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Mathematical Models for Control of Weld Bead Penetration in the GMAW Process

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This paper presents the effects of welding process parameters on weld bead penetration for the gas metal arc welding (GMAW) process. Welding process parameters included wire diameter, gas flow rate, welding speed, arc current and welding voltage. The experimental results have shown that weld bead penetration increased as wire diameter, arc current and welding voltage increased, whereas an increase in welding speed was found to decrease the weld bead penetration. However,the weld bead penetration is not affected significantly by gas flow rate changes. Mathematical equations for study of the relationship between welding process parameters and weld bead penetration have also been computed by employing a standard statistical package program, SAS.

Keywords: Adaptive control system; Gas metal arc welding; Multiple regression analysis; Procedure optimisation; Weld bead penetration

1. Introduction

The gas metal arc welding (GMAW) process is generally accepted today as the preferred joining technique and is commonly chosen for assembling large metal structures such as bridges, automobiles, aircraft, aerospacecraft, ships and rolling stock owing to its joint strength, reliability, and low cost compared to other joint processes. The demand to increase productivity and quality, the shortage of skilled labour and the strict health and safety requirements have led to the development of the automated and/or robotic welding process to deal with many of the present problems of welded fabrication. In the past decades, several effective and reliable welding processes have been developed into mechanised welding machines which include power sources, wire feeders and welding control units. Robotic welders have replaced human welders in many welding applications, and reasonable seam tracking systems are commercially available, but fully adequate process control systems have not been developed owing to a lack of reliable sensors and mathematical models that correlate welding process parameters to weld bead penetration for automated and/or robotic welding processes.

To make effective use of the automated and/or robotic arc welding process, it is imperative that a mathematical model, which can be programmed easily and fed to the robot, should be developed. It should give a high degree of confidence in predicting the weld bead dimensions and shape to achieve the desired mechanical properties of the weldment. The mathematical model should also cover a wide range of material thicknesses and be applicable for all attitude welding. In the GMAW, the welding process parameters are known to include arc current, polarity, welding voltage, welding speed, electrode extension, electrode orientation, weld joint position, wire diameter, shielding gas composition, gas flow rate, material composition and material thickness. The parameters are interdependent and a change in one parameter might affect another. Relationships between welding process parameters and the weld bead penetration are generally complex and the required control system will be dependent on a realistic model of the welding process.

Many attempts [1–12] have been made to predict and understand the effects of welding process parameters on weld bead penetration. Jackson and Shrubsall [1], Apps et al. [2] and McGlone and Chadwick [3] investigated the effects of weld process parameters on weld bead penetration and developed a mathematical model to predict weld bead penetration for submerged-arc welding (SAW) from results of their systematic experimental work. Giedt and Tallerico [4] studied the relationship between electron beam welding machine settings and weld bead penetration, and concluded that the priority for obtaining accurate results should be optimum beam focus current, beam voltage or current, welding velocity, and focus coil to work distance. Metzbower [5] presented two methods, a 2^3 factorial experimental method and a dimensional anlaysis method, for calculating the weld bead penetration in high-power-density welding as a function of laser power, welding speed, and focal distance, and compared the calculations with sets of experimental data. Both techniques displayed a good correlation

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between calculated and measured weld bead penetration. Similar mathematical models relating welding process variables to weld bead penetration for gas tungsten arc (GTA) welding have also been reported [6–8].

Hinata et al. [9] described a method for obtaining a narrow bead and deep penetration by stationary GTA welding for application to SU304 and examined how the method is affected by factors such as the trace elements in the base metal, the electrode setting conditions and the shielding gas. Dilthey et al. [10] analysed the effects of the electrical, thermodynamic and hydrodynamic processes on metal-arc active gas welding for creating a welding expert system and introduced technology and process data into the dialogue with the computer. Lübbert [11] investigated and tested sensor-controlled influencing of the process with a visual output sensor for observing the shape of the fusion bath in order to influence the geometrical dimensions such as weld width, reinforcement and penetration.

The objectives of this paper are to discuss the results obtained in a detail experimental study on the effects of welding process parameters such as wire diameter, gas flow rate, welding speed, arc current, and welding voltage on weld bead penetration, and to develop mathematical equations for evaluating the effects of welding process parameters on the weld bead penetration and studying a relationship between input variables and weld bead penetration. Included in this work is the characterisation of GMAW process and the identification of the various problems that result from the GMAW process and establishment of guidelines and criteria for most effective joint design.

2. Experimental Procedure

The experimental materials were 200 × 75 × 12 mm AS 1204 mild steel plates with a composition of C 0.25%, Si 0.4% and P 0.04% on which welds were laid adopting the bead-on-plate technique. To optimise the GMAW process, two samples were taken for observation after discarding 50 mm on each side to eliminate the end effects, and both surfaces were cleaned to eliminate any dirt and oxides. The selection of the welding electrode wire was based principally upon matching the mechanical properties and physical characteristics of the base metal, weld size, and existing electrode inventory. Steel wires with diameters of 0.9, 1.2 and 1.6 mm with composition of C 0.07–0.15%, Mn 1.00–1.50%, Si 0.60–0.85%, S 0.035% max, P 0.025% max and Cu 0.5% max, were used as welding consumables.

The welding process parameters included in this study were three sizes of wire diameter (0.9, 1.2 and 1.6 mm), three levels of gas flow rate (6, 10 and 14 $\min^{-1}$), three levels of welding speed (250, 330 and 410 mm $\min^{-1}$) and three levels of welding voltage (20, 25 and 30 V). The arc current levels selected for 0.9 mm diameter wire were 90, 190, 250 amp, whereas the levels for 1.2 and 1.6 mm diameter wires were 180, 260, 360 amp. All other parameters except these parameters under consideration remained unaltered.

The welding facility at the Centre for Advanced Manufacturing and Industrial Automation (CAMIA) was chosen as the basis for the data collection and evaluation. The facility consists of a Lincoln gas metal arc welding unit (which includes a welding power source, a welder remote control unit, and a wire torch) and a Hitachi robot manipulator that has a robot control unit and robot teach box. Torch positioning and motion control were obtained using the Hitachi six-axis robot controller (M6060II).

Experimental test plates were located in the fixture jig by the robot controller and the required input weld conditions were selected for the particular weld steps in the robot path. With welder and argon shield gas turned on, the robot was initialised and welding was carried out. This continued until the predetermined experimental runs were completed. To measure the weld bead penetration, the transverse section of each weld were cut using a power hacksaw from the midlength position of welds and the end faces were machined. Specimen end faces were polished and etched using a 2.5% nital solution to display the weld bead penetration.

The measurements of weld bead penetration, as shown in Fig. 1, were made using a metallurgical microscope interfaced with an image anlaysis system. Images are represented by a 256 level grey scale and the program can be used to identify areas of the same shade and calculate the distance between them to calculate their individual areas. The fractional factorial matrix was assumed to link the mean values of the measured results with changes in the five welding process parameters for determining weld bead penetration. The experimental results were analysed on the basis of the relationship between welding process parameters and weld bead penetration of the GMAW process.

3. Experimental Results

The summary of experimental results in terms of weld process parameters and weld bead penetration is given in Table 1. The metallurgical sections of four welds made using various welding process parameters are presented in Fig. 2. Figures 3–6 represent the results of the effect on each welding process parameter on the weld bead penetration.

Figure 3 shows the effect of gas flow rate and wire diameter on average weld bead penetration. The average weld bead penetrations were adjusted by taking the average of all measured values with the same gas flow rate for a particular wire diameter, but without considering the effects of welding speed, arc current and welding voltage. It can be seen here that a higher weld bead penetration is obtained with a larger wire diameter, but the effect of gas flow rate on weld bead penetration seems to have little significance.

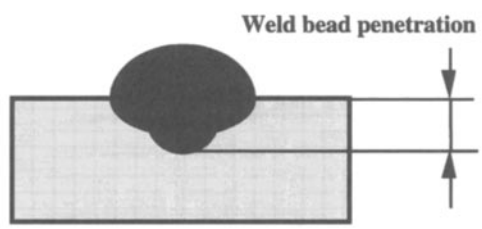

Fig. 1. Schematic representation of weld bead penetration.

Table 1. Welding process parameters and weld bead penetration.

Trial number	Wire diameter	Gas flow rate	Welding speed	Arc current	Welding voltage	Weld bead penetration
1	0.9	6	250	180	20	1.65
2	0.9	6	250	260	30	1.74
3	0.9	6	250	360	25	1.96
4	0.9	6	330	180	30	0.64
5	0.9	6	330	260	25	1.26
6	0.9	6	330	360	20	1.45
7	0.9	6	410	180	25	0.93
8	0.9	6	410	260	20	1.33
9	0.9	6	410	360	30	1.94
10	0.9	10	250	180	30	0.66
11	0.9	10	250	260	25	1.79
12	0.9	10	250	360	20	1.55
13	0.9	10	330	180	25	0.72
14	0.9	10	330	260	20	1.18
15	0.9	10	330	360	30	2.21
16	0.9	10	410	180	20	1.05
17	0.9	10	410	260	30	0.97
18	0.9	10	410	360	25	2.16
19	0.9	14	250	180	25	1.01
20	0.9	14	250	260	20	1.31
21	0.9	14	250	360	30	2.61
22	0.9	14	330	180	20	1.17
23	0.9	14	330	260	30	0.96
24	0.9	14	330	360	25	2.07
25	0.9	14	410	180	30	0.8
26	0.9	14	410	260	25	1.51
27	0.9	14	410	360	20	1.52
28	1.2	6	250	180	30	1.24
29	1.2	6	250	260	25	2.53
30	1.2	6	250	360	20	1.59
31	1.2	6	330	180	25	1.33
32	1.2	6	330	260	20	2.05
33	1.2	6	330	360	30	5.39
34	1.2	6	410	180	20	1.84
35	1.2	6	410	260	30	2.03
36	1.2	6	410	360	25	4.37
37	1.2	10	250	180	25	1.78
38	1.2	10	250	260	20	1.87
39	1.2	10	250	360	30	5.65
40	1.2	10	330	180	20	1.59
41	1.2	10	330	260	30	1.72
42	1.2	10	330	360	25	2.97
43	1.2	10	410	180	30	1.41
44	1.2	10	410	260	25	2.04
45	1.2	10	410	360	20	1.32
46	1.2	14	250	180	20	2.04
47	1.2	14	250	260	30	3.08
48	1.2	14	250	360	25	5.93
49	1.2	14	330	180	30	1.26
50	1.2	14	330	260	25	3.36
51	1.2	14	330	360	20	5.42
52	1.2	14	410	180	25	1.66
53	1.2	14	410	260	20	2.97
54	1.2	14	410	360	30	5.81
55	1.6	6	250	180	25	1.72
56	1.6	6	250	260	20	2.56
57	1.6	6	250	360	30	4.71
58	1.6	6	330	180	20	2.18
59	1.6	6	330	260	30	2.47

Table 1. Continued.

Trial number	Wire diameter	Gas flow rate	Welding speed	Arc current	Welding voltage	Weld bead penetration
60	1.6	6	330	360	25	3.65
61	1.6	6	410	180	30	1.35
62	1.6	6	410	260	25	2.46
63	1.6	6	410	360	20	3.04
64	1.6	10	250	180	20	1.69
65	1.6	10	250	260	30	2.64
66	1.6	10	250	360	25	4.69
67	1.6	10	330	180	30	1.38
68	1.6	10	330	260	25	2.63
69	1.6	10	330	360	20	3.5
70	1.6	10	410	180	25	1.71
71	1.6	10	410	260	20	2.03
72	1.6	10	410	360	30	5.7
73	1.6	14	250	180	30	2.0
74	1.6	14	250	260	25	2.28
75	1.6	14	250	360	20	3.85
76	1.6	14	330	180	25	1.89
77	1.6	14	330	260	20	2.45
78	1.6	14	330	360	30	4.5
79	1.6	14	410	180	20	1.75
80	1.6	14	410	260	30	2.33
81	1.6	14	410	360	25	3.32

(a)
(b)

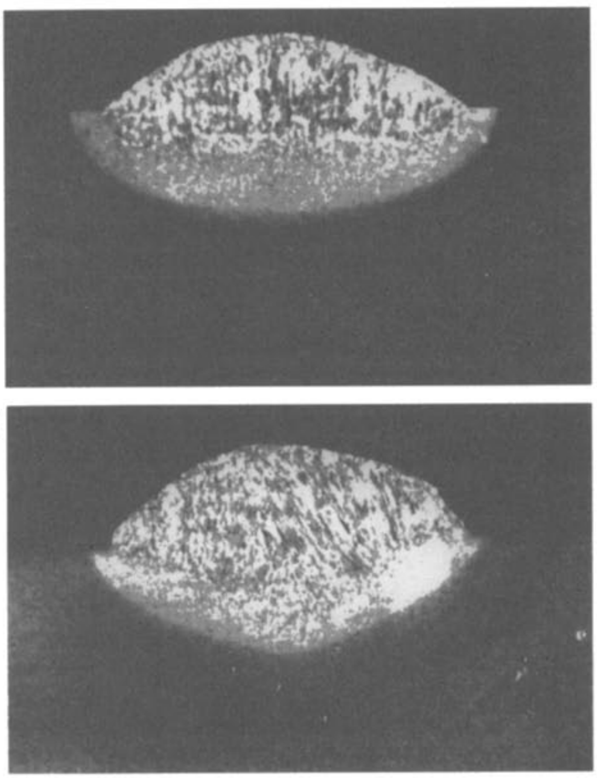

Fig. 2a–b. Metallurgical sections of welds according to the various combinations of welding process parameters. (*a*) $D = 1.2$ mm, $G = 6$ l, $S = 250$ mm min^{-1}, $I = 180$ amp, $V = 30$ volt. (*b*) $D = 1.2$ mm, $G = 6$ l, $S = 330$ mm min^{-1}, $I = 360$ amp, $V = 30$ volt.$S = 410$ mm min^{-1}, $I = 360$ amp, $V = 20$ volt.

(a)
(b)

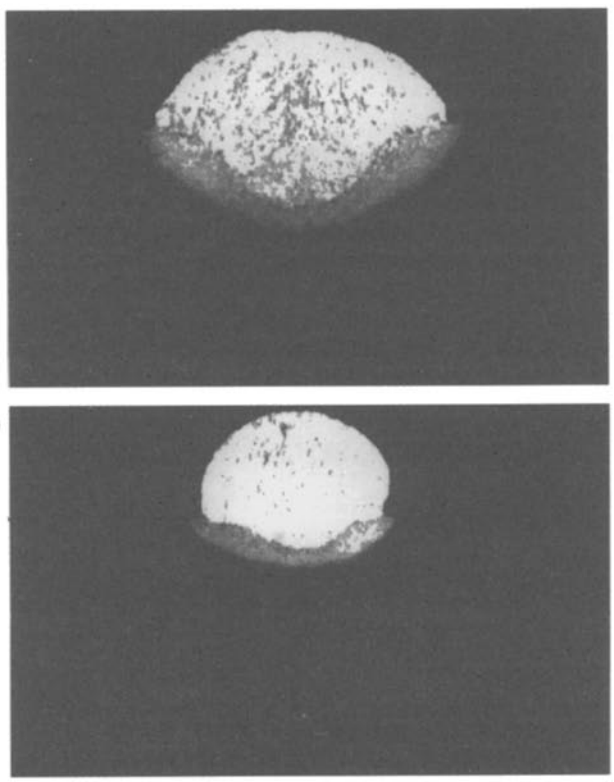

Fig. 2c–d. Metallurgical sections of welds according to the various combinations of welding process parameters. (*c*) $D = 1.6$ mm, $G = 6$ l, $S = 410$ mm min^{-1}, $I = 360$ amp, $V = 20$ volt. (*d*) $D = 0.9$ mm, $G = 6$ l, $S = 330$ mm min^{-1}, $I = 250$ amp, $V = 20$ volt.

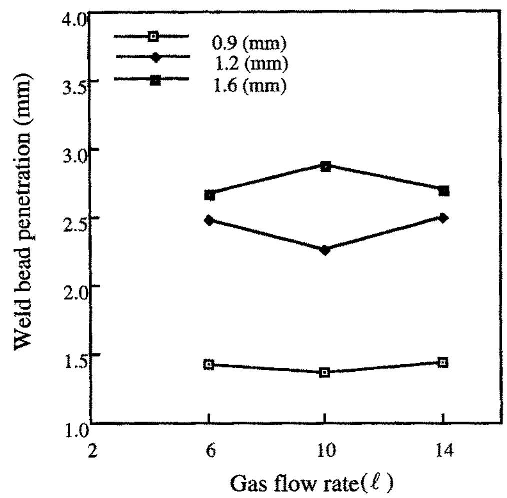

Fig. 3. The effect of gas flow rate on average weld bead penetration.

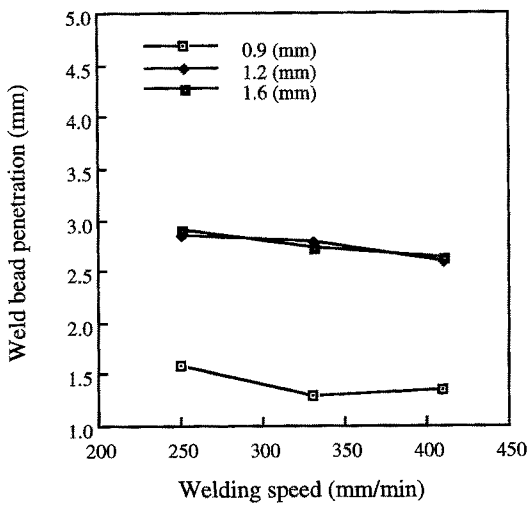

Fig. 4. The effect of welding speed on average weld bead penetration.

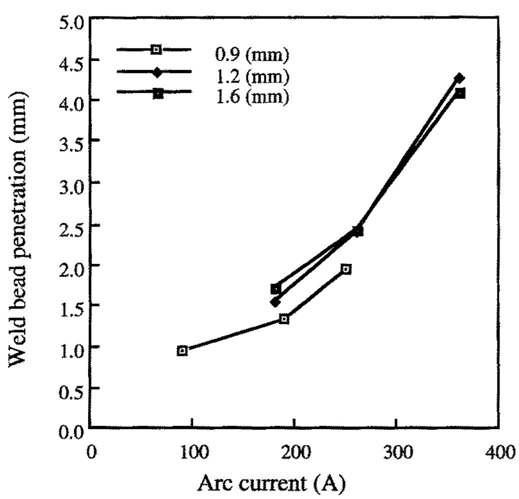

Fig. 5. The effect of arc current on average weld bead penetration.

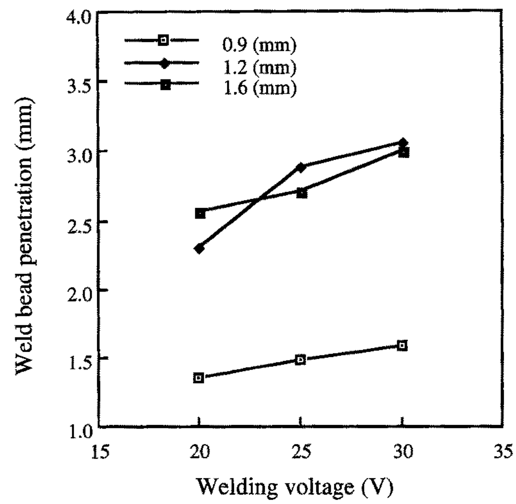

Fig. 6. The effect of welding voltage on average weld bead penetration.

Figure 4 indicates that there is a decrease in weld bead penetration as welding speed increases. The average weld bead penetrations were produced by taking the average of all values for welds deposited with the same welding speed for a specific wire diameter, ignoring the effects of gas flow rate, arc current and welding voltage.

When arc current increases, the weld bead penetration increases as can be seen in Fig. 5. The average weld bead penetrations were found by taking the average of all measured values with the same arc current for a given wire diameter, without taking into account the effects of gas flow rate, welding speed and welding voltage.

Figure 6 displays the effect of welding voltage on weld bead penetration for a given wire diameter. Weld bead penetrations were produced by taking the average of all measured values for welds deposited with the same welding voltage for a specific wire diameter, yet ignoring the effects of gas flow rate, welding speed and arc current. It seems from Fig. 6 that there is an increase in weld bead penetration when welding voltage increases.

4. Development of Mathematical Models

4.1 Selection of a Mathematical Model

The experimental results have shown that weld bead penetration is influenced by wire diameter, gas flow rate, arc current and welding voltage, and that a mathematical model can be effectively applied for prediction of the optimal welding conditions in the GMAW process. It was therefore thought that a formalised approach to procedure optimisation could successfully establish combinations of welding process parameters which would produce welds of a given quality standard. With five welding process parameters, the response parameter (Y) could be the weld bead penetration under consideration and is represented as follows:

$$Y = f(D, G, S, I, V) \quad (1)$$

where D = wire diameter
G = gas flow rate
S = welding speed
I = arc current
V = welding voltage

The empirical mathematical model can be divided into three parts; curvilinear, polynomial and linear equations. McGlone and Chadwick [3] introduced a curvilinear formula which assumed a linear relationship for close ranges and considered all the main effects together with the two factor interactions represented as follows:

$$Y = a(D)^b(G)^c(S)^d(I)^e(V)^f \quad (2)$$

Where a, b, c, d, e and f are constants.

The first procedure of this technique was the analysis of variance (ANOVA) which quantified the effects of welding process parameters on weld bead penetration in order to verify the significance of each welding process parameter on the optimisation variable and to detect whether there were any interaction effects among the welding process parameters themselves. Secondly, the multiple correlation coefficient and the Fisher's F-ratio (F) were employed to gauge goodness-of-fit and indicate significance at the 1% level of Fisher's F-ratio for including the physical considerations about the logical shape of the equations. As a result, a function based on the analysis of variance was developed for describing the experimental results using the method of least squares.

Raveendra and Parmar [12] also proposed a portion of the power series-algebraic polynomial which includes the main effects of welding process parameters and first-order interactions, and is presented as follows:

$$\begin{aligned} Y = a_1 &+ a_2D + a_3G + a_4S + a_5I + a_6V + a_7DG \\ &+ a_8DS + a_9D + a_{10}DV + a_{11}GS + a_{12}GI + a_{13}GV \\ &+ a_{14}SI + a_{15}SV + a_{16}IV \end{aligned} \quad (3)$$

where $a_1, \ldots, a_{16}$ are constants.

Finally, the linear equation [13,14] could be expressed as follows:

$$Y = c_1 + c_2D + c_3G + c_4S + c_5I + c_6V \quad (4)$$

where $c_1, \ldots, c_6$ are constants.

4.2 Mathematical Models Developed

To predict particular weld bead penetration and to establish the interrelationship between weld process parameters to weld bead penetration, mathematical models can be proposed as the basis for a control system for the automatic GMAW process. Best-fit equations from the study of the relationship between the five welding process parameters and weld bead penetration were obtained employing statistical techniques such as multiple regression analysis. These analyses were performed with the help of a standard statistical package program, SAS, using an IBM compatible PC [15].

The linear, polynomial and curvilinear equations in order to quantitatively evaluate the effects of welding process parameters on weld bead penetration were obtained from the experimental outputs, as shown below.

$$P_{(\log)} = \frac{D^{0.3945}G^{0.1187}I^{0.9557}V^{0.1529}}{10^{2.296}} \quad (5)$$

$$\begin{aligned} P_{(\text{pol})} = 6.1870 &- 1.0168D + 0.0605G - 0.00157S \\ &- 0.0235I - 0.2202V \\ &- 0.0136DG + 0.0066DI + 0.0011IV \end{aligned} \quad (6)$$

$$\begin{aligned} P_{(\text{lin})} = -&\ 1.9504 + 0.3428D + 0.0438G \\ &- 0.0016S + 0.0121I + 0.0417V \end{aligned} \quad (7)$$

where P = weld bead penetration
subscript (log) = curvilinear regression analysis
subscript (pol) = polynomial regression analysis
subscript (lin) = linear regression analysis

The adequacy of the mathematical models and the significance of coefficients were tested by applying the analysis of variance technique and student's test (t), respectively. Table 2 presents the standard error of estimates, the coefficient of multiple correlations and the coefficient of determinations for the three equations. Coefficients of multiple correlations of these equations are 0.8411, 0.8824 and 0.8273, respectively.

5. Discussion

5.1 Interpretation of the Regression Equations

The standard error of estimates denotes the standard deviation for the difference between the measured and the predicted values for all the experimental data. The standard error of estimates can be used to estimate the extent to which future results can be expected to differ from the prediction made by using the equation. Otherwise, the coefficient of multiple correlation and coefficient of determination denote the percentage of the total variability observed in the dependent variable that is the result of the weld process parameters included in the equations.

The validity of equation (6) can be judged from the value of the coefficient of multiple correlation (>0.85) given in Table 2 and Fig. 7 which presents the plot of the measured versus the calculated values of weld bead penetration obtained using polynomial regression analysis. A straight line which indicates a one-to-one relationship between the measured and calculated weld bead penetration has been added for clarity. Similarly, the value of the coefficient of multiple correlation and Fig. 8, which displays a plot of the measured weld bead penetration versus calculated values obtained using curvilinear

Table 2. Analysis of variance tests for model.

Equation	Standard error of estimates	Coefficient of multiple correlations	Coefficient of determinations
5	0.7263	0.8411	70.75
6	0.6509	0.8824	77.87
7	0.7617	0.8273	68.44

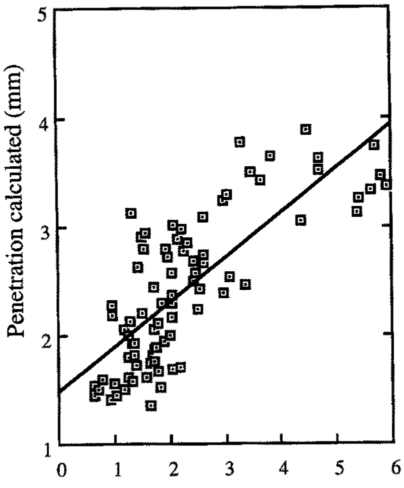

Fig. 7. Measured vs. calculated values of weld bead penetration.

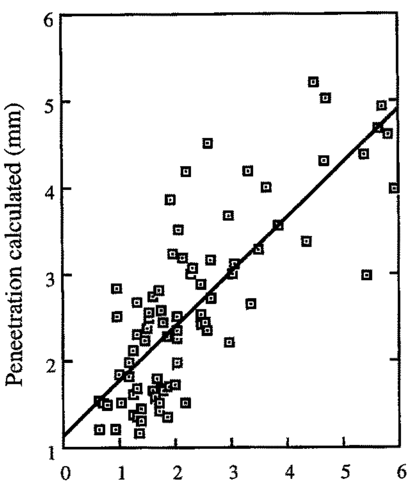

Fig. 8. Measured vs. calculated values of weld bead penetration.

regression analysis, is shown in the variability of equation (5). Finally, the validity of equation (7) can be determined from the value of the coefficient of multiple correlation (>0.8) and Fig. 9 which shows a plot of the measured weld bead penetration versus the calculated values obtained using linear regression analysis. Examination of these equations shows a reasonable correlation between measurement and calculation, although the polynomial equation provides better correlation results as shown in Figs 7 to 9.

5.2 Comparison of Theoretical Results with Experimental Data

Theoretical anlaysis can be carried out using a 3D conductive heat transfer configuration for thick plates or a 2D conductive

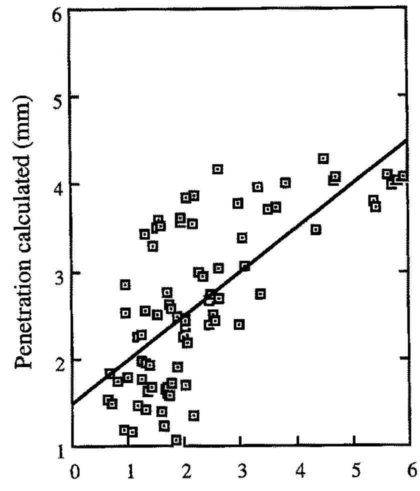

Fig. 9. Measured vs. calculated values of weld bead penetration.

heat transfer configuration for thin sheets. Christensen et al. [16] put the Rosenthal equation into a dimensionless form and showed that the overall size of the weldment is related to "operating parameters", n, where $n = QS/4\pi\lambda kT$, where Q = the rate of heat input to plate (J s^{-1}); S = welding speed; T = (melting temperature − ambient temperature); k = thermal conductivity; and λ = thermal diffusivity. Their model is claimed to be suitable for all combinations of materials and welding conditions with the limitations and assumptions combined with the point source equation. The experimental results were plotted using the same non-dimensional parameters, and compared with the theoretical results obtained by Christensen et al., who assumed a 3D conductive heat transfer configuration. Values of the material parameters were taken to be: λ = 0.091 $cm^2\,s^{-1}$, $T = 1500°C$, $k = 0.41$ J $cm^{-1}°C$ s. In the GMAW, Q is given by the product of η, V and I where η is the arc efficiency. The arc efficiency for the GMAW process employed to weld steel plates depends on welding process parameters such as welding voltage, arc current, electrode extension and type of shielding gas, and has been found to be in the range 66%–71%. It was assumed to be 70% for the comparisons made below.

Figure 10 presents the non-dimensional weld bead penetration. From Fig. 10, it appears that the theoretical equation overestimates the weld bead penetration, as has been noted by Roberts and Wells [17] and presents a considerable error. A straight line has been added for comparison. In addition, it is quite evident from the above comparison that prediction of weld bead penetration with reasonable accuracy, based on various models, requires adjustments in order to achieve better agreement with experimental results. Since conductive, convective and radiative heat transfer and mass transfer in the GMAW process are all involved, the development of an accurate analytical model can be complicated and perhaps inappropriate for either closed loop or adaptive control purposes. Instead, a regression model for weld bead geometry should be considered.

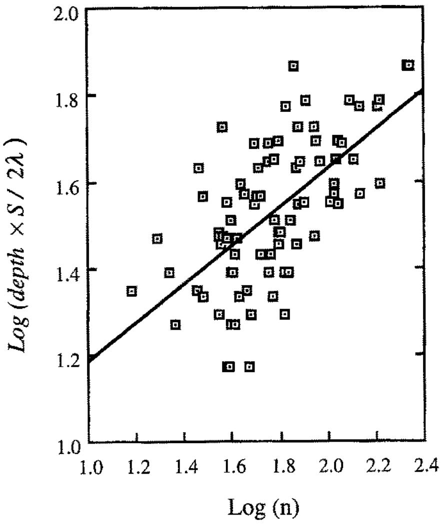

Fig. 10. Non-dimensional weld bead measurements vs. operating parameter for weld bead penetration.

5.3 Empirical Equation for Weld Bead Penetration

Chandel [18] presented the following equation to predict the weld bead penetration for the GMAW process:

$$P = \frac{I^{1.74}}{D^{0.46}\, L^{0.063}\, S^{0.366}\, V^{0.142}} \times \frac{1}{(10)^{3.25}} \tag{8}$$

where L is the wire extension.

The process input conditions employed to produce the 81 weld runs for fitting the above equation were input into Chandel's equations to provide theoretical results for weld bead penetration. This allowed the accuracy of Chandel's equations to be validated using experimental findings extracted during the course of this study. Results were plotted using a scatter graph for weld bead penetration. Figure 11 was produced for experimental versus theoretical results using Chandel's equations. The line of best fit for the plotted points was drawn

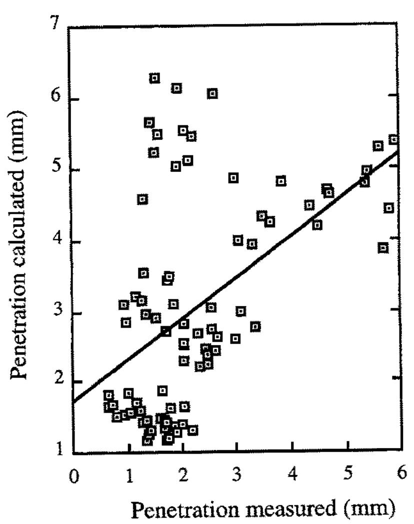

Fig. 11. Measured vs. calculated values of weld bead penetration.

using regression computation. It is evident from these results that the model's accuracy is questionable and its universal applicability is limited.

6. Conclusions

The effects of welding process parameters on weld bead penetration when bead-on-plate welds are deposited using the GMAW process have been studied, and the following conclusions reached.

1. Welding process parameters, such as wire diameter, gas flow rate, welding speed, arc current and welding voltage, influence the weld bead penetration for the GMAW process.
2. Weld bead penetration increased as wire diameter, arc curent and welding voltage increased, whereas an increase in welding speed was found to decrease the weld bead penetration.
3. Gas flow rate shows no significant effect on the weld bead penetration.
4. Comparison between the experimental findings and those proposed by the 3D conductive heat transfer model showed that the theoretical anlaysis generally overestimated the depth of penetration. Note that considerable scatter was found in the overall results.
5. Mathematical models developed from experimental results can be used to control the welding process parameters in order to achieve the desired weld bead penetration based on weld quality criteria.
6. These equations may prove useful and applicable for automatic control systems and expert systems.

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