REPORT OF THE IAU WORKING GROUP ON CARTOGRAPHIC COORDINATES AND ROTATIONAL ELEMENTS OF THE PLANETS AND SATELLITES

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Abstract. This paper is the entire report of the IAU Working Group on Cartographic Coordinates and Rotational Elements of the Planets and Satellites, including three annexes. Tables give the recommended values for the directions of the north poles of rotation and the prime meridians of the planets and satellites. Reference surfaces for mapping these bodies are described. The annexes discuss the guiding principles, given in the body of the report, present explanatory notes, and provide a bibliography of the rotational elements and reference surfaces of the planets and satellites, definitions, and algebraic expressions of relevant parameters.

1. Introduction

The IAU Working Group on Cartographic Coordinates and Rotational Elements of the Planets and Satellites was established as a consequence of the adoption of the following resolution at the IAU General Assembly at Grenoble in 1976:

"Commissions 4 and 16 noting that

(a) confusion exists regarding the present rotational elements of some of the planets

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(b) extensive amounts of new data from radar observations and by direct imaging from spacecraft have made cartography of the surfaces of the Moon, Mercury, Venus, and Mars a reality

(c) there will be an extension of these techniques to the mapping of larger satellites of Jupiter and Saturn in the near future

assert that

(a) to avoid a proliferation of inconsistent cartographic and rotational systems, there is a need to define the rotational elements of the planets and satellites on a systematic basis and to relate the new cartographic coordinates rigorously to the rotational elements

and therefore recommend that

(1) Commission 4 (Ephemerides) and Commission 16 (Physical Study of Planets and Satellites) establish a Joint Working Group to study the cartographic coordinates and rotational elements of the planets and satellites and to report recommendations thereon at the next general assembly of the IAU" (*Trans. IAU 16B*, p. 144, 1977).

In preparing the recommendations given in this report, the Working Group adopted the guiding principles that have been previously adopted by Commission 16 at the IAU General Assembly at Brighton in 1970, namely:

"1. The rotational pole of a planet or satellite which lies on the north side of the invariable plane shall be called north, and northern latitudes shall be designated as positive.

2. The planetographic longitude of the central meridian, as observed from a direction fixed with respect to an inertial coordinate system, shall increase with time. The range of longitudes shall extend from 0° to 360°" (*Trans. IAU 14B*, p. 128, 1971).

The technical arguments in support of, and in opposition to, both of these principles have been reviewed; these arguments were considered at the time of the adoption in the preparation of numerous maps of both planets and satellites, and the Group considers the advantages that are claimed for other principles are not sufficient to justify the adoption of new principles. Because of historical usage, longitudes on the Moon and Earth are measured from 0° to 180° east and west of the prime meridian. Thus these bodies are exceptions to the general rule. The Group does, however, recommend that the rotational elements and cartographic coordinate systems be specified more simply and uniformly than in the past.

The rotational elements define the direction of the axis of rotation and the rate of rotation relative to an inertial coordinate system. The values of the elements given later in this report are based, where possible, on recent observational determinations. These elements, especially those for the satellites, vary with time, but it is sufficiently accurate to adopt simplified models of these motions; in particular, short-period nutations are ignored.

Each cartographic coordinate system is defined by reference to the adopted axis of rotation and arbitrarily chosen prime meridian, whose position on the surface is specified, where possible, by the adoption of the longitude of a suitable observable feature. For some of the planets and most of the satellites it is sufficient, at present, to assume that the reference surface is spherical, but for others it is necessary to adopt a reference spheroid, with the principal axis of inertia along the axis of rotation.

The following sections of this report describe the ways in which the rotational elements and cartographic coordinate systems are defined. The recommended values are given in a series of tables.

2. Definition of Rotational Elements

The rotational elements of a planet or satellite specify the direction of the north pole and the orientation of its prime meridian as functions of time in the following manner:

The north pole is that pole of rotation which lies on the north side of the invariable plane of the solar system. The direction of the north pole is given with respect to the standard celestial equator and equinox of 1950.0, i.e., in effect with respect to the system of the fundamental catalog FK4. Variable quantities are expressed in units of ephemeris days (or Julian ephemeris centuries of 36 525 days) from the standard epoch of 1950 January 1.0, ET, or JED2 433 282.5; this epoch is denoted J1950 and is slightly different from the epoch 1950.0, which refers to the beginning of the Besselian year and corresponds to JED2 433 282.423. The values will be given with respect to the new standard equator, equinox and epoch of J2000, i.e., of 2000 January 1.5 or JED2 451 545.0, when the relationship between the systems of the new catalog FK5 and that of FK4 is precisely defined.

The direction of the north pole is specified by the value of its right ascension α_0 and declination δ_0, while the orientation of the prime meridian is specified by the angle W that is measured *along* the planet's equator in the positive sense with respect to the planet's north pole (i.e., in an easterly direction on the planet's surface) *from* the ascending node Q of the planet's equator on the standard equator *to* the point B where the prime meridian crosses the planet's equator (see Figure 1). (The point Q is the node at which a point moving around the planet's equator in a positive sense would cross the standard equator from south to north; the right ascension of the point Q is $90° + \alpha_0$ and the inclination of the planet's equator to the standard equator is $90° - \delta_0$.) The prime meridian is assumed to rotate uniformly with the planet, and so W varies linearly with time due to this rotation. In addition, α_0, δ_0, and W may vary with time due to a precession of the axis of rotation of the planet (or satellite). If W increases with time, the planet has a *direct* (or prograde) rotation relative to the invariable plane; if W decreases with time, the rotation is said to be *retrograde.*

In the absence of other information, the axis of rotation is assumed to be normal to the mean orbital plane; Mercury and most of the satellites are in this category. For

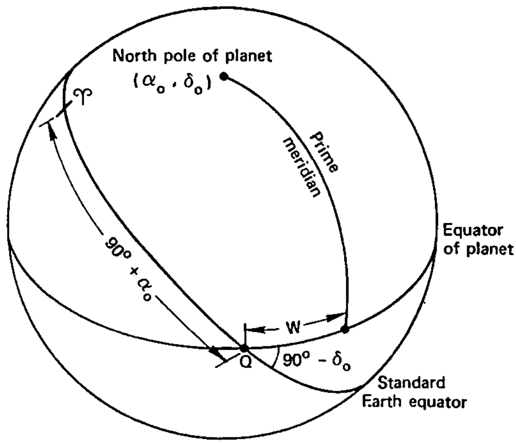

Fig. 1. Reference system used to define orientation of the planet.

many of the satellites it is assumed that the rotation rate is equal to the mean period of orbital revolution.

The angle W specifies the ephemeris position of the prime meridian, and for planets or satellites without any accurately observable fixed surface features the adopted expression for W defines the prime meridian and is not subject to correction. Where possible, however, the cartographic position of the prime meridian is defined by a suitable observable feature and so the constants in the expression $W = W_0 + \dot{W}d$, where d is the interval in days from the standard epoch, are chosen so that the ephemeris position follows the motion of the cartographic position as closely as possible; in these cases the expression for W may require emendation in the future.

For the planets on which no suitable features have been observed, W has been given the value of 360° at the standard epoch (Saturn, Uranus, Neptune, Pluto); in other cases W has been chosen so that the planetographic longitude in the central point of the apparent disk as seen from the center of the Earth (i.e., the longitude of the central meridian) has an arbitrary value at some adopted epoch (Venus, Jupiter). In general, the values given here are compatible with those currently in use.

For satellites on which no suitable features have been observed, the expression for W has been chosen so that the ephemeris position of the prime meridian passes through the subplanetary intersection of the satellite's equator and the plane containing the centers of the satellite, the planet, and the Sun at the time of the first superior heliocentric conjunction of the satellite and the planet after the standard epoch (*Trans. IAU 15B*, p. 108, 1974). An exception is the Moon, whose prime meridian passes through the mean sub-Earth direction.

Recommended values of the constants in the expressions for α_0, δ_0, and W are given for the planets in Table I and for the satellites in Table II. Expressions for the Sun, Earth, and Moon are given to a similar precision as those of the other bodies of the solar system for comparative purposes only.

TABLE I

Recommended values for the direction of the north pole of rotation and the prime meridian of the Sun and planets (1979)

Sun	$\alpha_0 = 286^\circ.0$	Jupiter	$\alpha_0 = 268^\circ.00 - 0^\circ.008T$	
	$\delta_0 = 63^\circ.8$		$\delta_0 = 64^\circ.50 + 0^\circ.003T$	
	$W = 240^\circ.9 + 14^\circ.184\,40d$		$W_I = 17^\circ.7 + 877^\circ.900d$	System I[d]
			$W_{II} = 16^\circ.8 + 870^\circ.270d$	System II
Mercury	$\alpha_0 = 280^\circ.9 - 0.033T$		$W_{III} = 80^\circ.6 + 870^\circ.536d$	System III
	$\delta_0 = 61^\circ.4 - 0^\circ.005T$			
	$W = 184^\circ.74 + 6^\circ.138\,502\,5d$[a]	Saturn	$\alpha_0 = 38^\circ.50 - 0^\circ.034T$	
			$\delta_0 = 83^\circ.31 - 0^\circ.004T$	
Venus	$\alpha_0 = 272^\circ.8$		$W_I = 360^\circ.0 + 844^\circ.300$	System I[d]
	$\delta_0 = 67^\circ.2$		$W_{III} = 360^\circ.0 + 822^\circ.857d$	System III
	$W = 213^\circ.63 - 1^\circ.481\,420\,5d$			
		Uranus	$\alpha_0 = 256^\circ.72$	
Earth	$\alpha_0 = 0^\circ.0 - 0^\circ.640\,32T$		$\delta_0 = -15^\circ.04$	
	$\delta_0 = 90^\circ.0 - 0^\circ.556\,69T$		$W = 360^\circ.0 - 554^\circ.913d$	
	$W = 99^\circ.87 + 360^\circ.985\,612d$[b]			
		Neptune	$\alpha_0 = 294^\circ.91$	
Mars	$\alpha_0 = 317^\circ.342 - 0^\circ.108T$		$\delta_0 = 40^\circ.53$	
	$\delta_0 = 52^\circ.711 - 0^\circ.061T$		$W = 360^\circ.0 + 468^\circ.750d$	
	$W = 11^\circ.50 + 350^\circ.891\,983d$[c]			
		Pluto	$\alpha_0 = 305^\circ$	
			$\delta_0 = 5^\circ$	
			$W = 360^\circ.0 - 56^\circ.367d$	

Note: α_0, δ_0 are standard equatorial coordinates of 1950.0. Approximate coordinates of the north pole of the invariable plane are $\alpha_0 = 272^\circ.40$, $\delta_0 = 66^\circ.99$. T = interval in Julian ephemeris centuries (of 36 525 days) from the standard epoch. d = interval in ephemeris days from the standard epoch. The standard epoch is 1950 January 1.0 ET, i.e., JED2 433 282.5.

[a]The 20° meridian is defined by the crater Hun Kal.
[b]The 0° meridian is defined by the transit circle at Greenwich, England.
[c]The 0° meridian is defined by the crater Airy-0; its longitude in the system of the American Ephemeris, 1968 to present, (de Vaucouleurs' NA3), was $358^\circ.4 \pm 0.3$ (m.e.) on January 15.5, 1909 UT.
[d]System I refers to the atmospheric equatorial rotation; System III, to rotation derived from radio emissions. System II refers to atmospheric rotation north of the south component of the north equatorial belt, and south of the north component of the south equatorial belt.

TABLE II

Recommended values for the direction of the north pole of rotation and the prime meridian of the satellites (1979)

Earth

Moon

$$\alpha_0 = 270^\circ. - 3^\circ.878 \sin E_1 - 0^\circ.120 \sin E_2 + 0^\circ.070 \sin E_3 - 0^\circ.017 \sin E_4$$

$$\delta_0 = 66^\circ.534 + 1^\circ.543 \cos E_1 + 0^\circ.024 \cos E_2 - 0^\circ.028 \cos E_3 + 0^\circ.007 \cos E_4$$

$$W = 244^\circ.375 + 13^\circ.176\,358\,31d + 3^\circ.558 \sin E_1 + 0^\circ.121 \sin E_2 - 0^\circ.064 \sin E_3 + 0^\circ.016 \sin E_4 + 0^\circ.025 \sin E_5$$

TABLE II (*continued*)

where $E_1 = 12°.112 - 0°.052\,992\text{d}$
$E_2 = 24°.224 - 0°.105\,984\text{d}$
$E_3 = 227°.645 + 13°.012\,000\text{d}$
$E_4 = 261°.105 + 13°.340\,716\text{d}$
$E_5 = 358°.00 + 0°.985\,600\text{d}$

Mars

Phobos

$\alpha_0 = 317°.329 + 1°.674 \sin M_1$
$\delta_0 = 52°.717 + 1°.014 \cos M_1$
$W = 270°.202 + 1128°.844\,483\text{d} - 3°.310 \sin M_2 - 1°.332 \sin M_1$

Deimos

$\alpha_0 = 316°.307 + 3.051 \sin M_3$
$\delta_0 = 53°.367 - 1°.821 \cos M_3$
$W = 70°.832 + 285°.161\,807\text{d} - 2°.448 \sin M_3$

where $M_1 = 201°.605 - 0°.435\,427T$
$M_2 = 93°.440 + 1128°.409\,143T$
$M_3 = 23°.054 - 0°.018\,143T$

Jupiter

Amalthea

$\alpha_0 = 268°.0$
$\delta_0 = 64°.5$
$W = 50°.2 + 722°.630\,374\,6\text{d}$

Io

$\alpha_0 = 268°.002 - 0°.0085T + 0°.094 \sin J_1 + 0°.024 \sin J_2$
$\delta_0 = 64°.504 + 0°.0033T + 0°.040 \cos J_1 + 0°.011 \cos J_2$
$W = 262°.7 + 203°.488\,953\,8\text{d} - 0°.085 \sin J_1 - 0°.022 \sin J_2$

Europa

$\alpha_0 = 268°.029 - 0°.0085T + 1°.086 \sin J_2 + 0°.060 \sin J_3 + 0°.015 \sin J_4 + 0°.009 \sin J_5$
$\delta_0 = 64°.516 + 0°.0033T + 0°.468 \cos J_2 + 0°.026 \cos J_3 + 0°.007 \cos J_4 + 0°.002 \cos J_5$
$W = 156°.9 + 101°.374\,723\,5\text{d} - 0.980 \sin J_2 - 0°.054 \sin J_3 - 0°.014 \sin J_4 - 0°.008 \sin J_5$

Ganymede

$\alpha_0 = 268°.149 - 0°.0085T - 0°.037 \sin J_2 + 0°.431 \sin J_3 + 0°.091 \sin J_4$
$\delta_0 = 64°.574 + 0°.0033T - 0°.016 \cos J_2 + 0°.186 \cos J_3 + 0°.039 \cos J_4$
$W = 195°.8 + 50°.317\,608\,1\text{d} + 0°.033 \sin J_2 - 0°.389 \sin J_3 - 0°.082 \sin J_4$

Callisto

$\alpha_0 = 268°.678 - 0°.0085T - 0°.068 \sin J_3 + 0.590 \sin J_4 + 0°.010 \sin J_6$
$\delta_0 = 64°.830 + 0°.0033T - 0°.029 \cos J_3 + 0°.254 \cos J_4 - 0.004 \cos J_6$
$W = 158°.0 + 21°.571\,071\,5\text{d} + 0°.061 \sin J_3 - 0°.533 \sin J_4 - 0°.009 \sin J_6$

TABLE II (*continued*)

where
$J_1 = 19^\circ.2 + 4850^\circ.7T$
$J_2 = 120^\circ.8 + 1191^\circ.3T$
$J_3 = 349^\circ.5 + 262^\circ.1T$
$J_4 = 198^\circ.3 + 64^\circ.3T$
$J_5 = 241^\circ.6 + 2382^\circ.6T$
$J_6 = 317^\circ.7 + 6070^\circ.0T$

Saturn

Mimas
$\alpha_0 = 38^\circ.5 + 13^\circ.1 \sin S_1$
$\delta_0 = 83^\circ.3 - 1^\circ.5 \cos S_1$
$W = 207^\circ.6 + 381^\circ.995\,288\,7\mathrm{d} - 13^\circ.0 \sin S_1$

Enceladus
$\alpha_0 = 38^\circ.5$
$\delta_0 = 83^\circ.3$
$W = 301^\circ.8 + 262^\circ.731\,530\,2\mathrm{d}$

Tethys
$\alpha_0 = 38^\circ.5 + 9^\circ.4 \sin S_2$
$\delta_0 = 83^\circ.3 - 1^\circ.1 \cos S_2$
$W = 33^\circ.7 + 190^\circ.698\,168\,2\mathrm{d} - 9^\circ.3 \sin S_2$

Dione
$\alpha_0 = 38^\circ.5$
$\delta_0 = 83^\circ.3$
$W = 121^\circ.6 + 131^\circ.534\,717\,9\mathrm{d}$

Rhea
$\alpha_0 = 38^\circ.2 + 3^\circ.0 \sin S_3$
$\delta_0 = 83^\circ.3 - 0^\circ.4 \cos S_3$
$W = 14^\circ.1 + 79^\circ.690\,094\,4\mathrm{d} - 3^\circ.0 \sin S_3$

Titan
$\alpha_0 = 34^\circ.3 + 2^\circ.6 \sin S_4$
$\delta_0 = 83^\circ.7 - 0^\circ.3 \cos S_4$
$W = 79^\circ.1 + 22^\circ.576\,973\,4\mathrm{d} - 2^\circ.6 \sin S_4$

Hyperion
$\alpha_0 = 33^\circ.4 + 4^\circ.9 \sin S_5 + 2^\circ.7 \sin S_4$
$\delta_0 = 83^\circ.8 - 0^\circ.6 \cos S_5 - 0^\circ.3 \cos S_4$
$W = 336^\circ.0 + 16^\circ.919\,948\,9\mathrm{d} - 4^\circ.9 \sin S_5 - 2^\circ.7 \sin S_4$

Iapetus
$\alpha_0 = 320^\circ.2 - 3^\circ.9T$, $\delta_0 = 75^\circ.4 - 1^\circ.1T$ rocky or $\alpha_0 = 289^\circ.3$, $\delta_0 = 78^\circ.7$ icy
$W = 275^\circ.5 + 4^\circ.537\,958\,9\mathrm{d}$

where
$S_1 = 68^\circ.6 - 36\,504^\circ.9T$
$S_2 = 314^\circ.5 - 7226^\circ.0T$
$S_3 = 134^\circ.9 - 1017^\circ.7T$
$S_4 = 57^\circ.4 - 53^\circ.5T$
$S_5 = 22^\circ.6 - 239^\circ.2T$

TABLE II (*continued*)

Uranus

Miranda

$\alpha_0 = 256^\circ\!.7$
$\delta_0 = -15^\circ\!.0$
$W = 59^\circ\!.2 - 254^\circ\!.596\,888\,3d$

Ariel

$\alpha_0 = 256^\circ\!.7$
$\delta_0 = -15^\circ\!.0$
$W = 47^\circ\!.3 - 142^\circ\!.835\,604\,7d$

Umbriel

$\alpha_0 = 256^\circ\!.7$
$\delta_0 = -15^\circ\!.0$
$W = 146^\circ\!.4 - 86^\circ\!.868\,813\,6d$

Titania

$\alpha_0 = 256^\circ\!.7$
$\delta_0 = -15^\circ\!.0$
$W = 202^\circ\!.0 - 41^\circ\!.351\,362\,3d$

Oberon

$\alpha_0 = 256^\circ\!.7$
$\delta_0 = -15^\circ\!.0$
$W = 3^\circ\!.2 - 26^\circ\!.739\,437\,5d$

Neptune

Triton

$\alpha_0 = 294^\circ\!.89 - 20^\circ\!.087 \sin N$
$\delta_0 = 36^\circ\!.93 + 15^\circ\!.264 \cos N$
$W = 132^\circ\!.3 - 61^\circ\!.257\,514\,7d + 10^\circ\!.521 \sin N$

where $N = 158^\circ\!.3402 + 61^\circ\!.9803T$

Pluto

1978, P1

$\alpha_0 = 305^\circ$
$\delta_0 = 5^\circ$
$W =$

Note: α_0, δ_0 are standard equatorial coordinates of 1950.0. Approximate coordinates of the north pole of the invariable plane are $\alpha_0 = 272^\circ\!.40$, $\delta_0 = 66^\circ\!.99$. T = interval in Julian ephemeris centuries (of 36 525 days) from the standard epoch. d = interval in ephemeris days from the standard epoch. The standard epoch is 1950 January 1.0 ET, i.e., JED2 433 282.5.

3. Definition of Cartographic Coordinate Systems

Both planetocentric and planetographic systems of coordinates are used in the study of the planets and satellites. These systems are based on the same fundamental reference axis but differ, as explained below, in the definitions of latitude and longitude. Planetocentric coordinates are used for general purposes and are based on a right-handed system of axes, whereas planetographic coordinates are used for cartographic purposes and depend on the adoption of additional parameters to define a reference surface, usually a spheroid, that approximates an equipotential surface of the planet.

For these systems, the fundamental reference z-axis is the mean axis of rotation and the planetary equator is the plane that is normal to this axis and passes through the center of mass of the planet. The x-axis is defined by the intersection of the equatorial plane with the plane of the prime meridian, whose position is defined in an arbitrary manner. The y-axis of planetocentric rectangular coordinates is defined so as to form a right-handed system.

Latitude is measured north and south of the equator; north latitudes are designated as positive. The planeto*centric* latitude (ϕ) of a point is the angle between the equatorial plane and the line connecting the point to the center of mass. The planeto*graphic* latitude (ϕ') of a point on the reference surface is the angle between the equatorial plane and the normal to the reference surface at the point.

Longitude is measured around the equatorial plane from the prime meridian from 0° to 360°. Planeto*centric* longitudes (λ) are measured positively to the east, whereas planeto*graphic* longitudes (λ') are measured in the direction opposite to the rotation, i.e., positively to the west in the case of direct rotation. Planetocentric longitudes are measured from the ephemeris position of the prime meridian as defined by the adopted longitude of some clearly observable surface feature. These two positions may normally be assumed to coincide but it is conceivable that errors in the rotational elements may be such that the cartographic position may drift away from the ephemeris position by a small amount ΔW, where ΔW is measured positively to the east of the ephemeris position.

Planetocentric *radius* (R) is measured from the center of mass to the point concerned. In the planetographic system the position of a point (P) not on the reference surface is specified by the planetographic longitude and latitude of the point (P') on the surface at which the normal passes through P and by the *height* (h) of P above P'.

The reference surfaces for most of the planets are spheroids for which the radius of the equator (A) is larger than the polar semiaxis (C). For some planets and most satellites the reference surface is a sphere ($A = C$), and the planetocentric and planetographic latitudes are then numerically the same. The polar axis of each reference surface is assumed to be the mean axis of rotation as defined by the adopted rotational elements since the accuracy of measurement is, at present, such that a motion of the axis of rotation with respect to the axis of figure cannot be observed.

The recommended values of the parameters for the reference surfaces for planets and satellites are given in Table III. Radii for irregular-shaped satellites are given in Table IV.

It should be noted that east longitude on the Sun, Earth, and Moon is commonly considered to be in the positive direction.

TABLE III

Recommended reference spheroids for mapping the planets and major satellites (1979)

Planet	Satellite	Equatorial radius (km)	Flattening
Mercury		2 439	0
Venus		6 052	0
Earth		6 378.140	0.003 352 81
	Moon	1 738	0
Mars		3 393.4	0.005 186 5
Jupiter		71 398	0.064 808 8
	Io	1 816	0
	Europa	1 563	0
	Ganymede	2 638	0
	Callisto	2 410	0
Saturn		60 000	0.107 620 9
	Mimas	200	0
	Enceladus	275	0
	Tethys	520	0
	Dione	500	0
	Rhea	800	0
	Titan	2 900	0
	Hyperion	112	0
	Iapetus	725	0
Uranus		25 400	0.030
	Ariel	400	0
	Umbriel	275	0
	Titania	500	0
	Oberon	450	0
	Miranda	150	0
Neptune		24 300	0.0259
	Triton	1 600	0
Pluto		1 500	0
	1978, P1	600	0

Note: The equatorial radii for Mercury, Venus, Moon, and Mars are used in current mapping programs, and those for Jupiter and Saturn are used in sequencing and analyzing data from current flight missions. The values for Mars and Pluto differ from those recommended by the IAU in 1976 (*Trans. IAU 16B*, p. 60, 1977). The reference spheroid for Mars (3393.4 km radius) has been used in all mapping programs since 1973, although the IAU 1976 radius (3397.2 km) is probably a better value. In 1976 Pluto's satellite, 1978, P1, had not been discovered.

TABLE IV

Recommended reference shapes for mapping irregular satellites (1979)

Planet	Satellite	Equatorial radius, A (km)	Equatorial radius, B (km)	Polar radius, C (km)
Mars				
	Phobos	13.5	10.7	9.6
	Deimos	7.5	6.0	5.5
Jupiter				
	Amalthea	135	85	77.5

Annex 1: Arguments Concerning the Guiding Principles

As indicated in the introduction to this report, the Working Group considered the possibility of recommending the adoption of changes to the guiding principles that were adopted in 1970, but concluded that the balance of argument favors their retention. The following notes indicate briefly the considerations that led to this conclusion.

1. Definition of north pole

The north pole of a planet might be defined with respect to (a) the invariable plane of the solar system (recommended in guiding principles), or (b) the orbital plane of the planet, or (c) the direction for which the rotation of the planet is in the positive, counterclockwise, or right-handed sense, or (d) the equator of the Earth. Choice (a) is appropriate for discussions of general characteristics of the orbital and rotational motions of the systems of planets, whereas (b) is more appropriate for the study of an individual planet. In practice, however, (a) and (b) are equivalent for the major planets. Choice (c) eliminates negative rotation rates, and is alleged to simplify the mathematical treatment, but it gives rise to confusion when the pole of positive rotation of the planet (e.g., Venus) lies in the southern celestial hemisphere. Choice (d) would eliminate completely the possibility that the north pole of a planet could have a southern declination, but the equator of the Earth has no relevance to the physical nature of the problem.

It is also possible to define the rotational elements and coordinate systems with respect to the pole of positive rotation while defining the north pole with respect to the invariable plane of the solar system. Such a system would require that northern latitudes be negative when the rotation is retrograde with respect to the north pole, thus contravening a convention that is in almost universal use and is expressed in the second part of the first guiding principle.

The considerations for satellites and asteroids are similar to those for the major planets. In particular, the benefits of choosing the north pole on the basis of the dynamics of the individual system (e.g., by specifying the rotational axis with respect to the orbital plane or the equatorial plane of the planet) appear to be less significant than of those using a single, unambiguous criterion for all bodies in the solar system.

In practice, there would be little difference between using the invariable plane and the ecliptic to define the north poles; the ecliptic may be specified more precisely, but like the Earth's equator, it is not of direct relevance. The position of the invariable plane is specified with respect to the standard reference frame in the footnote to Table I of this report.

2. Measure of longitude

Two methods of defining the direction of increasing planetographic longitude were considered. The first is in accordance with the 1970 IAU guiding principle (namely, so that the longitude of the central meridian shall increase with time when observed from a fixed direction) or in accordance with the conventional, right-handed, spherical polar coordinate system in which longitude is always measured positively to the east. The 1970 IAU guiding principle implies that for a direct rotation (i.e., rotation in the positive sense about the north pole), which is the most common direction of rotation in the solar system, planetographic longitude is measured positively to the west. The guiding principle is contrary to the current convention in use for the Sun, Earth, and Moon, where east longitude is positive and west longitude is negative. The practice of measuring geographic longitude positively to the west is common in astronomy, but is contrary to the recommendation of the 1884 conference for adoption of the Earth's prime meridian (*Explanatory Supplement*, page 7). A change at this time would lead to both confusion and extra cost.

The guiding principle does clearly remind the user of the direction of rotation of the body – direct rotation, west longitude is positive; retrograde rotation, east longitude is positive. This principle has been in common usage on Mars and Jupiter for more than a century and, since 1970, many planetwide and detailed maps have been published.

Planetographic coordinates, which refer to an adopted reference spheroid, are used primarily for mapping the surfaces of planets and satellites, whereas planetocentric coordinates, which correspond directly to spherical polar coordinates, are used for more general purposes, such as the study of the external gravitation field of the planet. In such studies, it is desirable to use right-handed coordinate systems in which the azimuthal coordinate (in this case, planetocentric longitude) is measured positively to the east, i.e., in the opposite sense to planetographic longitude for bodies in direct rotation. (It may be noted that IAU Commission 7 (*Trans. IAU 11B*, p. 174, 1962) has already recommended that geocentric longitude be measured positively to the east in the formulas for the Earth's gravitational potential.) There is clearly a risk of confusion arising from the use of different sign conventions for planetocentric and planetographic longitudes, but this risk can be reduced if authors follow the practice of specifying carefully their notation and conventions in papers and reports; for example, if the sense in which *longitude* is measured is ambiguous, the terms *west longitude* and *east longitude* should be used.

Annex 2: Notes on the Tables of Rotational Elements and Reference Surfaces

The recommended values of the parameters that specify the rotations and cartographic reference surfaces for the planets and satellites are given in Tables I to IV in the main body of this report. The chosen parameters are in accordance with the definitions and recommendations given in Sections 2 and 3. Since these expressions only approximately describe the motions of the planets and satellites, some error must be associated with their use; for most applications, however, the accuracy of these convenient equations will be adequate. Moreover, the values of many of the parameters will change with the acquisition of new information and analysis. Tables I and II contain the rotation rates in degrees per day; the rotation period, a more familiar form of the same information, is given in Table V.

Rotational elements are given in Table II for only those satellites whose tidal evolution times are short compared with the age of the solar system. The tidal torques of these satellites should have had time to dissipate their rotational energies and to achieve principal axis rotation; thus in Table II the rotation periods are assumed equal to the satellite's orbital period (Peale, 1977). Within a degree or so, the rotation axes of these satellites should be normal to their orbital planes (as assumed in Table II). This will be the case provided $m \ll (C-A)/C$, where mn is the rate of regression of the node of the satellite orbit, n is its mean motion, and A and C are principal moments of inertia of the satellite, with C about the axis of rotation and A about the axis pointing towards the planet. For Iapetus, this condition can only hold if the body is not in hydrostatic equilibrium (if it is composed mostly of rock). If, however, Iapetus is composed mostly of ice, then it will be close to hydrostatic equilibrium, and $m \gg (C-A)/C$. In this case, the rotation pole will be approximately normal to the Lapalacian plane of Iapetus, which is inclined about 8° to the instantaneous orbit plane (Peale, 1977). Both cases are given in Table II.

The method of defining the prime meridian of all of the satellites except the Moon is described in Section 2. J. G. Beerer (1976) of JPL used this definition to compute the values of W for all of the satellites at the standard epoch.

1. Mercury

A sidereal rotation period of 59 ± 5 days for Mercury was discovered by Pettengill and Dyce (1965) using radar measurements. A rotation period exactly equal to two-thirds of the mean orbital period (58.6462 days) was suggested by Colombo (1965) and confirmed by telescopic photographs (Smith and Reese, 1968) and spacecraft pictures (Klaasen, 1976). Dynamic considerations (Peale, 1972) and analysis of Mariner 10 pictures (Klaasen, 1976) suggest that the axis of rotation is very nearly normal to the orbital plane. Thus the spin axis of Mercury is assumed to be normal to its orbital plane and the sidereal rotation rate is assumed to be exactly two-thirds of the mean orbital period. The prime meridian is located so that the center of the crater Hun Kal is at planetographic longitude 20° (Davies and Batson, 1975; Davies and Katayama, 1976).

TABLE V

Rotation periods of the planets and satellites

Planet		Satellite	Rotation period (days)	Rotation period (d)	(h)	(m)	(s)
Mercury			58.6462	58	15	30	33.9
Venus			243.01	243	0	14	24.0
Earth			0.9973	0	23	56	4.1
		Moon	27.3217	27	7	43	11.2
Mars			1.0260	1	0	37	22.7
		Phobos	0.3189	0	7	39	13.8
		Deimos	1.2624	1	6	17	54.9
Jupiter	(System I)		0.4101	0	9	50	30.0
	(System II)		0.4137	0	9	55	40.6
	(System III)		0.4135	0	9	55	29.7
		Amalthea	0.4982	0	11	57	22.8
		Io	1.7691	1	18	27	33.5
		Europa	3.5512	3	13	13	42.0
		Ganymede	7.1546	7	3	42	33.4
		Callisto	16.6890	16	16	32	11.2
Saturn	(System I)		0.4264	0	10	14	0
	(System III)		0.4375	0	10	30	0
		Mimas	0.9424	0	22	37	5.1
		Enceladus	1.3702	1	8	53	7.0
		Tethys	1.8878	1	21	18	25.9
		Dione	2.7369	2	17	41	9.9
		Rhea	4.5175	4	12	25	12.0
		Titan	15.9454	15	22	41	26.9
		Hyperion	21.2767	21	6	38	23.4
		Iapetus	79.3308	79	7	56	22.8
Uranus			0.6488	0	15	34	12.0
		Miranda	1.4140	1	9	56	9.6
		Ariel	2.5204	2	12	29	20.8
		Umbriel	4.1442	4	3	27	37.2
		Titania	8.7059	8	16	56	28.0
		Oberon	13.4633	13	11	7	5.7
Neptune			0.7680	0	18	25	55.2
		Triton	5.8768	5	21	2	38.1
Pluto			6.3867	6	9	16	52.2

The reference surface for the Mercury mapping program is a sphere with a radius of 2439 km (Davies and Batson, 1975; Davies and Katayama, 1976). This radius is consistent with early radar measurements (Ash *et al.*, 1971) and is close to the Mariner 10 occultation radii measurement of 2439.6 km, 2438.3 km (Howard *et al.*, 1974b).

2. Venus

The direction of the north pole and the retrograde rotation rate of Venus have been determined by analyses of radar observations from Earth taken over an extended period. Recent results confirm that the rotation period is slightly shorter than the Earth resonance period of 243.16d. The following table summarizes some of the results:

Direction of North Pole α	δ_0	Rotation period (days)	Reference
278°0 ± 5°0	69°0 ± 2°0	242°6 ± 0.6	Goldstein (1966)
270°3 ± 0°6	66°7 ± 0°4	245.1 ± 0.7	Dyce *et al.* (1967)
274°0 ± 3°0	64°0 ± 2°0	241.0 ± 1.0	Shapiro (1967a)
275°3 ± 1°8	65°8 ± 1°2	243.09 ± 0.18	Shapiro (1967b)
272°7 ± 0°1	65°3 ± 1°0	243.0 ± 0.7	Jurgens (1970)
273°0	66°0	243.0	*Trans. IAU 14B*, p. 128 (1971)
272°8 ± 0°05	67°2 ± 0°3	243.01 ± 0.03	Shapiro *et al.* (Table 1) (1979)
272	67°2	243.01	Recommended

The prime meridian is defined so that the planetographic longitude of the central meridian of Venus as observed from the center of the Earth was 320°0 at 0^h on 20 June 1964 (JED2 438 566.5) (*Trans. IAU 14B*, p. 128, 1971).

The reference surface for the mapping of Venus is a sphere with a radius of 6052 km. This radius was based upon a combined analysis of Earth-based radar and Mariner 5 tracking data (Howard *et al.*, 1974a).

3. Earth

The values for the rotational elements and reference surface of Earth are given a similar precision to those of the other planets. The values are for comparative purposes only and are consistent with the IAU system of astronomical constants (*Trans. IAU 16B*, p. 58, 1977).

4. Earth's satellite

The values for the rotational elements of the Moon were derived by J. G. Williams (1979) and agree with those of the LURE 2 program to a few hundredths degree. The reference surface for the Moon is a sphere with a radius of 1738 km; this datum has been used in most mapping projects since 1960.

5. Mars

The direction of the north pole and the rotation rate of Mars have been determined very accurately by an analysis of range and doppler measurements from the *S*-band radio transmitters on the Viking lander spacecraft (Mayo *et al.*, 1977; Michael, 1979). The prime meridian passes through the center of the crater Airy-0 (de Vaucouleurs *et al.*, 1973); the most recent measurement of *W* was made by Davies *et al.* (1978). The reference surface for Mars was defined in de Vaucouleurs *et al.* (1973) and has been used in all mapping programs in the United States and the USSR (Tjuflin, 1978).

6. Mars' satellites

The values for the rotational elements of Phobos and Deimos were derived by T. C. Duxbury (1979) from the orbital elements obtained from Mariner 9 (Born and Duxbury, 1975) modified to incorporate the direction of rotation of Mars' axis determined from Viking data (Mayo *et al.*, 1977). The reference surfaces of these satellites came from Mariner 9 measurements (Pollack *et al.*, 1973).

7. Jupiter

Three rotational systems are in common use for Jupiter. System I applies to all equatorial atmospheric points between the north component of the south equatorial belt and the south component of the north equatorial belt (*Explanatory Supplement*, p. 339). System II applies to atmospheric points north of the south component of the north equatorial belt and south of the north component of the south equatorial belt (*Explanatory Supplement*, p. 339). System III was derived from analysis of radio signals received from Jupiter (Riddle and Warwick, 1976). The rotational periods of these systems are I $9^h\,50^m\,30\overset{s}{.}0034$, II $9^h\,55^m\,40\overset{s}{.}6322$, and III $9^h\,55^m\,29\overset{s}{.}711$. The longitude coordinates for the three systems are defined in Seidelman and Divine (1977). The direction of the spin axis was derived from Pioneer 10 and 11 tracking data (Null, 1976).

The equatorial radius of the reference surface came from the *Trans. IAU 16B*, p. 60, 1977; the flattening was derived by Anderson (1976).

8. Jupiter's satellites

The values for the rotational elements of the Galilean satellites were derived by J. H. Lieske (1979) and agree with those based on the Lieske E2 ephemeris to better than $0\overset{\circ}{.}01$. The reference surfaces of Amalthea (Smith *et al.*, 1979) and Galilean satellites (Davies *et al.*, 1979) were derived from measurements made on pictures taken by the Voyager spacecraft.

9. Saturn

Because few reliable measurements of Saturnian clouds have been made, it is not possible to determine one rotation rate for the equatorial zone and another for higher latitudes (Reese, 1971) as was possible with Jupiter. The equatorial rotation period (System I) of $10^h\,14^m$ is from the *Explanatory Supplement*. The rotational system derived from radio signals (System III) has a period of $10^h\,30^m$ (Brown, 1975). The direction of the rotational axis of Saturn was derived by D. Pascu (1978) and is based upon the work of G. Struve, as reported in the *Explanatory Supplement*.

The equatorial radius of the reference surface came from the *Trans. IAU 16B*, p. 60, 1977; the flattening was obtained from measurements by Reese (1971).

10. Saturn's satellites

The values for the rotational elements of the satellites of Saturn were derived by A. T. Sinclair (1978) and were based on work reported by Kozai (1957, 1976), Garcia (1972), Sinclair (1974, 1977), and Woltjer (1928).

The reference surfaces for the satellites of Saturn are estimates from Cruikshank (1978) and are based upon measurements from many sources.

11. Uranus

Because recent measurements of the rotation period of Uranus vary greatly, it is not possible at this time to have confidence in any particular period. Some of the measurements are

Rotation Period	*Reference*
24 ± 3 hr	Hayes and Belton (1977)
23.923 ± 0.003 hr	Smith and Slavsky (1979)
23^{+5}_{-2} hr	Trafton (1977)
16.6 ± 0.5 hr	Franklin *et al.* (1980)
15.57 ± 0.80 hr	Brown and Goody (1977)
$15.0^{+40}_{-2.6}$ hr	Munch and Hippelein (1979)
13.0 ± 1.3 hr	Trauger *et al.* (1978
12.8 ± 1.7 hr	Elliot *et al.* (1979)

For this report a rotation period of 15.57 hr has been selected. The direction of the rotational axis was determined by Dunham (1971).

The equatorial radius (25 400 km) of the reference surface came from the *Trans. IAU 16B*, p. 60, 1977, and agrees with the measurement of Dollfus (1970). Other interesting measurements are 25 900 km (Danielson *et al.*, 1972) and 26 228 km (Elliot *et al.*, 1979). The flattening (0.030) was taken from Dollfus (1970); other measurements were 0.033 (Elliot *et al.*, 1979), 0.022 (Franklin *et al.*, 1980), and 0.010 (Danielson *et al.*, 1972).

12. Uranus' satellites

It is assumed that the axes of rotation of the satellites are parallel to that of Uranus.

13. Neptune

Recent measurements of the rotation period of Neptune vary greatly, so at this time it is not possible to have much configurence in any particular value. Some of the measurements are

Rotation Period	*Reference*
19.583 ± 0.005 hr	Cruikshank (1978)
18.173 ± 0.005 hr	Cruikshank (1978)
18.44 ± 0.01 hr	Slavsky and Smith (1978)
18.432 hr	Smith and Slavsky (1978)
15.4 ± 3 hr	Belton *et al.* (1979)
$11.2^{+1.8}_{-1.2}$ hr	Munch and Hippelein (1979)

For this report a rotation period of 18.432 hr has been selected. The direction of the rotational axis was determined by Gill and Gault (1968).

The equatorial radius (24 300 km) of the reference surface came from the *Trans. IAU 16B*, p. 60, 1977, and agrees with the measurement of Dollfus (1970). The flattening (0.0259) came from analysis of occultation observations by Freeman and Lyngå (1970).

14. NEPTUNE'S SATELLITE

The values of the rotational elements of Triton were derived by Sinclair and Davies (1979) and were based on the reports of Gill and Gault (1968) and Eichelberger and Newton (1926). The radius of Triton is that given by Morrison *et al.* (1977).

15. PLUTO

The rotation period of Pluto was determined photometrically by Anderson and Fix (1973) as 6.3867 days. The direction of the axis of rotation is poorly determined (see Anderson and Fix, 1973; Christy and Harrington, 1978); the direction used was estimated by Harrington and Christy (1980) and implies retrograde rotation. The estimate of the radius was made by Harrington and Christy (1980) and Cruikshank *et al.* (1976).

16. PLUTO'S SATELLITE

The axis of rotation of the satellite of Pluto is assumed to be parallel to that of Pluto. The radius of the satellite is an estimate by Harrington and Christy (1980).

Annex 3: Notes and Definitions

1. PLANETOGRAPHIC AND PLANETOCENTRIC COORDINATES

The planetocentric radius (R) of a point (P) on the surface of a planet or satellite is measured from the point (P) to the center of mass (O – the intersection of the mean axis of rotation and the equator). The planetocentric latitude (ϕ) of the point (P) is the angle between the line (PO) and the planetary equator. The planetographic latitude (ϕ') of the point (P) is the angle between the normal to the reference surface (at P') and the planetary equator (see Figure 2). The height (h) of the point (P) is the distance (PP'). The reference surface is normally a spheroid with an equatorial radius (A) and polar radius (C). The polar flattening (f) is then

$$f = \frac{A-C}{A}.$$

Because h is small compared to R, it is usually sufficiently accurate to relate ϕ and ϕ' by the following expression

$$\tan\phi = \left(\frac{C}{A}\right)^2 \tan\phi' = (1-f)^2 \tan\phi'.$$

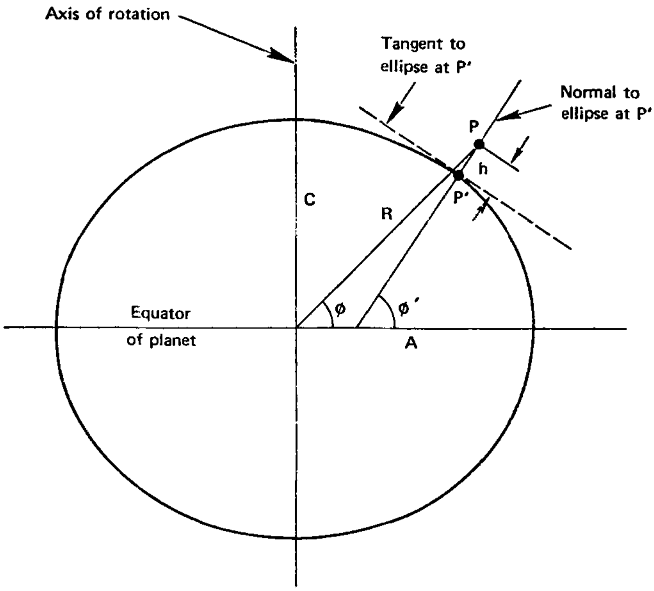

Fig. 2 Planetocentric latitude (ϕ) and planetographic latitude (ϕ') of the point (P).

The planetocentric longitude (λ) and the planetographic longitude (λ') of the point (P) are related by

$$\lambda = (360 - \lambda') \qquad \text{if rotation is direct}$$

$$\lambda = \lambda' \qquad \text{if rotation is retrograde}\,.$$

2. Observations from spacecraft

In most cases, spacecraft navigation data are available in planet-centered, standard equatorial coordinates of 1950.0. For this reason, it is a particularly convenient inertial coordinate system for reducing planetary data.

A point (P) on the surface of the planet has planetocentric coordinates R, ϕ, λ and planet-fixed rectangular coordinates X, Y, Z, where

$$X = R\cos\phi\cos\lambda$$

$$Y = R\cos\phi\sin\lambda$$

$$Z = R\sin\phi\,.$$

The X axis lies on the equator at 0° longitude, the Y axis lies on the equator at 90° longitude, and the Z axis is the axis of rotation of the planet. The standard equatorial coordinates of 1950.0 of point P_x, P_y, P_z can be expressed

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = MV \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

where

$$M^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(90° - \delta_0) & \sin(90° - \delta_0) \\ 0 & -\sin(90° - \delta_0) & \cos(90° - \delta_0) \end{bmatrix} \begin{bmatrix} \cos(\alpha_0 + 90°) & \sin(\alpha_0 + 90°) & 0 \\ -\sin(\alpha_0 + 90°) & \cos(\alpha_0 + 90°) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$V = \begin{bmatrix} \cos W & -\sin W & 0 \\ \sin W & \cos W & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

If coordinates of the spacecraft are S_x, S_y, S_z, their body-fixed coordinates can be computed as

$$V^T M^T \begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix}.$$

from which their planetocentric coordinates can be found. The values of α_0, δ_0, W are determined from the expressions in Tables I and II evaluated at the proper epoch. If a picture containing P is taken by the spacecraft at S, the coordinates x_c, y_c of P on the picture are given by

$$x_c = \frac{\xi}{\zeta} f, \qquad y_c = \frac{\eta}{\zeta} f,$$

where

$$\begin{bmatrix} \xi \\ \eta \\ \zeta \end{bmatrix} = C \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} - C \begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix},$$

f is the calibrated principal distance (focal length), and C is the transformation matrix from standard equatorial coordinates of 1950.0 into the camera coordinate system.

3. Observations from Earth

3.1. *Precession to Equator of Date*

For observations of planets from the Earth, the most useful reference plane is the Earth's equatorial plane of date. It is thus necessary to precess the planet's rotation elements from the equator and equinox of 1950.0. Then the position angle of the axis of rotation and the longitude of the central meridian can be calculated.

First, evaluate the expressions given in Table I or Table II for α_0, δ_0, and W at the time required. It is necessary to antedate the time required by the light time from the planet to the Earth when evaluating W, and also to make the appropriate correction from UT to ET if the time required is specified in UT.

Denote by α_1, δ_1, W_1 the values of α_0, δ_0, W when referred to the equator and equinox of date. These are given by

$$\cos\delta_1 \sin(\alpha_1 - z) = \cos\delta_0 \sin(\alpha_0 + \zeta)$$

$$\cos\delta_1 \cos(\alpha_1 - z) = -\sin\delta_0 \sin\theta + \cos\delta_0 \cos\theta \cos(\alpha_0 + \zeta)$$

$$\sin\delta_1 = \sin\delta_0 \cos\theta + \cos\delta_0 \sin\theta \cos(\alpha + \zeta)$$

and

$$W_1 = W - \eta ,$$

where $\sin\eta = \sin\theta \sin(\alpha_0 + \zeta)/\cos\delta_1$.

ζ, z, and θ are given by:

$$\zeta = 0''.005 + 2304''.997\ T + 0''.302\ T^2 + 0''.018\ T^3$$

$$z = \zeta + 0''.791\ T^2$$

$$\theta = 0''.004 + 2004''.298\ T - 0''.426\ T^2 - 0''.042\ T^3 ,$$

where T is time in Julian centuries from JED2 433 282.5 (J1950).

These formulas are valid for all values of the declination. In practice it is generally sufficiently accurate to apply the precession correction as a linear function of time. The appropriate rate of change is most easily calculated by determining the precesssion corrections using the above formulas at, say, epoch J2000 (JED2 451 545.0), and applying these corrections linearly over the intermediate 50 years. Note that the precession correction at epoch J1950 is not quite zero, as this epoch differs from 1950.0 by 0.077 days. For practical purposes, this difference can be ignored, and the constant terms in ζ, z, and θ can be neglected.

3.2. *The Planetocentric Sphere*

At the time required, the RA and Dec of the north pole of the planet and the position of its prime meridian relative to the Earth's equator of date are α_1, δ_1, W_1. Let the planet have RA and Dec α, δ relative to the Earth (apparent coordinates referred to the equator of date). The Earth then has RA and Dec $180° + \alpha$, $-\delta$ relative to the planet.

Denote by K and D coordinates of the Earth relative to the equator of the planet, where K is measured from Q_1 towards the east of the planet (see Figure 3). (Q_1 is the ascending node of the planet's equator on the Earth equator of date.)

Denote by P the *position angle of the central meridian*, measured from the north point of the disk to the observer's east (i.e. counterclockwise). Note that P is measured to the part of the central meridian between the sub-Earth point and the planet's *north* pole.

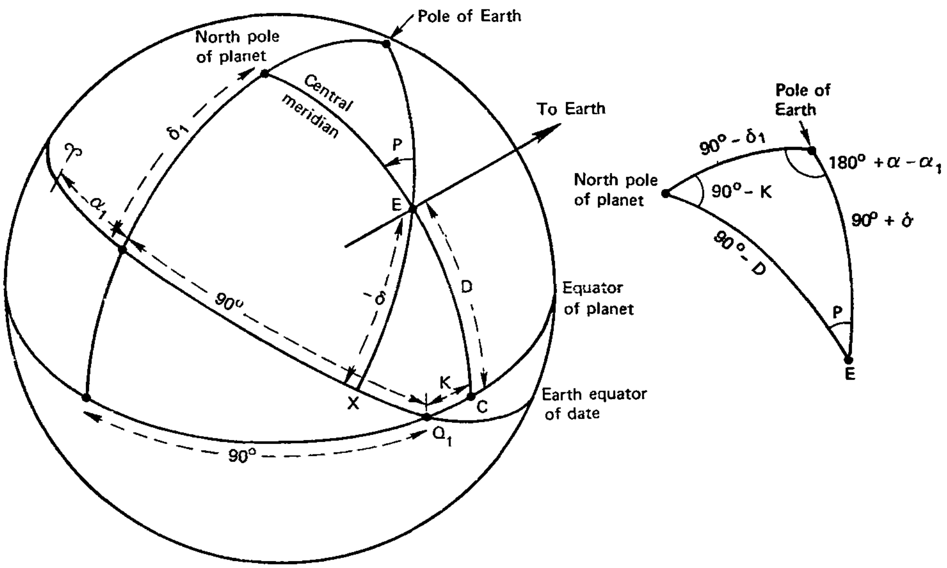

Fig. 3. The planetocentric sphere, and spherical triangle formed by poles of Earth and planet and sub-Earth point $\Upsilon X = 180° + \alpha$.

3.2.1. *Position angle of central meridian P* (also called *Position Angle Axis*) is given by

$$\cos D \sin P = \cos \delta_1 \sin (\alpha_1 - \alpha)$$

$$\cos D \cos P = \sin \delta_1 \cos \delta \cos \delta_1 \sin \delta \cos (\alpha_1 - \alpha) ,$$

where $\cos D > 0$ (since $-90° < D < 90°$).

3.2.2. *Planetographic longitude of central meridian* ω is measured in the direction opposite to the rotation direction, and is given by (see Figures 4 and 5)

$$\omega = W_1 - K \qquad \text{if } \dot{W} > 0$$

$$\omega = K - W_1 \qquad \text{if } \dot{W} < 0$$

and K is given by

$$\cos D \sin K = -\cos \delta_1 \sin \delta + \sin \delta_1 \cos \delta \cos (\alpha_1 - \alpha)$$

$$\cos D \cos K = \cos \delta \sin (\alpha_1 - \alpha) ,$$

where $\cos D > 0$.

3.3. *Sky Coordinates of Point A on the Planet's Surface*

Let A have planetographic longitude and latitude λ' and ϕ'. For precise work, the planetocentric latitude is calculated from

$$\tan \phi = (1 - f)^2 \tan \phi' ,$$

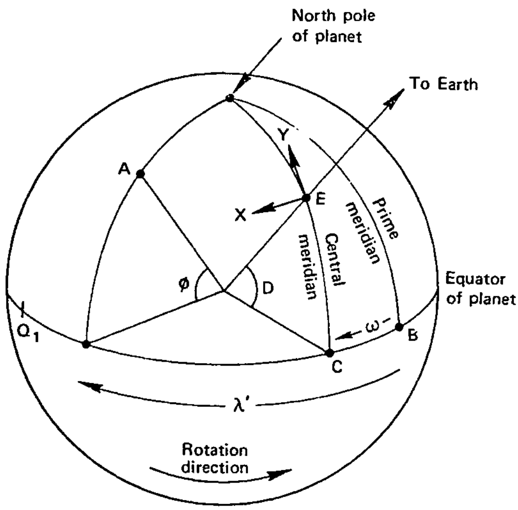

Fig. 4. Direct rotation ($\dot{W} > 0$). $Q_1B = W_1$, $Q_1C = K$.

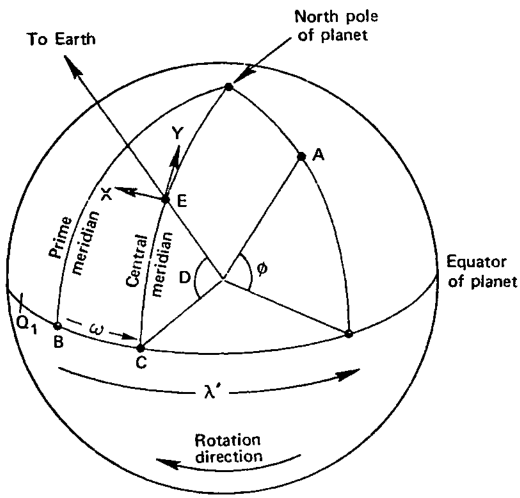

Fig. 5. Retrograde rotation ($\dot{W} < 0$). $Q_1B = W_1$, $Q_1C = K$.

where f is the flattening of the reference spheroid used for the planet. Usually it is sufficiently accurate to take $\phi = \phi'$. Let distance of A from the center of mass of the planet be ρ.

Define axes EX, EY drawn on the apparent disk of the planet as seen from the Earth. E is the center of the disk, EY is towards the north pole of the planet, and EX is towards the west of the planet, at right angles to EY. See Figures 4 and 5 for the cases of direct and retrograde rotation. The coordinates X, Y of point A are given by

$$X = \begin{bmatrix} \rho \cos\phi \sin(\lambda' - \omega) & \text{if } \dot{W} > 0 \\ -\rho \cos\phi \sin(\lambda' - \omega) & \text{if } \dot{W} < 0 \end{bmatrix}$$

$$Y = \rho\,[\sin\phi \cos D - \cos\phi \sin D \cos(\lambda' - \omega)]\,,$$

where $\sin D = -\sin\delta_1 \sin\delta - \cos\delta_1 \cos\delta \cos(\alpha_1 - \alpha)$.

3.4. *Differential Right Ascension and Declination of Point A*

The center of the disk has RA and Dec α and δ. Let the point A have RA and Dec $\alpha + \Delta\alpha$ and $\delta + \Delta\delta$. Then $\Delta\alpha$ and $\Delta\delta$ are given by

$$\Delta\alpha \cos\delta = X \cos P + Y \sin P$$

$$\Delta\delta = -X \sin P + Y \cos P\,.$$

Note that the planetocentric distance ρ that appears in the expressions for X and Y must be expressed in suitable angular units for use in the above equations. Usually it will be sufficiently accurate to take ρ equal to the angular semidiameter of the planet at the time of observation.

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