfectly competitive permit market is relaxed. A plausible alternative would be that both firms, which are the only suppliers of the produced good, are also the only demanders for emission permits. The arising bargaining over emission permits in such a framework may well lead to completely different results than in the current analysis.

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