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Erratum

The Rotation Number for Almost Periodic Potentials

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It has been pointed out to us by B. Simon that the proof of Proposition 3.3, p. 413, is incorrect. The statement, however, is valid as it stands and we want to present a proof of that statement:

According to Proposition 2.1, we have to show that $m_+(x+t_n)$ converges uniformly for any sequence t_n for which $q(x+t_n)$ does. We show first the convergence for $x=0$ and subsequently verify the uniform convergence for all $x \in \mathbb{R}$.

We change the notation and write for fixed z in $\operatorname{Im} z \neq 0$,

$$m[q] = m_+(0, z; q).$$

Proposition. *If q_n, q are bounded continuous functions and $q_n \to q$ uniformly on compact sets, then $m[q_n] \to m[q]$.*

The proof can be based on H. Weyl's construction of the limit point $m[q]$. (See, e.g., in [6, Chap. IX, Sect. 2] of our paper.) If $\varphi = \varphi_b(x, z, \theta)$ is a nontrivial solution of $(L-z)\,\varphi = 0$ satisfying $\varphi' \cos\theta + \varphi \sin\theta = 0$ for $x = b$, then

$$C_b = \left\{ \frac{\varphi'}{\varphi}\bigg|_{x=0}, 0 \leqq \theta \leqq 2\pi \right\}$$

defines a circle in the complex plane which for $b \to \infty$ shrinks to the limit point $m[q]$. Denoting by D_b the closed disc bounded by C_b, one has $D_{b'} \subset D_b$ for $b' > b$ (strict containment). In particular, $m[q]$ is contained in all D_b.

If now D_b^n denote the corresponding discs for q_n, we can, for $\varepsilon > 0$, choose b so large that the radius of D_b is $< \varepsilon/2$. Fix $b' > b$ and note that clearly $C_{b'}^n \to C_{b'}$ as $n \to \infty$, i.e. for $n > N$ we have

$$m[q_n] \in D_{b'}^n \subset D_b.$$

Hence both $m[q_n]$ and $m[q]$ are contained in D_b, i.e. satisfy $|m[q_n] - m[q]| < \varepsilon$ for $n > N$, proving the proposition.

We apply this proposition to the translates $q_t = q(x+t)$ of $q \in \mathscr{A}(\mathscr{M})$, and observe that

$$m[q_t] = m_+(t, z), \quad \operatorname{Im} z \neq 0.$$

Thus if $\|q_{t_n} - q^*\|_\infty \to 0$, we have $m[q_{t_n}] \to m[q^*]$, and it follows that $m[\cdot]$ can be extended to a continuous functional on the hull $E(q)$, the closure of $\{q_t\}$. Since $E(q)$ is compact, $m[\cdot]$ is actually uniformly continuous and therefore

$$m[q_{t+t_n}] - m[q_t^*] = m(t+t_n, z) - m^*(t, z)$$

tends to zero uniformly in $t \in \mathbb{R}$, since

$$\|q_{t+t_n} - q_t^*\|_\infty = \|q_{t_n} - q^*\|_\infty \to 0.$$

There are a number of different ways to prove Proposition 3.3, for example, by using Scharf's argument, [23]. Also B. Simon suggested another simple proof and we thank him for this communication as well as for pointing out our mistake. Finally we note that the above proposition is stronger than necessary since it does *not* require uniform convergence of $q_n \to q$ on the whole axis.

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