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Social Security and Intergenerational Equity

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This paper considers the implications of social security for intergenerational equity. It shows that a balanced-budget unfunded system can be optimal even in a dynamically efficient economy without uncertainty and externalities. The relevant criteria for the optimality of the public transfer program are equity among generations and time consistency. The scheme can survive adverse shocks if the well-being of the elderly at each point in time is sufficiently valued.

Keywords: social security, overlapping generations, intergenerational equity.

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1 Introduction

All the major industrialized countries switched to some form of social-security scheme following World War II. The sustainability of such social pacts is now under question. In particular, pay-as-you-go schemes, whereby pensions are paid by those currently at work, are bound to be seriously affected by structural changes. The productivity slowdown experienced by the major economies together with the ageing of population, determined by the decline in the birth rate and the increase in life expectancy, have cast serious doubt on the possibility of maintaining social-security plans, especially when the reduction of high deficits and debt is very high on the policy agenda (see, e.g., OECD, 1995). In particular, it is argued that the sustainability of social security may require drastic corrections of excessively generous programs, following the adverse shocks (see Marchand and Pestieau, 1991).

Reforms have already been enacted or are under way, often accompanied by social discontent and political turmoil. The reluctance to accept reductions in the social-security coverage probably rests on the same rationale behind the very existence of social security.

The first step towards an understanding of the issues should be, in our view, to assess which predictions can be derived from intergenerational-equity considerations alone, in order to make a logical distinction between ethical issues and other important aspects such as adverse selection and uncertainty. The latter have been examined by, for example, Gordon and Varian (1988), who have clearly shown that social security provides insurance against economy-wide shocks: adversely hit generations are helped out by the luckier ones. Additional motivations for social security are chronic market failures or myopic governments, concerned with short-term electoral outcomes rather than with the well-being of future generations.

None of the above assumptions, strictly speaking, is needed to explain the basic features of public pension programs. This has been shown by Veall (1986), who provides a persuasive motive for the existence of social security in a model with intergenerational altruism and consumption externalities. Partial altruism is also at the heart of the results by Hansson and Stuart (1989), who demonstrate that living generations sign a social contract consisting of transfers from the young to the old for the current and subsequent periods; this is partially at the expense of future generations, who will not find it optimal to change the contract.

The classic rationale for the existence of social security in a deterministic context is based on paternalism (Samuelson, 1975), however. Individuals act on the basis of a utility function which is not their true one. They end up saving too little for their old age, possibly because they underestimate their life expectancy. Social security can avoid this "Samaritan's dilemma" (Buchanan, 1975) by forcing the young to compulsory saving in order to pay for the consumption of the elderly. Other possible rationales for the government to act paternalistically include unwillingness to accept growing old and "deliberate" myopia (see Diamond, 1977; Atkinson, 1987).

One aim of our paper is to demonstrate that social-security schemes can be justified on purely ethical grounds in deterministic, optimizing models without externalities, myopic behavior, or partial altruism. Equity among generations is sufficient to motivate a public transfer program. The objective of the government is to design an intergenerationally equitable pension plan. The scheme should be time consistent, that is, there should be no incentive for the government to renege ex post on its ex ante decisions. Intuitively, the most appealing way to design a time-consistent plan is to treat all generations alike, in a perfectly symmetric fashion. The appropriate analytical framework can be developed by adapting to social security the procedure suggested by Calvo and Obstfeld (1988), who provide a general methodology to treat different generations in an equitable manner. The main requirement will be that utilities are discounted back to the birth date and not to the current date.

Alternative systems and/or partial modifications to a given pension program can also be evaluated in terms of their welfare implications, given the social-loss function to be minimized by a farsighted government. This approach enables us to verify the ethical underpinnings of the Aaron rule (Aaron, 1966), according to which the superiority of a funded over an unfunded scheme crucially hinges on the difference between the real interest rate and the sum of population and productivity growth rates.

The ethical case for positive social discounting is argued in detail. The discount rate plays a dual role: it should not simply be interpreted as the rate at which the felicities of future generations are discounted, since it also reflects the social concern for the elderly within each generation. The reverse discounting procedure necessary to achieve time consistency must imply, in fact, that at each point in time the benevolent planner attaches greater weight to the currently old. Therefore, even under dynamic efficiency and total absence of both uncertainty and externalities, social-security plans may be the optimal choice for a benevolent planner who is concerned with the welfare of the old.

This theoretical finding should, of course, be interpreted in the correct perspective. It implies that the viability and optimality of social security crucially depend on how the welfare of the elderly is valued by society. A deeply felt care for the old can provide a strong motive for maintaining social security in the face of adverse shocks. The reluctance to switch to fully funded pension schemes can thus be justified on ethical grounds. Unfunded social-security programs can still be optimal despite declines in the population-growth rate. However, they should be suitably modified to reflect the impact of the negative demographic shock.

We finally investigate the robustness of our findings with respect to the explicit modeling of the supply side, along the lines suggested by the recent literature on growth. The qualitative results remain unaffected.

The paper is organized as follows. Section 2 adapts the Calvo–Obstfeld criterion and considers a pure-endowment overlapping-generations model of the variety typically employed in the classic works on the topic (see, among others, Samuelson, 1958, 1975; Diamond, 1965; Veall, 1986; Hansson and Stuart, 1989). Section 3 analyzes the desirability of unfunded social-security programs and the welfare consequences of adjustment to adverse shocks. The main results are summarized in the concluding Sect. 4.

2 Overlapping Generations and Equity

We consider an overlapping-generations economy formed by identical consumers, each of whom lives for two periods. The aim is to analyze the intergenerational-welfare consequences of social-security programs. In order to concentrate on the pure transfer effects of such schemes, we consider an endowment economy without capital accumulation (Samuelson, 1975; Veall, 1986). We analyze how social welfare should properly be defined under the requirement that all generations must be treated in a symmetric fashion.

Consumers have intertemporally separable preferences given by

$$U(c_1^s, c_2^s) = u(c_1^s) + \frac{1}{1+\rho} u(c_2^s) \ , \tag{1}$$

where the superscript s denotes the date of birth and the subscript $(1, 2)$ denotes the age of the consumer. Without loss of generality, individuals receive their lifetime endowment when they are young. The government implements a social-security transfer scheme to the elderly, financed by levying taxes on the young. The individual lifetime budget constraint is therefore

$$c_1^s + \frac{c_2^s}{1+r} = y - \tau + \frac{\beta}{1+r} \ , \tag{2}$$

where y is income when young, r is the rate of interest, τ are taxes levied on young consumers, and β are the social-security transfers received when old. Intertemporal utility maximization requires that the marginal rate of intertemporal substitution be equalized to the marginal rate of intertemporal transformation, which yields the usual first-order condition:

$$\frac{u'(c_2^s)}{u'(c_1^s)} = \frac{1+\rho}{1+r} \ . \tag{3}$$

Consumption when young and when old is a function of lifetime wealth and can be written as

$$c_1^s = c\left(y - \tau + \frac{\beta}{1+r}, r\right) \ , \tag{4a}$$

$$c_2^s = (1+r)(y - \tau - c_1^s) + \beta \ , \tag{4b}$$

where $c'(\cdot) > 0$.

In conformity with the literature (Samuelson, 1958, 1975; Diamond, 1977; Feldstein, 1985) we consider an unfunded balanced-budget social-security transfer program. The relationship between taxes and contributions must satisfy the following constraint:

$$(1 + n)\tau = \beta \, , \tag{5}$$

where n is the constant population-growth rate. The transfer scheme must also satisfy the feasibility constraint, which prescribes that taxes do not exceed income: $\tau \leq y$.

The problem for the government is to design a social-security program which maximizes social welfare. An appropriate criterion for social optimization must achieve intergenerational fairness. A necessary requirement for the social-welfare function is thus that all generations be treated symmetrically. If this were not the case, some generations would enjoy an advantage over others. An additional requirement is that the social-welfare function is time consistent. The government must have no incentive to renege ex post on its commitments. Time consistency is closely related to symmetry, and hence to fairness. If generations are ex ante treated symmetrically according to the plan, any deviations from the pre-announced policy would result in ex post asymmetry among generations. Equity and time consistency require that there must be no incentive to such deviations.

The requirements of symmetry across generations and of time consistency imply that the welfare of each generation must be discounted back to the date of birth, rather than to the current date. In order to see this, suppose that one were to discount future utilities to the present. In this case, the welfare of the generations yet to be born would be assigned different weights, depending on their date of birth. However, all generations currently alive would be given the same weight. Hence, there would be an asymmetry between the generations which are not yet born and those which are presently alive. This asymmetry would give rise to time inconsistencies. In the design of social security, future generations would be distinguished according to their date of birth, whereas the generations currently alive would all be treated in the same fashion. Thus, the policies to which the government had originally committed itself would cease to be optimal as time elapses, since the system of relative weights changes over time.

A social-welfare function that meets both the requirements of symmetry and of time consistency in an optimizing framework had been suggested by Calvo and Obstfeld (1988) for a stationary economy to investigate the effects of fiscal policy. We examine the issue of social security in an economy with population growth.[1] It is well-known that positive social discounting need not be associated with selfishness by the present generations at the expenses of future generations, but could be essential for a fairer intergenerational allocation of resources (see Mirrlees, 1967, p. 112; Chakravarty, 1969, sect. 3.4; Dasgupta and Heal, 1979, p. 261; Marini and Scaramozzino, 1997). Let δ ($> n$) be the pure social discount rate, representing the decrease in the weight given to future generations. The time-consistent social-objective function is

$$\Omega(t) = \sum_{s=t}^{\infty}\left(\frac{1+n}{1+\delta}\right)^{s-t}\left[u(c_1^s) + \frac{1}{1+\rho}u(c_2^s)\right] + \left(\frac{1+n}{1+\delta}\right)^{-1}\frac{1}{1+\rho}u(c_2^{t-1}),$$

which can be written as

$$\Omega(t) = \sum_{s=t}^{\infty}\left(\frac{1+n}{1+\delta}\right)^{s-t}\left[u(c_1^s) + \frac{1+\delta}{(1+\rho)(1+n)}u(c_2^{s-1})\right]. \quad (6)$$

In Eq. (6), the lifetime utility of each generation is discounted back to its date of birth. This has two important consequences. Firstly, there is perfect symmetry amongst all generations. The factor $(1+n)/(1+\delta)$ applies to the utilities of all agents alive at any given time. Secondly, there must be reverse discounting of the welfare of the elderly. The discounted utility index of the old is multiplied by $(1+\delta)(1+n)$. Since $\delta > n$, agents receive a smaller weight when young in the social-welfare function. As younger generations come into being and then grow older, their relative weight steadily increases. Hence, symmetry is preserved both over time and across generations.

The social rate of time preference thus exerts a dual role. On the one hand, it places a weight on the utility of future generations. On the other, it captures the social concern about the well-being of the elderly of each generation. The resulting social contract is time consistent, since all agents in each generation know that they are being treated in a fair and equitable fashion.[2]

1 See also Marini and Scaramozzino (1995) for an extension and an application to environmental issues.

2 Equation (6) is also consistent with Koopmans's (1960) ethical-preference ordering over the set of well-being paths. This ordering is derived under a general set of axioms, requiring continuity and stationarity of the utility function, absence of intertemporal complementarity, and the existence of a best and a worst program.

3 Optimal Social Security

The time-consistent social-welfare function (6) forms the basis for an optimal social-security program. The main issue is whether the introduction of tax-funded social security can increase social welfare. If the government introduces an (unexpected) balanced-budget social-security scheme at time t and if this is perceived as permanent by the economy, then its effects on consumption will be

$$\left.\frac{\partial c_1^s}{\partial \beta}\right|_{(1+n)\tau=\beta} = \frac{c'(n-r)}{(1+r)(1+n)}, \quad s \geq t\,, \tag{7a}$$

$$\left.\frac{\partial c_2^s}{\partial \beta}\right|_{(1+n)\tau=\beta} = \begin{cases} 1 & \text{if } s = t-1, \\ \dfrac{(1-c')(n-r)}{(1+n)} & \text{if } s \geq t, \end{cases} \tag{7b}$$

where c' denotes the marginal propensity to consume by the young out of their lifetime income [Eq. (4a)]. Both young consumers and all future generations would smooth the effects of the transfer program over their life cycle, the net effect on consumption depending on the sign of the difference $n - r$. Older consumers of the current generation, by contrast, would always consume the entire transfer they receive. Thus, when $n > r$ everybody would benefit from the transfer policy: this would be the case in the presence of dynamic inefficiency. On the other hand, when $n < r$ the currently old will benefit but all future generations will experience lower consumption. In this pure-endowment economy there is therefore a trade-off between the welfare of different generations.

According to the social-welfare function (6), and using (7a) and (7b), the marginal effect on social welfare of the transfer scheme is as follows:

$$\begin{aligned}\left.\frac{\partial}{\partial \beta}\Omega(t)\right|_{(1+n)\tau=\beta} &= \frac{1+\delta}{(1+\rho)(1+n)}u'(c_2^{t-1}) \\ &+ \frac{c'(n-r)}{(1+n)(1+r)}\sum_{s=0}^{\infty}\left(\frac{1+n}{1+\delta}\right)^s u'(c_1^{t+s}) \\ &+ \frac{(1-c')(n-r)}{(1+n)(1+\rho)}\sum_{s=0}^{\infty}\left(\frac{1+n}{1+\delta}\right)^s u'(c_2^{t+s})\,.\end{aligned} \tag{8}$$

The net impact of the introduction of social security depends on the parameters δ, ρ, r, and n. In order to establish the effect of the social-security program let us assume that the utility index can be expressed as $u(c) = \ln(c)$.[3] Consumption when young and when old [Eqs. (4a) and (4b)] becomes

$$c_1^s = \frac{1+\rho}{2+\rho}\left(y - \tau + \frac{\beta}{1+r}\right), \tag{9a}$$

$$c_2^s = \begin{cases} \frac{1}{2+\rho}[(1+r)(y-\tau)+\beta] & \text{if } s \geq t, \\ \frac{1}{2+\rho}(1+r)y+\beta & \text{if } s = t-1. \end{cases} \tag{9b}$$

Given the balanced-budget requirement, we must set $\tau = \beta/(1+n)$, whence we obtain

$$c_1^s = \frac{1+\rho}{2+\rho}\left[y + \frac{\beta(n-r)}{(1+r)(1+n)}\right], \tag{10a}$$

$$c_2^s = \begin{cases} \frac{1}{2+\rho}\left[(1+r)y + \frac{\beta(n-r)}{(1+n)}\right] & \text{if } s \geq t, \\ \frac{1}{2+\rho}(1+r)y+\beta & \text{if } s = t-1. \end{cases} \tag{10b}$$

Consumption for the old age is provided for by the unfunded transfer and by a fully funded component which is given by

$$(1+r)(y-\tau-c_1^s) = \frac{1+r}{2+\rho}\left[y - \frac{(1+r)+(1+n)(1+\rho)}{(1+n)(1+r)}\beta\right], \tag{11}$$

using (10a). The change in consumption of the young following the introduction of the social-security scheme is

$$c' = (1+\rho)/(2+\rho)\,. \tag{12}$$

Substituting (10a), (10b), and (12) into (8) we obtain (see Appendix 1):

$$\frac{\partial}{\partial\beta}\Omega(t)\bigg|_{(1+n)\tau=\beta} = A^{-1}B\,, \tag{13}$$

where

$$A = (1+\rho)(1+n)(\delta-n)[(1+r)y+(2+\rho)\beta]\cdot \\ \cdot[(1+r)(1+n)y+\beta(n-r)][(1+\delta)(2+\rho)]^{-1}\,, \tag{13a}$$

3 Samuelson (1969, p. 242) gives justifications for the use of the logarithmic functional form. Feldstein (1985) and Veall (1986) profitably apply it to the analysis of pension programs.

$$B = (\delta - n)[(1+n)(1+r)y + \beta(n-r)] + (n-r)(1+n)[(1+r)y + (2+\rho)\beta] \,. \tag{13b}$$

It is possible to establish that $A > 0$ since $\delta > n$. The impact on social welfare of the social-security scheme, therefore, critically depends on the sign of the coefficient B.

An interesting special case occurs when the pure social discount rate is very close to the population-growth rate, i.e., in a neighborhood of $\delta = n$. The welfare of future generations is now discounted at the population-growth rate. This would correspond to the same weight being assigned to each generation, irrespective of its size. In this case, $\text{sign}(B) = \text{sign}(n - r)$. The critical parameter is thus the difference between the demographic growth rate and the rate of interest. The former increases the tax base and makes it possible to pay for the social-security program. The latter measures the competitive rate of return to savings in capital markets. Therefore, when $\delta = n$ we have:

$$\frac{\partial}{\partial \beta}\Omega(t) \gtrless 0 \iff n \gtrless r \,. \tag{14}$$

The intuition is as follows. Individuals can save when young and invest in the capital market at the rate of interest r. Alternatively, the government could implement a social-security scheme whereby resources are transferred across generations at the rate n. When $\delta = n$ all generations (both current and future) are given the same weight by the government, irrespective of their size. The balanced-budget social-security program therefore increases social welfare when $n > r$, and reduces it when $n < r$. This confirms the original findings by Aaron (1966), according to which it is optimal to introduce a social-security program when $n > r$. In this case, the contributions paid by the young enable all generations to enjoy a higher income during retirement and a higher lifetime welfare.

However, the above results crucially rely on the welfare of future generations being discounted at a rate equal to the population-growth rate. If instead $\delta > n$ (that is, if the government discounts more heavily the lifetime utilities of future generations or, equivalently, attaches greater weight to the elderly at any given time), then it could still be true that $\partial\Omega/\partial b > 0$ even if $r > n$. The critical condition is that the social discount rate is sufficiently greater than the rate of interest. This can be seen by rewriting (13b) as

$$B = (\delta - r)[(1+n)(1+r)y - (r-n)[(\delta - n) + (1+n)(2+\rho)]\beta \, . \qquad (13b')$$

In a neighborhood of $\beta = 0$, $\text{sign}(B) = \text{sign}(\delta - r)$. In general, $B > 0$ provided $\delta \gg r$. In this case, among the generations alive at any given time the elderly would be given a greater weight in the social-welfare function (because of the reverse discounting of their utility), thus shifting the balance of optimal policies in their favor. The weights attached to future generations will also decline rapidly, and the net gains to the current generations from the introduction of social security more than offset the anticipated losses to the future generations. By symmetry, the relative weights of the older versus the younger generations remain constant over time, and thus it is always optimal and time consistent to implement the transfer scheme.

The optimal benefit ratio when $[\partial\Omega/\partial\beta]_{\beta=0} > 0$ can be obtained by setting $\partial\Omega/\partial\beta = 0$ in Eq. (13). This requires $B = 0$ in Eq. (13b), which yields

$$\frac{\beta}{y} = \frac{(\delta - r)(1+n)(1+r)}{(r-n)[(\delta - n) + (1+n)(2+\rho)]} \, . \qquad (15)$$

The critical condition for a positive transfer scheme to be optimal is thus that the social discount rate be sufficiently greater than the rate of interest. The main intuition for this result can be understood as follows. The social discount rate δ is the correct welfare measure of intertemporal preferences. This must be compared with the rate of interest, which measures the intertemporal terms of trade in private capital markets. A social-security program is socially optimal when the former is sufficiently greater than the latter, i.e., when the social benefits from the transfer scheme exceed the private returns from savings. Thus, the introduction of the social-security program can be optimal even under dynamic efficiency. Only when the economy is characterized by severe undercapitalisation and the social discount rate is not sufficiently high can one conclude that the social-security program would not be optimal.[4] Thus the case for social security can be quite strong even under dynamic efficiency.

Our framework can be employed to formulate policy recommendations for social-security reforms following adverse demographic shocks. Serious concern has been expressed about the impact on government budgets of

4 Simulations reported by Feldstein (1985) show that the introduction of a social-security program reduces utility in the presence of very high real interest rates.

the decline in fertility rates and the consequent ageing of the population structure (see, e.g., OECD, 1995). If younger generations shrink in size relative to the older ones, the social-security system becomes unbalanced. The contributions by the young are no longer sufficient to pay for the benefits to the elderly. In order to preserve a balanced-budget program it could become necessary to redesign the social-security scheme and to reduce the benefits to the older generations.

The decline in the population-growth rate can be represented in our model as an unanticipated fall in n to n'. This modifies the structure of the population by increasing the proportion of the elderly. The balanced-budget condition becomes $(1+n')\tau' = \beta'$, where τ' and β' are the new contributions and benefits to the program. The relevant policy issue is how the burden of adjustment should be shared amongst the different generations in such a way that symmetry and time consistency be preserved. Suppose the unanticipated fall in the population-growth rate takes place at time t. The social-objective function is now

$$\Omega_{(n')}(t) = \frac{1+\delta}{(1+n')(1+\rho)} u(c_2^{t-1}) + u(c_1^t) + \sum_{s=t+1}^{\infty} \left(\frac{1+n'}{1+\delta}\right)^{s-t} \left[u(c_1^s) + \frac{1+\delta}{(1+\rho)(1+n')} u(c_2^{s-1})\right]. \tag{16}$$

Consumption of the elderly is given by

$$c_2^{t-1} = \frac{1}{2+\rho}\left[(1+r)y + \frac{\beta(n-r)}{1+n}\right] - (\beta - \beta'), \tag{17a}$$

whereas consumption of the younger generations is

$$c_1^s = \frac{1+\rho}{2+\rho}\left[y + \frac{\beta'(n'-r)}{(1+r)(1+n')}\right], \quad s \geq t, \tag{17b}$$

$$c_2^s = \frac{1}{2+\rho}\left[(1+r)y + \frac{\beta'(n'-r)}{(1+n')}\right], \quad s \geq t. \tag{17c}$$

Substituting (17a)–(17c) into (16) and differentiating with respect to β' we obtain (see Appendix 1):

$$\frac{\partial}{\partial \beta} \Omega_{(n')}(t) = C^{-1} D, \tag{18}$$

where $C > 0$ and D are defined as

$$C \equiv (1+n')(1+\rho)(\delta - n') \cdot \\ \cdot [(1+r)(1+n)y + \beta(n-r) - (2+\rho)(1+n)(\beta - \beta')] \cdot \\ \cdot [(1+r)(1+n')y + \beta'(n'-r)] , \tag{19a}$$

$$D \equiv (1+\delta)(2+\rho)(1+n)(\delta - n')[(1+r)(1+n')y + \beta'(n'-r)] \\ + (1+\delta)(n'-r)(2+\rho)(1+n') \cdot \\ \cdot [(1+r)(1+n)y + \beta(n-r) - (2+\rho)(1+n)(\beta - \beta')] . \tag{19b}$$

Hence, $\partial\Omega_{(n')}/\partial\beta' = 0$ iff $D = 0$, which yields

$$\frac{\beta'}{y} = \frac{(1+n')\{(1+r)(1+n)(\delta - r) - (r-n') \cdot \cdot [(r-n) + (1+n)(2+\rho)]\beta/y\}}{(1+n)(r-n')[(\delta - n') + (1+n')(2+\rho)]} . \tag{20}$$

In general, $\partial\Omega_{(n')}(t)/\partial\beta' > 0$ if $D > 0$. The optimal ratio β'/y is smaller than β/y, for $n' < n$ (see Appendix 1). The value of the coefficient D is proportional to

$$(\delta - r)(1+n)(1+r)y - (r-n')[(r-n) + (1+n)(2+\rho)]\beta . \tag{21}$$

Expression (21) is very similar to the definition of B in Eq. (13b′). It is apparent that $D < 0$ if $r \gg n'$ (i.e., under strong dynamic efficiency), or also if β/y happened to be large before the fall in the population-growth rate. These results conform to intuition. A decline in population growth requires a downward adjustment of both contributions and benefits. This implies a necessary increase in the fully funded component of social security, as given by Eq. (11). Indeed, maintenance of a social-security program may no longer be optimal. The crucial role of population growth is now even more evident: in a world characterized by high real interest rates, a high social discount rate is required to maintain the optimality of social-security schemes in the presence of a falling population-growth rate. The social-security program may also cease to be optimal if the original scheme was excessively generous, involving a high benefit ratio β/y. Following adverse shocks, no drastic change or complete dismantling of social security need necessarily take place. What is required to maintain the optimality of the program is that structural adjustment should be enacted when permanent shocks are perceived to occur. In particular, the fully funded share of the pension program needs to be immediately increased following an adverse permanent shock.

Our analysis has been restricted to the pure-endowment overlapping-generations economy, in order to focus on purely intergenerational-equity issues and to compare results with the classic works in the area. However, we can explicitly investigate the robustness of our findings in the light of the recent literature on economic growth (see Appendix 2). The validity of our analysis can therefore also be extended to this class of models.

4 Conclusions

Social-security reforms are arguably the most debated fiscal-policy issue in the major developed countries due to the occurrence of permanent demographic and productivity shocks. We have addressed this crucial problem in overlapping-generations models of the variety typically employed in the classic contributions to the literature.

In order to rigorously analyze the ethical consequences we have adapted the utilitarian framework devised by Calvo and Obstfeld (1988) to resolve time-inconsistency issues. This entails that generations be treated in a perfectly symmetric fashion and implies that utilities must be discounted back to the birth date rather than to the current date.

Social-security programs are shown to be optimal when the economy is characterized by dynamic inefficiency, thus confirming the existing results in the literature dating back to Aaron (1966). However, we have demonstrated that the case for social security may also be rather strong under dynamic efficiency, even when both uncertainty and externalities are assumed not to exist. A farsighted benevolent government may, in fact, find it optimal to implement social-security schemes even when the rate of interest exceeds the (sum of productivity and) population-growth rate(s). The case for balanced-budget unfunded social-security schemes is thus strengthened when intergenerational-equity issues are properly taken into account.

The system can also be reformed to survive the occurrence of serious adverse shocks if the welfare of the elderly at each point in time is sufficiently valued: the relevant policy issue is to design an adjustment rule for the benefit ratio. In particular, our proposed framework can be employed to implement optimal reforms following the occurrence of a negative demographic shock. Both the benefit and the contribution ratios must decline when the shock occurs. The fully funded component of the pension program has to be increased in order to preserve intergenerational equity.

Appendix

1 Proofs

Proof of Eq. (13): By replacing Eqs. (10a), (10b), and (12) into (8) we obtain

$$
\begin{aligned}
&\frac{\partial \Omega(t)}{\partial \beta} = \\
&= \frac{(1+\delta)}{(1+\rho)(1+n)} \frac{2+\rho}{(1+r)y+(2+\rho)\beta} \\
&\quad + \frac{(1+\rho)(n-r)}{(2+\rho)(1+r)(1+n)} \frac{1+\delta}{\delta-n} \frac{2+\rho}{1+\rho} \frac{(1+r)(1+n)}{(1+r)(1+n)y+\beta(n-r)} \\
&\quad + \frac{(1+n)(2+\rho)(n-r)}{(1+\rho)(2+\rho)(1+n)} \frac{1+\delta}{\delta-n} \frac{1}{(1+r)(1+n)y+\beta(n-r)} \\
&= (1+\delta)\left\{ \frac{2+\rho}{(1+n)(1+\rho)[(1+r)y+(2+\rho)\beta]} \right. \\
&\qquad \left. + \frac{n-r}{\delta-n}\left(1+\frac{1}{1+\rho}\right) \frac{1}{(1+r)(1+n)y+(n-r)\beta} \right\} \\
&= \frac{(1+\delta)(2+\rho)}{1+\rho} \left\{ \frac{1}{(1+n)[(1+r)y+(2+\rho)\beta]} \right. \\
&\qquad \left. + \frac{n-r}{\delta-n} \frac{1}{(1+r)(1+n)y+(n-r)\beta} \right\} \\
&= A^{-1}B \, ,
\end{aligned}
$$

where A and B are as defined in Eqs. (13a) and (13b). □

Proof of Eq. (18): The social-objective function, Eq. (16), can be written using (17a)–(17c) as

$$
\begin{aligned}
&\Omega_{(n')}(t) = \\
&= \frac{1+\delta}{(1+n')(1+\rho)} \ln\left\{ \frac{1}{2+\rho}\left[(1+r)y + \frac{\beta(n-r)}{1+n}\right] - (\beta-\beta') \right\} \\
&\quad + \ln\left\{ \frac{1+\rho}{2+\rho}\left[y + \frac{\beta'(n'-r)}{(1+r)(1+n')}\right] \right\} +
\end{aligned}
$$

$$+\frac{1+n'}{\delta-n'}\ln\left\{\frac{1+\rho}{2+\rho}\left[y+\frac{\beta'(n'-r)}{(1+r)(1+n')}\right]\right\}$$
$$+\frac{1+\delta}{(1+\rho)(\delta-n')}\ln\left\{\frac{1}{2+\rho}\left[(1+r)y+\frac{\beta'(n'-r)}{1+n'}\right]\right\}.$$

The derivative with respect to β is

$$\frac{\partial}{\partial\beta'}\Omega_{(n')}(t)=$$
$$=\frac{1+\delta}{(1+n')(1+\rho)}\cdot$$
$$\cdot\frac{(2+\rho)(1+n)}{(1+r)(1+n)y+\beta(n-r)-(2+\rho)(1+n)(\beta-\beta')}$$
$$+\frac{1}{y+\beta'(n'-r)/[(1+r)(1+n')]}\frac{n'-r}{(1+r)(1+n')}\frac{1+\delta}{\delta-n'}$$
$$+\frac{1+\delta}{(1+\rho)(\delta-n')}\frac{1}{(1+r)y+\beta'(n'-r)/(1+n')}\frac{n'-r}{1+n'}$$
$$=\frac{(1+\delta)(2+\rho)(1+n)}{(1+n')(1+\rho)}\cdot$$
$$\cdot\frac{1}{(1+r)(1+n)y+\beta(n-r)-(2+\rho)(1+n)(\beta-\beta')}$$
$$+\frac{(1+\delta)(n'-r)}{\delta-n'}\frac{1}{(1+r)(1+n')y+\beta'(n'-r)}$$
$$+\frac{(1+\delta)(n'-r)}{(1+\rho)(\delta-n')}\frac{1}{(1+r)(1+n')y+\beta'(n'-r)}$$
$$=C^{-1}D\,,$$

where C and D are as defined in Eqs. (19a) and (19b). □

Proof that $\beta'/y<\beta/y$: The denominator of β'/y [Eq. (20)] is always greater than the denominator of β/y [Eq. (15)], since

$$(r-n')(\delta-n')(1+n)+(r-n')(1+n)(1+n')(2+\rho)$$
$$>(r-n)(\delta-n)+(r-n)(1+n)(2+\rho)\,.$$

The numerator of β/y is greater than the numerator of β'/y, iff

$$(\delta - r)(1+n)(1+r) > (\delta - r)(1+n)(1+r)(1+n') - (r-n')(1+n')[(r-n)+(1+n)(2+\rho)] \cdot \cdot \frac{(\delta - r)(1+n)(1+r)}{(r-n)[(\delta - n)+(1+n)(2+\rho)]},$$

which is equivalent to

$$(1+n')\frac{r-n'}{r-n}\frac{(r-n)+(1+n)(2+\rho)}{(\delta-n)+(1+n)(2+\rho)} > n',$$

which is likely to be satisfied when $n' < n$. □

2 *Social Security and Growth*

Following Buiter (1993) a standard model of endogenous growth is employed, where the constant-returns-to-scale production function of the representative firm is modeled as

$$Y_{it} = F\left(K_{it}, \frac{K_t}{L_t} \cdot L_{it}\right) = \frac{K_t}{L_t} \cdot L_{it} f(k_{it}), \quad k_{it} \equiv \frac{K_{it}}{(K_t/L_t) \cdot L_{it}}, \tag{22}$$

where K_{it} is capital of firm i, L_{it} is labor input, and where $K_t \equiv \sum_i K_{it}$ and $L_t \equiv \sum_i L_{it}$. Equation (22) is consistent with external effects from learning-by-doing of the variety studied by Sheshinski (1967). Assuming, for simplicity, no physical capital depreciation the first-order conditions yield:

$$r_t = f'(k_t), \tag{23}$$

$$w_t = (K_t/L_t)[f(k_t) - k_t f'(k_t)], \tag{24}$$

where w_t is the wage rate. The consumption function can be derived by replacing the exogenous endowment in Eqs. (10a)–(10b) with the endogenous wage earnings.[5]

In a symmetric equilibrium, the aggregate production function becomes

$$Y_t = K_t f(1) \equiv \alpha K_t, \quad 0 < \alpha, \tag{25}$$

5 Elsewhere we analyze in detail the dynamic implications of social-security programs in endogenous-growth models (Marini and Scaramozzino, 1996).

which implies

$$r = f'(1) \equiv \alpha', \quad 0 < \alpha' < \alpha \,, \tag{26a}$$

$$w_t = (K_t/L_t) \cdot (\alpha - r) \,. \tag{26b}$$

Substituting (26a) and (26b) into (6) and differentiating, we can rewrite the Calvo–Obstfeld functional as

$$\frac{\partial \Omega(t)}{\partial \beta} =$$

$$= \frac{1+\delta}{(1+n)(1+\rho)} \frac{1}{\frac{1+r}{2+\rho}(\alpha - r)\frac{K_{t-1}}{L_{t-1}} + \beta}$$

$$+ \sum_{s=t}^{\infty} \left(\frac{1+n}{1+\delta}\right)^{s-t} \frac{1}{\frac{1+\rho}{2+\rho}\left[(\alpha - r)\frac{K_s}{L_s} + \frac{\beta(n-r)}{(1+r)(1+n)}\right]} \frac{n-r}{1+n} \frac{1+\rho}{2+\rho}$$

$$+ \frac{1}{1+\rho} \sum_{s=t}^{\infty} \left(\frac{1+n}{1+\delta}\right)^{s-t} \frac{1}{(1+r)(\alpha - r)\frac{K_s}{L_s} + \frac{\beta(n-r)}{(1+n)}} \frac{n-r}{1+n} \frac{1}{2+\rho}$$

$$= \frac{1}{(1+n)(1+\rho)} \left\{ \frac{1+\delta}{\frac{1+r}{2+\rho}(\alpha - r)\frac{K_{t-1}}{L_{t-1}} + \beta} \right.$$

$$+ (n-r) \sum_{s=t}^{\infty} \left(\frac{1+n}{1+\rho}\right)^{s-t} \left[\frac{1+\rho}{(\alpha - r)\frac{K_s}{L_s} + \frac{\beta(n-r)}{(1+r)(1+n)}} \right.$$

$$\left.\left. + \frac{1}{(1+r)(\alpha - r)\frac{K_s}{L_s} + \frac{\beta(n-r)}{(1+n)}} \right] \right\}$$

$$= \frac{1}{(1+n)(1+\rho)} \left\{ \frac{1+\delta}{\frac{1+r}{2+\rho}(\alpha - r)\frac{K_{t-1}}{L_{t-1}} + \beta} + (n-r)[(1+r) \cdot \right.$$

$$\left. \cdot (1+\rho) + 1] \sum_{s=t}^{\infty} \left(\frac{1+n}{1+\delta}\right)^{s-t} \frac{1}{(1+r)(\alpha - r)\frac{K_s}{L_s} + \frac{\beta(n-r)}{1+n}} \right\} . \tag{27}$$

The first term in brackets in expression (27) measures the effect of the transfer scheme on the welfare of the currently old, and the terms in the summation the effect on the welfare of each of the following generations (born at time s). Since $\alpha > r$, the impact of the scheme on the currently old is always positive. Under dynamic efficiency $n < r$, and the impact on future generations is negative if $(1+r)(\alpha - r)K_s/L_s > \beta(r-n)/(1+n)$. However, the rate of growth of the capital stock is greater than the population-growth rate (Marini and Scaramozzino, 1996), and therefore the negative impact on future generations declines monotonically towards zero.

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