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Stationary States of Quantum Dynamical Semigroups*

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Abstract. We study the class of stationary states and the domain of attraction of each of them, for a dynamical semigroup possessing a faithful normal stationary state. We give applications to the approach to stationarity of an open quantum system, and to models of the quantum measurement process.

1. Introduction

Quantum dynamical semigroups provide a convenient mathematical framework for the study of approach to equilibrium of an open quantum system. In some recent works of several authors, conditions have been found for a dynamical semigroup to possess a unique stationary state and to induce approach to it, in the cases of N-level systems [1, 2] and of dynamical semigroups with a faithful normal stationary state [3, 4]; the related problem of irreducibility has been treated in [5–7].

In this note we extend the results of [3] in several respects. Under the only assumptions that the dynamical semigroup T_t under consideration acts on a von Neumann algebra $\mathcal{M}$ and possesses (at least) a faithful normal stationary state, which imply, by [3], that the fixed point set of T_t in $\mathcal{M}$ is a von Neumann subalgebra of $\mathcal{M}$, we give in Section 2 a classification of the normal stationary states of T_t. Under an additional condition of sufficient dissipativity, which is less restrictive than the ones of [3], we prove in Section 3 that any normal state tends to a limit as $t \to \infty$, under the action of T_t, and we characterize the domain of attraction of each normal stationary state. The results apply to the discussion of approach to stationarity for a spatially confined quantum system weakly coupled to several heat reservoirs at different nonzero temperatures [8] and to some models of the quantum measurement process (cf. [9]). For a restricted class of dynamical semigroups, we prove in Section 4 a stronger property of approach to equilibrium, which should be of interest in the study of infinitely extended quantum systems,

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in order to discriminate real dissipation of local disturbances from their migration to infinity (cf. [10]).

2. Classification of Stationary States

We recall some definitions and preliminaries.

A dynamical semigroup [11–13, 6] of a von Neumann algebra $\mathcal{M}$ is a weakly* continuous one-parameter semigroup $\{T_t : t \geqq 0\}$ of completely positive identity preserving normal maps of $\mathcal{M}$ into itself, with T_0 the identity map I. Assume that T_t possesses a faithful normal stationary state ω. Then $\mathcal{M}$ can be identified with its representation $\pi_\omega(\mathcal{M})$ on the Hilbert space $\mathcal{H} = \mathcal{H}_\omega$, with cyclic and separating vector Ω. There exists [11, 14] a strongly continuous contraction semigroup $\hat{T}_t$ on $\mathcal{H}$ such that $\hat{T}_t(A\Omega) = T_t(A)\Omega$ for all A in $\mathcal{M}$, $t \geqq 0$. The set $\mathcal{V}$ of vector functionals, defined by

$$\mathcal{V} = \{\psi \in \mathcal{M}_* ; \psi(A) = (\Psi, A\Omega), \Psi \in \mathcal{H}, \forall A \in \mathcal{M}\}$$

is uniformly dense in $\mathcal{M}_*$ by [15] Example 5 (see also [10], Theorem 4.2). The fixed point set of T_t in $\mathcal{M}$ is a von Neumann subalgebra of $\mathcal{M}$, which we denote by $\mathcal{M}(T)$ ([3], Lemma 3; using [7], Theorem 3.1).

Theorem 2.1. *Let T_t be a dynamical semigroup of a von Neumann algebra $\mathcal{M}$, with a faithful normal stationary state ω. Then there exists a unique T_t-invariant normal conditional expectation E of $\mathcal{M}$ onto the fixed point subalgebra $\mathcal{M}(T)$ given by*

$$E(A) = w^* - \lim_{\lambda \to 0} \lambda \int_0^\infty dt\, e^{-\lambda t} T_t(A), \quad A \in \mathcal{M}, \tag{2.1}$$

the integral being a weak Riemann integral.*

Proof (*cf.* [16]). The net $\left\{\lambda \int_0^\infty dt e^{-\lambda t} T_t(A)\right\}_\lambda$ is bounded in norm by $\|A\|$, hence, by the weak* compactness of the unit ball of $\mathcal{M}$, it converges in the weak* topology as $\lambda \to 0$ if and only if it has exactly one weak* limit point. Let A_0 be one of such limit points. By [17], Theorems 18.6.2 and 18.7.3, $s - \lim_{\lambda \to 0} \lambda \int_0^\infty dt\, e^{-\lambda t} \hat{T}_t$ exists and is the projection P onto the subspace of $\hat{T}_t$-invariant vectors in $\mathcal{H}$. Then $PA\Omega = A_0\Omega = PA_0\Omega$. This proves that A_0 is uniquely determined by A and is in $\mathcal{M}(T)$, since Ω is separating for $\mathcal{M}$. Let $A_0 = E(A)$. It is clear that E is linear and completely positive, and that $E(A) = A$ if and only if A is in $\mathcal{M}(T)$. If $A \in \mathcal{M}, B_1, B_2 \in \mathcal{M}(T)$, then $E(B_1 A B_2) = B_1 E(A) B_2$, since $T_t(B_1 A B_2) = B_1 T_t(A) B_2$ for all t, by [7], Theorem 3.1. Hence E is a conditional expectation, which is normal since it has a predual map E_*, whose explicit expression on the dense set $\mathcal{V}$ is

$$E_* \psi(A) = (P\Psi, A\Omega) \text{ if } \psi(A) = (\Psi, A\Omega).$$

Moreover, E is T_t-invariant by construction. If E' is another T_t-invariant normal conditional expectation of $\mathcal{M}$ onto $\mathcal{M}(T)$, then $E' = E E' = E' E = E$.

Corollary 2.2. *A normal functional* φ *on* $\mathcal{M}$ *is* T_t*-invariant if and only if it is of the form*

$$\varphi = (\varphi \upharpoonright \mathcal{M}(T)) \circ E.$$

In particular, $\omega = (\omega \upharpoonright \mathcal{M}(T)) \circ E$, *and* E *is faithful since* ω *is.*

By Corollary 2.2, the study of normal T_t-invariant states on $\mathcal{M}$ is reduced to the study of normal states on $\mathcal{M}(T)$. The fact that $\mathcal{M}(T)$ is the range of a faithful normal conditional expectation E with $\omega \circ E = \omega$ gives restrictions on $\mathcal{M}(T)$, and hence on the set of T_t-invariant states in $\mathcal{M}_*$. Indeed, by [18], $\mathcal{M}(T)$ is globally invariant under the modular automorphism group σ_t associated to ω by Tomita-Takesaki theory [19], and if $\mathcal{M}$ is a type I factor, then $\mathcal{M}(T)$ is of type I and its centre is totally atomic [20].

It is clear that ω is the unique stationary state for T_t in $\mathcal{M}_*$ if and only if $\mathcal{M}(T) = \mathbb{C}\mathbb{1}$ (cf. [3, 4]). In the more general situation, we have the following

Proposition 2.3. *A state* φ *in* $\mathcal{M}_*$ *is* T_t*-invariant and majorized by a scalar multiple of* ω *if and only if it is of the form*

$$\varphi(A) = \omega(B)^{-1}(JB\Omega, A\Omega),\ A \in \mathcal{M}, \tag{2.2}$$

for some positive B *in* $\mathcal{M}(T)$, J *being the antiunitary involution on* $\mathcal{H}$ *such that* $J\Omega = \Omega$, $J\mathcal{M}J = \mathcal{M}'$.

Proof. The general form of such a state φ is

$$\varphi(A) = (\Omega, X\Omega)^{-1}(X\Omega, A\Omega)$$

for some positive X in $\mathcal{M}'$, with $X\Omega$ in $P\mathcal{H}$ (see e.g. [21], Proposition 2.5.I). Then $B = JXJ$ is a positive element of $\mathcal{M}$, and $B\Omega = JX\Omega$ is in $P\mathcal{H}$, since $\mathcal{M}(T)$ is stable under the modular automorphism group and $P\mathcal{H} = \overline{\mathcal{M}(T)\Omega}$. Hence B is in $\mathcal{M}(T)$ since Ω is separating for $\mathcal{M}$, and φ can be written in the form (2.2).

Remark. The state φ of the above Proposition is extremal T_t-invariant if and only if B is a minimal projection in $\mathcal{M}(T)$.

An adaptation of the arguments of [22], Theorem 6.4.1 gives the following

Proposition 2.4. *If* $\mathcal{M}(T)$ *is abelian, there exists a unique maximal measure* μ *on the set of* T_t*-invariant states on* $\mathcal{M}$, *with resultant* ω *and such that*

$$\int \prod_{i=1}^{n} \varphi(A_i)d\mu(\varphi) = \omega\left(\prod_{i=1}^{n} E(A_i)\right)$$

for all self-adjoint A_i *in* $\mathcal{M}$, $i = 1, \ldots, n$, *and for all* n.

3. Weak Approach to Equilibrium

In this Section, we discuss a condition under which any state ψ in $\mathcal{M}_*$, acted upon by $(T_t)_*$, converges weakly as $t \to \infty$ to a limit, which is given by $(\psi \upharpoonright \mathcal{M}(T)) \circ E$.

Definition. The integrated form of the dissipation function $D_t(.,.)$ is defined on $\mathcal{M} \times \mathcal{M}$, with values in $\mathcal{M}$, by

$$D_t(A,B) = T_t(A^*B) - T_t(A^*)T_t(B). \tag{3.1}$$

$D_t(A,A) \geqq 0$ for all A in $\mathcal{M}$, $t \geqq 0$ by the Kadison-Schwarz inequality [22]. Denote by $\mathcal{N}(T)$ the null space of $\{D_t : t \geqq 0\}$, i.e.

$$\mathcal{N}(T) = \{A \in \mathcal{M}\,; D_t(A,A) = 0 \text{ for all } t \geqq 0\}. \tag{3.2}$$

In general, $\mathcal{N}(T)$ contains $\mathcal{M}(T)$.

Theorem 3.1. *Let T_t be a dynamical semigroup of a von Neumann algebra $\mathcal{M}$, with a faithful normal stationary state ω. If $\mathcal{N}(T) = \mathcal{M}(T)$, then*

$$w^* - \lim_{t\to\infty} T_t(A) = E(A) \text{ for all } A \text{ in } \mathcal{M}. \tag{3.3}$$

Proof. For all A, B in $\mathcal{M}$, we have

$$\begin{aligned}\lim_{t\to\infty} \omega(D_t(A,B)) &= \lim_{t\to\infty} (A\Omega, [\mathbb{1} - \hat{T}_t^*\hat{T}_t]B\Omega) \\ &= (A\Omega, [\mathbb{1} - Q^2]B\Omega),\end{aligned}$$

where $Q^2 = s - \lim_{t\to\infty} \hat{T}_t^*\hat{T}_t$ ([23], p. 41). Then the expression

$$\begin{aligned}\omega(D_t(T_s(A), T_s(A))) &= (A\Omega, [\hat{T}_s^*\hat{T}_s - \hat{T}_{t+s}^*\hat{T}_{t+s}]A\Omega) \\ &= \omega(D_{t+s}(A,A)) - \omega(D_s(A,A))\end{aligned}$$

tends to zero as $s \to \infty$, for all t and for all A. Hence, using the Schwarz inequality for the positive sesquilinear form $\omega(D_t(.\,,.))$, we find that $\omega(D_t(T_s(A), B))$ tends to zero as $s \to \infty$, for all $t \geqq 0$ and for all A, B in $\mathcal{M}$. This proves that any weak* limit point A_∞ as $s \to \infty$ of the norm-bounded net $\{T_s(A)\}_s$ is in $\mathcal{N}(T)$.

Since E is a normal T_t-invariant conditional expectation, $T_s(A) = E(A) + (I - E)T_s(A)$ for all s, and $A_\infty = E(A) + (I - E)A_\infty$. If $\mathcal{N}(T) = \mathcal{M}(T)$, $(I - E)A_\infty = 0$, and $A_\infty = E(A)$. This holds for any weak* limit point, hence (3.3) follows from the weak* compactness of the unit ball of $\mathcal{M}$.

Remark. In particular, if $\mathcal{N}(T) = \mathbb{C}\mathbb{1}$, every normal state ψ tends to ω as $t \to \infty$, under the action of $(T_t)_*$. The same result has been derived by Albeverio and Høegh-Krohn [4] under the (less restrictive) assumptions that $\mathcal{M}(T) = \mathbb{C}\mathbb{1}$ and T_t has no (proper) eigenvalue on the unit circle besides 1. Although less powerful from the mathematical point of view, our result is perhaps easier to use in applications to the approach to stationarity of open quantum systems, moreover, we are able to study situations when the stationary state is not unique, such as the quantum measurement process.

For the rest of this Section, we consider the case $\mathcal{M} = \mathcal{B}(\mathcal{H}^0)$, the algebra of all bounded operators on a separable Hilbert space $\mathcal{H}^0$, and $T_t = \exp Lt$, L being a bounded linear map of $\mathcal{B}(\mathcal{H}^0)$ into itself. The general form of L, given in [13] is

$$L(A) = \sum_i V_i^* A V_i + K^*A + AK, \tag{3.4}$$

where K, V_i are in $\mathcal{B}(\mathcal{H}^0)$, $\sum_i V_i^* V_i + K^* + K = 0$, and the series converge ultraweakly. If A is in $\mathcal{N}(T)$, then

$$\dot{D}(A,A) \equiv L(A^*A) - L(A^*)A - A^*L(A) = \frac{d}{dt}D_t(A,A)\big|_{t=0} = 0.$$

Thus $\mathcal{N}(T) \subset \operatorname{Ker} \dot{D}$. When L has the form (3.4), $\operatorname{Ker} \dot{D} = \{V_i\}'$, the commutant being taken in $\mathcal{B}(\mathcal{H}^0)$ [2, 7].

We have the following

Theorem 3.2. *Let $T_t = \exp Lt$ be a dynamical semigroup of $\mathcal{B}(\mathcal{H}^0)$, with L of the form (3.4). Assume that* $\operatorname{lin}\{V_i\}$ *is a self-adjoint set, and $\{V_i\}' = \mathbb{C}\mathbb{1}$. Then, if T_t has a normal stationary state ω, ω is faithful and* $w^* - \lim_{t\to\infty} T_t(A) = \omega(A)\mathbb{1}$ *for all A in $\mathcal{B}(\mathcal{H}^0)$.*

Proof. If $\operatorname{lin}\{V_i\}$ is self-adjoint, $\{V_i\}' = \mathbb{C}\mathbb{1}$ is equivalent to "there is no proper closed subspace of $\mathcal{H}^0$ which is stable under all V_i's" (see e.g. [21], 2.2.4 and 2.3.I). Then T_t is irreducible in the sense of Davies and ω is faithful [5]. Now Theorem 3.1 applies, with $\mathcal{N}(T) = \mathbb{C}\mathbb{1}$.

Remark. Notice that, in contrast to the corresponding theorems for the case of a finite-dimensional $\mathcal{H}^0$ [2, 5], the existence of a normal stationary state must be assumed in advance. Counterexamples when no stationary state exists are discussed in [5, 7].

Theorem 3.1 can be applied to the study of the reduced dynamics of a quantum system weakly coupled to several heat reservoirs at nonzero temperatures, for in that case L is known and $\operatorname{lin}\{V_i\}$ is a self-adjoint set [2, 8].

Assume now that T_t has a faithful stationary state which is not unique. It follows from the discussion of Section 2 that there exists a family $\{P_n\}$ of mutually orthogonal T_t-invariant projections in $\mathcal{B}(\mathcal{H}^0)$, with $\sum_n P_n = \mathbb{1}$, such that the centre $\mathcal{Z}$ of $\mathcal{M}(T)$ is $\{P_n\}''$. We consider two cases:

(i) $\mathcal{M}(T) = \mathcal{Z}$: then $E(A) = \sum_n \omega(P_n)^{-1}\omega(P_nAP_n)P_n$ (cf. [24]);

(ii) $\mathcal{M}(T) = \mathcal{Z}'$: then $E(A) = \sum_n P_nAP_n$ (cf. [6], p. 59).

Indeed, it is easily checked that the maps defined above are T_t-invariant normal conditional expectations onto $\mathcal{Z}$ (resp. $\mathcal{Z}'$), by using the fact that $P_nT_t(A)P_n = T_t(P_nAP_n)$ by [7] Theorem 3.1. The two maps are possible choices for the "reduction of the wave-packet" occurring when an observable $B = \sum_n b_nP_n$ is measured. When all P_n's are one-dimensional, the cases (i) and (ii) coincide, otherwise, one usually chooses (ii) on the basis of a hypothesis of minimum disturbance. However, the analysis of Accardi [25] should lead to prefer (i). The condition $\mathcal{N}(T) = \mathcal{M}(T)$ is satisfied when

$\{V_i\}' = \{V_i, V_i^*, K, K^*\}'$ (see [7]).

In this situation, the asymptotic effect of T_t is the measurement of B. If one requires minimum disturbance, then $\{V_i\}' = \{P_n\}'$, and $V_i = \sum_n c_{in}P_n$. This proves that the model of [9] is essentially unique.

Finally, we remark that, if ω is not assumed to be faithful, the proof of Theorem 3.1 can be adapted to show that, if $\omega(D_t(A,A))=0$ for all t implies $A\Omega$ to be a multiple of Ω, then $\lim_{t\to\infty}\omega(AT_t(B))=\omega(A)\omega(B)$ for all A, B in $\mathcal{M}$.

4. Strong Approach to Equilibrium

For an infinitely extended quantum system, a property of approach to equilibrium like (3.3) can hold even if T_t is a group of automorphisms of $\mathcal{M}$. A possible characterization of true dissipation of local disturbances is [10]

$$\lim_{t\to\infty}\|\psi\circ T_t-\psi\circ E\|=0 \text{ for all } \psi \text{ in } \mathcal{M}_*. \tag{4.1}$$

Lemma 4.1. *A sufficient condition for (4.1) to hold is*

$$s-\lim_{t\to\infty}\hat{T}_t^*\upharpoonright(\mathbb{1}-P)\mathcal{H}=0, \tag{4.2}$$

where P is the projection onto the subspace of $\hat{T}_t$-invariant vectors in $\mathcal{H}$.

Proof. Let ψ be in $\mathcal{V}$, $\psi(A)=(\Psi,A\Omega)$, $\Psi\in\mathcal{H}$. Then

$$\begin{aligned}|\psi(T_t(A))-\psi(E(A))|&=|(\Psi,\hat{T}_tA\Omega)-(\Psi,PA\Omega)|\\&=|(\hat{T}_t^*(\mathbb{1}-P)\Psi,A\Omega)|\leq\|\hat{T}_t^*(\mathbb{1}-P)\Psi\|\cdot\|A\|,\end{aligned}$$

which tends to zero as $t\to\infty$, uniformly in $A\in\mathcal{M}$, if (4.2) holds. Since $\mathcal{V}$ is uniformly dense in $\mathcal{M}_*$, (4.1) follows.

Conditions ensuring the validity of (4.2) are discussed in [23] Chapter II, Theorem 2.1 and Proposition 6.7; however, they are not suitable for applications in concrete cases. We are able to prove the property (4.1) under the assumption $\mathcal{N}(T)=\mathcal{M}(T)$ for the particular class of dynamical semigroups defined below.

Definition. A dynamical semigroup T_t of a von Neumann algebra $\mathcal{M}$ will be said to be normal w.r.t, a faithful normal state ω if there exists another dynamical semigroup T_t^+ of $\mathcal{M}$ such that

$$\omega(T_t^+(A)B)=\omega(AT_t(B)) \text{ for all } A, B \text{ in } \mathcal{M}, t\geqq 0 \tag{4.3}$$

and

$$T_tT_s^+=T_s^+T_t \text{ for all } t,s\geqq 0. \tag{4.4}$$

Remark. In particular, ω is stationary under T_t and T_t^+. Conditions (4.3) and (4.4) hold for instance when T_t satisfies detailed balance w.r.t. ω in the sense of [26] and the Hamiltonian part of T_t is the modular automorphism group associated to ω (cf. also [27]); for the reduced dynamics of a spatially confined quantum system in the weak coupling limit [28] this situation occurs when the energy level shift is proportional to the original Hamiltonian.

Theorem 4.2. *For a dynamical semigroup T_t which is normal w.r.t. ω, the following are equivalent*

(i) $\mathcal{N}(T)=\mathcal{M}(T)$;

(ii) *for all ψ in $\mathcal{M}_*$, $\psi\circ T_t$ converges in the norm topology of $\mathcal{M}_*$ as $t\to\infty$;*

and if the above equivalent conditions hold, the limit of $\psi \circ T_t$ *as* $t \to \infty$ *is* $\psi \circ E$.

Proof. Denote by Q the positive square root of $s - \lim_{t\to\infty} \hat{T}_t^* \hat{T}_t$ ([23], p. 41). It follows from (4.4) that $\{\hat{T}_t, \hat{T}_t^*\}$ is an abelian family of operators, hence Q is a projection commuting with $\hat{T}_t^*$ and $\hat{T}_t$, and also $s - \lim_{t\to\infty} \hat{T}_t \hat{T}_t^* = Q$. Then $Q\mathscr{H}$ reduces both $\hat{T}_t$ and $\hat{T}_t^*$, and by ([23] p. 40–41) the restrictions of $\hat{T}_t$ and $\hat{T}_t^*$ to $(\mathbb{1} - Q)\mathscr{H}$ contract strongly to zero as $t \to \infty$, whereas their restrictions to $Q\mathscr{H}$ are isometric. Now Q maps $\mathscr{M}\Omega$ into $\mathscr{M}\Omega$ (just let A_∞ be a weak* limit point as $t \to \infty$ of the norm bounded net $\{T_t^+ T_t(A)\}_t$, then $QA\Omega = A_\infty \Omega$) and $\|(\mathbb{1} - Q)A\Omega\|^2 = \lim_{t\to\infty} \omega(D_t(A, A))$ for all A in $\mathscr{M}$; hence $Q\mathscr{H}$ is the closure of $\mathscr{N}(T)\Omega$. It follows that $\mathscr{N}(T)$ is globally invariant under T_t and T_t^+, and

$$T_t^+ T_t \restriction \mathscr{N}(T) = T_t T_t^+ \restriction \mathscr{N}(T) = I \restriction \mathscr{N}(T). \tag{4.5}$$

(i) $\Rightarrow$ (ii): (i) implies that $P = Q$, hence Lemma 4.1 can be applied and (ii) follows, with $\lim_{t\to\infty} \psi \circ T_t = \psi \circ E$.

(ii) $\Rightarrow$ (i): Let ψ be in $\mathscr{M}_*$, and take φ in $\mathscr{M}_*$ such that $\|\psi \circ T_t - \varphi\| \to 0$ as $t \to \infty$. Then φ is T_t-invariant. Denote by $\hat{\psi}, \hat{\varphi}$ the restrictions of ψ, φ to $\mathscr{N}(T)$. Taking into account Equation (4.5) we have

$$\|\psi \circ T_t - \varphi\| = \|(\psi - \varphi) \circ T_t\| \geqq \|(\hat{\psi} - \hat{\varphi}) \circ T_t\| = \|\hat{\psi} - \hat{\varphi}\|.$$

Then, for any ψ in $\mathscr{M}_*$, there exists $\varphi = \varphi(\psi)$ in $\mathscr{M}_*$ such that

$$\hat{\psi} = \hat{\varphi} = [(\varphi \restriction \mathscr{M}(T)) \circ E] \restriction \mathscr{N}(T).$$

This can only hold if $\mathscr{M}(T) = \mathscr{N}(T)$, which proves (i).

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Note Added in Proof

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