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A rigidity theorem for submanifolds with parallel mean curvature in a sphere

By

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1. Introduction. It seems interesting to generalize the famous Chern-do Carmo-Kobayashi Rigidity Theorem [1] for minimal submanifolds to general cases. Let M^n be an n-dimensional compact submanifold with parallel mean curvature in a unit sphere $S^{n+p}(1)$, and h its second fundamental form. It follows from the Gauss equations that the square norm of h is given by

$$S = n(n-1) - R + n^2 H^2,$$

where R and H are the scalar curvature and the mean curvature of M respectively. It was proved by Okumura [3, 4] that if the normal bundle of M is flat, $n \geqq 3$, and $S < 2 + \frac{n^2}{n-1} H^2$, then M is totally umbilical. Yau [6] proved that if $p > 1$, and $S < \frac{n}{3 + n^{\frac{1}{2}} - (p-1)^{-1}}$, then M lies in a totally geodesic $S^{n+1}(1)$. In [5], the author improved Yau's result above. More precisely we proved that if $p > 1$, and $S < \min\left\{\frac{2n}{1 + n^{\frac{1}{2}}}, \frac{n}{2-(p-1)^{-1}}\right\}$, then M is a totally umbilical sphere. In this paper, we shall prove a rigidity theorem for submanifolds with parallel mean curvature in $S^{n+p}(1)$ by using a different method, which generalizes the main theorems in [1, 2], and also improves the results in [3, 4, 5]. Our pinching constant in Theorem 3 is sharp. Finally, I would like to thank Professor An-Min Li for his valuable suggestions.

2. Preliminaries. Let M^n be an n-dimensional compact manifold immersed in an $(n+p)$-dimensional unit sphere $S^{n+p}(1)$. We shall make use of the following convention on the range of indices:

$$1 \leqq A, B, C \ldots \leqq n+p,\ 1 \leqq i, j, k, \ldots \leqq n,\ n+1 \leqq \alpha, \beta, \gamma, \ldots \leqq n+p.$$

Choose a local orthonormal frame field $\{e_A\}$ in $S^{n+p}(1)$ such that, restricted to M, the e_i's are tangent to M. Let $\{\omega_A\}$ and $\{\omega_{AB}\}$ be the dual frame field and the connection 1-forms

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of N respectively. Restricting these forms to M, we have

$$\omega_{\alpha i} = \sum_j h_{ij}^\alpha \omega_j, \quad h_{ij}^\alpha = h_{ji}^\alpha, \tag{2.1}$$

$$h = \sum_{\alpha, i, j} h_{ij}^\alpha \omega_i \otimes \omega_j \otimes e_\alpha, \quad \xi = \frac{1}{n} \sum_{\alpha, i} h_{ii}^\alpha e_\alpha, \tag{2.2}$$

$$R_{ijkl} = \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk} + \sum_\alpha (h_{ik}^\alpha h_{jl}^\alpha - h_{il}^\alpha h_{jk}^\alpha), \tag{2.3}$$

$$R_{\alpha\beta kl} = \sum_i (h_{ik}^\alpha h_{il}^\beta - h_{il}^\alpha h_{ik}^\beta), \tag{2.4}$$

where h, ξ, R_{ijkl} and $R_{\alpha\beta kl}$ are the second fundamental form, the mean curvature vector, the curvature tensor and the normal curvature tensor of M respectively. We set

$$S = \|h\|^2, \quad H = \|\xi\|, \quad H_\alpha = (h_{ij}^\alpha)_{n \times n}. \tag{2.5}$$

Definition 1. M is called a submanifold with parallel mean curvature if ξ is parallel in the normal bundle of M. In particular, M is called minimal if H vanishes identically.

Now we assume that M is a submanifold with parallel mean curvature ($H \neq 0$). We choose e_{n+1} such that $e_{n+1} // \xi$, $\operatorname{tr} H_{n+1} = nH$ and $\operatorname{tr} H_\beta = 0$, $n + 2 \leqq \beta \leqq n + p$. Set

$$S_H = \sum_{i,j} (h_{ij}^{n+1})^2, \quad S_I = \sum_{\substack{i,j \\ \beta \neq n+1}} (h_{ij}^\beta)^2. \tag{2.6}$$

We have the following proposition immediately from the definition.

Proposition 1. *M is a submanifold with parallel mean curvature in $S^{n+p}(1)$ if and only if either $H \equiv 0$, or H is constant and $H_{n+1} H_\alpha = H_\alpha H_{n+1}$, for all α.*

We denote the covariant derivatives of h_{ij}^α by h_{ijk}^α and h_{ijkl}^α, etc. The Laplacian Δh_{ij}^α of h is defined by $\Delta h_{ij}^\alpha = \sum_k h_{ijkk}^\alpha$. Following [6], we have

$$\Delta h_{ij}^{n+1} = \sum_{k,m} h_{mk}^{n+1} R_{mijk} + \sum_{k,m} h_{im}^{n+1} R_{mkjk}, \tag{2.7}$$

$$\Delta h_{ij}^\beta = \sum_{k,m} h_{mk}^\beta R_{mijk} + \sum_{k,m} h_{im}^\beta R_{mkjk} + \sum_{\substack{k \\ \alpha \neq n+1}} h_{ki}^\alpha R_{\alpha\beta jk}, \quad \beta \neq n+1. \tag{2.8}$$

By using Lagrange multiplier method, we have the following

Lemma 1. *Let $a_1, \ldots, a_n$ be real numbers satisfying $\sum_i a_i = 0$ and $\sum_i a_i^2 = a$. Then*

$$\Big|\sum_i a_i^3\Big| \leqq (n-2)\,[n(n-1)]^{-\frac{1}{2}} a^{\frac{3}{2}}, \tag{2.9}$$

and the equality holds if and only if at least $n - 1$ numbers of the a_i's are same with each other.

For a matrix $A=(a_{ij})_{n\times n}$, we denote by $N(A)$ the square norm of A, i.e., $N(A)=\operatorname{tr}(A^tA)=\sum_{i,j}a_{ij}^2$. Then $N(A)=N(TA^tT)$, for each orthogonal $(n\times n)$-matrix T.

Lemma 2 (See [1, 2]). *Let $A_{n+1},\dots,A_{n+p}$ be symmetric $(n\times n)$-matrices. Set $S_{\alpha\beta}=\operatorname{tr}(A_\alpha^tA_\beta)$, $S_\alpha=S_{\alpha\alpha}=N(A_\alpha)$, $S=\sum_\alpha S_\alpha$. Then*

$$\sum_{\alpha,\beta}N(A_\alpha A_\beta-A_\beta A_\alpha)+\sum_{\alpha,\beta}S_{\alpha\beta}^2\leqq\left(1+\tfrac{1}{2}\operatorname{sgn}(p-1)\right)S^2,\tag{2.10}$$

where $\operatorname{sgn}(\cdot)$ *is the standard sign function. Moreover, the equality holds if and only if at most two matrices A_α and A_β are not zero and these two matrices can be transformed simultaneously by an orthogonal matrix into scalar multiples of $\widetilde{A}_\alpha$ and $\widetilde{A}_\beta$ respectively, where*

$$\widetilde{A}_\alpha=\left(\begin{array}{cc|c}0&1&\\1&0&0\\\hline 0&&0\end{array}\right),\quad \widetilde{A}_\beta=\left(\begin{array}{cc|c}1&0&\\0&-1&0\\\hline 0&&0\end{array}\right).$$

3. Main results. First of all, we define our pinching constants as follows

$$\alpha(n,H)=n+\frac{n^3}{2(n-1)}H^2-\frac{n(n-2)}{2(n-1)}\sqrt{n^2H^4+4(n-1)H^2},\tag{3.1}$$

$$C(n,p,H)=\begin{cases}\alpha(n,H), & \text{for } p=1,\text{ or } p=2 \text{ and } H\neq0,\\ \min\{\alpha(n,H),\tfrac{1}{3}(2n+5nH^2)\}, & \text{for } p\geqq3,\text{ or } p=2\text{ and } H=0.\end{cases}\tag{3.2}$$

Theorem 1. *Let M^n be a compact submanifold with parallel mean curvature ($H\neq0$) in $S^{n+p}(1)$. If $S\leqq\alpha(n,H)$, then either M is pseudo-umbilical, or $S=S_H=\alpha(n,H)$ and M is the isoparametric hypersurface $S^{n-1}\left(\frac{1}{\sqrt{1+\lambda^2(n,H)}}\right)\times S^1\left(\frac{\lambda(n,H)}{\sqrt{1+\lambda^2(n,H)}}\right)$ in a totally geodesic $S^{n+1}(1)$, where $\lambda(n,H)=H+\sqrt{\frac{\alpha(n,H)-nH^2}{n(n-1)}}$.*

Proof. By (2.7) and Gauss equations, we have

$$\begin{aligned}\tfrac{1}{2}\Delta S_H&=\sum_{i,j,k}(h_{ijk}^{n+1})^2+\sum_{i,j}h_{ij}^{n+1}\Delta h_{ij}^{n+1}\\&=\sum_{i,j,k}(h_{ijk}^{n+1})^2+\sum_{i,j,k,m}h_{ij}^{n+1}h_{mk}^{n+1}\Big[\delta_{mj}\delta_{ik}-\delta_{mk}\delta_{ij}+\sum_\alpha(h_{mj}^\alpha h_{ik}^\alpha-h_{mk}^\alpha h_{ij}^\alpha)\Big]\\&\quad+\sum_{i,j,k,m}h_{ij}^{n+1}h_{im}^{n+1}\Big[\delta_{mj}\delta_{kk}-\delta_{mk}\delta_{jk}+\sum_\alpha(h_{mj}^\alpha h_{kk}^\alpha-h_{mk}^\alpha h_{jk}^\alpha)\Big]\\&=\sum_{i,j,k}(h_{ijk}^{n+1})^2+n\sum_{i,j}(h_{ij}^{n+1})^2-\Big(\sum_{i,j}(h_{ij}^{n+1})^2\Big)^2-n^2H^2\\&\quad+nH\sum_{i,j,k}h_{ij}^{n+1}h_{jk}^{n+1}h_{ki}^{n+1}-\sum_{\beta\neq n+1}\Big(\sum_{i,j}(h_{ij}^{n+1}-H\delta_{ij})h_{ij}^\beta\Big)^2.\end{aligned}\tag{3.3}$$

Let $\{e_i\}$ be a frame diagonalizing the matrix H_{n+1} such that $h_{ij}^{n+1} = \lambda_i^{n+1}\delta_{ij}$, for all i, j. Set

(3.4) $$f_k = \sum_i (\lambda_i^{n+1})^k,$$

(3.5) $$\mu_i^{n+1} = H - \lambda_i^{n+1}, \quad i = 1, 2, \ldots, n,$$

(3.6) $$B_k = \sum_i (\mu_i^{n+1})^k.$$

Then

(3.7) $$B_1 = 0, \quad B_2 = S_H - nH^2,$$

(3.8) $$B_3 = 3HS_H - 2nH^3 - f_3.$$

From (3.3), (3.7), (3.8) and Lemma 1, we get

$$\begin{aligned}
\tfrac{1}{2}\Delta S_H &= \sum_{i,j,k} (h_{ijk}^{n+1})^2 + nS_H - S_H^2 - n^2H^2 + nHf_3 - \sum_{\beta \neq n+1} \Big(\sum_i \mu_i^{n+1} h_{ii}^{\beta}\Big)^2 \\
&\geqq \sum_{i,j,k} (h_{ijk}^{n+1})^2 + nS_H - S_H^2 - n^2H^2 + nH\left[3HS_H - 2nH^3 - \frac{n-2}{\sqrt{n(n-1)}} B_2^{\frac{3}{2}}\right] - B_2 S_I \\
&= \sum_{i,j,k} (h_{ijk}^{n+1})^2 + B_2\left[n + 2nH^2 - S - \frac{n(n-2)}{\sqrt{n(n-1)}} H (S_H - nH^2)^{\frac{1}{2}}\right] \\
&\geqq \sum_{i,j,k} (h_{ijk}^{n+1})^2 - B_2\left[\sqrt{S - nH^2} + \frac{n(n-2)}{2\sqrt{n(n-1)}} H \right.\\
&\quad \left. + \frac{1}{2(n-1)} \sqrt{n^3(n-1)H^2 + 4n(n-1)^2}\right] \\
&\quad \times \left[\sqrt{S - nH^2} + \frac{n(n-2)}{2\sqrt{n(n-1)}} H - \frac{1}{2(n-1)} \sqrt{n^3(n-1)H^2 + 4n(n-1)^2}\right].
\end{aligned} \tag{3.9}$$

On the other hand, the assumption

$$S \leqq \alpha(n, H)$$

is equivalent to

(3.10) $$\sqrt{S - nH^2} + \frac{n(n-2)}{2\sqrt{n(n-1)}} H - \frac{1}{2(n-1)} \sqrt{n^3(n-1)H^2 + 4n(n-1)^2} \leqq 0,$$

which together with (3.9) shows that S_H is subharmonic on M. By the Hopf maximum principle, we see that S_H must be a constant. This together with (3.9) and (3.10) force that

(3.11) $$B_2(S_H - nH^2)^{\frac{1}{2}} = B_2(S - nH^2)^{\frac{1}{2}},$$

(3.12) $$B_2\left[\sqrt{S - nH^2} + \frac{n(n-2)}{2\sqrt{n(n-1)}} H - \frac{1}{2(n-1)} \sqrt{n^3(n-1)H^2 + 4n(n-1)^2}\right] = 0.$$

If $S_H = nH^2$, then M is a pseudo-umbilical submanifold.

If $S = S_H$ and

$$\sqrt{S - nH^2} + \frac{n(n-2)}{2\sqrt{n(n-1)}} H - \frac{1}{2(n-1)} \sqrt{n^3(n-1)H^2 + 4n(n-1)^2} = 0,$$

then $S = S_H = \alpha(n, H)$, and $S_I = 0$. Consequently M is a hypersurface in a totally geodesic $S^{n+1}(1)$. From (3.9) we have

$$B_3 = \frac{n-2}{\sqrt{n(n-1)}} B_2^{\frac{3}{2}}. \tag{3.13}$$

It follows from Lemma 1 that at least $n-1$ numbers of $\{\mu_i^{n+1}\}$ are same with each other. Without loss of generality, we assume that $\mu_k^{n+1} = \mu$, $k = 1, 2, \ldots, n-1$, and $\mu_n^{n+1} = \bar{\mu}$. Then

$$(n-1)\mu + \bar{\mu} = 0, \tag{3.14}$$

$$(n-1)\mu^2 + \bar{\mu}^2 = \alpha(n, H) - nH^2. \tag{3.15}$$

Substituting the solution of equations (3.14) and (3.15) with condition $(n-1)\mu^3 + \bar{\mu}^3 > 0$ into (3.5), we get

$$\lambda_i^{n+1} = H + \sqrt{\frac{\alpha(n, H) - nH^2}{n(n-1)}}, \quad i = 1, 2, \ldots, n-1,$$

$$\lambda_n^{n+1} = H - \sqrt{\frac{(n-1)(\alpha(n, H) - nH^2)}{n}}. \tag{3.16}$$

Hence M is the isoparametric hypersurface

$$S^{n-1}\left(\frac{1}{\sqrt{1 + \lambda^2(n, H)}}\right) \times S^1\left(\frac{\lambda(n, H)}{\sqrt{1 + \lambda^2(n, H)}}\right) \quad \text{in } S^{n+1}(1),$$

where $\lambda(n, H) = H + \sqrt{\dfrac{\alpha(n, H) - nH^2}{n(n-1)}}$. This proves Theorem 1.

Remark 1. It is clear that the pinching constant $\alpha(n, H)$ is best possible.

Corollary 1. *Let M^n be a compact hypersurface with constant mean curvature $(H \neq 0)$ in $S^{n+1}(1)$. If $S \leqq \alpha(n, H)$, then either M is the totally umbilical sphere $S^n\left(\frac{1}{\sqrt{1 + H^2}}\right)$ or the isoparametric hypersurface $S^{n-1}\left(\frac{1}{\sqrt{1 + \lambda^2(n, H)}}\right) \times S^1\left(\frac{\lambda(n, H)}{\sqrt{1 + \lambda^2(n, H)}}\right)$.*

If M is a pseudo-umbilical submanifold with nonzero parallel mean curvature and $p \geqq 2$, it is to see from a theorem of [6] that M is a minimal submanifold in

$S^{n+p-1}\left(\frac{1}{\sqrt{1+H^2}}\right)$ with second fundamental form H_α, $\alpha = n+2, \ldots, n+p$. Hence, we have the following

Theorem 2. *Let M^n be a compact submanifold with parallel mean curvature ($H \neq 0$) in $S^{n+p}(1)$. If $S \leqq \alpha(n, H)$, then either M is a totally umbilical sphere, a isoparametric hypersurface in a totally geodesic $S^{n+1}(1)$, or a minimal submanifold in a totally umbilical $S^{n+p-1}\left(\frac{1}{\sqrt{1+H^2}}\right)$.*

Theorem 3. *Let M^n be a compact submanifold with parallel mean curvature in $S^{n+p}(1)$. If $S \leqq C(n, p, H)$, then either M is the totally umbilical sphere $S^n\left(\frac{1}{\sqrt{1+H^2}}\right)$, the isoparametric hypersurface $S^{n-1}\left(\frac{1}{\sqrt{1+\lambda^2(n,H)}}\right) \times S^1\left(\frac{\lambda(n,H)}{\sqrt{1+\lambda^2(n,H)}}\right)$ in a totally geodesic $S^{n+1}(1)$, one of the Clifford minimal hypersurfaces $S^k\left(\sqrt{\frac{k}{n}}\right) \times S^{n-k}\left(\sqrt{\frac{n-k}{n}}\right)$, $k = 1, 2, \ldots, n-1$, in $S^{n+1}(1)$, the Clifford minimal surface*

$$S^1\left(\frac{1}{\sqrt{2(1+H^2)}}\right) \times S^1\left(\frac{1}{\sqrt{2(1+H^2)}}\right) \text{ in } S^3\left(\frac{1}{\sqrt{1+H^2}}\right),$$

or the Veronese surface in $S^4\left(\frac{1}{\sqrt{1+H^2}}\right)$.

P r o o f. (i) If $H = 0$, M is minimal. The assertion follows from the main theorems in [1, 2].

(ii) If $H \neq 0$ and $p = 1$, we know from Corollary 1 that either M is the hypersphere $S^n\left(\frac{1}{\sqrt{1+H^2}}\right)$ or the isoparametric hypersurface $S^{n-1}\left(\frac{1}{\sqrt{1+\lambda^2(n,H)}}\right) \times S^1\left(\frac{\lambda(n,H)}{\sqrt{1+\lambda^2(n,H)}}\right)$.

(iii) If $H \neq 0$ and $p \geqq 2$, it is straightforward to see from (2.8), Proposition 1 and Lemma 2 that

$$\begin{aligned}
\tfrac{1}{2}\Delta S_I = & \sum_{\substack{i,j,k\\ \beta \neq n+1}} (h^\beta_{ijk}) + \sum_{\beta \neq n+1} \operatorname{tr}(H_{n+1}H_\beta)^2 - \sum_{\beta \neq n+1} [\operatorname{tr}(H_{n+1}H_\beta)]^2 \\
& + nH \sum_{\beta \neq n+1} \operatorname{tr}(H_{n+1}H_\beta^2) - \sum_{\beta \neq n+1} \operatorname{tr}(H_{n+1}^2 H_\beta^2) + nS_I \\
& - \sum_{\alpha,\beta \neq n+1} \operatorname{tr}(H_\alpha H_\beta - H_\beta H_\alpha)^2 - \sum_{\alpha,\beta \neq n+1} [\operatorname{tr}(H_\alpha H_\beta)]^2 \\
\geqq & \sum_{\substack{i,j,k\\ \beta \neq n+1}} (h^\beta_{ijk})^2 + nH \sum_{\beta \neq n+1} \operatorname{tr}(H_{n+1}H_\beta^2) - \sum_{\beta \neq n+1} [\operatorname{tr}(H_{n+1}H_\beta)]^2 \\
& + nS_I - (1 + \tfrac{1}{2}\operatorname{sgn}(p-2))S_I^2 .
\end{aligned} \tag{3.17}$$

We know from Theorem 1 that either M is pseudo-umbilical or the isoparametric hypersurface $S^{n-1}\left(\frac{1}{\sqrt{1+\lambda^2(n,H)}}\right)\times S^1\left(\frac{\lambda(n,H)}{\sqrt{1+\lambda^2(n,H)}}\right)$ in a totally geodesic $S^{n+1}(1)$.

If M is pseudo-umbilical, then (3.17) becomes

$$\begin{aligned}\tfrac{1}{2}\Delta S_I &\geqq \sum_{\substack{i,j,k\\ \beta\neq n+1}}(h^\beta_{ijk})^2+(n+nH^2)S_I-(1+\tfrac{1}{2}\operatorname{sgn}(p-2))S_I^2\\ &\geqq \sum_{\substack{i,j,k\\ \beta\neq n+1}}(h^\beta_{ijk})^2+S_I[n+nH^2-(1+\tfrac{1}{2}\operatorname{sgn}(p-2))(S-nH^2)]\geqq 0.\end{aligned}\tag{3.18}$$

This shows that S_I is a constant, and the inequalities above become equalities. It is not hard to see that

$$S_I[n+nH^2-(1+\tfrac{1}{2}\operatorname{sgn}(p-2))(S-nH^2)]=0.\tag{3.19}$$

If $S_I=0$, then M lies in a totally geodesic sphere $S^{n+1}(1)$ and M is the totally umbilical sphere $S^n\left(\frac{1}{\sqrt{1+H^2}}\right)$.

If $n+nH^2-(1+\frac{1}{2}\operatorname{sgn}(p-2))(S-nH^2)=0$, namely

$$S=\left(n-\frac{n}{3}\operatorname{sgn}(p-2)\right)(1+H^2)+nH^2,\tag{3.20}$$

then $h^\alpha_{ijk}=0$ and

$$\sum_{\alpha,\beta\neq n+1}\operatorname{tr}(H_\alpha H_\beta-H_\beta H_\alpha)^2+\sum_{\alpha,\beta\neq n+1}[\operatorname{tr}(H_\alpha H_\beta)]^2=(1+\tfrac{1}{2}\operatorname{sgn}(p-2))S_I^2.$$

By Lemma 2 and the same argument as in [1], we conclude that $n=2$, and the second fundamental form h can be written as follows

(a) $n=2$ and $p=2$, $H_3=H\begin{pmatrix}1&0\\0&1\end{pmatrix}$, $H_4=\sqrt{1+H^2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$, or

(b) $n=2$ and $p\geqq 3$, $H_3=H\begin{pmatrix}1&0\\0&1\end{pmatrix}$, $H_4=\sqrt{\frac{1+H^2}{3}}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$, $H_5=\sqrt{\frac{1+H^2}{3}}\begin{pmatrix}0&1\\1&0\end{pmatrix}$, $H_\beta=0$, $\beta\geqq 6$.

By Theorem 2, we know that M is a minimal submanifold in $S^{1+p}\left(\frac{1}{\sqrt{1+H^2}}\right)$ with second fundamental form $H_4,\dots,H_{2+p}$. Therefore, M is the Clifford minimal surface $S^1\left(\frac{1}{\sqrt{2(1+H^2)}}\right)\times S^1\left(\frac{1}{\sqrt{2(1+H^2)}}\right)$ in $S^3\left(\frac{1}{\sqrt{1+H^2}}\right)$, or the Veronese surface in $S^4\left(\frac{1}{\sqrt{1+H^2}}\right)$. This completes the proof of Theorem 3.

Remark 2. The pinching constant $C(n,p,H)$ is sharp, which is larger than ones in [3, 4, 5, 6].

References

[1] S. S. CHERN, M. DO CARMO and S. KOBAYASHI, Minimal submanifolds of a sphere with second fundamental form of constant length. In: Functional analysis and related fields. Berlin-Heidelberg-New York 1970.
[2] A. M. LI and J. M. LI, An intrinsic rigidity theorem for minimal submanifolds in a sphere. Arch. Math. **58**, 582–594 (1992).
[3] M. OKUMURA, Hypersurfaces and a pinching problem on the second fundamental tensor. Amer. J. Math. **96**, 207–213 (1974).
[4] M. OKUMURA, Submanifolds and a pinching problem on the second fundamental tensor. Trans. Amer. Math. Soc. **178**, 285–291 (1973).
[5] H. W. XU, A pinching constant of Simons' type and isometric immersion. Chinese Ann. Math. Ser. A **12**, 261–269 (1991).
[6] S. T. YAU, Submanifolds with constant mean curvature I, II. Amer. J. Math. **96**, 346–366 (1974); **97**, 76–100 (1975).

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