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Note

Thermal boundary layers in magnetohydrodynamic flow over a flat plate in the presence of a transverse magnetic field

T. Watanabe, Morioka, Japan, and **I. Pop,** Cluj, Romania

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Summary. A boundary layer solution for the heat transfer of an electrically conducting fluid over a semi-infinite flat plate in the presence of a transverse magnetic field has been studied. The heat due to viscous dissipation and stress work were also included into the energy equation. The governing nonsimilar partial differential equations are transformed into ordinary differential ones by means of difference-differential method. The temperature profiles and heat transfer coefficient are obtained for various values of the parameters entering the problem.

1 Introduction

The boundary layer flow of an incompressible electrically conducting fluid over a semi-infinite flat plate in the presence of a transverse magnetic field has been studied by many researchers in the past. By means of the difference-differential method, Watanabe [1], [2] reduced the momentum partial differential equation to an ordinary one and obtained the solution in a form of integral equations. In some cases, the application of this method is more convenient than other mathematical techniques. However, the thermal field has not been considered in [1], [2].

The present research note tries to investigate the thermal boundary layers in magnetohydrodynamic flow over an isothermal flat plate in the presence of a transverse magnetic field using the previous proposed solution for the momentum equation described by Watanabe [1], [2]. The analysis includes the viscous dissipation and stress work effects, too. It was shown by Kuiken [3], Soundalgekar and Takhar [4], and Ingham [5] that the terms representing viscous dissipative heat and stress work are of equal importance in the case of air and hence they should both be considered or neglected in the energy equation. The effects of the Eckert number on the temperature field and heat transfer coefficient have been discussed for some values of the magnetic interaction parameter and Prandtl number.

2 Analysis

Let cartesian coordinates (x, y) be introduced for the description of the steady flow and heat transfer of an incompressible electrically conducting fluid over a semi-infinite flat plate. The plate is located in the $y = 0$ plane, and the oncoming flow U_∞ is constant and parallel to the x-axis.

A magnetic field with a constant magnetic flux density B_0 is applied normal to the plate. The temperature of the plate is held at a constant value T_w, which is higher than the ambient temperature T_∞. We also assume that the viscous dissipative heat and stress work terms are not neglected in the present study. By applying the usual boundary layer approximation, the basic equations under these assumptions can be transformed to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0{}^2}{\varrho}(U_\infty - u) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{\sigma B_0{}^2}{\varrho c_p}U_\infty u + \frac{\sigma B_0{}^2}{\varrho c_p}u^2 \tag{3}$$

subject to the boundary conditions

$$\begin{aligned} &y = 0 : u = 0, \quad v = 0, \quad T = T_w \\ &y \to \infty : u = U_\infty, \quad T = T_\infty . \end{aligned} \tag{4}$$

Here u and v are the components of fluid velocity in the x- and y-directions, respectively, T is the fluid temperature, α, σ, ϱ, ν, and c_p are the thermal diffusivity, electrical conductivity, density, kinematic viscosity and specific heat at constant pressure of the fluid.

To seek a solution to Eqs. (1) to (3), we introduce the variables

$$\psi = (U_\infty \nu x)^{1/2} f(x, \eta), \qquad g(x, \eta) = (T - T_\infty)/(T_w - T_\infty) \tag{5}$$

where

$$\eta = y\left(\frac{U_\infty}{\nu x}\right)^{1/2} \tag{6}$$

and ψ is the stream function defined in the usual way. In view of (5) and (6) these equations reduce to

$$\frac{\partial^3 f}{\partial \eta^3} + \left(\frac{1}{2}f + X\frac{\partial f}{\partial X}\right)\frac{\partial^2 f}{\partial \eta^2} - X\frac{\partial^2 f}{\partial X\,\partial \eta}\frac{\partial f}{\partial \eta} + X\left(1 - \frac{\partial f}{\partial \eta}\right) = 0 \tag{7}$$

$$\frac{1}{Pr}\frac{\partial^2 g}{\partial \eta^2} + \left(\frac{1}{2}f + X\frac{\partial f}{\partial X}\right)\frac{\partial g}{\partial \eta} - X\frac{\partial g}{\partial X}\frac{\partial f}{\partial \eta} - XEc\frac{\partial f}{\partial \eta}\left(1 - \frac{\partial f}{\partial \eta}\right) + Ec\left(\frac{\partial^2 f}{\partial \eta^2}\right)^2 = 0 \tag{8}$$

with the boundary conditions

$$\begin{aligned} &\eta = 0 : \frac{\partial f}{\partial \eta} = 0, \quad \frac{1}{2}f + X\frac{\partial f}{\partial X} = 0, \quad g = 1 \\ &\eta \to \infty : \frac{\partial f}{\partial \eta} = 1, \quad g = 0 . \end{aligned} \tag{9}$$

Here $Pr = \nu/\alpha$ is the Prandtl number, $Ec = U_\infty{}^2/[c_p(T_w - T_\infty)]$ is the Eckert number and $X = \sigma B_0{}^2 x/(\varrho U_\infty)$ is the magnetic interaction parameter. In Eq. (8), the fourth and last terms represent the stress work and viscous dissipative heat, respectively.

Solutions to Eqs. (7) and (8) subject to the boundary conditions (9) are found using the difference-differential method as described by Watanabe [1], [2]. The derivative for X at $X = X_i = ih\,(i = 0, 1, 2, \ldots)$, where h is a constant step size, can be approximated by using a four point formula of Gregory-Newton, for example. In use of this method, we can transform (7) and (8) into the following form of integral equations:

$$\frac{df_i}{d\eta} = \left\{1 - \int_0^\infty E(\eta) \int_0^\eta \frac{R(\eta)}{E(\eta)}\, d\eta\, d\eta\right\} \frac{G(\eta)}{G(\infty)} + \int_0^\eta E(\eta) \int_0^\eta \frac{R(\eta)}{E(\eta)}\, d\eta\, d\eta \tag{10}$$

$$f_i = \int_0^\eta \frac{df_i}{d\eta}\, d\eta \tag{11}$$

$$g_i = 1 + \int_0^\eta P(\eta) \int_0^\eta \frac{S(\eta)}{P(\eta)}\, d\eta\, d\eta - \left\{1 + \int_0^\infty P(\eta) \int_0^\eta \frac{S(\eta)}{P(\eta)}\, d\eta\, d\eta\right\} \frac{Q(\eta)}{Q(\infty)} \tag{12}$$

where

$$E(\eta) = \exp\left[\int_0^\eta \left\{-\frac{1}{2} f_i - \frac{i}{6}\left(11f_i - 18f_{i-1} + 9f_{i-2} - 2f_{i-3}\right)\right\} d\eta\right] \tag{13}$$

$$G(\eta) = \int_0^\eta E(\eta)\, d\eta \tag{14}$$

$$R(\eta) = \frac{i}{6}\left(11\frac{df_i}{d\eta} - 18\frac{df_{i-1}}{d\eta} + 9\frac{df_{i-2}}{d\eta} - 2\frac{df_{i-3}}{d\eta}\right)\frac{df_i}{d\eta} + ih\left(\frac{df_i}{d\eta} - 1\right) \tag{15}$$

$$P(\eta) = \exp\left[\int_0^\eta \left\{-\frac{1}{2} Pr\, f_i - \frac{i}{6} Pr\,(11f_i - 18f_{i-1} + 9f_{i-2} - 2f_{i-3})\right\} d\eta\right] \tag{16}$$

$$Q(\eta) = \int_0^\eta P(\eta)\, d\eta \tag{17}$$

$$S(\eta) = \frac{i}{6} Pr\,(11g_i - 18g_{i-1} + 9g_{i-2} - 2g_{i-3})\frac{df_i}{d\eta} + ih\, Ec\, Pr\, \frac{df_i}{d\eta}\left(1 - \frac{df_i}{d\eta}\right) - Ec\, Pr\left(\frac{\partial^2 f}{\partial \eta^2}\right)^2. \tag{18}$$

3 Results and discussion

The numerical integration of Eqs. (10) to (12) is performed by iterative numerical quadratures using a Simpson's rule. A full description of this method is given also in [6]–[8] and it is unnecessary to repeat the details here.

Since the velocity field is discussed in [1], [2], we shall limit here only to the heat transfer characteristics. The temperature profiles for $X = 0.2$, $Pr = 0.733$, 1.0 and $Ec = 0$, 0.5, 1.0 are shown in Figs. 1 and 2. It is seen that there is a rise in the temperature due to the heat created by

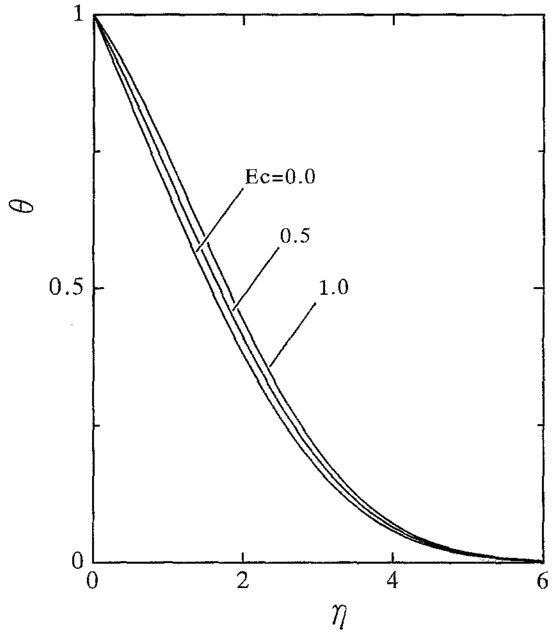

Fig. 1. Temperature profiles for $X = 0.2$, $Pr = 0.733$, $Ec = 0$, 0.5, and 1.0

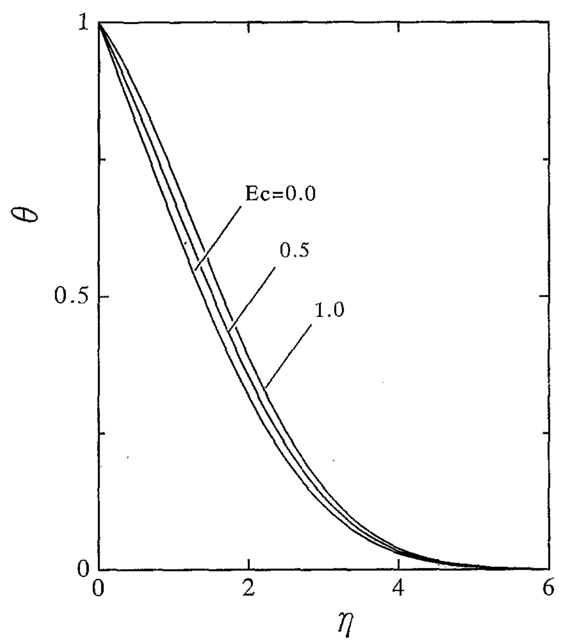

Fig. 2. Temperature profiles for $X = 0.2$, $Pr = 1.0$, $Ec = 0$, 0.5, and 1.0

viscous dissipation and compression work. On the other hand, these profiles become, as expected, steeper as Pr increases.

Once $g(X, \eta)$ is known for a set of parameters X, Pr, Ec, the local Nusselt number of our primary concern may be evaluated from

$$Nu = \frac{xq_w}{k(T_w - T_\infty)} \tag{19}$$

Table 1. Values of $Nu/Re_x^{1/2}$

X	$Pr = 0.733$			$Pr = 1.0$		
	Ec			Ec		
	0.0	0.5	1.0	0.0	0.5	1.0
0.0	0.29755	0.21075	0.12395	0.33206	0.24904	0.16603
0.5	0.35699	0.28285	0.20871	0.40280	0.30212	0.20144
1.0	0.38336	0.30532	0.22857	0.43446	0.32519	0.21727
1.5	0.39959	0.31986	0.24122	0.45413	0.34005	0.22710
2.0	0.41091	0.33011	0.25022	0.46798	0.35052	0.23401

where $q_w = -k(\partial T/\partial y)_{y=0}$ is the heat flux from the plate and k is the thermal conductivity of the fluid. Using (5), (6), and (12), we obtain

$$Nu/Re_x^{1/2} = -\left(\frac{\partial g}{\partial \eta}\right)_{\eta=0} = \frac{1}{Q(\infty)}\left\{1 + \int_0^\infty P(\eta) \int_0^\eta \frac{S(\eta)}{P(\eta)}\, d\eta\, d\eta\right\} \tag{20}$$

where $Re_x = U_\infty x/\nu$ is the local Reynolds number.

The values of $Nu/Re_x^{1/2}$ as given by Eq. (20) are given in Table 1 for $X = 0, 0.5, 1.0, 1.5, 2.0$; $Pr = 0.733, 1.0$; $Ec = 0, 0.5, 1.0$. To verify the proper treatment of the problem, we compare the present solution with known results from the literature. Thus, Ingham [5] has obtained for $Nu/Re_x^{1/2}$ the values of 0.3321 for $Ec = 0$ and $Pr = 1.0$, respectively, 0.1660 when $Ec = 1.0$ and $Pr = 1.0$ while our results are 0.33206 and 0.16603, which show an excellent agreement with those from [5].

Finally, the variation of $Nu/Re_x^{1/2}$ as a function of X is shown in Fig. 3 for $Pr = 0.733$, $Ec = 0, 0.5, 1.0$ and in Fig. 4 for $Pr = 1.0$, $Ec = 0, 0.5, 1.0$, respectively. We notice from Table 1 and these figures that the rate of heat transfer decreases due to heat created by both viscous dissipation and compression work ($Ec \neq 0$). It is also observed that the heat transfer increases

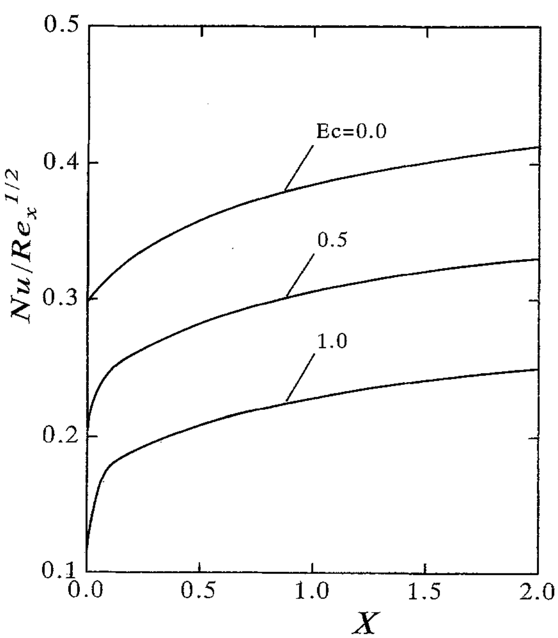

Fig. 3. Variation of the local Nusselt number as a function of X for $Pr = 0.733$, $Ec = 0, 0.5$, and 1.0

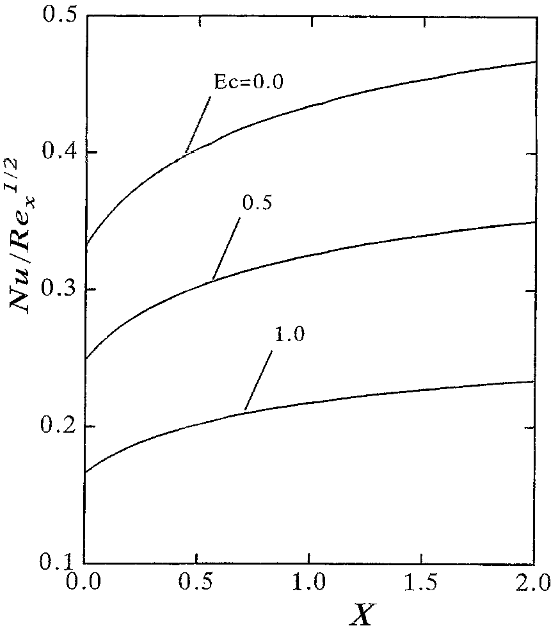

Fig. 4. Variation of the local Nusselt number as a function of X for $Pr = 1.0$, $Ec = 0, 0.5$, and 1.0

with Pr, because a higher Prandtl number fluid has a relatively lower thermal conductivity which reduces conduction and thereby increases the variations. This results in the reduction of the thermal boundary layer thickness and an increase in the heat transfer rate at the wall.

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Authors' addresses: Prof. Dr. T. Watanabe, Department of Mechanical Engineering, Faculty of Engineering, Iwate University, Morioka 020, Japan; Prof. Dr. I. Pop, Faculty of Mathematics, University of Cluj, R-3400 Cluj, CP 253, Romania