Structural Optimization 11, 244–251 © Springer-Verlag 1996

Evolutionary natural frequency optimization of thin plate bending vibration problems

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Abstract This paper extends the evolutionary structural optimization method to the solution for maximizing the natural frequencies of bending vibration thin plates. Two kinds of constraint conditions are considered in the evolutionary structural optimization method. If the weight of a target structure is set as a constraint condition during the natural frequency optimization, the optimal structural topology can be found by removing the most ineffectively used material gradually from the initial design domain of a structure until the weight requirement is met for the target structure. However, if the specific value of a particular natural frequency is set as a constraint condition for a target structure, the optimal structural topology can be found by using a design chart. This design chart describes the evolutionary process of the structure and can be generated by the information associated with removing the most inefficiently used material gradually from the initial design domain of a structure until the minimum weight is met for maintaining the integrity of a structure. The main advantage in using the evolutionary structural optimization method lies in the fact that it is simple in concept and easy to be included into existing finite element codes. Through applying the extended evolutionary structural optimization method to the solution for the natural frequency optimization of a thin plate bending vibration problem, it has been demonstrated that the extended evolutionary structural optimization method is very useful in dealing with structural topology optimization problems.

1 Introduction

Thin plate bending vibration problems exist in structures used in many engineering fields, such as civil engineering, aeronautical engineering, automotive engineering and so forth. The natural frequencies of thin plate structures usually represent the dynamic characteristics of the structures and therefore play an important role in the topology optimization design of a structure because the dynamic response of the structure is controlled by its dynamic characteristics. On the other hand, the minimum response of a structure generally leads to the minimum weight for a structural design. Thus, the natural frequency optimization of a thin plate structure can result in not only improving the dynamic characteristics of the structure, but also saving the material of the structure.

In terms of the numerical shape or sizing optimization of thin plate structures under either static or dynamic loading conditions, a great deal of research has been done during the last four decades (e.g. Kozlowski and Mróz 1969; Olhoff 1970, 1974; Banichuk 1982; Haftka and Prasad 1981; Armand and Lodier 1978; Cheng and Olhoff 1981, 1982; Rozvany *et al.* 1982, 1987; Ong *et al.* 1988). However, in terms of numerical topology optimization of thin plate structures, only a limited amount of work has been successfully carried out in recent years (e.g. Soto and Díaz 1993; Díaz *et al.* 1995). Therefore, there is a definite need for developing some numerical solutions to the topology optimization of vibrating thin plates with desired design constraints. Although the homogenization method (e.g. Bendsøe and Kikuchi 1988; Bendsøe 1989; Olhoff *et al.* 1991; Suzuki and Kikuchi 1991; Tenek and Hagiwara 1993; Díaz and Kikuchi 1992; Ma *et al.* 1993) and the discretized continuum type optimality criteria (DCOC) method (Zhou and Rozvany 1992, 1993) can be employed to deal with structural topology optimization problems, the evolutionary structural optimization method (Xie and Steven 1993, 1994a, 1994b) is used in this study to solve natural frequency optimization problems for vibrating thin plate structures because the evolutionary structural optimization method is simple in concept and very easy to be included into existing finite element codes.

In the evolutionary structural optimization method, structural natural frequency optimization problems can be classified in the following two categories. (1) If the purpose of structural optimization is to maximize a particular natural frequency of a target structure and the weight of the target structure is set as a constraint condition during the natural frequency optimization, the optimal topology of the structure can be found out by removing the most inefficiently used material gradually from the initial design domain until the weight requirement is met for the target structure. (2) If the purpose of the structural optimization is to minimize the weight of a target structure and the specific value of a particular natural frequency is set as a constraint condition for the target structure, the optimal topology of the structure can be found out by using a design chart, which describes the evolutionary process of the structure and can be plotted using the information associated with removing the most inefficiently used material gradually from the initial design domain until the minimum weight is met for maintaining the integrity of the structure. Two examples, namely the fundamental and the second natural frequency optimization of a thin plate, are given in this paper to show how the evolutionary structural optimization method is used to solve natural frequency optimization of thin plate bending vibration problems. The related results have demonstrated the feasibility of the evolutionary structural optimization method in dealing with structural topology optimization problems.

2 Basic criteria for evolutionary natural frequency optimization of a thin plate structure

Like other structural optimization methods, the main purpose of the evolutionary structural optimization method is to find out an optimal structural topology by using the structural material most smartly and efficiently under a condition of satisfying some basic requirements for a structural design. More specifically, the main purpose of the evolutionary natural frequency optimization of a structure is to find an optimal structural topology, which corresponds to the maximum value of a particular natural frequency of the structure and satisfies either a constrained weight requirement or a minimum weight requirement for the structure, depending on the problem category mentioned in the last section. Based on this consideration, the evolutionary structural optimization method has successfully been extended to the solution for the natural frequency optimization of two-dimensional structures with nonstructural lumped masses (Zhao *et al.* 1995). In the extended evolutionary structural optimization method, the design domain of a problem is fully filled with the expected structural material so that the total weight of the design domain is greater than the constrained weight of a target structure. By removing the most inefficiently used material gradually, the weight of the structure, which is initially equal to the total weight of the design domain, approaches either the constrained weight or the minimum weight of the target structure after several evolutionary cycles during the structural optimization. Although the extended evolutionary structural optimization method was previously developed for dealing with two-dimensional structural natural frequency optimization under plane stress conditions, it can be used to solve the natural frequency optimization problem for a thin plate structure. For this reason, the basic criteria for the evolutionary natural frequency optimization of a thin plate structure are summarized below.

2.1 The material efficiency criterion

For the natural frequency optimization problem, the efficiency of the material used in the design domain can be indicated by the contribution factor of an element to the concerned natural frequency of a system. Based on the energy conservation principle, the contribution factor of a thin plate element to the concerned natural frequency can be derived and expressed as follows:

$$\delta\omega_i = \sqrt{\omega_i^2 + \alpha} - \omega_i \,, \tag{1}$$

where

$$\alpha = \frac{\omega_i^2 \{\mathbf{d}_i^e\}^T [\mathbf{m}^e] \{\mathbf{d}_i^e\} - \{\mathbf{d}_i^e\}^T [\mathbf{k}^e] \{\mathbf{d}_i^e\} + g_1 - g_2}{M_i - g_3} \,, \tag{2}$$

where

$$g_1 = \omega_i^2 \{\delta\,\mathbf{d}_i\}^T ([\mathbf{M}]_0 - [\mathbf{M}^e]) \{\delta\,\mathbf{d}_i\} \,,$$
$$g_2 = \{\delta\,\mathbf{d}_i\}^T ([\mathbf{K}]_0 - [\mathbf{K}^e]) \{\delta\,\mathbf{d}_i\} \,,$$
$$g_3 = \{\delta\,\mathbf{d}_i\}^T [\mathbf{M}^e] \{\delta\,\mathbf{d}_i\} + g_1/\omega_i^2 \,, \tag{3}$$

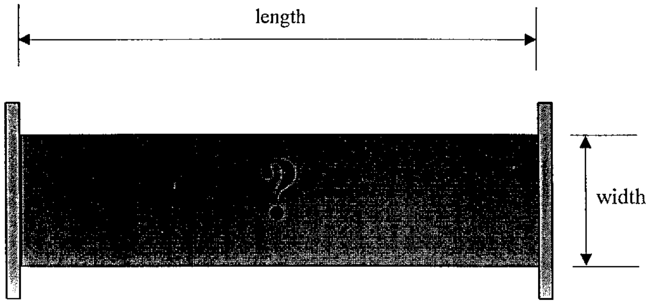


Fig. 1. Design domain for a thin plate with unknown topology

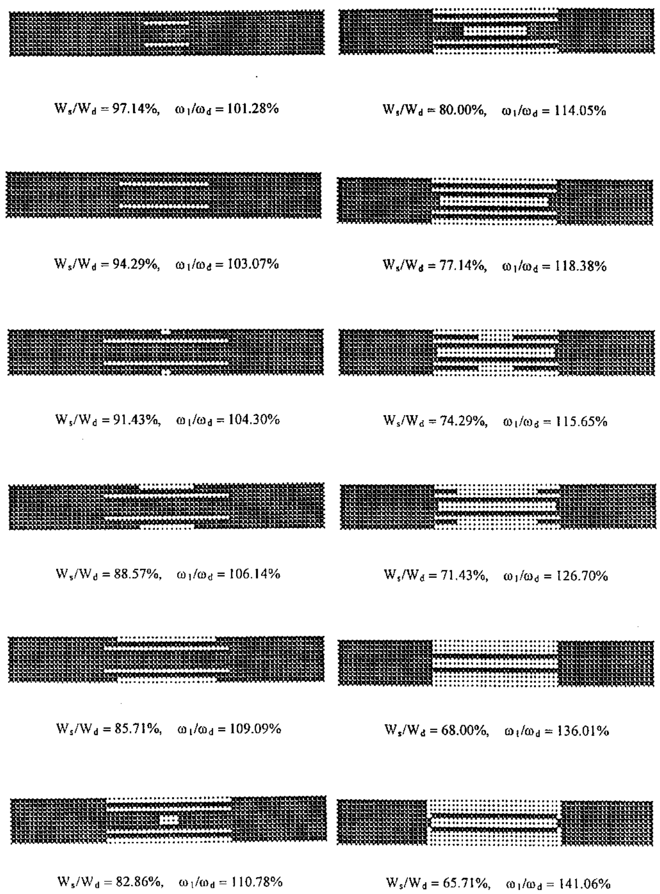


Fig. 2. First natural frequency optimization of the thin plate

where ω_i is the i-th natural frequency of the old system, $\delta\omega_i$ is the exact contribution factor of a thin plate element to the i-th natural frequency of the old system, $\{\mathbf{d}_i^e\}$ is the modal shape vector of the thin plate element in the old system, $\{\mathbf{d}_i\}$ is the nodal shape vector of the old system, $\{\delta\,\mathbf{d}_i\}$ is the i-th modal shape vector difference between the old and the new systems, $[\mathbf{m}^e]$ and $[\mathbf{k}^e]$ are the mass matrix and stiffness matrix of the thin plate element; $[\mathbf{M}^e]$ and $[\mathbf{K}^e]$ are the enlarged mass matrix and stiffness matrix of the thin plate element; $[\mathbf{M}]_0$ and $[\mathbf{K}]_0$ are the global mass matrix and global stiffness matrix of the old system; M_i is the modal mass corresponding to the i-th natural frequency of the system.

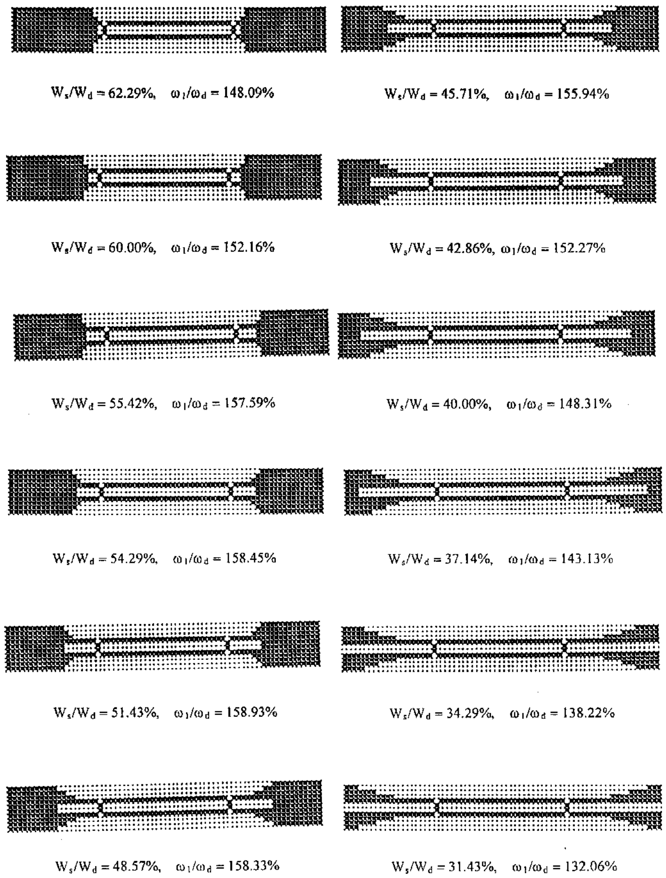


Fig. 3. First natural frequency optimization of the thin plate

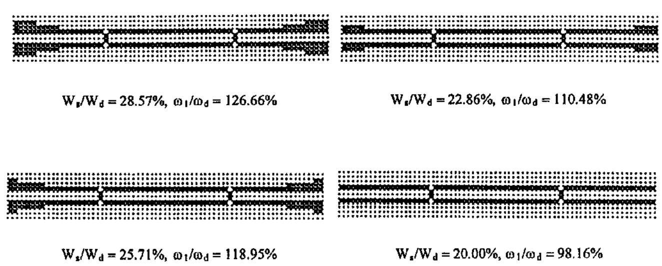
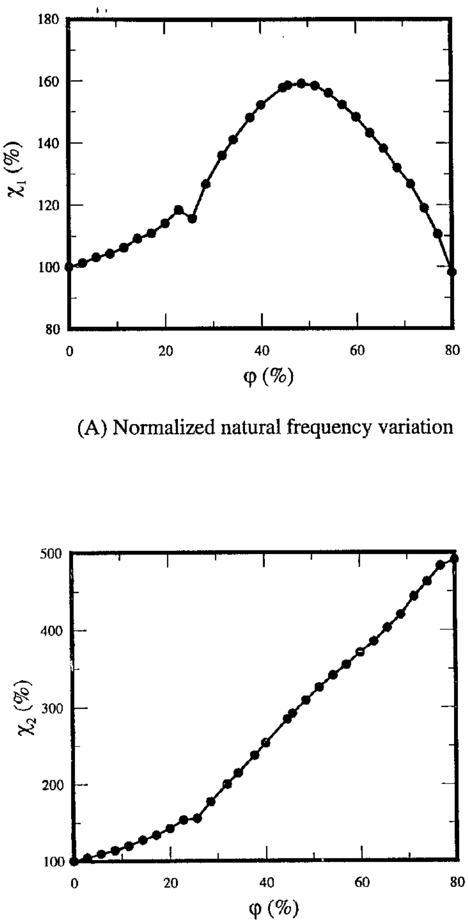


Fig. 4. First natural frequency optimization of the thin plate

The old system stands for a system before removing a thin plate element in the beginning of an iteration, while the new system stands for a system after removing the thin plate element at the end of the iteration. If the first-order variations are only considered, (2) can be further simplified as

$$\delta\omega_i^* = \frac{\omega_i^2\{\mathbf{d}_i^e\}^T[\mathbf{m}^e]\{\mathbf{d}_i^e\} - \{\mathbf{d}_i^e\}^T[\mathbf{k}^e]\{\mathbf{d}_i^e\}}{2\omega_i M_i}, \tag{4}$$

where $\delta\omega_i^*$ is the approximate contribution factor of a thin plate element to the i-th natural frequency of the old system.



(A) Normalized natural frequency variation

(B) Normalized material efficiency variation

Fig. 5. Normalized natural frequency and material efficiency variation during the first natural frequency optimization

2.2 *The positive definite structure criterion*

In the extended evolutionary structural optimization method, there is a need for establishing a structural connectivity criterion to maintain the integrity of the structure. From the structural point of view, violating the integrity of a structure may lead to a nonpositive definite structure. For this reason, the structural connectivity criterion can be called as the positive definite structure criterion. This criterion is expressed as

$$\omega_1 > 0, \tag{5}$$

where ω_1 is the fundamental natural frequency of the new system.

2.3 *The smooth change criterion*

In order to obtain an accurate solution for the evolutionary natural frequency optimization of a thin plate structure, a smooth change criterion needs to be included in the extended evolutionary structural optimization method. This smooth change criterion can be expressed as

$$\frac{|\omega_{\text{new}} - \omega_{\text{old}}|}{\omega_{\text{old}}} < \beta_1, \tag{6}$$

where ω_{old} and ω_{new} are the natural frequency of the old and the new systems, respectively, and β_1 is the maximum vari-

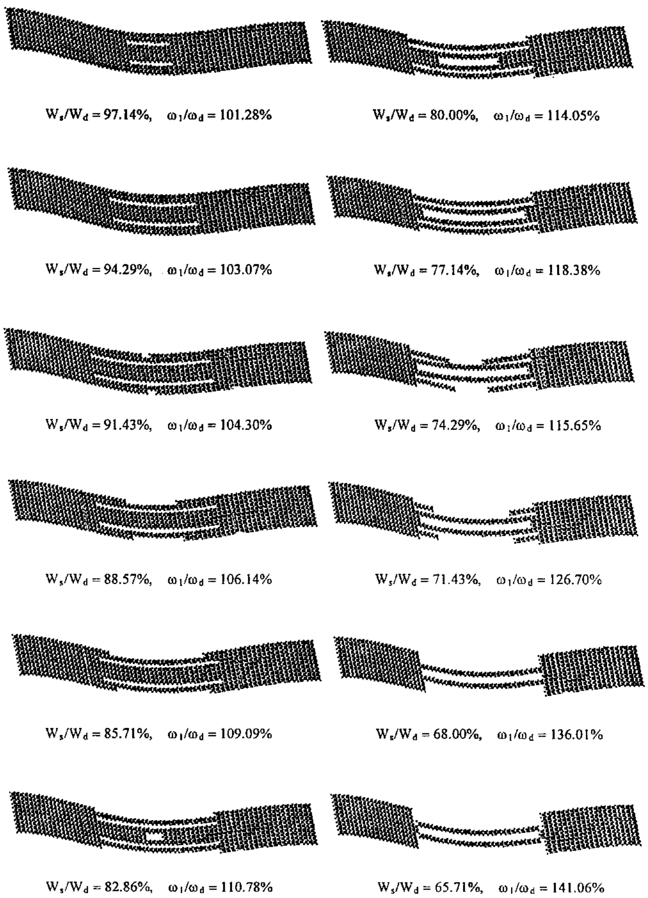


Fig. 6. First mode shapes of the thin plate during optimization

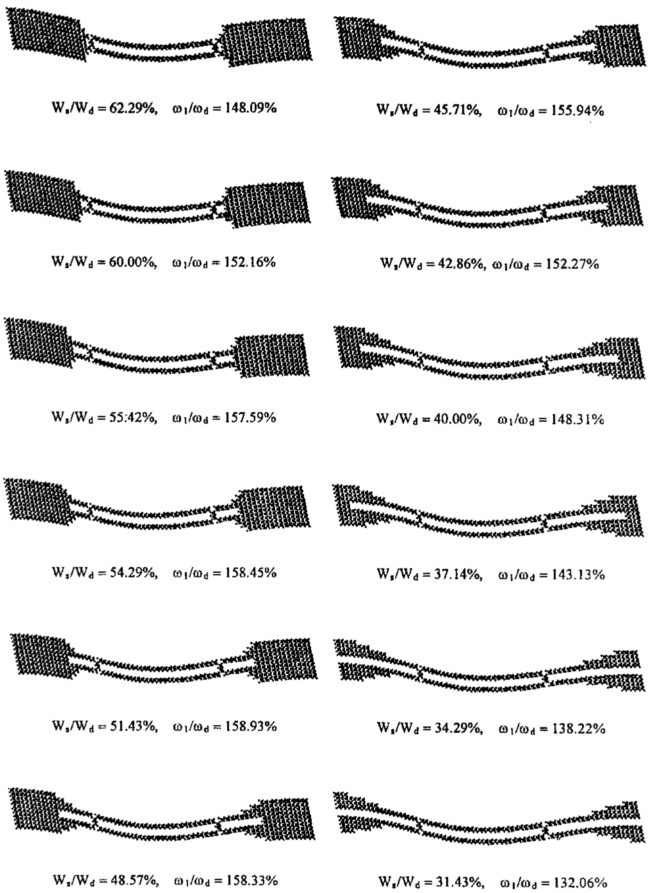


Fig. 7. First mode shapes of the thin plate during optimization

ation tolerance of the concerned natural frequency between the old and the new systems.

2.4 The average value criterion

In order to cope with a repeated natural frequency problem and/or a natural frequency order exchange problem which might be encountered in the process of the structural natural frequency optimization, the average value criterion needs to be introduced. For this purpose, the maximum number of the repeated natural frequency is determined by comparing the i-th natural frequency with its consecutive ones as follows:

$$\frac{\omega_{i+j} - \omega_i}{\omega_i} < \beta_2 \quad (j = 1, \ldots, n)\,, \tag{7}$$

where n is the maximum number of the repeated natural frequency, and β_2 represents the maximum allowable difference between the repeated natural frequencies.

If (7) does not hold true in a particular iteration, it indicates that there is not repeated natural frequency in this iteration so n should be set zero. If n is greater than zero in any iteration, then the average value criterion can be written as follows:

$$\delta\overline{\omega}_i^* = \frac{\overline{\omega}_i^2\{\overline{\mathbf{d}}_i^e\}^T[\mathbf{m}^e]\{\overline{\mathbf{d}}_i^e\} - \{\overline{\mathbf{d}}_i^e\}^T[\mathbf{k}^e]\{\overline{\mathbf{d}}_i^e\}}{2\overline{\omega}_i\overline{M}_i}\,, \tag{8}$$

where $\delta\overline{\omega}_i^*$ is the average contribution factor of a thin plate element to the i-th natural frequency due to all the repeated natural frequencies of the system, and $\overline{\omega}_i$, $\overline{M}_i$ and $\{\overline{\mathbf{d}}_i^e\}$ are

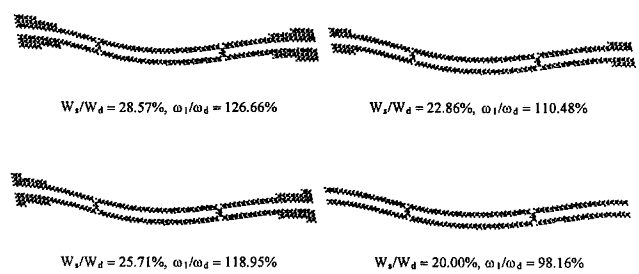


Fig. 8. First mode shapes of the thin plate during optimization

the average values of the related quantities due to all the repeated natural frequencies of the system. They are expressed as

$$\overline{\omega}_i = \frac{1}{n}\sum_{j=1}^{n}\omega_{i+j}\,, \quad \overline{M}_i = \frac{1}{n}\sum_{j=1}^{n}M_{i+j}\,,$$

$$\{\overline{\mathbf{d}}_i^e\} = \frac{1}{n}\sum_{j=1}^{n}\{\mathbf{d}_{i+j}^e\}\,. \tag{9}$$

2.5 The minimum number of elements removed criterion

If the design domain and the target structure are symmetric, there is a requirement for removing the minimum number of elements in a single iteration so as to maintain the symmetric nature of the system in the process of the evolutionary natural frequency optimization. This minimum number is

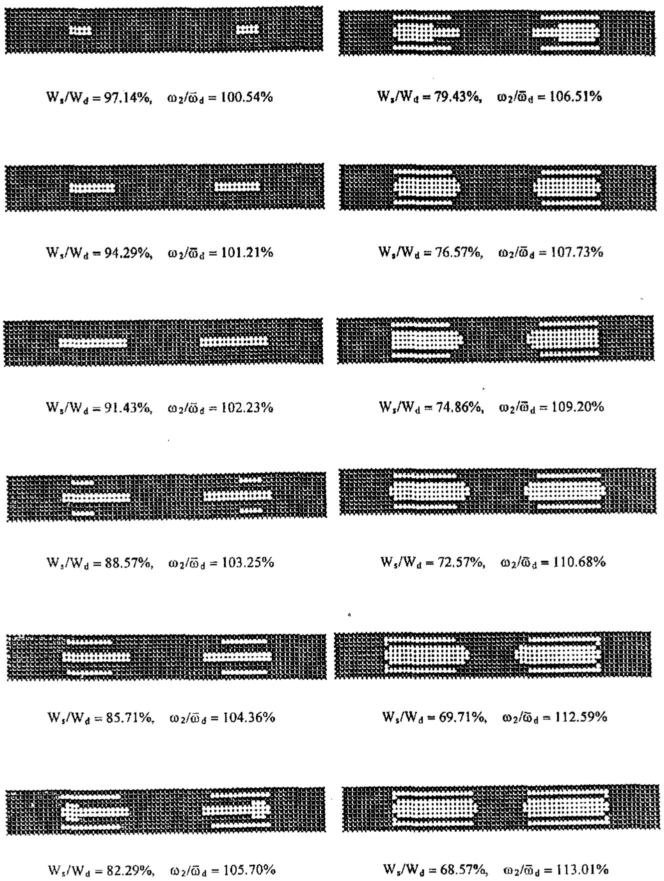


Fig. 9. Second natural frequency optimization of the thin plate

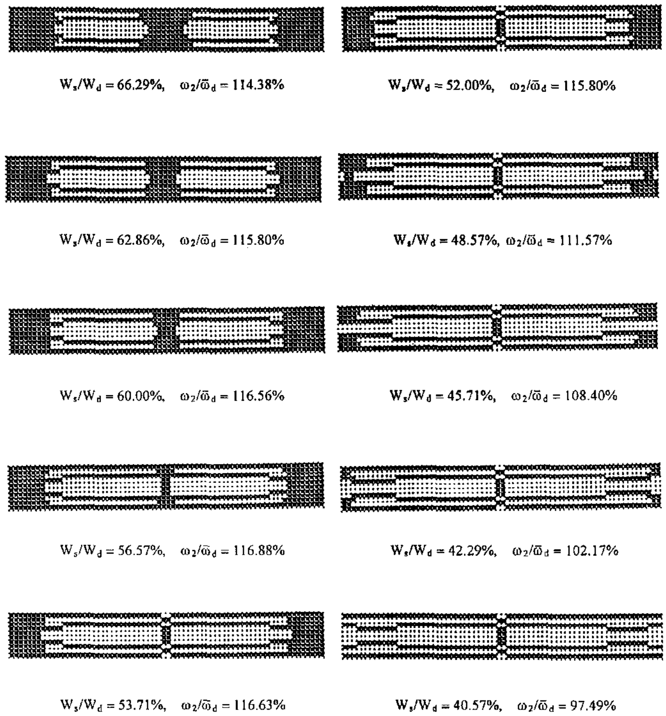


Fig. 10. Second natural frequency optimization of the thin plate

determined by considering the number of the symmetric axes in the system.

3 Evolutionary natural frequency optimization of a thin plate structure

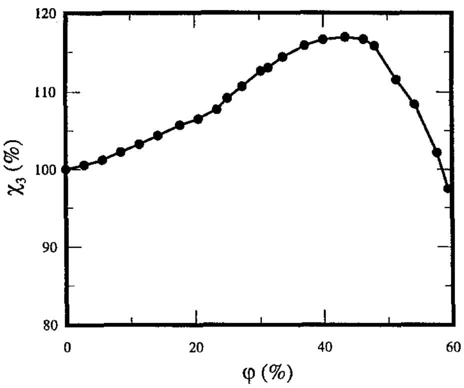
As shown in Fig. 1, the initial design domain of a thin plate, which is a rectangular with both the left and right sides clamped, is discretized into 700 4-node thin plate elements. The following parameters are used in the calculation: the length, width and thickness of the initial design domain are 14 m, 2 m and 0.1 m, the elastic modulus of the design domain materials is 25×10^6 kPa, Poisson's ratio is 0.3, the unit weight is 2.5×10^3 kg/m^3, and the weight of the initial design domain is 7000 kg. Based on these parameters, the fundamental natural frequency of the initial design domain before optimization is 10.60 rad/s. Moreover, in order to use the extended evolutionary structural optimization method for the natural frequency optimization of the structure, the following parameters are also used in the calculation: β_1 is equal to 0.2 and β_2 is equal to 0.03. Since there are two symmetric axes in the system, the minimum number of elements to be removed in an iteration is equal to four so as to maintain the symmetric nature of the target structure.

3.1 The fundamental natural frequency optimization

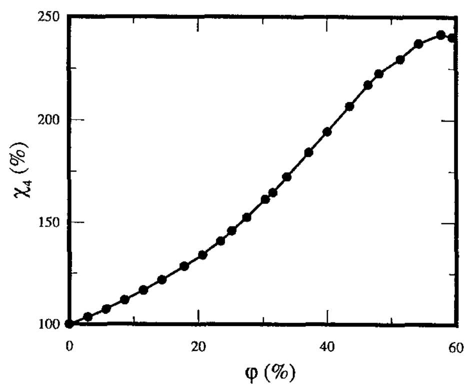
The first example of using the evolutionary structural optimization method in dealing with the natural frequency optimization of the thin plate structure is to maximize its fundamental natural frequency for any specified weight of the target structure. Generally, the specified weight of a structure may vary from the maximum possible structural weight, which is known as the weight of the initial design domain in this study, to the minimum possible structural weight required for maintaining the integrity of the structure.

Figures 2 to 4 show optimal structural topologies for different specified weights of the target structure. In these figures, W_s is the specified weight of the target structure; W_d is the weight of the initial design domain and is equal to 7000 kg in the calculation; ω_1 is the fundamental natural frequency of the target structure; ω_d is the fundamental natural frequency of the initial design domain. It is observed that the different weight of a target structure results in a different optimal topology for the target structure in the process of the fundamental natural frequency optimization.

As stated in the Introduction, for the first category of natural frequency optimization problem, the structural evolution process can be stopped just when the constrained weight requirement is satisfied for a target structure. For example, if the ratio of the constrained weight of the target structure to the weight of the initial design domain is equal to 60%, the structural evolution process can be stopped when 40% material of the initial design domain is removed in the process of the fundamental natural frequency optimization. This consideration will result in saving CPU time. For this particular example, the second structural topology from the top of the left column in Fig. 3 is the optimal structural topology, which satisfies the constrained weight requirement ($W_s/W_d = 60\%$) and leads to the maximum fundamental natural frequency of 16.13 rad/s for the target structure. This fact implies that the fundamental natural frequency of the structure can increase by 52% once 40% material is removed from the initial



(A) Normalized natural frequency variation



(B) Normalized material efficiency variation

Fig. 11. Normalized natural frequency and material efficiency variation during the second natural frequency optimization

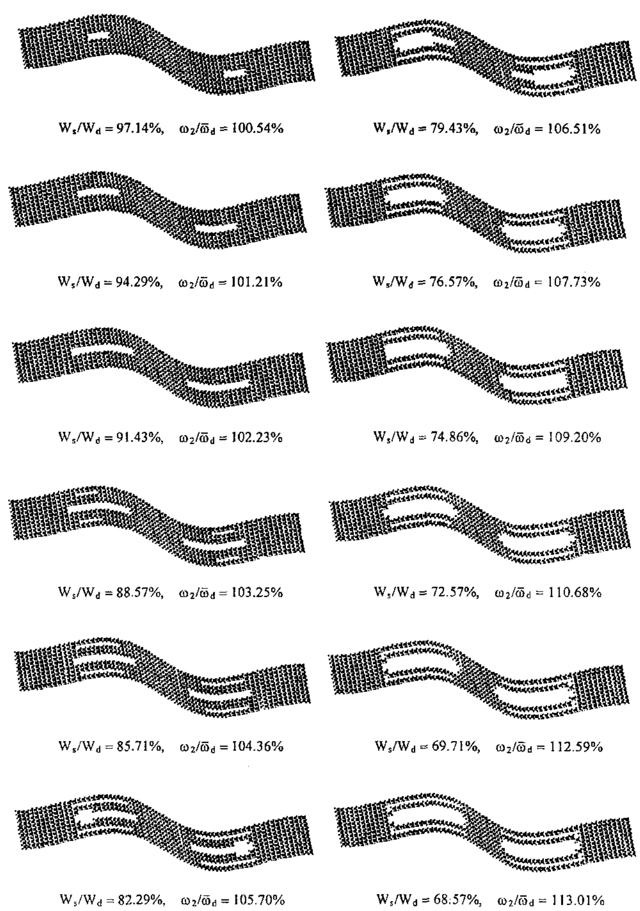


Fig. 12. Second mode shapes of the thin plate during optimization

design domain.

For the second category of natural frequency optimization problem, the structural evolution process needs to be carried out until the minimum weight is met for maintaining the integrity of a structure. This will provide enough information for plotting a design chart to express the relationship between the fundamental natural frequency and the constrained weight of the structure. Figure 5a shows such a design chart for choosing the optimal topology of the structure. In this figure, χ_1 stands for the ratio of the fundamental natural frequency of the target structure to that of the initial design domain, while φ stands for the weight of material removed when compared with the weight of the initial design domain. It is observed that for an expected value of the fundamental natural frequency, more than one optimal structural topology might be found in the process of the fundamental natural frequency optimization. Thus, the best optimal structural topology needs to be chosen using this design chart. For instance, if the desired value of χ_1 is set as 148%, there are two optimal structural topologies that satisfy the constrained value of the fundamental natural frequency for the target structure. The first one corresponds approximately to $W_s/W_d = 62.29\%$, while the second one corresponds approximately to $W_s/W_d = 40\%$. Thus, the best optimal structural topology in this case is the second one because it not only satisfies the constrained value of the fundamental natural frequency, but also leads to the minimum weight for the target structure.

In order to assess the general performance of a target structure, the normalized material efficiency indicator of a structure is defined as

$$\chi_2 = (\omega_1/W_s)/(\omega_d/W_d) = \frac{\omega_1 W_d}{\omega_d W_s}. \tag{10}$$

Figure 5b shows the normalized material efficiency of a target structure during fundamental natural frequency optimization. In this figure, $\varphi = \frac{W_d - W_s}{W_d}$ is the normalized weight of the material removed from the initial design domain. It is clear that as the material is removed gradually, the material efficiency of the structure increases accordingly.

Figures 6 to 8 show the first mode shapes of the thin plate structure during its fundamental natural frequency optimization. It is clear that although the weight and the fundamental natural frequency of the thin plate structure may vary from one structural generation to another, the type of the first mode shape does not change for all structural topologies in the process of the fundamental natural frequency optimization.

3.2 The second natural frequency optimization

The second example of using the evolutionary structural optimization method in dealing with the natural frequency opti-

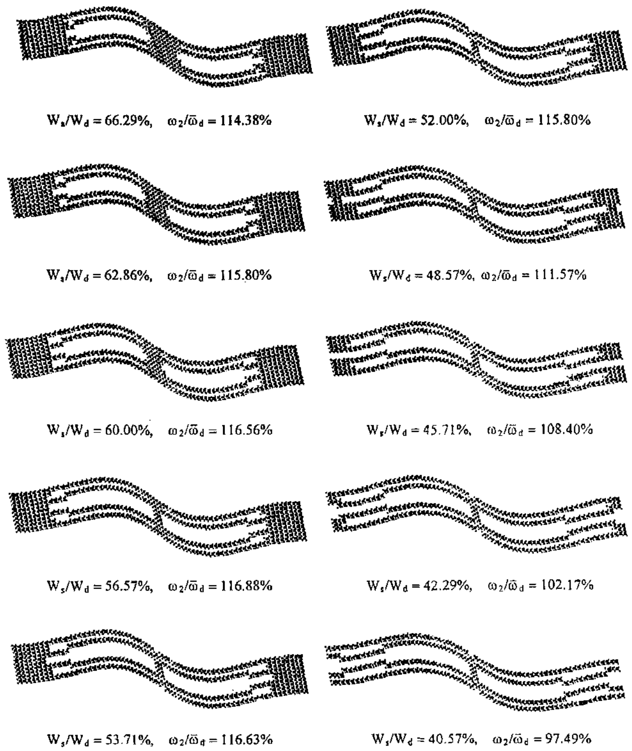


Fig. 13. Second mode shapes of the thin plate during optimization

mization of the thin plate structure is to maximize its second natural frequency. Figures 9 and 10 show optimal structural topologies for different specified weights of the target structure. In these figures, W_s and W_d are the specified weight of the target structure and the weight of the initial design domain, while ω_2 and $\overline{\omega}_d$ are the second natural frequency of the target structure and the initial design domain, respectively. Figure 11 shows the normalized natural frequency and material efficiency variation of the thin plate structure during its second natural frequency optimization. In this figure, φ is of the same meaning as in the first example, while χ_3 and χ_4 are defined as follows:

$$\chi_3 = \frac{\omega_2}{\overline{\omega}_d}, \quad \chi_4 = \frac{\omega_2 W_d}{\overline{\omega}_d W_s}, \tag{11}$$

where χ_3 is the normalized second natural frequency of the thin plate structure, and χ_4 is the normalized material efficiency indicator of the thin plate structure.

Also, Figs. 12 and 13 show the second mode shapes of the thin plate structure during its second natural frequency optimization. Clearly, similar conclusions as mentioned in the fundamental natural frequency optimization can be drawn from the related results shown in these figures.

4 Conclusions

In this paper, the evolutionary structural optimization method has successfully been extended to the solution for the natural frequency optimization of thin plate bending vibration problems. If the purpose of structural optimization is to maximize a particular natural frequency of a target structure and the weight of the target structure is set as a constraint condition during the natural frequency optimization, the optimal topology of the structure can be found by removing the most inefficiently used material gradually from the initial design domain until the weight requirement is met for the target structure. If the purpose of structural optimization is to minimize the weight of a target structure and the specific value of a particular natural frequency is set as a constraint condition for the target structure, the optimal topology of the structure can be found by using a design chart, which describes the evolutionary process of the structure and can be plotted using the information associated with removing the most inefficiently used material gradually from the initial design domain until the minimum weight is met for maintaining the integrity of the structure. Through applying the extended evolutionary structural optimization method to the solution for the fundamental and second natural frequency optimization of a thin plate bending vibration problem, it has been demonstrated that the extended evolutionary structural optimization method is very useful in dealing with strutcural topology optimization problems.

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Received Nov. 13, 1995