

## Symmetric Cournot oligopoly and economic welfare: a synthesis<sup>★</sup>

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**Summary.** Recently, Mankiw–Whinston (1986) and Suzumura–Kiyono (1987) have shown that socially excessive firm entry occurs in unregulated oligopoly. This paper extends this “excess entry” results by looking into strategic aspects of cost-reducing R&D investment that creates incentives towards socially excessive investments. In the first stage, firms decide whether or not to enter the market. In the second stage, firms make a commitment to cost-reducing R&D investment. In the third stage, firms compete in output quantities. It is shown that the excess entry holds even in the presence of strategic commitments.

### 1 Introduction

Contrary to “a widespread belief that increasing competition will increase welfare (Stiglitz (1981, p. 184)),” recent studies have revealed that competition may sometimes be socially “excessive”. In particular, Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) have shown that socially excessive firm entry may occur in unregulated oligopolistic markets.<sup>1</sup> This happens because entry is occasionally more desirable to entrants than to the society, as new entry creates an incentive for incumbent firms to reduce their outputs. This result was established in a partial equilibrium framework for symmetric Cournot oligopoly.<sup>2</sup>

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<sup>1</sup> See, also, Perry (1984) and Weizsäcker (1980a; 1980b).

<sup>2</sup> Konishi, Okuno-Fujiwara and Suzumura (1990) have generalized this result with general equilibrium interactions, whereas Lahiri and Ono (1988) have shown that this paradoxical result essentially survives with heterogeneous firms by proving that eliminating minor firms increases social welfare through the improvement of average production efficiency which overwhelms the undesirable effect of a change in market structure.

The purpose of the present paper is to add a new dimension to this literature by looking into strategic aspects of cost-reducing R&D investment that may create incentives towards socially excessive investment. We consider an oligopolistic competition played in three stages. In the first stage, firms simultaneously decide whether or not to enter the market. In the second stage, firms make an irrevocable commitment to R&D investment, which affects production cost in the third stage where firms compete in quantities. Since R&D investment is a fixed commitment, firms' investment decisions are affected by strategic considerations.

In the first half of the paper, we analyze the second and the third stage game, with a number of firms fixed. Brander and Spencer (1983) analyzed this game in a Cournot *duopoly* setting and showed that the level of investment is higher at the strategic equilibrium than that at the non-strategic equilibrium. They also showed that investment is sometimes socially excessive as it exceeds the level that maximizes second-best social welfare.<sup>3</sup> In this paper, we identify the causes of this excessive investment and generalize their results in several respects. First, we shall focus on the excessive investment *at the margin* and decompose the welfare effect of an additional investment into the *commitment effect* and the *distortion effect*. Second, by invoking the concept of *strategic substitutes* due to Bulow et al. (1985), we shall provide a clear interpretation of the excessive investment result.<sup>4</sup> Third, we shall establish an increase in the number of firms is likely to cause a socially excessive investment.

In the latter half of the paper, we shall consider the full-fledged three stage game. Under a set of rather weak assumptions, we shall show that the excessive entry *a la* Mankiw–Whinston and Suzumura–Kiyono is extended even with the existence of strategic investment.

The structure of the paper is as follows. In Sect. 2, our model is formulated. Section 3 considers the second and the third stage games with a fixed number of firms, and decomposes the welfare effect of a change in R&D investment into the commitment and the distortion effects. In this section, we shall show that, under fairly mild conditions, the strategic R&D investment is socially excessive at the margin if the actual number of firms exceeds a certain critical number. Section 4 extends our analysis to the full three stage model and a marginal reduction of the number of firms from the free entry level is shown to improve social welfare under a slightly more restrictive set of assumptions. Proofs are gathered in Sect. 5. Section 6 concludes the paper.

## 2 Distortion and commitment effects: The homogeneous product case

### 2.1 Consider an industry where operating firms produce a homogeneous product. Firms engage in three-stage competition. There are infinite number of potential

<sup>3</sup> Note, however, they assumed the Cournot competition with product differentiation, while in this paper we assume the Cournot competition with homogeneous products. See also d'Aspremont and Jacquemin (1988) and Suzumura (1990) which analysed the role of R&D spillovers and cooperative research associations in the framework of two-stage oligopoly models.

<sup>4</sup> Brander and Spencer (1983, p. 227) assumed, in effect, that products are strategic substitutes. See, also, Besley and Suzumura (1989), Eaton and Grossman (1986) and Fudenberg and Tirole (1984) for other contexts where this assumption plays an essential role.

entrants. In the first stage, firms decide whether or not to enter the market in a predetermined sequential order. In the second stage each firm makes a strategic commitment to cost-reducing R&D, whereas firms compete in terms of quantities in the third stage.

In this paper, we will utilize three different equilibrium concepts. Given any arbitrary number of firms and R&D investment profile, the *third stage Cournot-Nash equilibrium* is defined. Given an arbitrary number of firms, the *second stage subgame perfect equilibrium* is defined when the relevant game is defined by the second and the third stages of the entire game. Finally, the *first stage free entry equilibrium* is defined as a subgame perfect equilibrium of the entire game. The focus of our analysis is the welfare performances of the second stage symmetric subgame perfect equilibrium and that of the first stage free entry equilibrium.

2.2 The inverse demand function for the product is  $p = f(Q)$ , where  $p$  is the price and  $Q$  is the industry output. The cost-reducing R&D and the output level of firm  $i$  is denoted by  $x_i$  and  $q_i$ , respectively, and the variable cost function of firm  $i$  is represented by  $c(x_i)q_i$ , where the function  $c(\cdot)$  is assumed to be identical for all firms.

For each specified number of firms  $n \geq 2$  and each specified profile of R&D commitments  $\mathbf{x} = (x_1, x_2, \dots, x_n) > 0$ , the third stage payoff function of firm  $i$  is given by

$$(2.1) \quad \pi^i(\mathbf{q}; \mathbf{x}; n) \equiv \{f(Q) - c(x_i)\}q_i - x_i,$$

where  $\mathbf{q} = (q_1, q_2, \dots, q_n)$  and  $Q = \sum_{j=1}^n q_j$ . For notational simplicity, we assume that the R&D level  $x_i$  is measured by the expenditure for equipment installations. Let  $\mathbf{q}^N(\mathbf{x}; n)$  denote the *third stage Cournot-Nash equilibrium* corresponding to the specified  $(\mathbf{x}; n)$ .

We assume throughout that  $\mathbf{q}^N(\mathbf{x}; n)$  is unique, symmetric and positive if the R&D profile  $\mathbf{x}$  is symmetric and positive.<sup>5</sup> We also assume:

**A(1):**  $f(Q)$  is twice continuously differentiable with  $f'(Q) < 0$  for all  $Q \geq 0$  such that  $f(Q) > 0$ . Furthermore, there exists a constant  $\delta_0 > -\infty$  such that

$$(2.2) \quad \delta(Q) \equiv \frac{Qf''(Q)}{f'(Q)} \geq \delta_0 \quad \text{for all } Q \geq 0 \quad \text{with } f(Q) > 0.^6$$

**A(2):**  $c(x)$  is twice continuously differentiable and satisfies  $c(x) > 0$ ,  $c'(x) < 0$  and  $c''(x) > 0$  for all  $x \geq 0$ .

For any output profile  $\mathbf{q} = (q_1, q_2, \dots, q_n)$ , R&D profile  $\mathbf{x} = (x_1, x_2, \dots, x_n)$  and

<sup>5</sup> An  $n$ -vector  $\mathbf{y} = (y_1, y_2, \dots, y_n)$  is symmetric if  $y_i = y_j$  for all  $i, j = 1, 2, \dots, n$ , whereas  $\mathbf{y}$  is positive if  $y_i > 0$  for all  $i = 1, 2, \dots, n$ .

<sup>6</sup> The elasticity  $\delta$  of the slope of inverse demand function plays a crucial role in many contexts of oligopolistic interaction. See Besley and Suzumura (1989), Seade (1980a, 1980b), Suzumura (1990), and Suzumura and Kiyono (1987), among others.

the number of firms  $n$ , we define

$$\alpha_i(\mathbf{q}; \mathbf{x}; n) \equiv \frac{\partial^2}{\partial q_i^2} \pi^i(\mathbf{q}; \mathbf{x}; n)$$

$$\beta_{ij}(\mathbf{q}; \mathbf{x}; n) \equiv \frac{\partial^2}{\partial q_i \partial q_j} \pi^i(\mathbf{q}; \mathbf{x}; n) \quad (i \neq j; i, j = 1, 2, \dots, n).$$

Note that  $\beta_{ij}(\mathbf{q}; \mathbf{x}; n)$  is the crucial term that determines whether the second stage strategies are *strategic substitutes* ( $\beta_{ij}(\mathbf{q}; \mathbf{x}; n) < 0$ ) or *strategic complements* ( $\beta_{ij}(\mathbf{q}; \mathbf{x}; n) > 0$ ).<sup>7</sup> It will be assumed that:

**A(3):** *The second stage strategies are strategic substitutes so that  $\beta_{ij}(\mathbf{q}; \mathbf{x}; n) < 0$  holds for any  $(\mathbf{q}; \mathbf{x}; n)$  ( $i \neq j; i, j = 1, 2, \dots, n$ ).*

*Remark 1.* A(1) admits the following class of inverse demand functions with constant elasticity of  $f'(Q)$ :

$$(2.3) \quad f(Q) = \begin{cases} a - bQ^\gamma & \text{if } \gamma = \delta + 1 \neq 0 \\ a - b \cdot \log Q & \text{if } \delta = -1, \end{cases}$$

where  $a$  is a non-negative constant and  $b$  is a positive (resp. negative) constant if  $\gamma < 0$  (resp.  $\gamma > 0$ ). Note that (2.3) includes a linear demand ( $\gamma = 1$ ) as well as constantly elastic demand ( $a = 0$ ), so that it still accommodates a wide class of “normal” inverse demand functions.

*Remark 2.* The assumption of strategic substitutability is quite natural to require in our present context, since it is equivalent to assuming the downward sloping reaction functions in the third stage quantity game.

*Remark 3.* It is easy to verify that

$$(2.4) \quad \alpha_i(\mathbf{q}^N(\mathbf{x}; n); \mathbf{x}; n) = 2f'(Q^N(\mathbf{x}; n)) + q_i^N(\mathbf{x}; n) \cdot f''(Q^N(\mathbf{x}; n))$$

$$(2.5) \quad \beta_{ij}(\mathbf{q}^N(\mathbf{x}; n); \mathbf{x}; n) = f'(Q^N(\mathbf{x}; n)) + q_i^N(\mathbf{x}; n) \cdot f''(Q^N(\mathbf{x}; n))$$

hold, where  $Q^N(\mathbf{x}; n) = \sum_{j=1}^n q_j^N(\mathbf{x}; n)$ . If  $\mathbf{x}$  is symmetric  $\alpha_i$  and  $\beta_{ij}$  are identical for all  $i$  and  $j$ . In this case, invoking (2.2) we can rewrite (2.4) and (2.5) into

$$(2.6) \quad \alpha(\mathbf{x}; n) = n^{-1} \cdot f'(Q^N(\mathbf{x}; n)) \cdot (2n + \delta(Q^N(\mathbf{x}; n)))$$

$$(2.7) \quad \beta(\mathbf{x}; n) = n^{-1} \cdot f'(Q^N(\mathbf{x}; n)) \cdot (n + \delta(Q^N(\mathbf{x}; n)))$$

respectively, where  $\alpha(\mathbf{x}; n) \equiv \alpha_i(\mathbf{q}^N(\mathbf{x}; n); \mathbf{x}; n)$  and  $\beta(\mathbf{x}; n) \equiv \beta_{ij}(\mathbf{q}^N(\mathbf{x}; n); \mathbf{x}; n)$  for notational simplicity. Therefore, A(3) implies that:

$$(2.8) \quad n + \delta(Q^N(\mathbf{x}; n)) > 0$$

for any  $\mathbf{x}$  and  $n \geq 2$ , where use is made of A(1). Note that (2.8) is satisfied for any

<sup>7</sup> For the concept of strategic substitutes and complements, see Bulow, Geanakoplos and Klemperer (1985). See, also, Eaton and Grossman (1986) and Fudenberg and Tirole (1983).

$n \geq 2$  if and only if

$$(2.8^*) \quad 2 + \delta(Q^N(\mathbf{x}; n)) > 0$$

holds. Note also that A(1) and (2.8) guarantee that  $\alpha(\mathbf{x}; n) < 0$  holds for any  $(\mathbf{x}; n)$ .

2.3 Under the assumption of an interior optimum, the third stage Cournot-Nash equilibrium  $\mathbf{q}^N(\mathbf{x}; n)$  is characterized by

$$(2.9) \quad f(Q^N(\mathbf{x}; n)) + q_i^N(\mathbf{x}; n) \cdot f'(Q^N(\mathbf{x}; n)) = c(x_i) \quad (i = 1, 2, \dots, n).$$

The first aim of our analysis is to ascertain how the Cournot-Nash output  $q_i^N(\mathbf{x}; n)$  behaves in response to a change in  $x_i$ ,  $x_j$  ( $i \neq j$ ) and  $n$ . Defining  $\omega(\mathbf{x}; n) \equiv (\partial/\partial x_i)q_i^N(\mathbf{x}; n)$  and  $\theta(\mathbf{x}; n) \equiv (\partial/\partial x_j)q_i^N(\mathbf{x}; n)$  ( $i \neq j$ ), straightforward computations assert the following:

**Lemma 1.** For each symmetric  $\mathbf{x}$  and  $n$ ,

$$(2.10) \quad (\partial/\partial n)q_i^N(\mathbf{x}; n) = -\frac{q_i^N(\mathbf{x}; n) \cdot \beta(\mathbf{x}; n)}{\alpha(\mathbf{x}; n) + (n-1)\beta(\mathbf{x}; n)} < 0$$

$$(2.11) \quad \omega(\mathbf{x}; n) = \frac{c'(x_i)}{\Delta(\mathbf{x}; n)} \cdot \{\alpha(\mathbf{x}; n) + (n-2)\beta(\mathbf{x}; n)\} > 0$$

$$(2.12) \quad \theta(\mathbf{x}; n) = -\frac{c'(x_i)}{\Delta(\mathbf{x}; n)} \cdot \beta(\mathbf{x}; n) < 0$$

hold, where

$$(2.13) \quad \Delta(\mathbf{x}; n) \equiv \{\alpha(\mathbf{x}; n) - \beta(\mathbf{x}; n)\} \cdot \{\alpha(\mathbf{x}; n) + (n-1)\beta(\mathbf{x}; n)\} > 0.^8$$

2.4 We now turn to the second stage game. For each specified  $n$ , the first stage pay-off function of firm  $i$  is given by

$$(2.14) \quad \Pi^i(\mathbf{x}; n) = \pi^i(q^N(\mathbf{x}; n); \mathbf{x}; n).$$

If we denote the Nash equilibrium of the second stage game by  $\mathbf{x}^N(n)$ , it is clear that  $\{\mathbf{x}^N(n), \mathbf{q}^N(\mathbf{x}^N(n); n)\}$  is nothing other than the *second stage subgame perfect equilibrium* among  $n$  firms. We assume throughout that  $\mathbf{x}^N(n)$  is unique, symmetric and positive for each  $n$ .

Assuming an interior optimum,  $\mathbf{x}^N(n)$  is characterized by

$$(2.15) \quad \{f(Q^N(\mathbf{x}^N(n); n)) - c(x_i^N(n))\} \cdot (\partial/\partial x_i)q_i^N(\mathbf{x}^N(n); n) + q_i^N(\mathbf{x}^N(n); n) \cdot \{f'(Q^N(\mathbf{x}^N(n); n)) \cdot (\partial/\partial x_i)Q^N(\mathbf{x}^N(n); n) - c'(x_i^N(n))\} - 1 = 0 \quad (i = 1, 2, \dots, n).$$

<sup>8</sup> Since  $\beta(\mathbf{x}; n) < 0$  and  $\alpha(\mathbf{x}; n) < 0$  hold under A(1) and A(3), it follows that

$$(1^*) \quad \alpha(\mathbf{x}; n) + (k-1)\beta(\mathbf{x}; n) < 0 \quad (k = 0, 1, \dots, n)$$

holds. Note that (1<sup>\*</sup>) is a sufficient condition for the local stability of the myopic adjustment process

$$(2^*) \quad \dot{q}_i = \sigma \cdot \frac{\partial}{\partial q_i} \pi^i(\mathbf{q}; \mathbf{x}; n) \quad (i = 1, 2, \dots, n)$$

where  $\dot{q}_i$  denotes the time derivative of  $q_i$ , and  $\sigma > 0$  stands for the adjustment coefficient.

Invoking (2.9) for  $\mathbf{x} = \mathbf{x}^N(n)$ , (2.15) reduces into

$$(2.16) \quad -c'(x_i^N(n)) \cdot q_i^N(\mathbf{x}^N(n); n) - 1 \\ = \{f(Q^N(\mathbf{x}^N(n); n)) - c(x_i^N(n))\} \cdot \sum_{j \neq i} (\partial/\partial x_i) q_j^N(\mathbf{x}^N(n); n) \quad (i = 1, 2, \dots, n),$$

which proves to be crucially important in what follows.

2.5 Consider now the profits  $\Pi^i(\mathbf{x}^N(n); n)$  earned by firm  $i$  at the second stage subgame perfect equilibrium among  $n$  firms. According to the classical entry/exit dynamics, the number of firms  $n$  will increase (resp. decrease) whenever  $\Pi^i(\mathbf{x}^N(n); n) > 0$  (resp.  $< 0$ ), viz.,

$$(2.17) \quad \dot{n} > 0 \text{ (resp. } < 0) \Leftrightarrow \Pi^i(\mathbf{x}^N(n); n) > 0 \text{ (resp. } < 0),$$

where  $\dot{n}$  denotes the time derivative of  $n$ .

Let the *equilibrium number of firms*  $n_e$  be defined as the stationary point of the dynamic process specified by (2.17):

$$(2.18) \quad \Pi^i(\mathbf{x}^N(n_e); n_e) = 0 \quad (i = 1, 2, \dots, n_e).$$

Then  $\{n_e, \mathbf{x}^N(n_e), \mathbf{q}^N(\mathbf{x}^N(n_e); n_e)\}$  constitutes the *first stage free entry equilibrium*.

2.6 To gauge the welfare performance of the industry, we define the *net market surplus function* by

$$(2.19) \quad W(\mathbf{q}; \mathbf{x}; n) \equiv \int_0^Q f(R) dR - \sum_{j=1}^n \{c(x_j) q_j + x_j\},$$

where  $Q = \sum_{j=1}^n q_j$ .

If the government can control this industry in its entirety from the viewpoint of social welfare maximization, the best that can be done is to impose the socially first best R&D,  $x^F(n)$ , and the socially first best output,  $q^F(n)$ , on each incumbent firm and to choose the *first best number of firms*,  $n_f$ . These are defined by

$$(2.20) \quad f(nq^F(n)) - c(x^F(n)) = 0$$

$$(2.21) \quad -c'(x^F(n)) \cdot q^F(n) - 1 = 0$$

$$(2.22) \quad n_f \equiv \arg \max_{n \geq 1} W(\mathbf{q}^F(n), \mathbf{x}^F(n); n).$$

Realistically speaking, however, such a first best policy is hard to implement, since firms are thereby imposed to produce in deficit. If the government cannot control firms' competitive strategies, however, the best that can still be done may be to choose the *second best number of firms*:

$$(2.23) \quad n_s \equiv \arg \max_{n \geq 1} W(\mathbf{q}^N(\mathbf{x}^N(n); n); \mathbf{x}^N(n); n).$$

That is, let  $n_s$  firms freely compete to establish the second-stage subgame perfect equilibrium  $\{\mathbf{x}^N(n_s); \mathbf{q}^N(\mathbf{x}^N(n_s); n_s)\}$ .

In the short-run, however, the government may not be able to control the number of firms. It may be forced to control the R&D level of each incumbent firms to the second best level,  $x^S(n)$ , defined by

$$(2.24) \quad x^S(n) \equiv \arg \max_{x > 0} W(\mathbf{q}^N(\mathbf{x}; n); \mathbf{x}; n).$$

Despite its obvious relevance and appeal, such second best policies may still be difficult to implement. Because of uncertainty on the precise nature of the functions involved, it may be prohibitively hard to identify where exactly  $x^S(n)$  is located. What is required is a policy prescription which does not presuppose the availability of detailed knowledge on the nature of demand and cost functions involved. This is precisely what we look for in the next sections.

### 3 Commitment effect, distortion effect and the number of firms

3.1 In this section, we assume the number of firms,  $n$ , is uncontrollable but R&D investment is under the government's control. Let  $W^N(\mathbf{x}; n)$  be the net market surplus with outputs evaluated at the third stage Cournot-Nash equilibrium:

$$(3.1) \quad W^N(\mathbf{x}; n) \equiv \int_0^{Q^N(\mathbf{x}; n)} f(Q) dQ - \sum_{j=1}^n \{c(x_j)q_j^N(\mathbf{x}; n) + x_j\}.$$

Suppose  $(\partial/\partial x_i)W^N(\mathbf{x}^N(n); n) < (\text{resp. } >) 0$ . Then a marginal *decrease* (resp. a marginal *increase*) of firm  $i$ 's investment at the second stage subgame perfect equilibrium *increases* social welfare, so that the investment at the subgame perfect equilibrium is *socially excessive* (resp. *socially insufficient*) at the margin.

To understand what determines the crucial term  $(\partial/\partial x_i)W^N(\mathbf{x}^N(n); n)$ , it is useful to decompose it into the *commitment effect*  $C_i(\mathbf{x}^N(n); n)$  and the *distortion effect*  $D_i(\mathbf{x}^N(n); n)$ . To be concrete, the commitment effect is defined by<sup>9</sup>

$$(3.2) \quad C_i(\mathbf{x}^N(n); n) \equiv -c'(x_i^N(n)) \cdot q_i^N(\mathbf{x}^N(n); n) - 1,$$

which, in view of (2.16), can be reduced into

$$(3.3) \quad C_i(\mathbf{x}^N(n); n) = \mu_i(\mathbf{x}^N(n); n) \cdot \sum_{j \neq i} (\partial/\partial x_i)q_j^N(\mathbf{x}^N(n); n).$$

The distortion effect, on the other hand, is defined by

$$(3.4) \quad D_i(\mathbf{x}^N(n); n) \equiv \sum_{j=1}^n \mu_j(\mathbf{x}^N(n); n) \cdot (\partial/\partial x_i)q_j^N(\mathbf{x}^N(n); n),$$

where  $\mu_j(\mathbf{x}^N(n); n) \equiv f(Q^N(\mathbf{x}^N(n); n)) - c(x_j^N(n))$  denotes the *marginal distortion* of firm

<sup>9</sup> In the *absence* of strategic commitment, the problem of social welfare maximization takes the form of maximizing  $\int_0^Q f(R) dR - \sum_{j=1}^n \{c(x_j)q_j + x_j\}$  with respect to  $\{(q_i, x_i)\}_{i=1}^n$ . The first-order conditions are then  $f(Q) - c(x_i) = 0$  and  $-c'(x_i)q_i - 1 = 0$  ( $i = 1, 2, \dots, n$ ). Note that the latter condition suggests that  $C_i(\mathbf{x}^N(n); n)$  becomes non-zero *only by the presence of strategic commitment*, which motivates our terminology.

$j$ , which is independent of firm index  $j$  at the symmetric equilibrium. By simply adding  $C_i(\mathbf{x}^N(n); n)$  and  $D_i(\mathbf{x}^N(n); n)$ , we obtain the crucial term  $(\partial/\partial x_i)W^N(\mathbf{x}^N(n); n)$ .

In view of symmetry of  $\mathbf{x}$  and (2.11)–(2.13),

$$(3.5) \quad C_i(\mathbf{x}^N(n); n) = (n - 1) \cdot \mu(\mathbf{x}^N(n); n) \cdot \theta(\mathbf{x}^N(n); n) < 0,$$

$$(3.6) \quad D_i(\mathbf{x}^N(n); n) = \mu(\mathbf{x}^N(n); n) \cdot \{\omega(\mathbf{x}^N(n); n) + (n - 1) \cdot \theta(\mathbf{x}^N(n); n)\} > 0,$$

where  $\mu(\mathbf{x}^N(n); n) \equiv \mu_j(\mathbf{x}^N(n); n) > 0$ . Therefore  $(\partial/\partial x_i)W^N(\mathbf{x}^N(n); n)$  consists of two components with opposite signs.

3.2 It may be useful to illustrate our decomposition of the marginal welfare effect with the help of Fig. 1. At the original symmetric subgame perfect equilibrium, each firm produces  $q_i^* \equiv q_i^N(\mathbf{x}^N(n); n)$  with the marginal cost  $c^* \equiv c(x_i^N(n))$ , and the industry output is  $Q^* \equiv nq_i^*$ . If firm  $i$  unilaterally increases its investment by a small amount  $\varepsilon > 0$ , its marginal cost is reduced to  $c^{**} \equiv c^* - \varepsilon \{-c'(x_i^N(n))\}$ . Products being strategic substitutes, this increase in firm  $i$ 's aggressiveness reduces other firms' output, so that firm  $i$ 's residual demand curve shifts up. As a result, industry output increases to  $Q^{**}$ , and output of firm  $i$  increases to  $q_i^{**}$ .

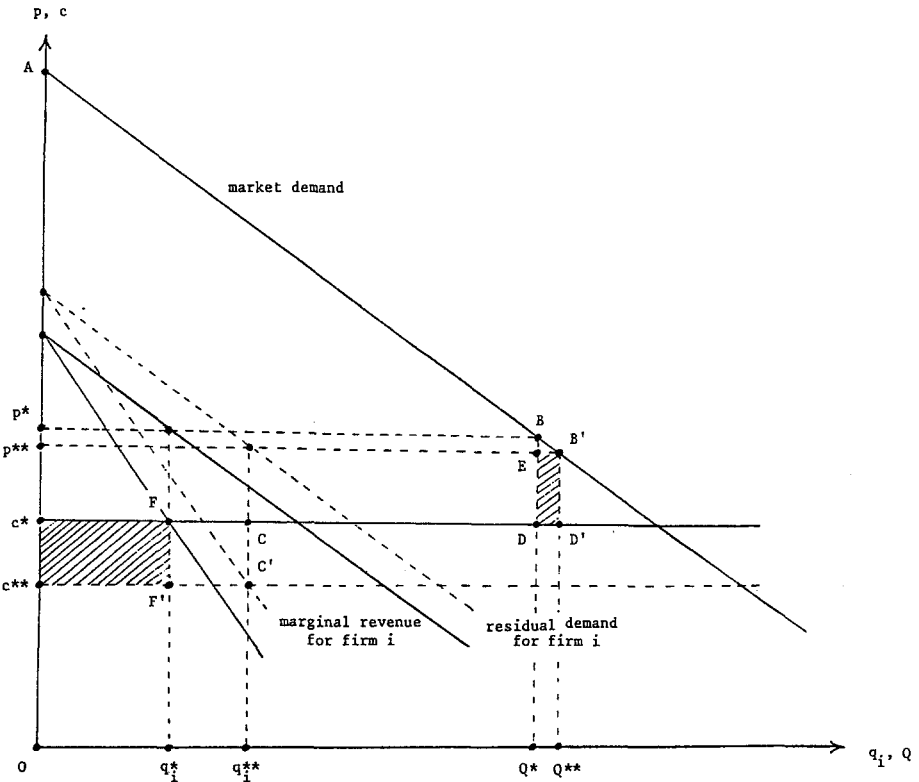


Fig. 1. Distortion effect and commitment effect.



The net welfare gain from this change consists of

$$\begin{aligned} \text{The change in consumers' surplus} &= \text{area } Ap^{**}B' - \text{area } Ap^*B \\ &= \text{area } Bp^*p^{**}B' \end{aligned}$$

and

$$\begin{aligned} \text{The change in profits} &= (\text{area } B'p^{**}c^*D' + \text{area } Cc^*c^{**}C' - \varepsilon) \\ &\quad - \text{area } Bp^*c^*D, \end{aligned}$$

which, after neglecting terms of second order infinitesimal, boils down to  $(\text{area } B'EDD') + (\text{area } Fc^*c^{**}F' - \varepsilon)$ . It is clear that the first term is nothing other than our distortion effect, whereas the second term corresponds precisely to our commitment effect defined by (3.2).

Thus, the distortion effect, which is nothing but the familiar sum of marginal distortions, represents the welfare loss caused by the exercise of firms' monopolistic power on consumers. Clearly, an increase in investment that increases the industry's total supply will generate a positive distortion effect. On the other hand, the commitment effect measures the extent to which a firm can extract additional profits by capturing other firms' market share by taking advantage of a better third stage game structure *via* an increase in investment in  $x_i$ . The total effect on economic welfare depends on the relative strength of these conflicting effects.

3.3 In the rest of this section, we shall elucidate that the commitment effect is likely to dominate the distortion effect, so that the term  $(\partial/\partial x_i)W^N(\mathbf{x}^N(n); n)$  is likely to become negative, if the number of firms is sufficiently large. In view of (3.5), (3.6), (2.6), (2.7), (2.11) and (2.12), and noting A(1) and (2.13), the condition for

$$(\partial/\partial x_i)W^N(\mathbf{x}^N(n); n) = C_i(\mathbf{x}^N(n); n) + D_i(\mathbf{x}^N(n); n) < 0$$

can be reduced into

$$(3.7) \quad 1 - (n-1) \cdot \left\{ - \frac{(\partial/\partial x_i)q_i^N(\mathbf{x}^N(n); n)}{(\partial/\partial x_i)Q^N(\mathbf{x}^N(n); n)} \right\} < 0,$$

which can be further reduced into

$$(3.8) \quad n^2 - 2n + (n-1)\delta(Q^N(\mathbf{x}^N(n); n)) > 0.$$

By virtue of A(1), (3.8) holds whenever  $\lambda(n) \equiv n^2 - 2n + (n-1)\delta_0 > 0$  is satisfied. Let  $N(\delta_0) > 0$  be the largest root of the quadratic equation  $\lambda(n) = 0$ . Then  $(\partial/\partial x_i)W^N(\mathbf{x}^N(n); n) < 0$  holds if  $n > N(\delta_0)$ . Thus:

**Theorem 1.** *Under A(1), A(2) and A(3), there exists a positive number  $N(\delta_0)$  such that  $(\partial/\partial x_i)W^N(\mathbf{x}^N(n); n) < 0$  holds, viz., the strategic cost-reducing investment is socially excessive at the margin if  $n > N(\delta_0)$ .*

An important question still remains. How large is the critical number  $N(\delta_0)$  which appears in Theorem 1? In the case of concave inverse demand functions, it is easy to see that  $N(\delta_0) = 2$ . In the case of constantly elastic inverse demand functions,  $N(\delta_0)$  will increase as the elasticity  $\eta$  of the inverse demand function increases, but for all values of  $\eta$  satisfying  $0 < \eta < 1$ , we have  $1 < N(\delta_0) < 2 + \sqrt{2}$ . Thus,  $N(\delta_0)$  remains fairly small for these important classes of situations.

3.4 It may be useful to graphically illustrate why the number of firms,  $n$ , plays an important role in deciding social excessiveness of investment. Define the third stage reaction function of firm  $i$  by

$$(3.9) \quad r_i(Q_{-i}; x_i^0) \equiv \arg \max_{q_i > 0} \{f(q_i + Q_{-i}) - c(x_i^0)\} q_i,$$

where  $Q_{-i} \equiv \sum_{j \neq i} q_j$ , and an investment profile  $\mathbf{x}^0 \equiv (x_1^0, x_2^0, \dots, x_n^0)$  is fixed. Then the cumulative reaction function  $R_i(Q; x_i^0)$  is defined by

$$(3.10) \quad q_i = R_i(Q; x_i^0) \quad \text{if and only if} \quad q_i = r_i(Q - q_i; x_i^0).$$

By construction, the industry output in the third stage Cournot-Nash equilibrium  $Q^N(\mathbf{x}^0; n)$  is the fixed point of the mapping

$$\sum_{j=1}^n R_j(Q; x_j^0), \text{ viz., } Q^N(\mathbf{x}^0; n) = \sum_{j=1}^n R_j(Q^N(\mathbf{x}^0; n); x_j^0).$$

Figure 2 describes the original third stage equilibrium  $E^0$  as a point where the curve  $\sum_{j=1}^n R_j(Q; x_j^0)$  cuts the 45° line.

Suppose now that firm  $i$  increases its investment marginally. Then the aggregate cumulative reaction curve will shift up to  $\sum_{j=1}^n R_j(Q; x_j^1)$ , where  $R_j(Q; x_j^1) = R_j(Q; x_j^0)$

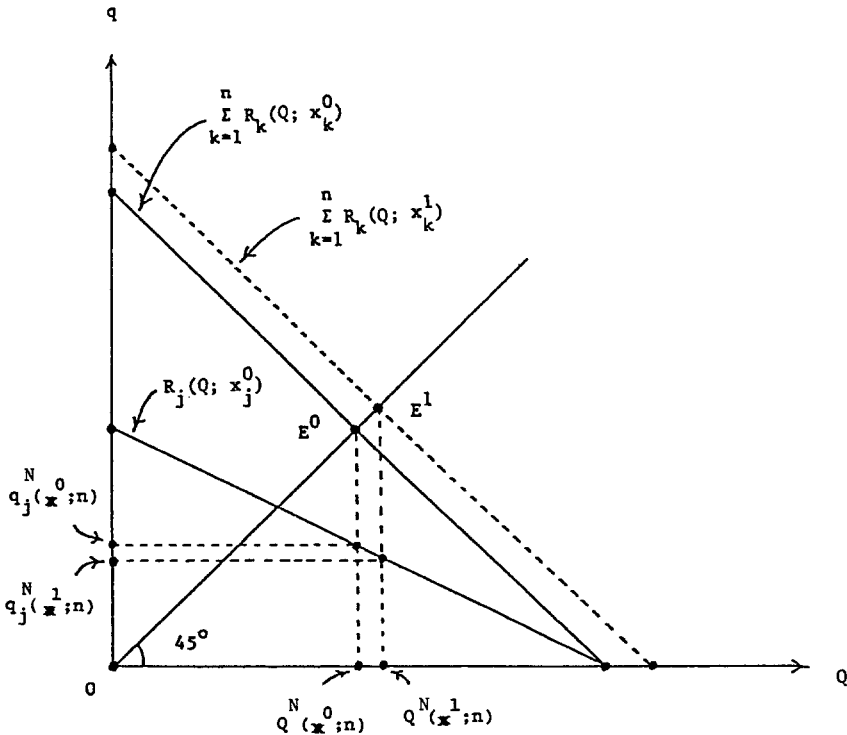


Fig. 2. Cumulative reaction curves.

for all  $j \neq i$ , so that the industry output increases by  $Q^N(\mathbf{x}^1; n) - Q^N(\mathbf{x}^0; n)$ , whereas the output of firm  $j$  ( $j \neq i$ ) decreases by  $q_j^N(\mathbf{x}^0; n) - q_j^N(\mathbf{x}^1; n)$ , where  $x_j^1 = x_j^0$  for all  $j \neq i$ . The ratio between the two,  $[Q^N(\mathbf{x}^1; n) - Q^N(\mathbf{x}^0; n)]/[q_j^N(\mathbf{x}^0; n) - q_j^N(\mathbf{x}^1; n)]$  which closely approximates  $-(\partial/\partial x_i)q_j^N(\mathbf{x}^0; n)/(\partial/\partial x_i)Q^N(\mathbf{x}^0; n)$  in (3.7) if an increase of firm  $i$ 's investment is small enough, is provided by the slope of the cumulative reaction curve.

Figure 2 describes a situation where the inverse demand function is linear, so that the reaction curve is also linear whose slope is independent of the number of firms  $n$ . In this case, as  $n$  becomes large,  $(\partial/\partial x_i)W^N(\mathbf{x}^0; n)$  clearly becomes negative, and the equilibrium investment becomes socially excessive at the margin.

3.5 Before closing this section, a final remark is in order. Since our welfare criterion need not be concave in general, a marginally welfare-improving investment may in fact be a "wrong" move from the global viewpoint. However, it is possible to compare level of the second-best investment directly with that of the second stage subgame perfect equilibrium if our model is parametrizable, viz., the inverse demand function as well as the cost function is constantly elastic. Quite consistent with our analysis so far, it can be shown that there exists a critical number of firms as a function of the elasticity  $\eta$  of the inverse demand function, say  $n^*(\eta)$ , such that *the subgame perfect equilibrium level of investment exceeds the second-best level if  $n > n^*(\eta)$* . The critical number is given by  $n^*(\eta) \equiv [(\eta + 3) + \sqrt{\{(\eta + 3)^2 - 4(\eta + 1)\}}]/2 < \eta + 3$ , which remains fairly small for a wide range of  $\eta$ .

#### 4 Excess entry in the long run

4.1 If the industry is left unregulated for a long time, the first stage free entry equilibrium with  $n_e$  firms, viz.,  $\{n_e, \mathbf{x}^N(n_e), \mathbf{q}^N(\mathbf{x}^N(n_e); n_e)\}$  will be attained. What, then, will be the welfare-improving policy that the government can enforce?

If the government can enforce the marginal cost pricing, it is easy to verify that the welfare-maximizing policy is to restrict the number of firms to either zero or one and impose the marginal cost pricing on the operating firm. Namely, we have the following:

**Theorem 2** (First-best excess entry). *Assume that A(2) holds. Then a small reduction in the number of firms  $n$  unambiguously improves first-best social welfare in the sense that*

$$(4.1) \quad (d/dn)W(\mathbf{q}^F(n); \mathbf{x}^F(n); n) < 0$$

holds as long as  $n \geq 2$ . Indeed, the first-best number of firms  $n_f$  is either 0 or 1.

4.2 Since enforcing the marginal cost principle is next to impossible for the actual government, we should examine how the second-best welfare function  $W(\mathbf{q}^N(\mathbf{x}^N(n); n); \mathbf{x}^N(n); n)$  will be affected when the number of firms changes by a small amount.

Differentiating  $W(\mathbf{q}^N(\mathbf{x}^N(n); n); \mathbf{x}^N(n); n)$  totally with respect to  $n$ , we obtain

$$(4.2) \quad (d/dn)W(\mathbf{q}^N(\mathbf{x}^N(n); n); \mathbf{x}^N(n); n) \\ = \Pi^i(\mathbf{x}^N(n); n) + n\{f(Q^N(\mathbf{x}^N(n); n)) - c(x_i^N(n))\} \cdot (\partial/\partial n)q_i^N(\mathbf{x}^N(n); n) \\ + n \cdot \{C(\mathbf{x}^N(n); n) + D(\mathbf{x}^N(n); n)\} \cdot x_i^{N'}(n).$$

Note that the first term in the RHS of (4.2) is zero when it is evaluated at  $n = n_e$  by virtue of the definition (2.18) of  $n_e$ , whereas the second term evaluated at  $n = n_e$ , viz,

$$(4.3) \quad \mu(\mathbf{x}^N(n_e); n_e) \cdot (\partial/\partial n)q_i^N(\mathbf{x}^N(n_e); n_e)$$

is always negative by virtue of A(1), (2.9) for  $\mathbf{x} = \mathbf{x}^N(n_e)$  and Lemma 1. Note also that (4.3) is the crucial term which leads to the excess entry theorem of Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) in the context of no strategic commitment.

The third term in the RHS of (4.2) is specific to the oligopoly models with strategic commitment. Its first component evaluated at  $n = n_e$ , viz.  $C(\mathbf{x}^N(n_e); n_e)$  is what we called the commitment effect in Sect. 3. Its second component evaluated at  $n = n_e$ , viz.  $D(\mathbf{x}^N(n_e); n_e)$  is the distortion effect.

As was shown in Sect. 3,  $C(\mathbf{x}^N(n_e); n_e) < 0$  and  $D(\mathbf{x}^N(n_e); n_e) > 0$ , so that the presence of strategic commitment seems to introduce some ambiguity in signing (4.2). If we replace A(1) by the following slightly stronger assumption, A(1\*), however, we can establish an unambiguous result.

**A(1\*):**  $f(Q)$  is twice continuously differentiable with  $f'(Q) < 0$  for all  $Q \geq 0$  such that  $f(Q) > 0$ . Furthermore, the elasticity of  $f'(Q)$  is constant, say,  $\delta(Q) = \delta$ .<sup>10</sup>

With this, we can establish:

**Theorem 3** (Second-best excess entry at the margin). *Assume that A(1\*), A(2) and A(3) hold. Then a small reduction in the number of firms at the first stage free-entry equilibrium unambiguously improves the second-best social welfare in the sense that*

$$(4.4) \quad (d/dn)W^N(\mathbf{q}^N(\mathbf{x}^N(n_e); n_e); \mathbf{x}^N(n_e); n_e) < 0$$

holds as long as  $n_e \geq 1 - \delta$ .

Thanks to Theorem 3, under the assumed conditions, the exit of an incumbent firm at the first stage free-entry equilibrium is welfare-improving at the margin in the second-best sense even if we do not know where exactly  $n_f$  and  $n_s$  are located. Note that the crucial inequality  $n_e \geq 1 - \delta$  is obviously satisfied if the inverse demand function is concave, so that  $\delta \geq 0$  holds.

## 5 Proofs

(a) *Proof of Lemma 1*

Differentiating (2.9) with respect to  $n$  and rearranging terms using  $\alpha(\mathbf{x}; n)$  and  $\beta(\mathbf{x}; n)$ , we obtain

$$(5.1) \quad \{\alpha(\mathbf{x}; n) + (n-1)\beta(\mathbf{x}; n)\} \cdot (\partial/\partial n)q_i^N(\mathbf{x}; n) = -q_i^N(\mathbf{x}; n) \cdot \beta(\mathbf{x}; n),$$

which yields (2.10). The negative sign of  $(\partial/\partial n)q_i^N(\mathbf{x}; n)$  is due to A(1) and A(3).

<sup>10</sup> See Remark 1 following the statement of A(1).

To prove (2.11) and (2.12), we differentiate (2.9) with respect to  $x_i$  and  $x_j$  ( $i \neq j$ ), respectively, and rearrange terms using  $\omega(\mathbf{x}; n)$  and  $\theta(\mathbf{x}; n)$  to obtain

$$(5.2) \quad \alpha(\mathbf{x}; n) \cdot \omega(\mathbf{x}; n) + (n-1) \cdot \beta(\mathbf{x}; n) \cdot \theta(\mathbf{x}; n) = c'(x_i)$$

$$(5.3) \quad \beta(\mathbf{x}; n) \cdot \omega(\mathbf{x}; n) + \{\alpha(\mathbf{x}; n) + (n-2)\beta(\mathbf{x}; n)\} \cdot \theta(\mathbf{x}; n) = 0.$$

Solving (5.2) and (5.3) for  $\omega(\mathbf{x}; n)$  and  $\theta(\mathbf{x}; n)$ , we obtain (2.11) and (2.12). The signs of  $\omega(\mathbf{x}; n)$ ,  $\theta(\mathbf{x}; n)$  and  $\Delta(\mathbf{x}; n)$  are determined by A(3), (2.6) and (2.7).  $\square$

(b) *Proof of Theorem 1*

The sketch of the proof is given in the main text and hence it is omitted.  $\square$

(c) *Proof of Theorem 2*

Differentiating  $W(\mathbf{q}^F(n); \mathbf{x}^F(n); n)$  totally with respect to  $n$ , we obtain

$$(5.4) \quad \begin{aligned} (d/dn)W(\mathbf{q}^F(n); \mathbf{x}^F(n); n) &= \{f(nq^F(n)) - c(x^F(n))\} \cdot q^F(n) - x^F(n) \\ &\quad + nq^{F'}(n) \cdot \{f(nq^F(n)) - c(x^F(n))\} \\ &\quad + nx^{F'}(n) \cdot \{-c'(x^F(n)) \cdot q^F(n) - 1\}. \end{aligned}$$

Invoking (2.20) and (2.21), we are then led to conclude that

$$(5.5) \quad (d/dn)W(\mathbf{q}^F(n); \mathbf{x}^F(n); n) = -x^F(n),$$

which is always negative, as was to be established.  $\square$

(d) *Proof of Theorem 3*

*Step 1.* By virtue of A(1), (2.9) for  $\mathbf{x} = \mathbf{x}^N(n_e)$ , (3.5) and (3.6), it can easily be verified that the sign of (4.2) coincides with that of

$$(5.6) \quad \Lambda(n) \equiv (\partial/\partial n)q_i^N(\mathbf{x}^N(n); n) + x_i^{N'}(n) \cdot \{\omega(\mathbf{x}^N(n); n) + 2(n-1) \cdot \theta(\mathbf{x}^N(n); n)\}$$

at  $n = n_e$ . Invoking Lemma 1,  $\Lambda(n)$  can be further reduced into

$$(5.7) \quad \Lambda(n) = \frac{1}{\alpha^N(n) + (n-1)\beta^N(n)} \cdot \left\{ -q_i^N(\mathbf{x}^N(n); n) \cdot \beta^N(n) + \frac{\alpha^N(n) - n \cdot \beta^N(n)}{\alpha^N(n) - \beta^N(n)} \cdot c'(x_i^N(n)) \cdot x_i^{N'}(n) \right\},$$

where  $\alpha^N(n) \equiv \alpha(\mathbf{x}^N(n); n)$  and  $\beta^N(n) \equiv \beta(\mathbf{x}^N(n); n)$  for short. By virtue of (2.6) and (2.7) for  $\mathbf{x} = \mathbf{x}^N(n)$  and A(1\*), it follows that

$$(5.8) \quad \text{sgn } \Lambda(n) = \text{sgn}[A + B \cdot x_i^{N'}(n)],$$

where

$$(5.9) \quad A = q_i^N(\mathbf{x}^N(n); n) \cdot f'(Q^N(\mathbf{x}^N(n); n)) \cdot (n + \delta) < 0$$

$$(5.10) \quad B = c'(x_i^N(n)) \cdot \{n(n-2) + \delta(n-1)\}.$$

Note that  $A > 0$  follows by virtue of (2.8), but the term  $\{n(n-2) + \delta(n-1)\}$  is positive only when  $n > N(\delta)$ , as was shown in (3.8).

*Step 2.* We examine some properties of the second stage payoff function  $\Pi^i(\mathbf{x}; n)$  with the purpose of evaluating  $x_i^{N^i}(n)$  which appears in (5.8). To begin with, simple yet complicated computation using (2.2), (2.4), (2.5), A(1\*) and Lemma 1 establishes that

$$(5.11) \quad \begin{aligned} \Pi_i^i(\mathbf{x}; n) &\equiv (\partial/\partial x_i)\Pi^i(\mathbf{x}; n) \\ &= -c'(x_i) \cdot q_i^N(\mathbf{x}; n) \cdot \xi(n) - 1 \end{aligned}$$

holds, where

$$(5.12) \quad \xi(n) \equiv 1 + \frac{n-1}{n} \cdot \frac{n+\delta}{1+n+\delta} > 0$$

in view of (2.8).

Differentiating (5.11) partially with respect to  $x_i$  and  $x_j$  ( $i \neq j$ ), respectively, we obtain

$$(5.13) \quad \begin{aligned} \Pi_{ii}^i(\mathbf{x}; n) &\equiv (\partial^2/\partial x_i^2)\Pi^i(\mathbf{x}; n) \\ &= -\xi(n) \cdot \{c''(x_i) \cdot q_i^N(\mathbf{x}; n) + c'(x_i) \cdot \omega(\mathbf{x}; n)\} < 0 \end{aligned}$$

$$(5.14) \quad \begin{aligned} \Pi_{ij}^i(\mathbf{x}; n) &\equiv (\partial^2/\partial x_i \partial x_j)\Pi^i(\mathbf{x}; n) \\ &= -\xi(n) \cdot c'(x_i) \cdot \theta(\mathbf{x}; n) \\ &= \xi(n) \cdot c'(x_i) \cdot \frac{c'(x_i) \cdot (n+\delta)}{f'(Q^N(\mathbf{x}; n)) \cdot n \cdot (n+1+\delta)} < 0 \quad (i \neq j) \end{aligned}$$

where the last equality of (5.14) is obtained in view of (2.6) and (2.7). Note that the second order condition for profit maximization at the second stage game requires that  $\Pi_{ii}^i(\mathbf{x}^N(n); n) < 0$  holds, while A(2), Lemma 1 and (5.14) ensure that  $\Pi_{ij}^i(\mathbf{x}; n) < 0$  ( $i \neq j$ ) holds for any  $(\mathbf{x}; n)$ . Therefore, the second stage strategies are warranted to be strategic substitutes if the third stage strategies are.<sup>11</sup>

Differentiating (5.11) partially with respect to  $n$  and noting that

$$(5.15) \quad \xi'(n) = \frac{2n^2 + 2\delta n + \delta(\delta + 1)}{n^2(1+n+\delta)^2}$$

follows from (5.12), we can finally obtain

$$(5.16) \quad (\partial/\partial n)\Pi_i^i(\mathbf{x}^N(n); n) = c'(x_i^N(n)) \cdot q_i^N(\mathbf{x}^N(n); n) \cdot \frac{(1-n)\{2(n+\delta)^2 + \delta\}}{n^2(1+n+\delta)^2}.$$

*Step 3.* By definition,  $\mathbf{x}^N(n)$  is characterized by

$$(5.17) \quad \Pi_i^i(\mathbf{x}^N(n); n) = 0 \quad (i = 1, 2, \dots, n).$$

<sup>11</sup> It is the latter half of A(1\*) that is responsible for this nice property. In general, this property does not necessarily hold. See Besley and Suzumura (1989) and Suzumura (1990).

Differentiating (5.17) totally and invoking symmetry, we obtain

$$(5.18) \quad x_i^{N'}(n) = -\frac{(\partial/\partial n)\Pi_i^i(\mathbf{x}^N(n); n)}{\Pi_{ii}^i(\mathbf{x}^N(n); n) + (n-1)\Pi_{ij}^i(\mathbf{x}^N(n); n)}.$$

In view of (5.18), (5.8) is reduced into

$$(5.19) \quad \text{sgn } \Lambda(n) = \text{sgn} \left[ \frac{A\Pi_{ii}}{\Pi_{ii} + (n-1)\Pi_{ij}} + \frac{(n-1)A \cdot \Pi_{ij} - B \cdot \Pi_{in}}{\Pi_{ii} + (n-1)\Pi_{ij}} \right]$$

where  $\Pi_{ii} \equiv \Pi_{ii}^i(\mathbf{x}^N(n); n)$ ,  $\Pi_{ij} \equiv \Pi_{ij}^i(\mathbf{x}^N(n); n)$  and  $\Pi_{in} \equiv (\partial/\partial n)\Pi_i^i(\mathbf{x}^N(n); n)$ .

It follows that (5.9), (5.13) and (5.14) assure the first term of the right hand side of (5.19) is unambiguously negative. Thus, for the sign of  $\Lambda(n)$  to be negative, a sufficient condition is

$$(5.20) \quad \Gamma(n) \equiv (n-1)A \cdot \Pi_{ij} - B \cdot \Pi_{in} < 0.$$

Invoking (5.9), (5.10), (5.14) and (5.16), a straightforward calculation yields

$$(5.21) \quad \Gamma(n) = q_i(n) \cdot \{c'(n)\}^2 \cdot (n-1) \cdot \Omega(n)$$

where

$$(5.22) \quad \Omega(n) \equiv \left\{ \frac{(n+\delta)^2}{n \cdot (n+1+\delta)} \cdot \xi(n) + \frac{\{n(n-2) + \delta(n-1)\} \cdot \{2(n+\delta)^2 + \delta\}}{n^2 \cdot (1+n+\delta)^2} \right\},$$

$q_i(n) \equiv q_i^N(\mathbf{x}^N(n); n)$  and  $c'(n) \equiv c'(x_i^N(n))$ .

In view of (5.21), if  $n > 1$ , the sufficient condition for  $\Lambda(n)$  to be negative boils down to the condition that  $\Omega(n)$  to be negative. In view of (5.12), a straightforward computation yields that

$$(5.23) \quad \Omega(n) = \phi_\delta(n) / [n^2 \cdot (n+1+\delta)^2],$$

where

$$(5.24) \quad \phi_\delta(n) \equiv -4n^3 - 8\delta n^2 - \delta(5\delta - 2)n - \delta^2(\delta + 1).$$

*Step 4.* The proof of Theorem 3 is complete if we can show that  $\phi_\delta(n) < 0$  as long as  $n \geq 1 - \delta$ . Since  $\phi_\delta(n) < 0$  holds for all  $n > 0$  if  $\delta \geq 0$ , we have only to examine the case where  $\delta < 0$ . With this goal in mind, let  $n^*(\delta)$  stand for the largest real root of the cubic equation  $\phi_\delta(n) = 0$ . The coefficient of the highest order term of this cubic equation being negative, we have  $1 - \delta > n^*(\delta)$  if all of  $\phi_\delta(n)$ ,  $\phi'_\delta(n)$  and  $\phi''_\delta(n)$  are negative at  $n = 1 - \delta$ . This is indeed the case, as we have

$$\phi_\delta(1 - \delta) = 2(\delta - 2) < 0,$$

$$\phi'_\delta(1 - \delta) = -\delta^2 + 6\delta - 12 < 0,$$

and

$$\phi''_\delta(1 - \delta) = -8(3 - \delta) < 0$$

for  $\delta < 0$ . If  $n \geq 1 - \delta$ , we have  $n > n^*(\delta)$ , so that we obtain  $\phi_\delta(n) < 0$ , as was to be verified.  $\square$

## 5 Concluding remarks

In this paper, we have examined the welfare performance of oligopoly with strategic commitments, which culminated into the excess entry results. The second best excess entry at the margin, which is the main result of this paper, is based on three explicit assumptions. The first assumption is on the admissible class of inverse demand functions. Despite its restrictive nature, we should note that a wide class of demand functions satisfies this assumption, as it does accommodate all linear inverse demand functions as well as all constantly elastic inverse demand functions. The second assumption is on the nature of cost reduction technology, which seems to be on the safe ground. The third assumption is on the nature of strategic inter-relatedness of competitive measures. Within a model of quantity competition, the assumed strategic substitutability seems to be widely recognized as a normal case. Despite its rather paradoxical implications, therefore, our welfare verdicts cannot be flatly discarded as pathological. The fact that our results hold even in the presence of strategic commitments seems to enhance its relevance rather substantially.

It goes without saying that there are other implicit assumptions on which our results hinge. To cite just a few, quantity competition rather than price competition, exclusive focus on the symmetric equilibria, no uncertainty in cost-reducing R&D, and no product differentiation and no R&D spillovers can be referred to. It is almost certain, and in some cases demonstrably certain, that the mileage of our excess entry results are severely limited by these implicit assumptions. Nevertheless, the fact remains that the arena where our results do have their bites is in no sense negligible. Presumably, we are in need for more careful analyses of the role of competition as an efficient allocator of resources. The purpose of this paper will be served if it succeeds in bringing this simple point home.

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