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The effect of inhomogeneity and anisotropy on Stoneley waves

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Summary. In this paper we study the dispersion equation of Stoneley waves that are travelling in an inhomogeneous elastic half-space over an anisotropic homogeneous elastic half-space.

The phase velocity is calculated as a function of the wave number. The results indicate that the effect of anisotropy on such waves is small and can be neglected, while the effect of inhomogeneity is very pronounced. The results show that Stoneley waves do not exist after some cut-off value of the wave number.

1 Introduction

The crust of the earth has three dimensional structures with continuous variation of elastic parameters [1]. These variations are caused by long term variation of temperature and pressure with depth. Inhomogeneity and anisotropy in the earth are regarded as the origin of attenuation and scattering of seismic waves [2], [3].

The classical problems of wave propagation in elastic homogeneous media posses a property such that they can be formulated in terms of two potentials, each one satisfying a second order differential equation. For problems in inhomogeneous media this is not always possible. Ravindra [4] proved that when the density is constant and Poisson's ratio equals 0.25 (i.e. $\lambda = \mu$) it is possible to have two separate second order wave equations for compressional and shear wave potentials. Acharya [5] used the results of [4] to study the reflection from the free surface of an inhomogeneous medium. Rao [6] considered the tensorial stress equation in heterogeneous media and showed that for plane strain and axisymmetric cases, the stresses are determined by a single function satisfying a fourth order differential equation. Rao and Goda [7] used this approach to study Lamb's problem for a class of heterogeneous elastic half-spaces. Also, Rao [8] studied the reflection and transmission coefficients from Epstein transition zones in nonhomogeneous isotropic elastic media.

Approximate theories have, also, been used in dealing with wave propagation in heterogeneous media as the geometrical ray theory. Most of the applications of ray theory have been limited to ray tracing because of some essential instabilities that appear in the numerical evaluation of ray amplitude. They are mostly due to the fact that ray theory is an infinite frequency method. However, there are several methods that have been proposed to avoid these instabilities in ray theory. Chapman [9] proposed to use the WKB method, Chapman and Drummond [10] used the method of Maslov for the calculation of high frequency fields in arbitrary heterogeneous media. Farra and Madariaga [11] studied ray propagation in three dimensional heterogeneous media. A general formalism based on Hamiltonian theory was used so that the results are independent of the coordinate system.

The effect of anisotropy on P and SV waves has not been discussed by many investigators. Isotropy is one of the assumptions which people usually make. Such an assumption is not right especially in sedimentary layers. Anderson and Harkrider [12] indicated that anisotropy is also present in the near surface layers. Abubakar and Hudson [13] studied the dispersive properties of liquid overlying an anisotropic half-space and compared the results with the case of an isotropic lower half-space. They found that anisotropy has little effect on the results. Sharma [14] discussed the propagation of Rayleigh waves in an elastic medium with two horizontal layers overlying a semi-infinite elastic medium above which lies a liquid layer. The second elastic layer was assumed to have an anisotropic behavior. Keith and Crampin [15] studied the characteristics of body wave propagation in anisotropic media and developed a means of calculating the plane wave reflection coefficient of body waves for layered anisotropic media. They, also, computed theoretical seismograms for the same model.

In this paper we investigate the effect of inhomogeneity and anisotropy on Stoneley waves. We use a model which consists of two half-spaces, the upper one is an isotropic inhomogeneous elastic medium, while the lower one is an anisotropic homogeneous elastic medium. We obtain the dispersion equation and investigate it for different cases.

2 Formulation of the problem

2.1 Model description

Choose a two dimensional Cartesian coordinate system (x, z) with the z-axis pointing upwards. Let two elastic half-spaces be in contact along the plane $z = 0$. The upper half-space is occupied by an isotropic inhomogeneous elastic medium, while the lower half-space is occupied by an anisotropic homogeneous medium. We assume that the quantities relating to the upper and lower half-spaces are denoted by the the subscripts 1 and 2, respectively. The densities ρ_1 and ρ_2 are constants. Lame's parameter λ_1, μ_1 are assumed to be equal, i.e. Poisson's ratio in the upper half equals 0.25. The velocities α_1 and β_1 of the compressional and shear wave, respectively, are assumed to be functions of depth only. Clearly, as $\lambda_1 = \mu_1$ then $\alpha_1 = (3)^{1/2}\,\beta_1$.

2.2 Equations of motion and boundary conditions

Let $S_j = (u_j, v_j)\, j = 1, 2$ be the displacements in the two half-spaces. According to Ravindra [4], it is possible to have separate wave equations for the compressional and shear wave potentials, ϕ_1 and ψ_1, respectively. The displacement S_1 is given by

$$S_1 = \mu_1 \nabla(\mu_1{}^{-1}\phi_1) + \mu_1{}^{-1}\nabla \times (\mu_1\psi_1) \tag{1}$$

where the potentials ϕ_1 and ψ_1 satisfy the equations

$$\nabla^2\phi_1 - \alpha_1{}^{-2}(z)\,\frac{\partial^2\phi_1}{\partial t^2} = 0, \tag{2}$$

$$\nabla^2\psi_1 - \beta_1{}^{-2}(z)\,\frac{\partial^2\psi_1}{\partial t^2} = 0. \tag{3}$$

The expressions for the stresses are given by

$$(\sigma_1)_{zz} = \mu_1 \left(\frac{\partial^2 \phi_1}{\partial x^2} + 2 \frac{\partial^2 \psi_1{}^y}{\partial x \, \partial z} - g \frac{\partial \psi_1{}^y}{\partial x} + 3 \frac{\partial^2 \phi_1}{\partial z^2} - 3g \frac{\partial \phi_1}{\partial z} - 3\phi_1 g' \right) \tag{4}$$

$$(\sigma_1)_{zx} = \mu_1 \left(2 \frac{\partial^2 \phi_1}{\partial x \, \partial z} - g \frac{\partial \phi_1}{\partial x} + \frac{\partial^2 \psi_1{}^y}{\partial x^2} - \frac{\partial^2 \psi_1{}^y}{\partial z^2} - g \frac{\partial \psi_1{}^y}{\partial z} - \psi_1{}^y g' \right), \tag{5}$$

where

$$g = \frac{\mu_1{}'}{\mu_1}, \tag{6}$$

$'$ denotes differentiation with respect to z. $\psi_1{}^y$ is the y-component of ψ_1. However, $\psi_1{}^y$ is the only component which we shall need. For this reason, from this point on, we shall write it as ψ_1.

As for the lower half-space, we assume that the properties of the medium are defined by the condition that its strain energy volume density function W_2 has the form [16]

$$2W_2 = A_2 e_{xx}^2 + C_2 e_{zz}^2 + 2F_2 e_{xx} e_{zz} + L_2 e_{xz}^2. \tag{7}$$

Since W_2 is a positive definite form,

$$A_2 > 0, \quad C_2 > 0, \quad L_2 > 0 \quad \text{and} \quad A_2 C_2 > F_2{}^2. \tag{8}$$

Following [13], we shall further assume that

$$A_2 > L_2 \quad \text{and} \quad C_2 > L_2. \tag{9}$$

The stresses can be derived from the strain energy volume density function by the formulae

$$(\sigma_2)_{ij} = \frac{\partial W_2}{\partial e_{ij}} \quad \text{and} \quad (\sigma_2)_{ii} = \frac{\partial W_2}{\partial e_{ii}} \quad \text{(no summation)}. \tag{10}$$

Thus we get

$$(\sigma_2)_{zz} = F_2 \frac{\partial u_2}{\partial x} + C_2 \frac{\partial v_2}{\partial z} \tag{11.1}$$

$$(\sigma_2)_{zx} = L_2 \left(\frac{\partial u_2}{\partial z} + \frac{\partial v_2}{\partial x} \right), \tag{11.2}$$

where e_{xx} etc. are replaced by there values in terms of displacements. The equations of motion, when A_2, C_2, L_2 and F_2 are constants, are given by

$$\rho_2 \frac{\partial^2 u_2}{\partial t^2} = A_2 \frac{\partial^2 u_2}{\partial x^2} + L_2 \frac{\partial^2 u_2}{\partial z^2} + (F_2 + L_2) \frac{\partial^2 v_2}{\partial z \, \partial x} \tag{12.1}$$

$$\rho_2 \frac{\partial^2 v_2}{\partial t^2} = L_2 \frac{\partial^2 v_2}{\partial x^2} + C_2 \frac{\partial^2 v_2}{\partial z^2} + (F_2 + L_2) \frac{\partial^2 u_2}{\partial z \, \partial x}. \tag{12.2}$$

If we substitute

$$A_2 = C_2 = \lambda_2 + 2\mu_2, \quad F_2 = \lambda_2, \quad L_2 = \mu_2, \tag{13}$$

Eqs. (12.1), (12.2) are reduced to the familiar equations for isotropic homogeneous elastic media [17].

The boundary conditions are

(i) continuity of the x and z components of the displacements at the surface of interface, i.e. at $z = 0$,

$$u_1 = u_2 \tag{14.1}$$

$$v_1 = v_2 \tag{14.2}$$

(ii) continuity of the normal and tangential stresses at $z = 0$,

$$(\sigma_1)_{zz} = (\sigma_2)_{zz} \tag{15.1}$$

$$(\sigma_1)_{zx} = (\sigma_2)_{zx}. \tag{15.2}$$

3 Solution of the problem

We assume that α_1, in the upper half-space, varies as

$$\alpha_1{}^2(z) = \alpha_\infty^2 + (\alpha_0{}^2 - \alpha_\infty^2)\, e^{-\varepsilon z}, \tag{16}$$

where the subscripts 0 and ∞ mean the values of the parameters at $z = 0, z = \infty$, respectively. ε is a positive constant which has a dimension of $(\text{length})^{-1}$.

The general solution of Eq. (2) is given by

$$\phi_1 = G_1(z)\, e^{i(\omega t - kx)}, \tag{17}$$

where ω is the frequency and k is the horizontal wave number. $G(z)$ satisfies the differential equation

$$\frac{d^2G_1(z)}{dz^2} + G_1(z)\left(\frac{\omega^2}{\alpha^2(z)} - k^2\right) = 0. \tag{18}$$

The solution of Eq. (18) can be obtained by introducing the new variable ζ, where

$$\zeta = e^{-\varepsilon z}. \tag{19}$$

The semi-infinite interval $0 \leqq z \leqq \infty$ will be transformed to the unit interval $0 \leqq \zeta \leqq 1$. From Eq. (19) we find that

$$\frac{d}{dz} = -\varepsilon\zeta D, \tag{20}$$

where $D = \frac{d}{d\zeta}$. From (16) and (19)

$$\alpha_1 = \alpha_\infty(1 - \Lambda\zeta), \tag{21}$$

where

$$\Lambda = 1 - \alpha_0{}^2/\alpha_\infty^2. \tag{22}$$

Also, as $\lambda_1 = \mu_1$ then $\mu_1 = \dfrac{1}{3}\rho_1\alpha_\infty^2(1 - \Lambda\zeta)$ and we find that

$$\mu_1' = \frac{\varepsilon}{3}\rho_1\alpha_\infty^2(\Lambda\zeta), \qquad g = \varepsilon\frac{\Lambda\zeta}{1 - \Lambda\zeta}, \qquad g' = -\varepsilon^2\frac{\Lambda\zeta}{(1 - \Lambda\zeta)^2}.$$

Equation (18) in terms of ζ takes the form

$$(1 - \Lambda\zeta)\,\zeta D\zeta D G_1(\zeta) + \frac{1}{\varepsilon^2}\left(\frac{\omega^2}{\alpha_\infty^2} - k^2(1 - \Lambda\zeta)\right)G_1(\zeta) = 0. \tag{23}$$

Equation (23) is in a suitable form for solution by using the Frobenius power series method around $\zeta = 0$, viz,

$$G_1(\zeta) = \sum_{n=0}^{n=\infty} a_n\zeta^{n+m}. \tag{24}$$

Equation (23) gives

$$(1 - \Lambda\zeta)\sum_{n=0}^{n=\infty} a_n(n + m)^2\,\zeta^{n+m} + \frac{1}{\varepsilon^2}\left(\frac{\omega^2}{\alpha_\infty^2} - k^2(1 - \Lambda\zeta)\right)\sum_{n=0}^{n=\infty} a_n\zeta^{n+m} = 0. \tag{25}$$

Equating the coefficient of ζ^{n+m} to zero

$$a_n\left[(n + m)^2 + \frac{1}{\varepsilon^2}\left(\frac{\omega^2}{\alpha_\infty^2} - k^2\right)\right] - \Lambda a_{n-1}\left[(n + m - 1)^2 - \frac{k^2}{\varepsilon^2}\right] = 0. \tag{26}$$

Putting $n = 0$ in Eq. (26) and letting $a_{-1} = 0$ and $a_0 = 1$, we get

$$m_{1,2} = \pm l\left(1 - \frac{c^2}{\alpha_\infty^2}\right)^{1/2}, \tag{27}$$

where $c = \omega/k$ is the phase velocity of Stoneley waves and $l = k/\varepsilon$.

The recurrence relation (26) is reduced to

$$a_n(n^2 + 2mn) - \Lambda a_{n-1}\big((n + m - 1)^2 - l^2\big) = 0. \tag{28}$$

The general solution for $G_1(\zeta)$ is given by

$$G_1(\zeta) = Q_1\sum_{n=0}^{n=\infty} a_n^{(1)}\zeta^{n+m_1} + Y_1\sum_{n=0}^{n=\infty} a_n^{(2)}\zeta^{n+m_2}, \tag{29}$$

where $a_n^{(1)}$, $a_n^{(2)}$ are given by Eq. (28) after replacing m by m_1 and m_2, respectively. Q_1 and Y_1 are arbitrary constants. However, for large values of z, i.e. for very small values of ζ the only important terms in the series solution are the zeroth terms, from which one can deduce that in order to satisfy the radiation conditions at $z = \infty$, we have to set $Y_1 = 0$ [7]. Therefore, the solution for $G_1(\zeta)$ takes the form

$$G_1(\zeta) = Q_1\sum_{n=0}^{n=\infty} a_n\zeta^{n+m}, \tag{30}$$

where $m = m_1$.

Following the same procedures, we find that the solution of Eq. (3) is given by

$$\psi_1 = T_1(\zeta)\, e^{i(\omega t - kx)}, \tag{31}$$

where $T_1(\zeta)$ is given by

$$T_1(\zeta) = H_1 \sum_{n=0}^{n=\infty} b_n \zeta^{n+\nu}. \tag{32}$$

H_1 is an arbitrary constant. b_n is given by a similar equation as Eq. (28) after replacing a_n by b_n and m by ν, where

$$\nu = l\left(1 - \frac{c^2}{\beta_\infty^2}\right)^{1/2}. \tag{33}$$

From Eq. (1) we find that

$$u_1 = \frac{\partial \phi_1}{\partial x} - g\psi_1 - \frac{\partial \psi_1}{\partial z}, \qquad v_1 = -g\phi_1 + \frac{\partial \phi_1}{\partial z} + \frac{\partial \psi_1}{\partial x}.$$

Substituting for ϕ_1 and ψ_1 we get (after dropping the term $e^{i(\omega t - kx)}$)

$$u_1 = -i\varepsilon l Q_1 \sum_{n=0}^{n=\infty} a_n \zeta^{n+m} + \varepsilon H_1 \sum_{n=0}^{n=\infty} b_n \left(-\frac{A\zeta}{1 - A\zeta} + (n+\nu)\right) \zeta^{n+\nu} \tag{34}$$

$$v_1 = -\varepsilon Q_1 \sum_{n=0}^{n=\infty} a_n \left(\frac{A\zeta}{1 - A\zeta} + (n+m)\right) \zeta^{n+m} - il\varepsilon H_1 \sum_{n=0}^{n=\infty} b_n \zeta^{n+\nu}. \tag{35}$$

Also, from Eqs. (4) and (5) we find that

$$\begin{aligned}(\sigma_1)_{zz} = {}& \mu_1 \varepsilon^2 Q_1 \sum_{n=0}^{n=\infty} a_n \left(-l^2 + 3(n+m)^2 + \frac{3A\zeta}{1 - A\zeta}(n+m) + \frac{3A\zeta}{(1 - A\zeta)^2}\right) \zeta^{n+m} \\ & + i\mu_1 \varepsilon^2 l H_1 \sum_{n=0}^{n=\infty} b_n \left(2(n+\nu) + \frac{A\zeta}{1 - A\zeta}\right) \zeta^{n+\nu}\end{aligned} \tag{36}$$

$$\begin{aligned}(\sigma_1)_{zx} = {}& i\mu_1 \varepsilon^2 l Q_1 \sum_{n=0}^{n=\infty} a_n \left(2(n+m) + \frac{A\zeta}{1 - A\zeta}\right) \zeta^{n+m} \\ & + \mu_1 \varepsilon^2 H_1 \sum_{n=0}^{n=\infty} b_n \left(-l^2 - (n+\nu)^2 + \frac{A\zeta}{1 - A\zeta}(n+\nu) + \frac{A\zeta}{(1 - A\zeta)^2}\right) \zeta^{n+\nu}.\end{aligned} \tag{37}$$

As for the lower medium, we seek solutions of Eqs. (12.1) and (12.2) of the form

$$u_2 = G_2(z)\, e^{i(\omega t - kx)} \tag{38.1}$$

$$v_2 = T_2(z)\, e^{i(\omega t - kx)}. \tag{38.2}$$

$G_2(z)$ and $T_2(z)$ satisfy the equations

$$-\rho_2 \omega^2 G_2(z) = -k^2 A_2 G_2(z) + L_2 G_2''(z) - ik(F_2 + L_2)\, T_2'(z) \tag{39}$$

$$-\rho_2 \omega^2 T_2(z) = -k^2 L_2 T_2(z) + C_2 T_2''(z) - ik(F_2 + L_2)\, G_2'(z), \tag{40}$$

if we assume

$$G_2(z) = Pe^{ksz}, \quad T_2(z) = Re^{ksz}, \tag{41}$$

where P and R are constants. From (39) and (40) we find that P and R satisfy the following two equations:

$$(A_2 - \rho_2 c^2 - L_2 s^2)\, P + is(F_2 + L_2)\, R = 0 \tag{42}$$

$$(L_2 - \rho_2 c^2 - C_2 s^2)\, R + is(F_2 + L_2)\, P = 0. \tag{43}$$

Clearly, in order to have a non zero solution for (42) and (43), we must have

$$(A_2 - \rho_2 c^2 - L_2 s^2)\,(L_2 - \rho_2 c^2 - C_2 s^2) + s^2(F_2 + L_2)^2. \tag{44}$$

Using the notations

$$q = (A_2 - \rho_2 c^2), \quad r = (L_2 - \rho_2 c^2), \quad j = (F_2 + L_2), \quad \Gamma = rL_2 + qC_2 - j^2, \tag{45}$$

Eq. (44) can be written in the form

$$L_2 C_2 s^4 - \Gamma s^2 + rq = 0. \tag{46}$$

This is a quadratic form in s^2. Let $s_1{}^2$ and $s_2{}^2$ be the roots of (46), then

$$s_{1,2}^2 = \{\Gamma \pm (\Gamma^2 - 4L_2 C_2 rq)^{1/2}\}/\{2L_2 C_2\}. \tag{47}$$

These two roots are both positive real for elastic media with slight departure from isotropic media and when $c^2 < L_2/\rho_2$ [13].

From (47) the solutions for $G_2(z)$ and $T_2(z)$ which are bounded at $z = -\infty$ are given by

$$G_2(z) = Pe^{ksz} + Ee^{k\delta z} \tag{48}$$

$$T_2(z) = Re^{ksz} + Ne^{k\delta z}, \tag{49}$$

where s and δ are the positive roots of $s_1{}^2$ and $s_2{}^2$, respectively. E and N are constants. P, R and also E, N are not independent, as they are related by Eq. (42) and (43). That is,

$$\frac{R}{P} = i\,\frac{q - L_2 s^2}{sj} = i\eta \quad \text{and} \quad \frac{N}{E} = i\,\frac{q - L_2 \delta^2}{\delta j} = i\gamma. \tag{50}$$

Therefore, the displacements in the lower half-space (after dropping the term $e^{i(\omega t - kx)}$) are given by

$$u_2 = Q_2 e^{ksz} + H_2 e^{k\delta z} \tag{51}$$

$$v_2 = i(\eta Q_2 e^{ksz} + \gamma H_2 e^{k\delta z}), \tag{52}$$

where Q_2 and H_2 are arbitrary constants.

From Eqs. (11.1) and (11.2) we can obtain the required stresses, without the term $e^{i(\omega t - kx)}$, in the form

$$(\sigma_2)_{zz} = ikQ_2(-F_2 + C_2\eta s)\, e^{ksz} + ikH_2(-F_2 + C_2\gamma\delta)\, e^{k\delta z} \tag{53}$$

$$(\sigma_2)_{zx} = L_2 kQ_2(s + \eta)\, e^{ksz} + L_2 kH_2(\delta + \gamma)\, e^{k\delta z}. \tag{54}$$

Applying the boundary conditions in Eqs. (14.1), (14.2), (15.1) and (15.2) and equating the determinant of the coefficients of Q_1, H_1, Q_2 and H_2 to zero, we obtain the dispersion equation for Stoneley waves in the form

$$\det A = 0, \tag{55}$$

where $A = \{A_{ij}\}$, i denotes rows and j denotes columns and

$$A_{11} = -k \sum_{n=0}^{n=\infty} a_n, \qquad A_{12} = \varepsilon \sum_{n=0}^{n=\infty} b_n \pi_{12}$$

$$A_{13} = -1, \qquad A_{14} = -1$$

$$A_{21} = -\varepsilon \sum_{n=0}^{n=\infty} a_n \pi_{21}, \qquad A_{22} = k \sum_{n=0}^{n=\infty} b_n$$

$$A_{23} = \eta, \qquad A_{24} = \gamma$$

$$A_{31} = \mu_0 \varepsilon^2 \sum_{n=0}^{n=\infty} a_n \pi_{31}, \qquad A_{32} = -\mu_0 \varepsilon^2 l \sum_{n=0}^{n=\infty} b_n \pi_{32}$$

$$A_{33} = k(-F_2 + C_2 \eta s), \qquad A_{34} = k(-F_2 + C_2 \gamma \delta)$$

$$A_{41} = \mu_0 \varepsilon^2 l \sum_{n=0}^{n=\infty} a_n \pi_{41}, \qquad A_{42} = \mu_0 \varepsilon^2 \sum_{n=0}^{n=\infty} b_n \pi_{42}$$

$$A_{43} = -L_2 k(s + \eta), \qquad A_{44} = -L_2 k(\delta + \gamma),$$

where μ_0 is the value of μ_1 at $z = 0$,

$$\pi_{12} = -\Omega 1 + (n + v), \ \pi_{21} = \Omega 1 + (n + m), \ \pi_{31} = -l^2 + 3(n + m)^2 + 3\Omega 1(n + m) + 3\Omega 2,$$

$$\pi_{32} = 2(n + v) + \Omega 1, \quad \pi_{41} = 2(n + m) + \Omega 1, \quad \pi_{42} = -l^2 - (n + v)^2 + \Omega 1(n + v) + \Omega 2,$$

$$\Omega 1 = A/(1 - A) \quad \text{and} \quad \Omega 2 = A/(1 - A)^2.$$

4 Numerical study of the dispersion equation

In this Section we study the dispersion equation of Stoneley waves for four models designated as $M1$, $M2$, $M3$ and $M4$. In all models we have taken $\alpha_0 = 3$, $\alpha_\infty = 5$ and $\rho_1 = 2$. For ε we considered $\varepsilon = 1$ for $M1$ and $M2$, $\varepsilon = 0.5$ for $M3$ and $\varepsilon = 10$ for $M4$. The elastic constants for the lower half-space for $M1$, $M3$ and $M4$ are taken as in [13], viz,

$$A_2 = 26.94, \quad C_2 = 23.63, \quad F_2 = 6.61, \quad L_2 = 6.53, \quad \rho_2 = 2.7$$

while for $M2$ we considered

$$A_2 = C_2 = 26.94, \quad F_2 = L_2 = 6.53, \quad \rho_2 = 2.7$$

meaning that the lower half-space in $M2$ is isotropic.

Figure 1 for $M1$ shows that with increasing the wave number k, i.e. with decreasing the wave length, the phase velocity c decreases indicating a normal dispersion behavior. The rate of decrease in c with respect to k is slow at very small values of k, then it increases with the values k until it becomes very sharp around $k \approx 1.82$. After this value of k, the dispersion equation has no roots which means that Stoneley waves for this model do not exist for values of k greater than

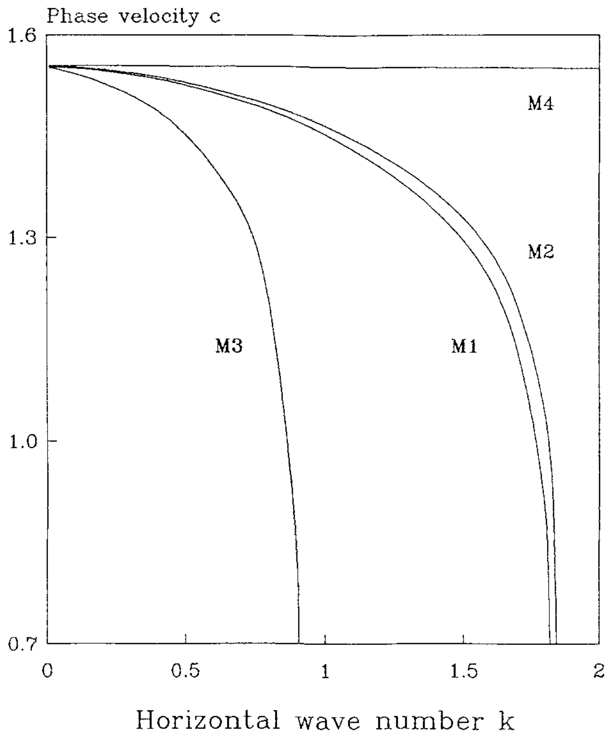

Fig. 1

1.82. In other words, this value of k is a cut-off value for the existence of Stoneley waves for $M1$, we denote it by k_c.

The results for $M2$ are almost the same as for $M1$ which means that the effect of anisotropy is very small on the propagation of Stoneley waves and for practical applications it can be neglected.

As for $M3$ we find that the dependence of the phase velocity on the wave number has the same features as the other two cases. Also, we find that k_c for this model $\approx .906$. The results for $M4$ are shown in the same figure up to $k = 2$, where it is clear that for this range of k the phase velocity of Stoneley waves is nearly constant. The full results for $M4$ are given in Table 1.

Table 1

k	2	6	10	14	16	18	18.2
c	1.55	1.55	1.49	1.4	1.28	0.98	0.82

Clearly, the general behavior is still the same and we find that k_c for $M4$ is about 18.2.

The comparison of the cut-off value for $M1$, $M3$ and $M4$ shows that k_c is nearly linearly dependent on ε. Also, we note that the variation in α_1, μ_1, etc. in the upper half-space takes place over a certain width due to the exponential variation which is given by Eq. (16). The width of this transition zone depends mainly on ε [7]. This means that the cut-off wave number depends on the width of the transition zone.

5 Concluding remarks

In this paper we investigated the effect of inhomogeneity and anisotropy on Stoneley waves. We found that the effect of anisotropy is very small and it can be neglected, while the effect of inhomogeneity is very strong. We found that for the kind of variations given in this paper, Stoneley waves do not exist after a certain value of the wave number. This value depends mainly on the width of the transition zone.

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