J Evol Econ (1996) 6: 1–30

Journal of
Evolutionary
Economics

Nonlinear stochastic effects of substitution – an evolutionary approach

E. Bruckner[1], W. Ebeling[2], M. A. Jiménez Montaño[3], A. Scharnhorst[1]

[1] Science Center, Reichpietschufer 50, D-10785 Berlin, Germany
[2] Humboldt University, Institute of Theoretical Physics, Unter den Linden 6, D-10099 Berlin, Germany
[3] Universidad de las Americas Puebla, Apartado Postal 47, Cholula 72820, Puebla, Mexico

Abstract. Technological innovations have been investigated by means of substitution and diffusion as well as evolution models, each of them dealing with different aspects of the innovation problem. In this paper we follow the well known research traditions on self-organisation models of complex systems. For the first time in the literature we show the existence of a specific niche effect, which may occur in the first stage of establishment of a new technology. Using a stochastic Master equation approach, we obtain analytical expressions for the survival probabilities of a new technology in smaller or larger ensembles. As a main result we demonstrate how a hyperselection situation might be removed in a stochastic picture and thresholds against the prevailing of a new technology in a step-by-step process can be overcome.

Key words: Innovation – Technology – Master equation – Survival probability – Evolution

JEL-classification: O3

Introduction

In physics, a tradition has developed (since the introduction of thermodynamic descriptions in the 19th century at the latest) where systems with large numbers of subsystems are considered. The many-subsystem approach has brought about a distinction between microscopic and macroscopic considerations, which emerged at about the same time as modern industrial society. The whole history of self-organisation and synergetics is focused on a surprising new understanding of the relationship between micro- and macro-level descriptions (see e.g. Nicolis and

Correspondence to: E. Bruckner

Prigogine 1987; Haken 1983). One of the main results of these historical findings was, that relatively independent of the nature of the sybsystems mainly the manner of their co-ordination is important for the demonstration of the well-known macroscopic phenomena of spontaneous structure formation. The macroscopic phenomenon results from the coherent behaviour of the subsystems, leading to a co-operative effect.

This fact generates a trend towards general models. To describe the manner of co-ordination, the mechanisms might be considered without providing them with substantial physical or biological meaning. Formally, mechanisms from different disciplinary contexts might be integrated. Thus, the self-organisation research tradition has led to a generalisation, which opens up new application areas. Interdisciplinary research tasks appeared within a disciplinary framework.

A few remarks to some general concepts

Following this line of research we use a modelling framework introduced in Bruckner et al. (1989), which allows a rather general view on an evolutionary process, especially useful to describe processes in social systems.

To specify the units of the evolutionary process, a general field concept is introduced. An enumerable set of fields is considered, each field being characterised by the number N_i and the properties of its representatives (occupying elements).

In the process of evolution, the occupation of new fields begins with a rather small number of representatives and is strongly influenced by stochastic effects. An adequate description of this initial phase of innovative instabilities is possible only on the basis of a stochastic model. The probability that the system at time t is in a particular state may then be described by the probability distribution function P. To describe the change of the probability distribution over time we assume that we can determine elementary processes, which, in one time step, change only the number of representatives of a single field ($N_i \rightarrow N_i \pm 1$) or simultaneously two occupation numbers (describing transitions to other fields). Introducing the corresponding transition probabilities as functions of the occupation numbers, we define a Markov process in the occupation number space.

To have a more sharp picture of the processes under consideration, we'll always, if it is convenient (or necessary), use a combined view of the stochastic and the deterministic picture of description.

If we observe the system over time, in the stochastic picture a sequence of occupation states for the entire system will be realised. Such a sequence can be interpreted as a realisation of the stochastic variables, e.g., as a result of a defined experiment (simulation experiment). In another experiment the resulting occupation numbers will be different from the first one according to the stochastic nature of the processes. The differences between the realisations express the effect of fluctuations within the system.

In large systems we can assume that, in general, the fluctuations are small compared with the realised value of the stochastic variable. Then, the different realisations will be grouped together. In such a case it is possible to consider an average realisation which expresses the expectation of the behaviour of the system. The average trajectory can be understood as a trend behaviour. The corresponding deterministic model allows to specify the conceptual framework of stationary states, stability or instability of states, constraints and competition, selection, attractors, multistability and thresholds (separatrices) in the state space.

Competition is introduced by global constraints concerning the growth of fields. Within a situation, in which each of the subsystems tries to follow a common goal, e.g., securing the population size by self-reproduction, the constraints do not enable a satisfying result for all fields, only a few are able to be successful. Then, some fields can only grow at the expense of other fields. Mostly, competition is accompanied with selection. Selection appears as a coherent behaviour which leads to the disappearance of at least one field.

One of the simplest models for competition is the well known Fisher–Eigen model of competing fields i which have different growth rates E_i (Eigen 1971). With the condition of a constant size of the total population a constraint for the growth of the fields is introduced, which leads to a competitive behaviour. The resulting dynamical equation shows that fields with growth rates better than the ensemble average $\langle E \rangle$ will succeed the competition and the others will fail. Asymptotically, the field with the largest growth rate will be the "winner". Therefore, in Eigen's concept the growth rate E_i can serve as selection value. With references to the Darwinian principle of the survival of the fittest, the parameter E_i serves as a quantitative expression of the qualitative property of fitness.

In general, competition and selection can be described by equations, which are generalised Lotka–Volterra-equations (see e.g. Bruckner et al. 1989). Especially, non-linear growth terms describing self-amplification and self-inhibition typically lead to the existence of thresholds (or separatrices) in state space. In difference to the simple Eigen model referred above in the presence of such thresholds the evolution of a field depends critically on the initial conditions (Mosekilde et al. 1988). In particular, in such a situation "hyperselection" can occur. "Hyperselection" means that due to the threshold new possibilities (mutants) never can succeed in the selection process if they start with a small initial number of representatives. As a consequence evolution stops.

In difference to the deterministic description in the stochastic picture the situation can change dramatically. If in the stochastic model the fluctuations reach a certain range (so-called critical fluctuations) due to co-operative processes in the system, the realisations can differ significantly. That means, by crossing a separatrix another behaviour of the system (e.g. another stable stationary state) can be obtained. In the case of occurrence of critical fluctuations averaging over an ensemble of trajectories is no longer a useful method. Thus, the crossing of separatrices has no deterministic analogue.

To combine the two pictures one can assume that in many cases the maxima of the probability function in a stationary behaviour (t tends to infinite) correspond to the location of attractors. In general, the probability function is a smooth varying function and sharp borders do not exist. A change in the attractor landscape of the deterministic picture corresponds to a change in shape of the stationary probability function in the stochastic picture. The deterministic picture thus gives important information on the system behaviour if a trend can be determined.

Our line of research in the present paper

Technological innovations have been investigated by means of substitution and diffusion as well as evolution models, each of them dealing with different aspects of the innovation problem. The evolutionary point of view has been proposed in earlier works on self-organisation models of complex systems by Allen (1975, 1976), Feistel and Ebeling (1976), Bruckner (1980, unpublished), Jiménez Montaño and

Ebeling (1980), Weidlich and Haag (1983), Silverberg (1984) and later demonstrated e.g. by Allen (1986), Bruckner et al. (1989, 1990), Silverberg (1992), Allen et al. (1992), Weidlich and Braun (1992), Troitzsch (1993), Saviotti and Mani (1993), Dosi and Kaniovski (1993) etc. In the present paper we concentrate on the competition of a new technology against an established one in a situation of hyperselection.

In section I.1. the general modelling framework has to be re-established in terms of technological evolution. Within this framework, the innovation process appears to be connected with other phenomenologically well-known processes like substitution or imitation. In every case a technology is linked to a firm, which experiences market conditions. Due to the importance of this connection, according to the approach of Nelson and Winter technological parameters are expressed by economic indicators (section I.2.). In section I.3. the deterministic analogue to the stochastic technological model is derived. Especially, we discuss how social averages govern the technological evolution process. In section I.4. We consider innovation in its first stage (as infection). We discuss whether and how the system structure then will change and how stability properties can be investigated in the deterministic as well as in the stochastic picture.

In section II. this considerations will be concentrated to the substitution problem and deterministic as well as stochastic substitution models will be considered. For the deterministic model a stability analysis of the stationary states will be carried out.

In section III. we investigate the time behaviour of the probability function for the stochastic substitution model. Assuming linear growth properties for the technologies (simple substitution model), an analytical solution of the Master equation can be obtained, including the stationary behaviour. Therefore, the probabilities of survival as well as of extinction are obtained for the long-term (as well as short-term) development. Thus, we can show that the short-term and long-term survival probabilities differ remarkable.

For more complex growth functions, we obtain analytical expressions for the survival probabilities of a new technology in the stationary behaviour. Therefore, we can show that in the case of hyperselection (Ayres 1991) (non-linear growth processes), the once-for-ever selection can be overcome in the stochastic picture.

In this way we can show that the results of Batten and others on conditions for change to a new technology are only valid within a certain limit and particularly not valid in a stochastic picture. Further, we can show that the survival probability not only depends on the growth rates, but also on the size of the ensemble and the initial conditions. Particularly in small ensembles, the survival probabilities change dramatically. On this basis we show a specific niche effect which is responsible for the remarkable change of the competition conditions in local areas.

I. Technological change

Modern economic growth is characterised by structural changes based on the introduction of new technologies in the economic sphere.

Following the original contributions of Schumpeter (1912) and Alchian (1950) a rapidly growing number of authors attempt to construct formal models of economic competition, growth, technological change, diffusion, and technologically induced fluctuations as an evolutionary process.

Beginning with the pioneering work of Nelson and Winter (1976, 1982) the methods of evolutionary theory have been successfully applied to innovation processes (see e.g. Dosi et al. 1988; Arthur 1989; Saviotti and Metcalfe 1991). To describe the process of evolution of technologies, the basic idea is that man has few elementary needs to be satisfied: food, clothing, shelter, transportation, communication, education, recreation, entertainment, etc.. Then, technological evolution consists on the microeconomic level mainly in substituting a new form of satisfaction for the old one.

To describe the technological evolution problem one has to determine the system, the subsystems, the elements, and their interactions relevant for the problem considered. We consider firms, plants (production units of firms) and technologies. The firms are introduced in the role of decisions carriers (represented by firm management) for what has to be done with plants (opening up, choosing a technology, closure) according to market (or non-market) conditions. The plants are on one hand thought to be units of survival according to market (or/and other) conditions. On the other hand, they play the role of users of a particular technology. Thus, firms (resp. plants) as well as technologies are considered as carriers of technological change.

In our view technological evolution cannot be described without such a close connection to the economic processes and the economic units which use the technologies. Of course, in such a conceptual framework the technological evolution process radically changes its pattern if the market itself changes significantly. But this is no argument against the method. Change for "better" and "better" technologies, which can be expected as the outcome of a "good" technological evolution process, is only imaginable on the base of a long-term functioning efficiency mechanisms.

1.1. Microeconomic carriers of technological change

Technological change usually is imagined as a macroeconomic change process, e.g. from the radio tube to the transistor. To describe it as an evolutionary process one has to identify its microeconomic carriers, which in a co-ordinated activity govern and cause the macroeconomic change process. In our view firms (resp. plants) as well as technologies are considered as microeconomic carriers of technological change. Therefore, to describe the process of technological change not only technological change has to be taken into account.

The new events appear within the system (a) by new microeconomic carriers; (b) by combination and re-combination of existing and new carriers.

Even a re-combination of existing carriers (e.g., existing plant re-combined with existing technology) represents from the point of view of the firms an innovation process, if the technology hasn't been in the firm before. Thus, concerning the innovation problem the macroeconomic level (system level) and the microeconomic level (firm level) has to be differentiated.

Macroeconomically an innovation appears in the system if a new technology has been produced. On the microeconomic level all combination and re-combination possibilities (excluding the process, which represents non-innovative firm extension) include an innovative effect.

Innovation matrix (firm level)

Creation and (re-)combination possibilities				Innovative processes
Existing		New		
		firm = unit,	techn.	Creation of a small firm (firm = unit) combined with creation of a new technology a)
	techn.	firm = unit		Creation of a small firm combined with choosing an existing technology b)
firm,		unit,	techn.	Creation of a new production unit combined with a new technology (firm extension, innovative) c)
firm,	techn.	unit		Creation of a new production unit combined with an existing technology on the system level (firm extension, innovative) d)
firm,	techn.	unit		Creation of a new production unit combined with an existing technology on the firm level (firm extension, non-innovative) e)
firm, unit,			techn.	Creation of a new technology, combined with an existing production unit f)
firm, unit, techn.				Re-combination of an existing production unit with an existing technology g)

To describe the technological evolution process a good model should be able to fit the creation as well as the (re-)combination possibilities of microeconomic carriers occurring in the system. An overview of the creation and (re-)combination processes for the two types of carriers of technological change is given in the Innovation matrix.

Now we have to find an application of the general model (Bruckner et al. 1989) which fits the creation and (re-)combination possibilities occurring in the system. To enable a useful application, the model must be re-constructed in terms of general features of technological change.

In reality, the multitude of possible technologies forms an infinite continuous set. Here we assume that the number of potential technologies is large but discrete and countable. Let us now specify our model by introducing an industrial state as a set of integers $(N_1, N_2, \ldots, N_i, \ldots)$ (1) where N_i denotes the number of production units (plants) using the technology i. The complete set of occupation numbers N_1, $N_2, \ldots, N_s$ determines the state of the system at a given time. Due to the large number of potential technologies most of the occupation numbers are zero.

The plants using technology i might belong to different firms. Furthermore, we assume that the system is open, e.g., that production units can be created (new plants enter the system) and closed down (plants leave the system). The production units may also change (plants change to a different technology). Such decision processes in the model occur as transitions of plants between technologies. If we assume that all decisions leading to a change of the set $\{N_i\}$ depend mostly on the present state, we can apply the concept of a Markov process. To be able to apply the general model we assume that the microscopic substitution and innovation activities may be considered as elementary processes (no more than two numbers can be changed simultaneously).

Then, re-construction of the general model, introduced in Bruckner et al. (1989), starts with the construction of the elementary processes in terms of the problem of technological change.

I.1.1. Entry of plants into the system

I.1.1.1. Self-reproduction. In economics the term reproduction implies self-maintenance of a firm on the market. Self-reproduction here means enlargement of the production unit (on the basis of extended reproduction) to the degree of creating a second production unit. If the firm has earnings remaining (gross profit) after paying wages and required dividends, such profit can be invested in the purchase of new capital. The capital stock, reduced by depreciation, is then increased by the firm's gross investment.

I.1.1.1.1. Creation of new production units. A firm may create new production units if it has a positive gross investment. In this case the remaining capital will be used as capital stock for a new plant (daughter plant) entering the system. In this way self-reproduction means entry into the market of a new production unit governed by one firm (this may also be described in terms of a concentration process).

In the following we assume that the probability that the new unit is equipped with technology i is proportional to N_i or powers of N_i. (That means the firms look for an attractive technology to equip the new production unit.) The process may therefore be understood as the entry of a plant into technology field i.

The firms are not acting in isolation from the development of other firms using other technologies, they are all part of a network of inter-relations. To describe a process in which a self-reproduction process of technology i might be sponsored by other firms we introduce a term proportional to N_i *and* N_j.

$$W(N_i + 1, N_j | N_i, N_j) = A_i^0 N_i + A_i^1 N_i^2 + B_{ij} N_i N_j \tag{2}$$

where A_i^0 is the coefficient of linear self-reproduction, A_i^1 measures self-amplification (second-order self-reproduction), and B_{ij} measures sponsoring from other firms.

I.1.1.1.2. Innovation of a new technology. A firm may use the remaining capital not only to create a new production unit but also to create a new technology by R&D. Then we have an entry of a plant into an empty technology field i governed by a firm. We will assume that this process of creation of a new technology in some way is oriented to an existing technology j which, although commonly used in the industry, is considered unattractive. Then, the probability of changing to a new technology i is proportional to N_j. If the firm invests capital in this way, we will call the process the *foundation of a daughter plant with new technology*. This is a second possibility for the entry of a plant into the system. In the model the probability is given by:

$$W(N_i + 1, N_j | N_i, N_j) = M_{ij}N_j \tag{3}$$

where the coefficient M_{ij} measures the rate at which a technology i is generated by self-reproduction of technology j.

Let us introduce the total rate A_i which describes all possible entries of new plants *linked to the number of existing plants using technology i*

$$A_i = A_i^0 + A_i^1 N_i + B_{ij}N_i + M_{ij}N_j/N_i \tag{4}$$

and the transition probability of the step $N_i \rightarrow N_i + 1$ as the sum of the processes mentioned above with:

$$W(N_i + 1, N_j | N_i, N_j) = A_i^0 N_i + A_i^1 N_i^2 + B_{ij}N_i N_j + M_{ij}N_j = A_i N_i. \tag{5}$$

I.1.1.2. Input. The creation of a small firm (=unit) may be combined with the creation of a new technology or with the take-over of an existing technology. Such a process appears as a spontaneous generation of an element of technology field i, which may already be occupied or not. In any case, the risk of survival here is carried entirely by the new firm.

There might be (or might not be) positive conditions for such a process, including influencing actors within the given system of technologies (e.g. investment, help, sponsoring by a given firm). The spontaneous generation process may be modelled as an inflow into the system from outside. Because such events are relatively seldom, the inflow is assumed to be small with a constant probability:

$$W(N_i + 1, N_j | N_i, N_j) = \Phi_o \tag{6}$$

where Φ_o is a constant rate.

I.1.2. Decline

Death (decline, bankruptcy). The dependence of firms on market developments implies the risk of closure that exists for every production unit. In this way, firms as well as plants are active only for a limited period of time. If the gross profit and the consequent gross investment of a firm are negative over a time period and the resulting capital stock declines then the production unit may be closed. This process can be described as the exit of plants from the technology field i. In the model, the corresponding probability is given by:

$$W(N_i - 1, N_j | N_i, N_j) = D_i N_i. \tag{7}$$

As in the case of A_i the process described by D_i includes the whole set of influencing factors for the survival probabilities of the production units of firms under consideration (among them saturation conditions). It follows that a production unit may close due to factors other than its use of a certain technology i. However, if a certain production unit is closed, a member of technology field i vanishes. If the technology i itself is a "good" one, this decline will be compensated by other growth processes such as self-reproduction or imitation. If it is a "bad" one, the decline will have a characteristic shape. We differentiate between two processes of decline D_i^0, D_i^1 with $D_i = D_i^0 + D_i^1 N_i$. Then we obtain

$$W(N_i - 1, N_j | N_i, N_j) = D_i^0 N_i + D_i^1 N_i^2 \tag{8}$$

where D_i^0 corresponds to the decline rate in a normal demand situation and D_i^1 describes processes of self-inhibition which become relevant for the description of saturation processes.

I.1.3. Transfer to other technologies

It is assumed that firms have the capability of changing the technologies used by their production units. Therefore, a plant representing a certain type of technology can make a transition to a different type and then become one of its representatives. This is a characteristic feature of elements in social systems. From a different perspective, such transitions can be interpreted as an exchange of elements (plants) between types of technologies (Bruckner et al. 1989).

I.1.3.1. Imitation. Firms always look for possible ways in which they can improve their gross return. If a firm has a gross return of less than the target level, it will begin to search for ways to achieve this (for example, by imitation). In the model, we consider the probability: $W(N_i + 1, N_j - 1 | N_i, N_j)$. In a first step, we assume that this transition probability is proportional to the number of plants which are able to carry out an imitation process, i.e., to N_j. On the other hand, a searching plant may examine the activities of other plants. The probability of finding a particular technology is proportional to the fraction of the total industrial output produced by that technology in the period in question (as found by Fisher, Pry). In terms of our model, we assume that the probability of finding a different technology i is proportional to N_i/N.

The transition rate W then depends on the possibility of carrying out this transition for the existing technology j and on the possibility of finding another technology i:

$$W(N_i + 1, N_j - 1 | N_i, N_j) = A_{ij}^1 N_j N_i / N. \tag{9}$$

The non-linearity of the r.h. term describes the mutual influence of technology i and technology j performing some kind of adaptation process within the system. Imitation is therefore described as a co-operative phenomenon oriented to the survival of the fittest. This process in particular may be connected with competition and selection.

I.1.3.2. Innovation of a new technology. An exchange process may not only be carried out by imitation but also by R&D. This is the case if field i is not already occupied. In this case R&D is done to provide an existing production unit with a new technology i. We assume that the search process is oriented on the technology j which the production unit already uses. Such a process will be carried out if technology j should be changed and if no appropriate technology seems to be available on the market. In the model this process corresponds to: $N_i = 0 \rightarrow N_i = 1$. For the probability we assume:

$$W(N_i = 1, N_j - 1 | N_i = 0, N_j) = A_{ij}^0 N_j \tag{10}$$

where A_{ij}^0 describes the ability to find new technologies and to apply them. Clearly, this process will be seldom and A_{ij}^0 will therefore be small. If a firm looks for a new technology by R&D as mentioned above this process is described by the coefficients M_{ij} as well as A_{ij}^0.

All coefficients in eqs. (2–10) have non-negative values. At least one element of the inflow rate must be non-zero. This guarantees that the system will not be absorbed by the zero state with all $N_i = 0$.

By means of the given transition probabilities (Eq. (2,3,6,8,9,10)) the stochastic process is completely defined. A corresponding representation in terms of probability distributions is the Master equation (see Bruckner et al. (1989)).

Innovation matrix

Creation and (re-)combination of carriers: Existing	New	Elementary process	Described by
	firm = unit, techn.	spontaneous generation	Φ_0
techn.	firm = unit	spontaneous generation	Φ_0
firm,	unit, techn.	self-reproduction	M_{ij}
firm, techn.	unit	self-reproduction	A_i^0, A_i^1
firm, unit,	techn.	non-co-operative exchange	A_{ij}^0
firm, unit, techn.		co-operative exchange	A_{ij}^1

Let us now summarise where and how new events occur in the framework of the general technological model. The main issue is that the innovation process must be differentiated and understood as connected with re-combination of carriers within the system. It follows that, within an evolutionary approach, different types of innovative behaviour and different types of innovative strategies must be taken into account. The main difference to a chemical or biological system is that the innovation and (re-) combination strategies are based on a to a high degree developed *rational* behaviour of the (microscopic) decision carriers. Probability and rationality thus meet in the interplay of microscopic and macroscopic levels of consideration.

I.2. Economic indicators and assumptions

I.2.1. Technology indicators

Nelson and Winter (1982) proposed to characterise a given technology i by the values of two state variables its coefficient of labour input per unit output $a_{la}^{(i)}$ and its coefficients of capital input per unit output $a_{ca}^{(i)}$.

As discussed, the capability of identical or creative self-reproduction is related to the amount of capital available for investment, which will be different for each individual plant according to its strategic behaviour. Otherwise, the gross return is related to the technology used by the plant. Technologies competing within a system can be differentiated by different rates of self-reproduction. Therefore, following Nelson and Winter A_i can be described as a function of the gross investment (gross return) as well as of economic factors characterising a given technology such as the coefficient of labour input per unit output $a_{la}^{(i)}$ and the coefficient of capital input per unit output $a_{ca}^{(i)}$. A simple Ansatz is:

$$A_i = (\text{Gross return})\, a_{la}^{(i)} a_{ca}^{(i)}\,. \tag{11}$$

I.2.2. Technology distance and R&D indicators

Since the search process of R&D will be mostly local, the transition probability is concentrated on technologies close to the current one and decreases rapidly with the "distance", which also can be modelled with help of economical factors.

Let us consider the process linked with A_{ij}^0. Nelson et al. (1976) proposed for the distance between technologies i and j the expression:

$$d(i,j) = (\mathrm{WTL})|\log a_{la}^{(i)} - \log a_{la}^{(j)}| + (\mathrm{WTK})|\log a_{ca}^{(i)} - \log a_{ca}^{(j)}| \tag{12}$$

where $(\mathrm{WTL}) + (\mathrm{WTK}) = 1$. The numbers (WTL) and (WTK) are the weights of labour and capital coefficients, respectively. Furthermore, they propose that the transition probability decreases linearly with $d(i,j)$, i.e., they assume that

$$A_{ij}^0 \propto (\mathrm{IN})(d_{crit} - d(i,j)) \tag{13}$$

where (IN) stands mnemonically for "case of INnovation" and where d_{crit} is the maximal technological distance which can still be crossed under reasonable assumptions.

Let us assume that R&D increases with gross return expressed by means of the rate A_j. On the other side, in the case where the currently calculated gross return exceeds a critical value, the plant shows in general a satisfying behaviour, i.e., it does not look for new technologies. Taking all this into account we assume the coefficients for local search and satisfying processes are

$$\begin{aligned} A_{ij}^0 &= (\mathrm{IN})(d_{crit} - d(i,j))A_j \qquad && \text{if } d(i,j) < d_{crit} \text{ and } A_j < A_{crit} \\ A_{ij}^0 &= 0 && \text{if } d(i,j) > d_{crit} \text{ and } A_j > A_{crit}\,. \end{aligned} \tag{14}$$

We have assumed here following Nelson et al. a step-like behaviour at d_{crit} and A_{crit}. It is easy to change to a smoother behaviour, e.g., by assuming $A_{ij}^0 = (\mathrm{IN})A_j q_{ij}$ where

$$q_{ij} = (\mathrm{IN}) \frac{d_{crit}}{\left(1 + \dfrac{d(i,j)}{d_{crit}}\right)} \frac{1}{\left[1 + \left(\dfrac{A_j}{A_{crit}}\right)^2\right]} \tag{15}$$

might be read as a research policy function.

The function q_{ij} determines the probability of finding a new technology i in a local R&D search process outgoing from the parameters of a used technology j. q_{ij} is of reasonable value if $d(i,j)$ is sufficiently small and if A_j is in a reasonable range between zero and A_{crit}: $0 < A_j < A_{crit}$. Then, a second function

$$Q_k = 1 - \sum_l (\mathrm{IN}) q_{lk} \tag{16}$$

is an expression of successful R&D activities of firms using technology k. We call Q_k the R&D influence factor in the system. $Q_k = 1$ if there is zero R&D activity of firms using technology k. The more successful R&D activities undertaken in the system the smaller is $Q_k < 1$ (any k).

1.3. Role of social averages

Technologies have to be chosen by firms to be adopted. According to it's qualitative properties a technology might be more or less attractive to a firm.

Denoting by Ω_i the attractiveness of a technology i we may assume that Ω_i might be expressed in terms of rates of self-reproduction and imitation behaviour in the system. A very simple Ansatz would be

$$A_i + A_{ij}^1 = (DE)\Omega_i; \quad \Omega_i = \frac{1}{(DE)}(A_i + A_{ij}^1), \tag{17}$$

with (DE) being a general DEmand factor which translates the attractiveness Ω_i of the technology i to the growth of the field i (number of firms using technology i).

To understand the role of the demand factor (DE) let's distinguish the two extreme cases:

$(DE) \gg 1$	due to the high demand also relatively "low" attractive technologies are chosen and
$(DE) \ll 1$	due to the low demand also relatively "high" attractive technologies are seldomly chosen.

One may assume that a technology i according to its qualitative parameters is *equally attractive* for the self-reproduction and the imitation process. That means, that it does not depend so much on the technology parameters (but on general market conditions) if a firm rather will introduce technology i by construction or re-construction of production units. Thus, we may assume:

$$A_i = (SR)\Omega_i, \quad A_{ij}^1 = (IM)\Omega_i, \tag{18}$$

with (SR) and (IM) being the demand factors for self-reproduction and imitation behaviour respectively. Assuming $\Omega_i \neq 0$ then, if $A_i = 0$, $(SR) = 0$ follows and, if $A_{ij}^1 = 0$, $(IM) = 0$ follows. If $(SR) = 0$, there is no demand for an existing technology being chosen to equip new production units and therefore no self-reproduction behaviour, if $M_{ij} = 0$. If $(IM) = 0$, there is no demand for an existing technology being chosen to equip existing production units and therefore no imitation behaviour. Equation (18) together with eq. (17) leads to $(DE) = (SR) + (IM)$. (19) E.g., for a demand factor $(DE) = 1$ (attractiveness Ω_i fully expressed in self-reproduction and exchange rates) we get: $\Omega_i = A_i + A_{ij}^1$ and $(SR) + (IM) = 1$. (20)

Now we introduce the attractiveness Ansatz (eqs. (17), (18)) into the transition probabilities (Eq. (2,3,6,8,9,10)) with the notations $A_i = A_i^0 + A_i^1(N_i - 1)$, $D_i = D_i^0 + D_i^1(N_i - 1) + D_i^1(N_i - 2)$ and setting Φ_o, M_{ij}, B_{ij}, C_{ij} equal to zero. By multiplying the resulting Master equation with N_k and performing the average

$$\langle N_k \rangle = \sum_{N_1}\sum_{N_2}\cdots\sum_{N_s} N_k P(N_1, \ldots, N_k, \ldots N_s, t)) \tag{21}$$

after some manipulations we finally obtain the following differential equation for the mean occupation numbers of the industrial state (using the assumption

$A^0_{kl} = (IN)A_l q_{kl}$):

$$\frac{d}{dt}\langle N_k\rangle = [((SR)+(IM))\Omega_k - (IM)\langle\Omega\rangle - D_k]\langle N_k\rangle + (IN)\sum_l [q_{kl}A_l\langle N_l\rangle - q_{lk}A_k\langle N_k\rangle]. \tag{22}$$

where

$$\langle\Omega\rangle = \sum_l \frac{\Omega_l\langle N_l\rangle}{\langle N\rangle}. \tag{23}$$

We denote the average attractiveness $\langle\Omega\rangle$ taken over all technologies in the system as a social average. Equation (22) is formally similar to the Eigen equation for the evolution of biopolymer species.

Including the R&D influence factor

$$Q_k = 1 - \sum_l (IN)q_{lk} \tag{24}$$

we may also write ($\langle N_k\rangle = N_k$)[1]:

$$\frac{d}{dt}N_k(t) = [(SR)\Omega_k Q_k - D_k]N_k + (IM)[\Omega_k - \langle\Omega\rangle]N_k + (IN)\sum_l q_{kl}A_l N_l. \tag{25}$$

A given technology k has a good chance to survive if it guarantees a high self-reproduction and/or high imitation behaviour. A sufficient number of firms (more than in average) must be interested to equip new and existing plants with technology k. The social average is taken over all technologies in the system. Thus, technologies compete for firms to use them.

If a firm's technology is weaker than the social average its only chance to survive is to move to a better technology by imitation or by R&D. Since a successful R&D needs already a relatively high value of gross return the best strategy for a "bad" firm is probably imitation. On the other hand, the best strategy for a "good" firm (with production units using technology k with ($\Omega_k > \langle\Omega\rangle$) might be self-reproduction.

Equation (25) shows the close connection between the imitation behaviour and the (increasing) attractiveness of technologies in the process of technological evolution. We see that the social average $\langle\Omega\rangle$ plays an important role as a threshold value which separates successful from unsuccessful technologies, if imitation is present in the system.

If $(IM) = 0$ and $(DE) = 1$, eq. (20) gives $(SR) = 1$ and Ω_k is expressed by A_k. Then the opening up of new production units must exceed the closure rate to have a positive growth for technology k. In this case of extension of the number of production units, the firms have no social limitation factor to take into account. If there is no demand (or no money) for an extension of production units, the firms are dependent on a re-construction of their existing production units.

If $(SR) = 0$ and $(DE) = 1$, eq. (20) gives $(IM) = 1$ and Ω_k is then expressed by A^1_{kl}. A positive growth rate for technology k requires in this case the re-construction of

[1] Let us note, that in the following we note $\langle N_k\rangle$ as N_k, if N_k represents a deterministic variable in the deterministic description. Only if N_k represents a stochastic variable, we use the brackets $\langle N_k\rangle$ to differentiate between the stochastic variable and the deterministic average.

existing production units (equipped with technology k) at a rate exceeding the closure rate. From the point of view of firms, a number of "bad" firms is fighting against closure of their production units by means of imitation.

If self-reproduction as well as imitation behaviour occur in the system, a survival strategy may be found by controlling the weight factors (IM) and (SR) of both processes according to eq. (20).

For the "bad" technologies ($\Omega_k < \langle\Omega\rangle$) there is no strategy at all, apart from bad firm management. Since the third and the second term on the r.h.s. of eq. (25) are always positive, the best survival strategy for a "good" technology ($\Omega_k > \langle\Omega\rangle$) probably consists mainly of taking measures to exceed the closure rate D_k by concentrating on self-reproduction processes and by diminishing the R&D influence. Then, imitation strengthens the effect of the difference ($\Omega_k - \langle\Omega\rangle$).

Equation (25) also shows the influence of R&D on the growth possibilities for established technologies. The last term on the r.h.s. of eq. (25) cannot be excluded but must be regarded as very small, if the competition area is sufficiently large. As $Q_k \leqslant 1$ always holds, the growth of established technologies due to self-reproduction processes is always diminished by successful R&D activities. If there are too many experimenting firms in the system, $\Sigma_l\,(\mathrm{IN})q_{lk}$ will be large and Q_k correspondingly small. If $Q_k \ll 1$, the self-reproduction chances of existing technologies are hardly influenced.

For a "good" technology, the creation of "better" technologies and the disappearance of "bad" technologies are two dangers, which might be overcome only in a limited time scale. Thus, attractiveness increases necessarily.

If a co-operative process begins within the system and leads to a spontaneous structure-formation process (transfer to a new stable stationary state), no averaging process can take place and therefore the social average loses its meaning as threshold value.

1.4. Infection

Innovation in its first stage may be understood as infection. Considering a technology which first appears in the system we can say, that the system consisting of s technologies has been infected with a new technology.

Let us first regard the infection problem in terms of the deterministic trend analysis, in which stability/instability concepts may be applied.

Let us assume that the system of the s existing technologies is in a stable state. That means we have certain relations (fractions) in which the technologies are represented in the system. We call this situation a given structure of the system. Then, the question occurs if and how by an infection with a new technology this structure will change.

The following cases are possible:

- the new technology will be integrated in the system without significant changes of the fraction of the old technologies (coexistence)
- the infection with a new technology performs a competition process in which (a) the new technology cannot survive. The old stable state will be reproduced (unsuccessful infection) or (b) one or several of the old technologies will disappear and the new technology will win the process (substitution: the old stable state becomes unstable, a new stable state will be adopted).

That means, that the approval of a new element in the game of evolution is connected with a test of the deterministic system for stability.

In this way, the question of the selection value of a new element with respect to the already existing population becomes a well-posed problem. Mathematically it reduces to an eigenvalue problem of rank $(s + 1)$ (Hofbauer and Sigmund 1984).

When we consider processes that include exchange between the firms i and j combined with non-linear growth functions of self-reproduction and decline, a calculation of the competition properties of an innovation becomes an extremely complicated mathematical problem, which requires a detailed stability analysis of the deterministic problem following the above mentioned method (Prigogine et al. 1972).

In this case a new technology is of higher selection value with respect to the existing occupation if the deterministic system is unstable with respect to a corresponding perturbation. In analogy to Eigen's concept one can say that the stability properties of the system represent the selection value of a new technology appearing by a transition process, by diffusion, or by innovation.

We have already noted that co-operative processes, as imitation, e.g., due to their property of mutual directedness (expressed by non-linear terms), may significantly influence the result of selection processes. In systems with non-linear interactions of the type considered here, the selection process no longer depends only on the growth properties of a new field but also on the existing configuration. We find multistability where the behaviour of the system depends on the initial conditions in such a way that a new "better" field is a potential possibility for the system but under the given conditions not a real one. Such a situation can describe the selection between two or more equivalent possibilities for the system (Eigen and Schuster 1977, 1978).

Infection of a stochastic system

In the stochastic description (e.g., by means of the Master equation formalism) the state of the system is characterised by the temporal development of a probability function $P(N_1, N_2, \ldots, N_i \ldots, t)$, which describes the probability of finding N_1 firms using a technology 1, N_2 firms using technology 2 etc. at time t. A stable stationary state is then related to a maximum of the stationary probability function, and changes of the location of stationary states and their stability properties correspond to changes in the landscape described by the probability function over the state space. To obtain answers on the behaviour of the system according to an infection with a new technology, one must determine the time-dependent behaviour of $P(N_1, N_2, \ldots, N_i \ldots, t)$ by solving the corresponding stochastic equations. In general, analytical solutions are hardly to obtain. In special cases, some expressions concerning the survival probability of the infecting technology can be obtained. This is analogous to an analysis of the stationary behaviour in the deterministic case. In this way, it can be shown, that in the stochastic picture the deterministic conclusions remain true in average.

By analysing the survival probability of new technologies, and also by computer experiments, we can show that the deterministic order can be destroyed or relativised by fluctuations. Separatrices, which in the deterministic picture separate different parts in the state space from each other, can be crossed. In this way, new channels of evolution are opened up by fluctuations.

II. Substitution

Under the presuppositions discussed in the former sections substitution may be described as an infection problem. As substitution is a replacement of an old technology for a new one only two technologies are taken into account. In our model only two fields are considered: $N_1 + N_2 = N$. The infection problem in such a substitution process occurs with the condition $N_2 \ll N$.

II.1. Earlier models re-interpreted

II.1.1. A logistic substitution model (The Fisher–Pry model)

In the early seventies Fisher and Pry (1971) developed their simple substitution model of technological change most studied and empirically tested. Since the work of Fisher and Pry is of fundamental importance to the whole range of later developments we will take the basic assumption of Fisher and Pry as our starting point. The main mathematical assumption is that: "The fractional rate of fractional substitution of new for old is proportional to the remaining amount of the old left to be substituted." (1971).

The corresponding model for the fractional growth rate per time unit reads:

$$\frac{d}{dt} f = af(1 - f). \tag{26}$$

The solution of this equation is well-known. If $a > 0$ the fractional rate increases according to the Verhulst-Pearl law up to the saturation point. This S-shaped growth curve is in good accordance with empirically obtained results concerning the substitution process of technologies. By generalising the growth rate as was, e.g., done by Sharif and Kabir (1976), Easingwood et al. (1981) the accuracy of forecasting can be improved.[2]

II.1.2. A generalised model of substitution

To enable improved understanding of the mechanisms of the substitution process several authors have proposed a two-dimensional system of coupled differential equations to describe the mutual influence between the new and the already existing technology (Mahajan and Peterson 1979; Batten 1982; Kwasnicka et al. 1983; Karmeshu et al. 1985; Bhargava 1989). In particular, we will refer here to an approach using generalised Lotka–Volterra-equations to describe the substitution process in terms of competition between technologies.

To introduce our model, we assume that the system is occupied by N_1 "master" producers with the reproduction rate E_1 and we infect the system with a few producers using a new technology with rate E_2. Further, we assume that the total number of producers is $N = N_1 + N_2 = \text{const.}$. This condition leads to competition where a new technology can only succeed if it replaces the old one. The dynamics is described by:

$$\frac{d}{dt} N_i = (E_i + B_i N_i) N_i - k_0 N_i, \quad i = 1, 2. \tag{27}$$

---

[2] For a classification of different generalizations of the Fisher–Pry model see Tingyan (1990).

The existence of different growth laws is expressed by help of a different choice for the parameters E_i and B_i. In the case of linear growth functions $E_i \neq 0$; $B_i = 0$ holds, while in the case of non-linear growth functions $E_i = 0$; $B_i \neq 0$, and in the case of growth functions containing a combination of linear and non-linear growth E_i, $B_i \neq 0$ is valid.

The condition of constant overall number N of production units leads to

$$k_0 = \frac{(E_2 + B_2N_2)N_2 + (E_1 + B_1N_1)N_1}{N}. \tag{28}$$

In the special case of linear growth this model contains the model proposed by Batten (1982) with a certain parameter choice.

Further, it should be noted that the model is in a certain sense located between the one-dimensional logistic substitution model and the two-dimensional Lotka–Volterra approach. Introducing the condition $N_1 + N_2 = N = \text{const.}$, we reduce the two-dimensional problem in a certain sense to a one-dimensional problem. Therefore, the substitution model of Fisher–Pry can be obtained from our model (case of linear growth) replacing N_1 with $(N - N_2)$ in the equation for N_2.

First, let us consider the states in which the system is in a stationary behaviour. The stationary points of the model (eq. 29–30) ($d/dt\ N_1^s = 0$, $dt/dt\ N_2^s = 0$) are:

$$\begin{aligned}
&\text{(i)}\ N_1^s = N; && N_2^s = 0\\
&\text{(ii)}\ N_1^s = 0; && N_2^s = N\\
&\text{(iii)}\ N_1^s = \frac{NB_2 + E_2 - E_1}{B_1 + B_2} && N_2^s = \frac{NB_1 + E_1 - E_2}{B_1 + B_2}.
\end{aligned} \tag{29}$$

For the case of linear growth, state (iii) is coincident with state (i).

Which one of these states can be realised depends on the stability analysis and is determined by the relationship of the various growth parameters. In the following, we use the growth rates E_i and B_i as indicators for the quality of a technology. In terms of our model the relation $\alpha = E_2/E_1$ in the case of linear growth, and $\alpha = B_2/B_1$ in the case of non-linear growth resp. determines a selection advantage. The new technology 2 in comparison to the master technology 1 may be characterised as a "good" one, if $\alpha > 1$ or as a "bad" one, if $\alpha < 1$.

As it is well known for $\alpha > 1$ in the linear case state (ii) is stable and state (i) unstable.[3] Thus, in the deterministic linear case "good" new technologies will substitute the master, "bad" ones will become extinct. This sharp selection behaviour is independent of the degree of infection (initial conditions) and has a direct effect.[4]

The (post) modern society is characterised by rapid changes. In particular, some industrial branches are characterised by very fast growth processes (up to saturation), which must be described by non-linear growth rates. In such cases the chances of survival of a new variant starting with a few elements are not very large.

In the case of quadratic growth, the system exhibits bistability. The stationary states ($N_1^s = N$; $N_2^s = 0$) and ($N_1^s = 0$; $N_2^s = N$) are stable. The state (iii), which lies

[3] This result is in accordance with the condition of successful substitution given in the Batten model.

[4] In the linear case the model is of the type of a Fisher–Eigen equation where the growth rate E_i serves as selection value and only the field with the highest growth rate will survive in the selection process.

between 0 and N, is unstable and separates the stable states. It is a separatrix. (The model in this case corresponds to the competition of hypercycles in the Eigen theory.) Then the fate of the new technology depends on the initial conditions.

If the initial number of the new firms is lower than the threshold

$$N_2(0) < \frac{N}{(\alpha + 1)}, \tag{30}$$

the new technology has no chance of survival within the deterministic picture. The threshold value corresponds to the state (iii). For example, for an initial condition below the unstable stationary point, technology 2 is located in the attractor basin of the stable state ($N_2^s = 0$ and $N_1^s = N$ correspondingly) and cannot leave it.

If N is 200 and $\alpha = 2$, the new technology must start with around 70 (exactly 67) firms, to substitute the old one with security. If $N_2(0)$ is smaller than 67 the new technology has no chance to substitute the old one. Such an initial condition is hardly to realise. On the other hand, if N is 200 and the new technology starts with around 10 firms, then α must be within the range of 20 for the new technology to survive.

We see that the hyperselection situation, which is in many cases not unrealistic is hard to overcome and thus we have to look for possibilities of a real substitution in a stochastic world.

In the case of mixed growth, bistability occurs again and analogous to eq. (30), a threshold can be determined as:

$$N_2(0) < \frac{NB_1 + E_1 - E_2}{B_1 + B_2}. \tag{31}$$

The location of the separatrix in relation to the two attractors depends again on the size of the ensemble. From eq. (30) and (31) it can be seen, that the greater N is the closer the unstable state will be to N, and the higher the threshold will be for a new technology to replace the old one.

II.2. Stochastic substitution models

In a stochastic dynamics which is based on integer particle numbers $N_i = 0, 1, 2, \ldots$ the picture changes completely. Let us assume that N_1 is the number of plants using the old technology and N_2 the number of plants using the new one and furthermore that $N_1 + N_2 = N = \text{const.}$.

The elementary stochastic process is assumed to be a substitution, i.e., one plant substitutes the new technology for the old one, or, in mathematical terms, we have the transition:

$$N_1 \to N_1 - 1, N_2 \to N_2 + 1 \tag{32}$$

with the transition probability

$$W(N_i + 1, N_j - 1 | N_i, N_j) = A_{ij}^1 N_j N_i / N. \tag{33}$$

Under the condition $N_1 + N_2 = N = \text{const.}$ in the state space only certain states resp. transitions are possible. Therefore, also in the stochastic description the two-dimensional problem can be reformulated as an one dimensional problem (Fig. 1).

To determine the probabilities W^+ and W^- let us start with the *case of linear growth* ($E_1, E_2 \neq 0$; $B_1, B_2 = 0$). According to the corresponding deterministic model

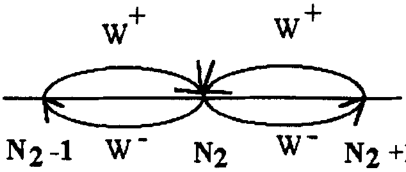

Fig. 1.

E_1 as growth rate of technology 1 can be understood as loss rate of technology 2 and vice versa. Then, $E_2 - E_1$ is the total growth rate of N_2. The corresponding transition probability ($E_i = A_{ij}^1$) is assumed to be

$$W^+ = W(N_2 + 1 | N_2) = E_2 N_2 \frac{(N - N_2)}{N}. \tag{34}$$

with $N_1 = N - N_2$. The opposite process has the transition probability

$$W^- = W(N_2 - 1 | N_2) = E_1 (N - N_2) \frac{N_2}{N}. \tag{35}$$

Let us assume now that the probability to find at time t the number of plants N_2 using the new technology is given by $P(N_2, t)$. Following the standard methods of the theory of Markov processes we describe the growth of the new technology as balance equation of loss and gain processes with the Master equation

$$\frac{\partial}{\partial t} P(N_2, t) = W^+(N_2 - 1) P(N_2 - 1) + W^-(N_2 + 1) P(N_2 + 1) - (W^+(N_2) + W^-(N_2)) P(N_2). \tag{36}$$

Introducing the approach for the W^+ and W^- in the case of linear growth we obtain (using $N_1 = N - N_2$):

$$\begin{aligned} \frac{\partial}{\partial t} P(N_2, t) = {} & \left(\frac{E_2}{N} (N_2 - 1)(N - N_2 + 1) \right) P(N_2 - 1, t) \\ & + \left(\frac{E_1}{N} (N_2 + 1)(N - N_2 - 1) \right) P(N_2 + 1, t) \\ & - \left(\frac{1}{N} (E_1 + E_2) N_2 (N - N_2) \right) P(N_2, t). \end{aligned} \tag{37}$$

The four terms on the r.h.s. correspond to the two gain processes $(N_2 - 1) \rightarrow N_2$ and $(N_2 + 1) \rightarrow N_2$ and to the two loss processes $N_2 \rightarrow (N_2 - 1)$ and $N_2 \rightarrow (N_2 + 1)$.

In order to investigate the relation to the Fisher–Pry model let us turn now to the mean value of $\langle N_2 \rangle$. We get after some manipulations

$$\frac{d}{dt} \langle N_2(t) \rangle = \frac{1}{N} (E_2 - E_1) \langle N_2 (N - N_2) \rangle). \tag{38}$$

With the approximation $\langle N_2 N_2 \rangle = \langle N_2 \rangle \langle N_2 \rangle$, the shortening $f = \langle N_2 \rangle / \langle N \rangle$, which is the fraction of new technology, and $a = (E_2 - E_1)$—the fractional growth rate per time unit, we get from eq. (38) the Fisher–Pry differential equation (comp. eq. 26). On the other side, eq. (38) corresponds to the equation for N_2 in the Eigen model resp. the Lotka–Volterra system, if we express in eq. (27) (with $B_i = 0$) N_1 by $(N - N_2)$.

In the case of non-linear growth processes ($E_i = 0$; $B_i \neq 0$ and $B_i N_i/V = A^1_{ij}$) we assume:

$$W^+ = W(N_1 - 1, N_2 + 1 | N_1, N_2) = B_2 \frac{(N - N_2)N_2^2}{NV} \tag{39}$$

$$W^- = W(N_1 + 1, N_2 - 1 | N_1, N_2) = B_1 \frac{(N - N_2)^2 N_2}{NV}. \tag{40}$$

Now, the transition probabilities contain a quadratic and cubic term of N_2, e.g., we consider growth processes accelerating with N_2. The variable V is an additional parameter which allows to control the strength of the non-linear growth.

In the case of mixed growth (linear as well as non-linear growth) we obtain combining eq. (34) and (39) (resp. (35) and (40)) the following transition probabilities:

$$W^+ = E_2 \frac{(N - N_2)N_2}{N} + B_2 \frac{(N - N_2)N_2^2}{NV} \tag{41}$$

$$W^- = E_1 \frac{(N - N_2)N_2}{N} + B_1 \frac{(N - N_2)^2 N_2}{NV}. \tag{42}$$

In the combination of linear and non-linear growth properties the parameter V can be used to control the strength of the non-linear growth properties in relation to the linear growth properties.

In the following section we consider analytical solutions of the corresponding Master equation in the cases introduced.

III. Results

III.1. Short-term effects

In general, the stochastic master equation is difficult to solve. A problem which may be studied by means of the stochastic theory is the time-dependent behaviour of a new technology 2 infecting an equilibrated industrial state where the technology 1 dominates. Let us assume that at the time $t = 0$ only M plants with $M \ll N$, e.g., $M = 1$ or 2, are *infected* with the new technology. In other words the initial condition reads

$$P(N_2, 0) = \begin{pmatrix} 1 & \text{if } N_2 = M \\ 0 & \text{if } N_2 \neq M \end{pmatrix}. \tag{43}$$

Due to our assumption that only a few plants are infected we will have $N_2 \ll N$ in the initial state of evolution and the master equation in the linear case (37) may be simplified by neglecting terms (N_2/N) resulting in

$$\frac{\partial}{\partial t} P(N_2, t) = E_2(N_2 - 1)P(N_2 - 1, t) + E_1(N_2 + 1)P(N_2 + 1, t) - (E_1 + E_2)N_2 P(N_2, t). \tag{44}$$

Following Bartholomay (1958, 1959; see also Eigen 1971) the solution of this equation $P(N_2, t)$ with the initial condition (43) can be derived. The most interesting application of the distribution is the determination of the probability of extinction

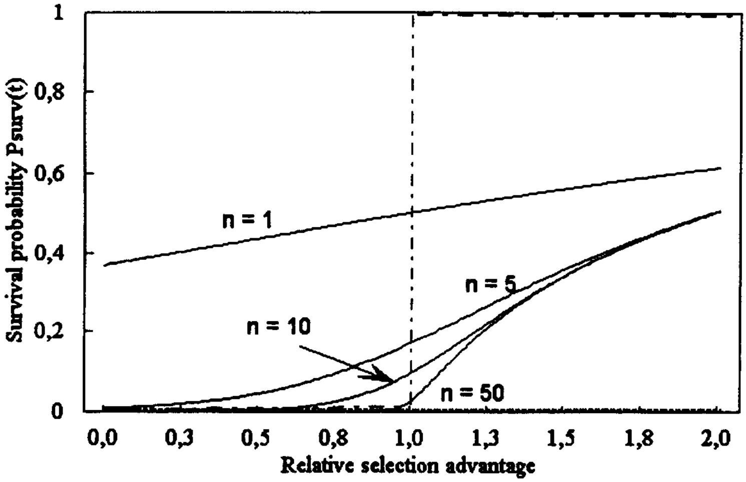

Fig. 2. Survival probability of a new infecting technology $P_{surv}(t)$ as function of the relative selection advantage κ after n generation

of the new technology after a time t which is given by (Allen and Ebeling 1983)

$$P(0,t) = \left(\frac{\exp((E_2 - E_1)t) - 1}{E_2/E_1 \exp((E_2 - E_1)t) - 1}\right)^M. \tag{45}$$

Correspondingly, the survival probability in the system after time t is

$$P_{surv}(t) = 1 - P(0,t). \tag{46}$$

Now, we ask for the survival probability of the new technology introduced by one production unit ($M = 1$). n generations after the appearance (denoted by $n = E_1 t$), P_{surv} can be written as:

$$P_{surv}(t) = \frac{\kappa}{1 + \kappa - \exp(-n\kappa)} \tag{47}$$

where κ is the relative selection advantage:

$$\kappa = \frac{E_2 - E_1}{E_1} = \alpha - 1. \tag{48}$$

According to eqs. (47, 48), the survival probability for the new technology depends on the relative selection advantage κ.

According to Darwin, selection leads to a survival of the fittest. Although valid in traditional approaches in a stochastic substitution model this sentence is only valid as a long-term effect. A short-term survival even of a "bad" technology infecting an equilibrated industry where a certain technology dominates cannot be excluded. The "bad" technology (with $E_2 < E_1$) may survive for some generations against the evolutionary trend towards "better" and "better" technologies. The trend is given by the sharp boundaries of the corresponding deterministic picture (indicated in Fig. 2 by the broken line).

To analyse the long-term behaviour of the survival probability within a definite time period, the survival probability in the stationary state (t tends to infinity) can be derived:

$$P_{surv}(t \to \infty) = 0 \qquad \text{for } E_2 < E_1 \tag{49}$$

and

$$P_{surv}(t \to \infty) = 1 - \left(\frac{E_1}{E_2}\right) \qquad \text{for } E_2 > E_1. \tag{50}$$

The survival probability in the stationary state will be zero if the new technology is a "bad" one ($E_2 < E_1$) and depends on the selection advantage in the other case. Thus, selection in a stochastic model shows stringency in the long run only.

III.2. Ensemble size and niches

If an analytical solution of the master equation as a whole is not accessible, we can investigate analytically the behaviour of the system in the limit $t \to \infty$. The calculation of the stationary distribution $P(N, t \to \infty)$ corresponds to the analysis of the stationary states in the deterministic description. For a birth and death process in the case of a two-dimensional system (with $N_1 + N_2 = N = \text{const.}$ and two absorber states $N_2 = 0$ and $N_2 = N$) the final stationary distribution must have the shape (Schimansky-Geier 1980; Ebeling et al. 1981; Ebeling and Feistel 1982):

$$P(N_2, t \to \infty) = \sigma\delta(N, N_2) + (1 - \sigma)\delta(0, N_2) \tag{51}$$

where σ is the survival probability of the new technology 2 in the stationary state and δ stands for the Kronecker symbol ($\delta(N, N_2) = 1$ for $N_2 = N$ and zero else).

In this case the survival probability σ can be derived from just one constant of motion. In the most general case (eq. (36) for a system with two absorber states), the result is

$$\sigma_{N_2(0),N} = \frac{1 + \sum\limits_{j=1}^{N_2(0)-1} \prod\limits_{i=1}^{j} \frac{W_i^-}{W_i^+}}{1 + \sum\limits_{j=1}^{N-1} \prod\limits_{i=1}^{j} \frac{W_i^-}{W_i^+}} \qquad \text{for } 0 < N_2(0) < N$$

and

$$\sigma_{N_2(0),N} = 1 \qquad \text{for } N_2(0) = N \tag{52}$$

where $W_i^+ = W(i+1|i)$ and $W_i^- = W(i-1|i)$, with $i = N_2$. σ depends not only on the parameters of the system but also on the initial conditions and the ensemble size N.

Assuming the corresponding probabilities are
in the linear case

$$W_i^+ = E_2\frac{(N-i)i}{N} \qquad W_i^- = E_1\frac{(N-i)i}{N} \tag{53}$$

in the quadratic case

$$W_i^+ = B_2\frac{(N-i)i^2}{NV} \qquad W_i^- = B_1\frac{(N-i)^2 i}{NV} \tag{54}$$

and in the case of mixed growth

$$W_i^+ = E_2 \frac{(N-i)i}{N} + B_2 \frac{(N-i)i^2}{NV}$$

and

$$W_i^- = E_1 \frac{(N-i)i}{N} + B_1 \frac{(N-i)^2 i}{NV}, \tag{55}$$

after some manipulations, we get the following survival probabilities for the new technology.

In the linear case we obtain:

$$\sigma_{N_2(0),N} = \frac{1 - \left(\frac{1}{\alpha}\right)^{N_2(0)}}{1 - \left(\frac{1}{\alpha}\right)^{N}}. \tag{56}$$

For large systems (infection problem) eq. (56) reduces to:

$$\sigma_{N_2(0),N\to\infty} = 0 \qquad \text{for } E_2 < E_1$$

and (57)

$$\sigma_{N_2(0),N\to\infty} = 1 - \left(\frac{1}{\alpha}\right)^{N_2(0)} \qquad \text{for } E_2 > E_1$$

which is equal to eqs. (49, 50) for $N_2(0) = M = 1$.

In the quadratic case we obtain

$$\sigma_{N_2(0),N} = \frac{1 + \sum_{j=1}^{N_2(0)-1} \left(\frac{B_1}{B_2}\right)^j \binom{N-1}{j}}{\left(1 + \frac{B_1}{B_2}\right)^{N-1}} \tag{58}$$

and in the mixed case:

$$\sigma_{N_2(0),N} = \frac{1 + \sum_{j=1}^{N_2(0)-1} \prod_{i=1}^{j} \dfrac{E_1 + B_1 \frac{N-i}{V}}{E_2 + B_2 \frac{i}{V}}}{1 + \sum_{j=1}^{N-1} \prod_{i=1}^{j} \dfrac{E_1 + B_1 \frac{N-i}{V}}{E_2 + B_2 \frac{i}{V}}} \tag{59}$$

The survival probabilities now depend not only on the selection advantage but also on the degree of infection (initial conditions of the new technology) and the *size of the ensemble in which the competition takes place.*

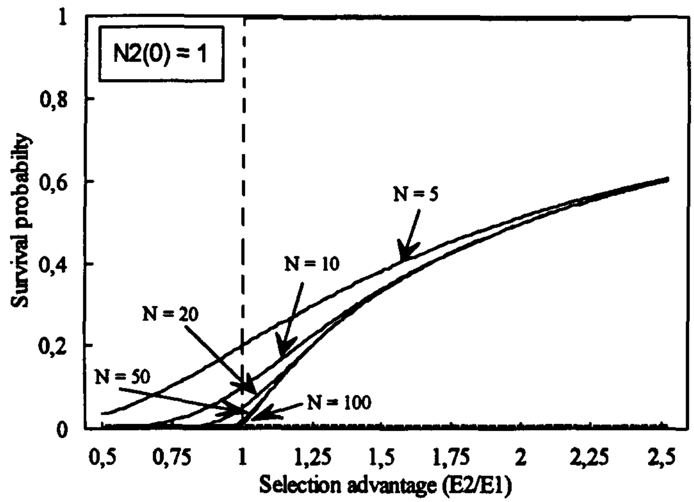

Fig. 3. Survival probability σ of a new technology in the *case of linear growth* as function of the selection advantage α depending on the ensemble size N ($N_2(0) = 1$)

The most important result obtained so far is that the probability of survival is in general a smooth function which is continuously increasing with the relative advantage. There is no sharp difference between the better and the worse technologies. In contrast to the deterministic picture, a new better technology will not automatically survive. In the linear case (eq. 56) we observe in the limit of $N \to \infty$ a kinetic phase transition of second order. Figure 3 shows that in reality, this situation will practically be realised for ensembles of a size greater then $N = 50$. Then, for $\alpha < 1$, the new technology has no chance of survival.

Besides the time-dependence of selection explained in section III.1., within a stochastic picture a "space" dependence of selection also occurs which we have called a specific niche effect.

Selection shows its stringency only in a sufficiently large competition area. On the other hand, a sufficiently small competition area acts as a niche in which the stringency of selection is softened.

For systems with a linear growth law, the niche effect means that any improvement (positive or negative) less than 10% has no significant effect on survival. As Fig. 3 shows, the new technology must be twice as good as the old one to get a survival probability exceeding 50%. On the other hand, a new technology which is worse by about 10% still has a certain chance of survival provided that no exceptionally good technology is produced (consider, e.g., case $N = 5$). This region of +/− 10% is neutral with respect to selection since stochastic effects here allow a variety of possibilities. The situation of the new technology can be improved decisively when the system is initially infected with more than one representative of the new technology (Fig. 4).

Locally developed niches may play a decisive role in a two-stage strategy of establishing a new technology. Although within the niche the Hamlet question (temporarily) is solved, in the case of linear growth rates the competition process is very unspecific and contains no recognition of the true quality of the new variant. Since recognition is a consequence of successful usage consequently the next step of establishment of a good new variant is the enlargement of the competition area.

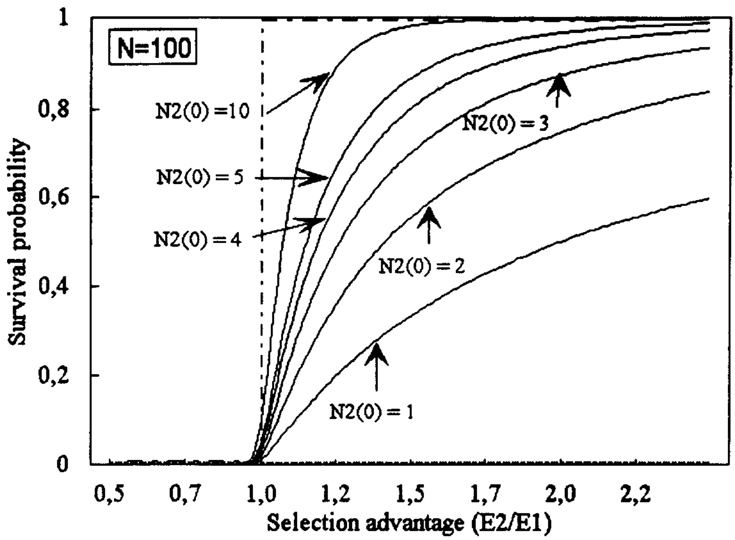

Fig. 4. Survival probability σ of a new technology in the *case of linear growth* as function of the selection advantage α depending on the initial condition $N_2(0)$ ($N = 100$)

In the case of quadratic growth laws, in the deterministic model (eq. 30), the survival of a new technology initially may require a number of firms within a range of 1/3 of the total existing population of firms, which is in economic standard situations impossible to fulfil. In the more realistic stochastic model represented by eq. (58) the picture is somewhat softer but the chances might be considered to be still worse. If N is 200, $\alpha = 2$ and the new technology starts with 67 firms, the survival probability is around 50%. To have a survival probabiltiy of 90%, one must start in this case with 75 firms. On the other hand, in contrast to the deterministic model, if the new technology starts with fewer than 67 firms, it still has a certain survival probability (with 65 firms 40%, with 60 firms 10%). If N is around 200, the new technology starts with 10 firms ($N_2(0) = 10$) and α is in the range of 20, the new technology has a survival chance of around 50%. To have a survival probability of 90%, α must be in the range of around 30.

As it is seen, the only real possibility of overcoming the hyperselection effect is to create a niche by limiting the competition area. As Fig. 5 shows, a finite population size in any case improves the survival probability of the new technology. This effect is essential since it guarantees the survival of mutants in hypercyclic systems with finite population size (Fig. 5).

Also in the non-linear growth case locally developed niches may play a constructive role in the technological evolution process. The new variants in the niche may first grow to considerable numbers and thus afterwards in the global system, the hyperselection situation may be overcome.

Figure 6 shows in a simulation experiment how a new technology can "tunnel" through the separatrix (Fig. 6). Also in this case the survival probability of the new technology increases if the degree of infection of the substitution process is increased (Fig. 7).

In the case of mixed growth (eq. 59), the survival probability depends on both the relations of E_1/E_2 and B_1/B_2 and is also determined by the strength of the

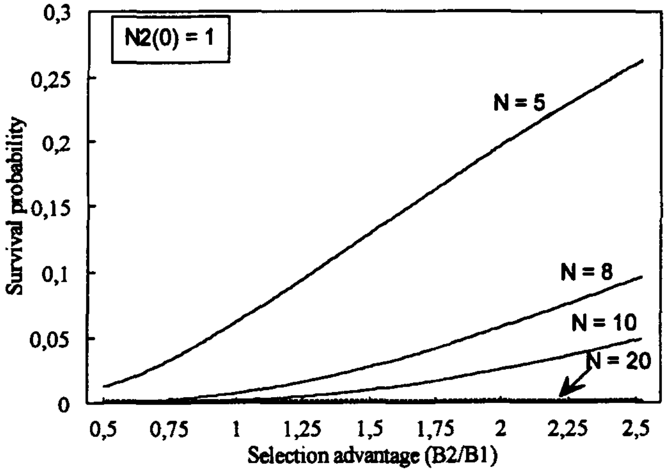

Fig. 5. Survival probability σ of a new technology in the *case of quadratic growth* as function of the selection advantage α depending on the ensemble size N ($N_2(0) = 1$)

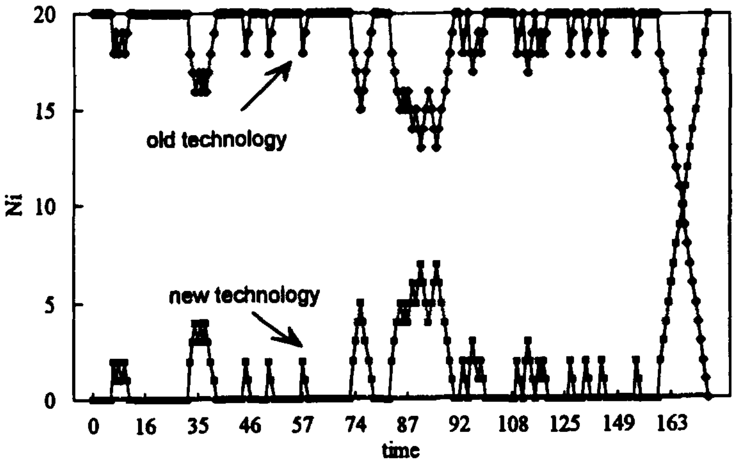

Fig. 6. Tunneling through the separatrix after several unsuccessful trials (Parameters $B_2/B_1 = 3$, $N = 20$, $N_2(0) = 1$)

quadratic growth relative to the linear growth represented in the model by the parameter V. A technology can significantly improve its position if it can develop growth properties different from the old one. It follows that a technology which is weaker in relation to the linear growth properties can nevertheless substitute the old one if it can develop other growth properties. As shown in Fig. 8, a technology which must in the linear case be twice as good as the old one arrives at the same survival probability (50%) already with only 50% of the linear reproduction rate but with a quadratic growth characteristics ($B_2 = 1.0$) (Fig. 8). In the case of mixed growth characteristics with the same properties of linear growth, the survival

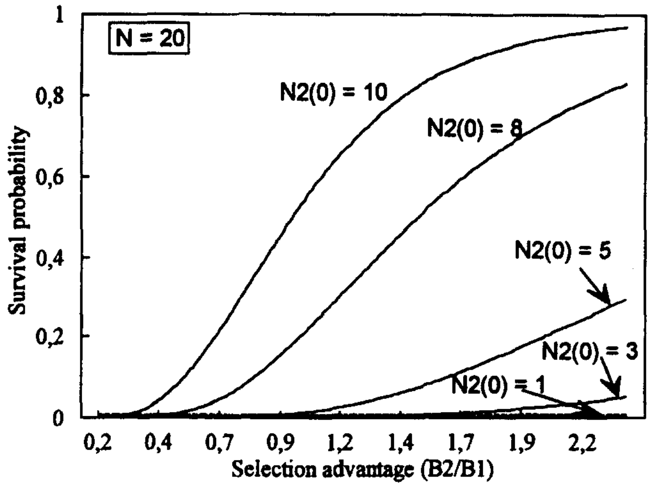

Fig. 7. Survival probability σ of a new technology in the *case of quadratic growth* as function of the selection advantage α depending on the initial conditions $N_2(0)$ ($N = 20$)

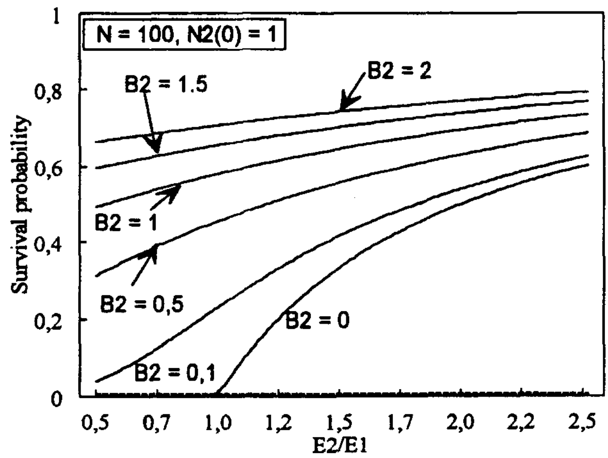

Fig. 8. Survival probability σ of a new technology in the *case of mixed growth* as function of the quotient (E_2/E_1) depending on B_2 ($N = 100$, $N_2(0) = 1$, $B_1 = 0$)

probability of the new technology will be lowered for bad technologies and improved for better technologies in relation to the linear case. This means that the competition will be tougher in the existence of quadratic growth laws.

IV. Discussion

Evolution of technologies is a complex dynamic process which is connected with innovations, competition, and selection. In this paper, some basic elements of a stochastic evolutionary theory of technological change were presented. It is proposed that this theory provides the framework for a deeper analysis of the processes of technological change and dynamic competition. Competition is introduced by imposing constraints, which limit the number of firms sharing a market. Therefore, technologies (as fields) "are competing" for firms resp. plants (as elements). In other words, firms using different technologies compete for higher capital gross return.

The model reflects the fact that any new achievement in technology is due to research and development in the same way the origin of any progress in biology is due to mutations. A new element in the evolution of technologies in comparison to the molecular evolution seems to be imitation of successful technologies. The importance of this process has been underlined by Nelson and Winter.

We consider as an interesting result that substitution, realised by imitation, leads to an increasing attractiveness of technologies in the process of technological evolution. In the case of a non-zero imitation rate the social average of attractiveness plays the role of a threshold which marks a border between successful and unsuccessful technologies. The stochastic evolutionary process selects in principle from the infinite reservoir of potential technologies only those which have an attractiveness above this average. In this way, the (average) attractiveness of developed technologies increases monotonously. This process will never stop as long as research and development continue to introduce new technologies.

In traditional substitution models, selection leads with certainty to the survival of the fittest. In a more realistic stochastic picture, this statement is only true with a certain probability, which also implies conversely that the "weaker" variants have a limited chance of survival.

In particular, a short-term analysis demonstrates reasonable deviation from the deterministic trend. While in the short-term weaker new technologies (with $E_2 < E_1$) may survive and consequently remain present in the system, in the long-term the quality factors of the technologies become efficient.

It is a well-known empirical fact that it needs some time to establish a good technology on the market.

A fundamental role for the evolution of technologies is played by the behaviour of new participants in the game. This leads to the basic importance of random effects for technological evolutionary processes. Deterministically, a sharp distinction is drawn between advantage and disadvantage and the decision concerning the fate of new technologies is definite.

We consider an important result the fact that limited competition areas act as niches for the survival of variants which are present in the system in small numbers. In such niches the global selection rules are neutralised to some extent. This fact leads in the case of linear growth to a temporary coexistence of "good" and "bad" new technologies. In the non-linear growth case the niche is the only possibility for a "good" technology to overcome the once-for-ever selection predicted by the deterministic theory. Better new technologies can win the competition if the competition area is sufficiently small, say $N \ll 100$.

The competition area in large systems including economic and social ones may be kept in a local domain simply because of the Markovian character of the process.

In the niche, the new but not yet established quality is protected against extinction for a limited time scale. After winning the competition in a small group, the new technology may infect the whole system. In this way, the stochastic effects open up new channels for the evolutionary process and may deeply influence the perspectives of economic systems.

Finally, we express our belief that the deep and not only formal analogy between biological and technological processes which has been discussed already by many authors, may be very fruitful for the development of mathematical models of technological change.

References

Alchian A (1950) Uncertainty, Evolution and Economic Theory. J Political Econ 58: 211–221

Allen PM (1975) Darwinian Evolution and a Predator-Prey Ecology. Bull Math Biol 37: 389–405

Allen PM (1976) Evolution, Population Dynamics, and Stability. Proc National Academy of Sciences of the USA 73: 665–668

Allen PM, Ebeling W (1983) Evolution and the Stochastic Description of Simple Ecosystems. BioSystems 16: 113–126

Allen PM (ed) (1986) Management and Modelling of Dynamic Systems. Eur J Oper Research 25: 1

Allen PM, Clark N, Perez-Trejo F (1992) Strategic Planning of Complex Economic Systems. Rev Political Econ 4: 275

Arthur BW (1989) Competing Technologies, Increasing Returns, and Look-In by Historical Events. Economic J 99: 116

Ayres RU (1991) Evolutionary Economics and Environmental Imperatives. Structural Change Econ Dynamics 2: 255

Bartholomay AF (1958) On the Linear Birth and Death Processes of Biology as Markoff Chains. Bull Math Biophysics 20: 97–118

Bartholomay AF (1958) Stochastic Models for Chemical Reactions. I. Theory of the Unimolecular Process. Bull Math Biophysics 20: 175–190

Bartholomay AF (1959) Stochastic Models for Chemical Reactions. II. The Unimolecular Rate Constant. Bull Math Biophysics 21: 363–373

Batten D (1982) On the Dynamics of Industrial Evolution. Regional Sci Urban Econ 12: 449–462

Bhargava SC (1989) Generalized Lotka–Volterra Equations and the Mechanism of Technological Substitution. Technol Forecasting Social Change 35: 319–326

Bruckner E, Ebeling W, Scharnhorst A (1989) Stochastic Dynamics of Instabilities in Evolutionary Systems. System Dynamics Rev 5: 176–191

Bruckner E, Ebeling W, Scharnhorst A (1990) The Application of Evolutionary Models in Scientometrics. Scientometrics 18: 21–41

Dosi G, Freeman C, Nelson R, Silverberg G (eds) (1988) Technological Change and Economical Theory. Francis Pinter, London

Dosi G, Kaniovski Yu (1993) The Method of Generalized Urn Scheme in the Analysis of Technological and Economic Dynamics. IIASA Working Paper WP-93-17

Easingwood C, Mahajan V, Muller E (1981) A Nonsymmetric Responding Logistic Model for Forecasting Technological Substitution. Technol Forecasting Social Change 20: 199–213

Ebeling W, Feistel R (1982) Physik der Selbstorganisation und Evolution. Akademie-Verlag, Berlin

Ebeling W, Sonntag I, Schimansky-Geier L (1981) On the Evolution of Biological Macromolecules. II. Catalytic networks: Studia Biophysica 84: 87

Eigen M (1971) The Self-organization of Matter and the Evolution of Biological Macromolecules. Naturwissenschaften 58: 465

Eigen M, Schuster P (1977, 1978) The Hypercycle. Naturwissenschaften 64: 541–565, 65: 341

Feistel R, Ebeling W (1976) Dynamische Modelle zum Selektionsverhalten offener Systeme. Wisse Z Wilhelm-Pieck-Universität Rostock 25: 507–513

Fisher JC, Pry RH (1971) A Simple Substitution Model of Technological Change. Technol Forecasting Social Change 3: 75–88

Haken H (1983) Advanced Synergetics. Springer, Berlin Heidelberg New York

Hofbauer J, Sigmund K (1984) Evolutionstheorie und dynamische Systeme. Paul Parey, Hamburg

Jiménez Montaño MA, Ebeling W (1980) A Stochastic Evolutionary Model of Technological Change. Collective Phenomena 3: 107–114

Karmeshu, Bhargava SC, Jain VP (1985) A Rationale For Law of Technological Substitution. Regional Sci Urban Econ 15: 137–141

Kwasnicka H, Galar R, Kwasnicki W (1983) Technological Substitution Forecasting with a Model Based on Biological Analogy. Technol Forecasting Social Change 23: 41–53

Mahajan V, Peterson RA (1979) First-Purchase Diffusion Models of New-Product Acceptance. Technol Forecasting Social Change 15: 127–146

Mosekilde E, Aracil J, Allen PM (1988) Instabilities and Chaos in Non-linear Dynamical Systems. System Dynamics Rev 4: 14–55

Nelson R, Winter SG, Schuette H (1976) Technical Change in an Evolutionary Model. Q J Econ 90: 90

Nelson R, Winter SG (1982) An Evolutionary Theory of Economic Change. The Belknap Press of Harvard University Press, Cambridge, Mass, London

Nicolis G, Prigogine I (1987) Die Erforschung des Komplexen: auf dem Weg zu einem neuen Verständnis der Naturwissenschaft. Piper, München Zürich

Prigogine I, Nicolis G, Babloyantz A (1972) Thermodynamics and Evolution. Physics Today 25: 38

Saviotti PP, Mani GS (1993) A Model of Technological Evolution Based on Replicator Dynamics. International Conference on New Developments in Technology Studies: Evolutionary Economics and Chaos Theory. Amsterdam

Saviotti PP, Metcalfe JS (eds) (1991) Evolutionary Theories of Economic and Technological Change. Harwood Academic, Chur

Schimansky-Geier L (1980) Stochastische Theorie der Nichtgleichgewichtsphasenübergänge in einkomponentigen bistabilen chemischen Reaktionssystemen. Dissertation, Humboldt-Universität, Berlin

Schumpeter JA (1912) Theorie der wirtschaftlichen Entwicklung. Duncker-Humblot, Leipzig

Sharif MN, Kabir CA (1976) Generalized Model for Forecasting Technological Substitution. Technol Forecasting Social Change 8: 353–364

Silverberg G (1984) Embodied Technical Progress in a Dynamic Economic Model: The Self-Organization Paradigm. In: Goodwin RM et al. (Eds.) Non-linear Models of Fluctuating Growth. Springer, Berlin Heidelberg New York

Silverberg G (1992) On the Complex Dynamics of Technical Change and Economic Evolution. J Scientific Industrial Res 51: 151–156

Tingyan X (1990) A Combined Growth Model for Trend Forecasting. Technol Forecasting Social Change 38: 175–186

Troitzsch KG (1993) Evolution of Production Processes. In: Haag G, Troitzsch KG (eds) Economic Evolution and Demographic Change. Springer, Berlin Heidelberg New York, pp 96–112

Weidlich W, Haag G (1983) Concepts and Models in Quantitative Sociology. Springer, Berlin Heidelberg New York

Weidlich W, Braun M (1992) The Master Equation Approach to Nonlinear Economics. J Evol Econ 2: 233–265