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Unsteady forced convection laminar boundary layer flow over a moving longitudinal cylinder

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Summary. The unsteady nonsimilar forced convection flow over a longitudinal cylinder, which is moving in the same or in the opposite direction to the free stream, has been investigated. The unsteadiness is due to the free stream velocity, cylinder velocity, surface temperature of the cylinder and the mass transfer, and nonsimilarity is due to the transverse curvature. The partial differential equations, governing the flow, have been solved numerically, using an implicit finite-difference scheme in combination with a quasilinearization technique. The results show that both, skin friction and heat-transfer, are appreciably affected by the free stream velocity distributions and by the cylinder velocity. Also, skin friction as well as heat-transfer are found to increase as the transverse curvature or the suction increases, but the effect of injection is just the opposite. The heat-transfer is significantly affected by the viscous dissipation and variation of surface temperature with time. It is observed that results of this problem are crucially dependent on the parameter α, which is the ratio of the velocity of the cylinder to the velocity of the free stream. In particular, it is found that solutions for the upstream moving cylinder exist only for a certain range of this parameter (α), and they are nonunique in a small range of α too.

List of symbols

A	constant
C_f	skin friction coefficient, Eq. (21)
Nu_x	local Nusselt number (heat-transfer coefficient), Eq. (22)
Ec	Eckert number (dissipation parameter)
c_p	specific heat at constant pressure
k	thermal conductivity
q	heat transfer rate per unit area
F, G	dimensionless velocity and normalized temperature respectively
T	temperature of the fluid
T_w	surface temperature of the cylinder
μ, ν, ϱ	coefficient of viscosity, kinematic viscosity and density respectively
Pr	Prandtl number ($\mathrm{Pr} = \mu c_p/k$)
r, x	radial and axial co-ordinates
Re_x	local Reynolds number
t^*, t	dimensionless and dimensional times respectively
u, v	velocity components of the fluid along x- and r-directions respectively
U	free stream velocity
η, ξ	transformed coordinates
τ	shear stress
R, ϕ	functions of t^*
$\bar{R}$	radius of the cylinder
ψ, f	dimensional and dimensionless stream functions
$\varepsilon, \varepsilon_1$	constants

$u_w(t)$ time-dependent cylinder velocity
α ratio of the velocity of the cylinder to the velocity of the free stream
F_w', G_w' skin friction and heat-transfer parameter respectively

Superscripts

' derivative with respect to η

Subscripts

e, w conditions at the edge of the boundary layer and on the surface of the cylinder respectively
w_0 conditions on the surface of the cylinder at $t^* = 0$
i initial condition
r, t, t^*, x, ξ derivatives with respect to r, t, t^*, x, ξ respectively
∞ free stream value

1 Introduction

Boundary-layer flows over a moving or stretching surface are of great importance in view of their relevance to a wide variety of technical applications, particularly in the manufacture of fibres in glass and polymer industries. The investigation of drag, heat and mass transfer in such situations belongs to a separate class of problems in boundary-layer theory, distinguishing itself from the study of flows over static surfaces. The first and foremost work regarding the boundary-layer behavior on moving surfaces in a quiescent fluid was considered by Sakiadis [1]. Subsequently, several investigators [2]–[13] have worked on the problem of a moving or stretching surface (flat plates) under different situations. Simultaneous studies on boundary-layer flows over moving cylindrical surfaces alone have also been considered by various researchers. The work of Sakiadis [14] who restricted his study only to momentum transfer in the boundary-layer on a continuous cylindrical surface was extended by Bourne and Elliston [15], to include heat-transfer also. The Karman-Pahlhausen integral technique was adopted in their analysis. The accuracy of their integral solutions was tested by Karnis and Pechoc [16] who obtained exact solutions of the boundary-layer equations on a continuous isothermal cylinder for $\Pr \gtrless 1$ by a power series method. Choi [17] has considered the boundary-layer flow on a moving longitudinal cylinder, taking into account the effect of the variable properties of air. The solution has been obtained by both, the momentum integral method and the finite-difference scheme. All the aforementioned studies [14]–[17] are related to moving cylinders in a fluid at rest. Recently, Pop et al. [18] have studied the problem of steady forced convection boundary-layer of non-Newtonian fluids on a continuously moving cylinder, using the box method due to Keller and Cebeci [19]. The problem of a longitudinal cylinder moving in a fluid having a time-dependent free stream, has not been considered so far.

The aim of the present analysis is to study the unsteady nonsimilar laminar and incompressible forced flow over a moving longitudinal cylinder when the free stream velocity, cylinder velocity and surface temperature of the cylinder vary arbitrarily with the time. The cylinder is assumed to move in the same direction ($\alpha > 0$) or in the opposite direction ($\alpha < 0$) to the free stream. The transverse curvature of the cylinder brings nonsimilarity into the flow. The influence of mass transfer and viscous dissipation has been included in the analysis. It may be noted, that here the cylinder is moving as a rigid boundary. The partial differential equations, involving three independent variables governing the flow, are solved using an implicit finite-difference scheme in combination with a quasilinearization technique. The results are found to be dependent on the parameter α, which is the ratio of cylinder velocity to the free stream velocity.

2 Analysis

We consider the unsteady, laminar boundary-layer forced flow of a viscous incompressible fluid over a long thin longitudinal cylinder of radius $\bar{R}$, moving axially with a time-dependent velocity $u_w(t)$ (see Fig. 1). We assume that the free stream velocity, the surface temperature of the cylinder and the surface mass transfer vary arbitrarily with time. The free stream temperature is taken to be constant. We also assume that the radius of the cylinder is large compared with the boundary-layer thickness, so that the boundary-layer curvature can be neglected. Under the foregoing assumptions, the boundary-layer equations, governing the flow, can be expressed as

$$(ru)_x + (rv)_r = 0 \tag{1}$$

$$u_t + uu_x + vu_r = (\nu/r)\,(ru_r)_r + (u_e)_t \tag{2}$$

$$T_t + uT_x + vT_r = \mathrm{Pr}^{-1}\,(\nu/r)\,(rT_r)_r + (\mu/\varrho c_p)\,(u_r)^2. \tag{3}$$

The relevant initial and boundary conditions are given by

$$u(x, r, 0) = u_i(x, r), \quad v(x, r, 0) = v_i(x, r), \quad T(x, r, 0) = T_i(x, r) \tag{4}$$

$$u(x, \bar{R}, t) = u_w(t) = u_{w0}\phi(t^*), \quad v(x, \bar{R}, t) = v_w$$

$$T(x, \bar{R}, t) = T_w(t) = T_\infty + (T_{w0} - T_\infty)\,R(t^*) \tag{5}$$

$$u(x, \infty, t) = u_e(t) = U\phi(t^*), \quad T(x, \infty, t) = T_\infty.$$

Here $\phi(t^*)$ and $R(t^*)$ are arbitrary functions of t^*, representing the nature of the unsteadiness in the free stream velocity (cylinder velocity) and in the surface temperature of the cylinder, respectively.

Applying the transformations

$$\xi = (4/\bar{R})\,(\nu x/U)^{1/2}, \quad \eta = (U/\nu x)^{1/2}\,[(r^2 - \bar{R}^2)/4\bar{R}], \quad t^* = (\nu/\bar{R}^2)\,t \tag{6}$$

$$\psi(x, r, t) = \bar{R}(\nu Ux)^{1/2}\,\phi(t^*)\,f(\xi, \eta, t^*) \tag{7}$$

$$G(\xi, \eta, t^*) = (T - T_{w0})/(T_\infty - T_{w0}) \tag{8}$$

$$F = f' \tag{9}$$

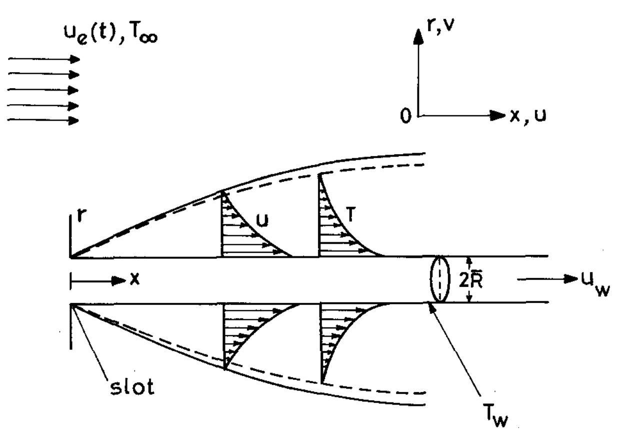

Fig. 1. Physical model and co-ordinate system

to Eqs. (1)–(3), we find that Eq. (1) is identically satisfied and Eqs. (2) and (3) reduce, respectively, to

$$(1 + \xi\eta) F'' + (\xi + \phi f) F' - (\xi^2/4) [F_{t^*} + \phi^{-1}\phi_{t^*}(F - 2)] = \xi\phi(FF_\xi - f_\xi F') \tag{10}$$

$$\mathrm{Pr}^{-1} (1 + \xi\eta) G'' + (\xi \,\mathrm{Pr}^{-1} + \phi f) G' - (\xi^2/4) G_{t^*} + (1 + \xi\eta) \,\mathrm{Ec}\, \phi^2(F')^2$$
$$= \xi\phi(FG_\xi - f_\xi G'), \tag{11}$$

where

$$u = (\psi_r/r) = U\phi(t^*) F(\xi, \eta, t^*)/2 \tag{12}$$

$$v = -(\psi_x/r) = -\bar{R}\phi(t^*) (\nu U/x)^{1/2} \{f + \xi f_\xi - \eta F\}/2r \tag{13}$$

$$f(\xi, \eta, t^*) = \int_0^\eta F \, d\eta + f_w \tag{14}$$

$$f_w = -(\bar{R}/2\nu\phi\xi) \int_0^\xi \xi v_w \, d\xi \tag{15}$$

$$\mathrm{Ec} = U^2/[4c_p(T_\infty - T_{w0})]. \tag{16}$$

The boundary conditions (5) are accordingly transformed to

$$F = \alpha, \quad G = 1 - R(t^*) \quad \text{at} \quad \eta = 0$$
$$F \to 2, \quad G \to 1 \quad \text{as} \quad \eta \to \infty \tag{17}$$

for $\xi \geqq 0$ and $t^* \geqq 0$, where $\alpha = 2(u_w/u_e)$.

The flow is assumed to become unsteady only if $t^* > 0$, and being steady at $t^* = 0$. Thus, the initial conditions for $F(\xi, \eta, t^*)$ and $G(\xi, \eta, t^*)$ at $t^* = 0$ are given by the steady flow equations obtained by putting

$$\phi(t^*) = R(t^*) = 1, \quad \phi_{t^*} = F_{t^*} = G_{t^*} = 0 \tag{18}$$

in Eqs. (10) and (11). Consequently, the initial conditions can be written as

$$(1 + \xi\eta) F'' + (\xi + f) F' = \xi(FF_\xi - f_\xi F') \tag{19}$$

$$\mathrm{Pr}^{-1} (1 + \xi\eta) G'' + (\xi \,\mathrm{Pr}^{-1} + f) G' + (1 + \xi\eta) \,\mathrm{Ec}\, (F')^2 = \xi(FG_\xi - f_\xi G'). \tag{20}$$

Equations (19) and (20), in the absence of viscous dissipation, are precisely the equations governing a steady forced convection flow, which have been studied by Sparrow and Yu [20], Wanous and Sparrow [21] and Sparrow et al. [22] for $\alpha = 0$ (i.e., when the cylinder is at rest). It may also be noted that Eqs. (19) and (20) for $\alpha \geqq 0$ reduce to those of Abdelhafez [7] if we put $\mathrm{Ec} = \xi = 0$, and replace f by $(f/2)$ in them. Abdelhafez studied the skin friction and heat-transfer on a continuous flat surface, moving in a parallel free stream. It may be remarked here that Chappidi and Gunnerson [9] have obtained a closed form solution for the flow and the thermal transport from a flat surface ($\xi = 0$), moving through a flowing fluid. They used an integral technique with a perturbation procedure. However, the governing equations, considered by Chappidi et al. are in integral form. Furthermore, Eqs. (19)–(20), for $\alpha = 2$, are easily recognized to be those of Karnis and Pechoc [16], who analysed the thermal laminar boundary-layer on a continuous moving cylinder, neglecting the viscous dissipation. It is noted here, that $\xi = 0$

corresponds to the flat plate case and $\xi > 0$ represents the transverse curvature effect. Also, it provides a measure of the distance along the cylinder.

The local skin friction and the local heat-transfer coefficients can be expressed as

$$C_f = 2(\tau_w/\varrho(u_e^2)_{t^*=0}) = \phi(t^*)\,(\mathrm{Re}_x)^{-1/2}\,F_w'/2 \tag{21}$$

$$\mathrm{Nu}_x = (x/k)\,(q_w/(T_{w0} - T_\infty)) = (\mathrm{Re}_x)^{1/2}\,G_w'/2 \tag{22}$$

where

$$\tau_w = \mu(\partial u/\partial r)_w, \quad \mathrm{Re}_x = (Ux/v) \quad \text{and} \quad q_w = -k(\partial T/\partial r)_w. \tag{23}$$

For computation, we have considered three unsteady free stream velocity distributions, which are given by

$$\phi(t^*) = 1 + \varepsilon t^{*2} \quad \text{(accelerating flow)} \tag{24}$$

$$\phi(t^*) = 1 - \varepsilon t^{*2}, \quad \varepsilon > 0 \quad \text{(decelerating flow)} \tag{25}$$

and

$$\phi(t^*) = 1 - a[1 - \exp(-ct^{*2})], \quad a > 0, \quad c > 0 \quad \text{(exponentially decelerating flow)}. \tag{26}$$

Further, we have taken the surface temperature distribution with time in the form:

$$R(t^*) = 1 + \varepsilon_1 t^*, \quad \varepsilon_1 > 0. \tag{27}$$

We have also considered two types of surface mass transfer distributions viz., v_w = constant or $v_w = v_0 x^{-1/2}$. Accordingly, the surface mass transfer parameter f_w, given in Eq. (15) becomes

$$f_w = A\xi/\phi(t^*), \quad A = -\bar{R}v_w/4v, \quad \text{when } v_w \text{ is constant} \tag{28}$$

$$f_w = A/\phi(t^*), \quad A = -v_0/(vU)^{1/2}, \quad \text{when } v_w = v_0 x^{-1/2} \tag{29}$$

where $A > 0$ is for suction and $A < 0$ for injection.

3 Results and discussion

The partial differential equations (10) and (11), with the boundary conditions (17) and initial conditions (19) and (20), have been solved using an implicit finite-difference scheme in combination with a quasilinearization technique. Since the method is described in detail in [23], its description is omitted here. To produce grid independent numerical results, the step sizes $\Delta\eta$, $\Delta\xi$ and Δt^* have been optimized. For this purpose, the computed values of physical parameters with a step size $\Delta\eta$ (keeping $\Delta\xi$ and Δt^* fixed), are compared with those obtained using reduced step sizes viz., $(\Delta\eta/2)$, $(\Delta\eta/4)$ and so on. The percentage of the difference in all these values is less than 0.05%. Further, these computed values have been extrapolated[1], using Richardson's extrapolation [24], to obtain higher order accuracy. The extrapolated values are again compared with those obtained using different step sizes and the percentage of difference is found to be less than a maximum of 0.03%. Consequently, the step sizes $\Delta\eta = \Delta\xi = \Delta t^* = 0.05$ have been used

[1] Tables containing Richardson's extrapolated values of skin friction and heat-transfer, for various values of step sizes, can be obtained from the authors.

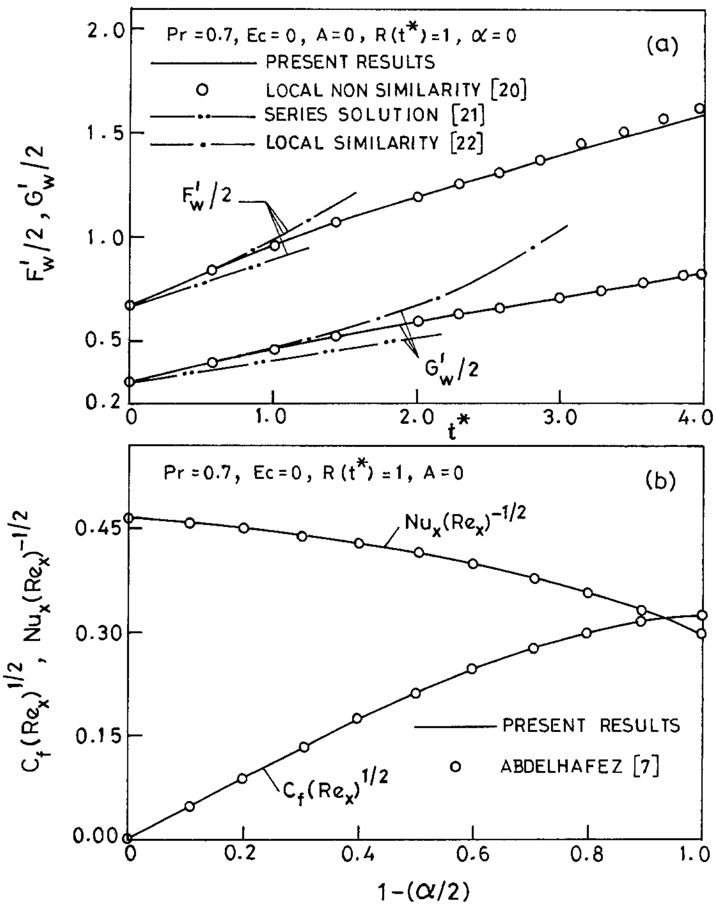

Fig. 2. Comparison of steady-state results

for the computation. The value of η_∞ (i.e., the edge of the boundary-layer) has been chosen as 5.0. All computations were performed in double precision on VAX-8810, for various values of α: $(-2 < \alpha \leqq 2)$. A typical case takes 1.05 minutes CPU time, when the computation is done from $\xi = 0$ to $\xi = 1.5$.

In order to assess the accuracy of our method, particular cases of our results have been obtained and these are compared in Fig. 2 with those available in literature. The skin friction and heat-transfer results (F_w', G_w'), obtained for the steady case with $\alpha = A = \mathrm{Ec} = 0$, have been compared with those given in [20]–[22]. The comparison is shown in Fig. 2a. The heat-transfer result $(G_w'/2)$ is found to be in good agreement with the local nonsimilarity method of Sparrow and Yu [20] while the skin friction result $(F_w'/2)$ for $\xi > 3.0$ differs from corresponding results of [20] in the maximum about 1%. The series solution of Wanous and Sparrow [21] and the local similarity results of Sparrow et al. [22] differ very much from the present results for $\xi \geqq 0.5$, indicating the inadequacy of these approximation methods. Results for the steady state skin friction and heat transfer coefficients $[C_f(\mathrm{Re}_x)^{1/2}, \mathrm{Nu}_x(\mathrm{Re}_x)^{-1/2}]$ have also been obtained for a continuous moving flat surface $(\xi = 0)$. These are compared with those of Abdelhafez [7]. The results, as seen in Fig. 2b, are in excellent agreement. The analogous steady state heat-transfer results $[\mathrm{Nu}_x(\mathrm{Re}_x)^{-1/2}]$, obtained for a moving flat surface $(\xi = 0)$, have also been compared with those of Chappidi and Gunnerson [9]. The comparison is shown in Table 1. The heat-transfer

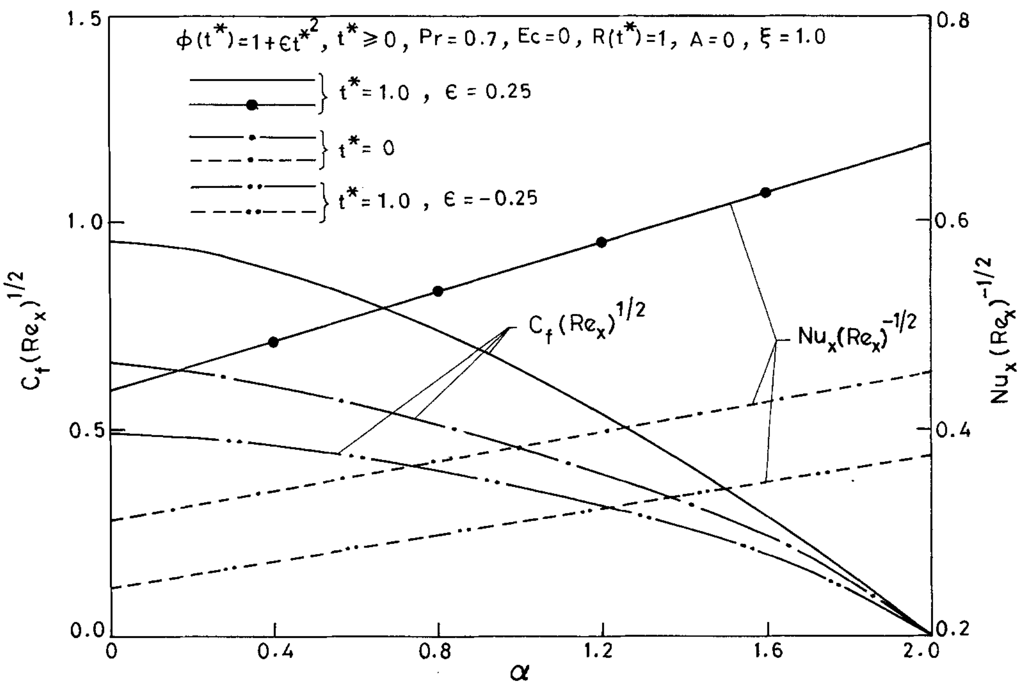

Fig. 3. Skin friction and heat-transfer for downstream moving cylinder

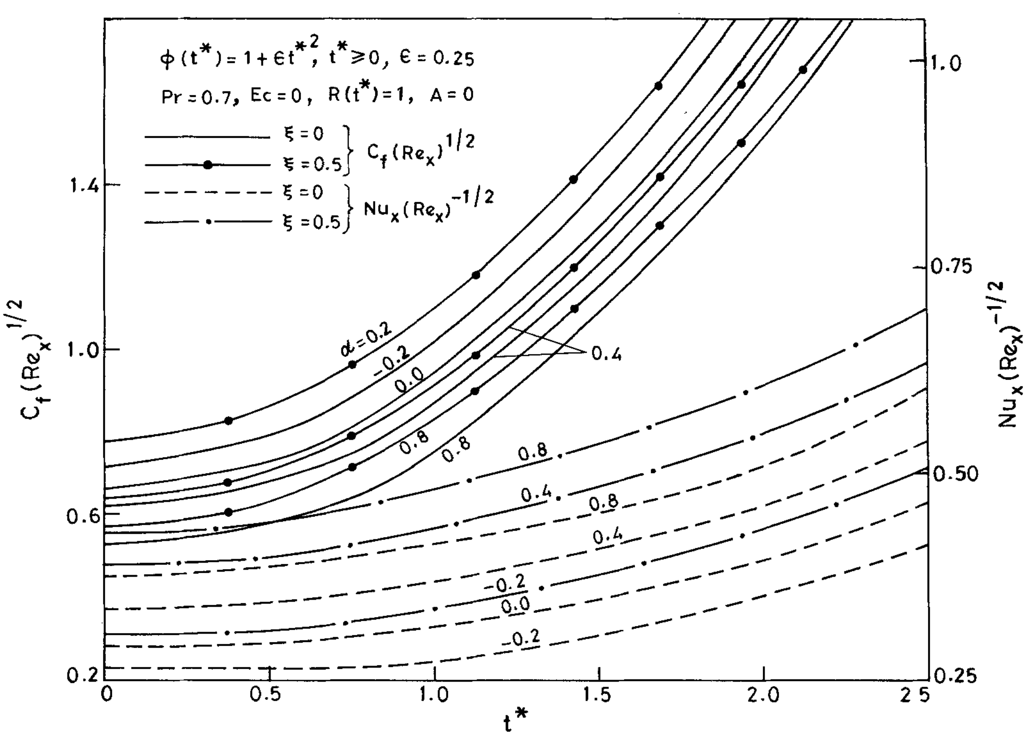

Fig. 4. Skin friction and heat-transfer for various values of α

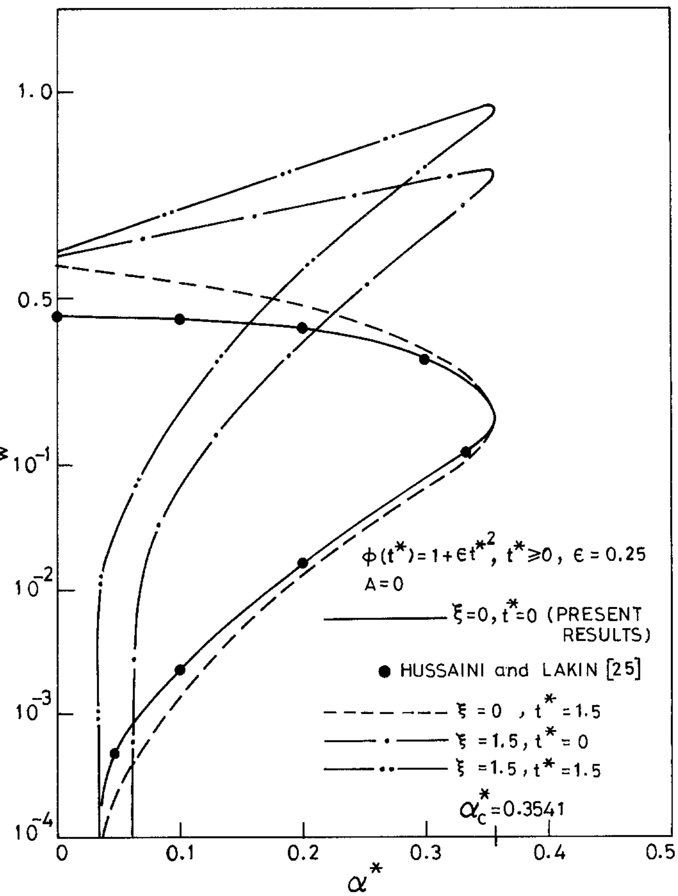

Fig. 5. Skin friction as a function of α^* for upstream moving cylinder

Table 1. Comparison of $Nu_x(Re_x)^{-1/2}$ with those of Chappidi and Gunnerson [9] ($Pr = 0.7$)

$R = 1 - \alpha/2$	Present results	Ref. [9]
0	0.4722	0.4583
0.2	0.4434	0.4362
0.4	0.4120	0.4132
0.6	0.3778	0.3872
0.8	0.3385	0.3572
1.0	0.2922	0.3226

results of [9] are within 4% of the present numerical solutions for $R = 0$, where $R = 1 - (\alpha/2)$ ($0 \leqq \alpha \leqq 2$, $0 \leqq R \leqq 1$). But, as $R \to 1$, the percentage of the difference increases to nearly 10%. This could be attributed to the fact that the closed form results of [9] are obtained by an approximate method. Furthermore, the comparison of our steady state heat-transfer results [$Nu_x(Re_x)^{-1/2}$] for the moving cylinder ($\alpha = 2$) with that of Karnis and Pechoc [16] is shown in Table 2. The agreement is good for small curvature, i.e., when $\xi \ll 1$. However, the series expansion results of [16] seem to be inaccurate for higher values of ξ.

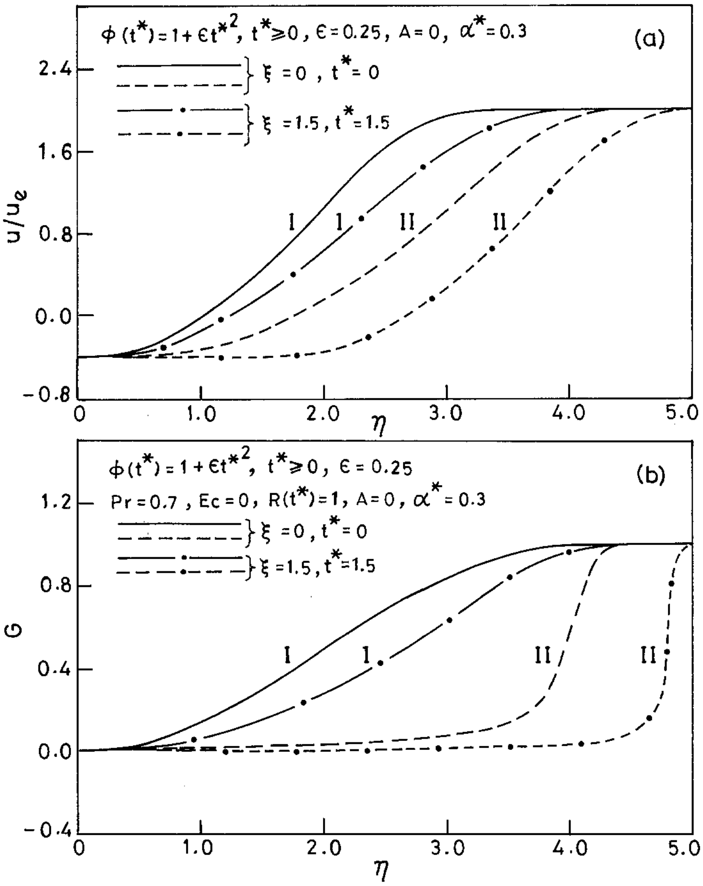

Fig. 6. a Velocity profiles **b** Temperature profiles (upstream moving cylinder)

Table 2. Comparison of $Nu_x(Re_x)^{-1/2}$ with those of Karnis and Pechoc [16]

ξ	Pr = 0.7		Pr = 1.0	
	Present results	Ref. [16]	Present results	Ref. [16]
0.0001	0.35144	0.35288	0.44411	0.44754
0.0005	0.35290	0.35736	0.44612	0.45221
0.001	0.35434	0.36070	0.44761	0.45570
0.005	0.36261	0.37467	0.45613	0.47026
0.01	0.37054	0.38499	0.46427	0.48103
0.04	0.40504	0.41930	0.49895	0.51682
0.05	0.41443	0.42719	0.50817	0.52506
0.06	0.42328	0.43425	0.51667	0.53243
0.07	0.43158	—	0.52467	0.53915

The effect of α (the ratio of the velocity of the cylinder to the velocity of the free stream) on $C_f(Re_x)^{1/2}$ and $Nu_x(Re_x)^{-1/2}$ in the range $0 \leqq \alpha \leqq 2$ is shown in Fig. 3. This figure contains results for the steady, the accelerating as well as for the decelerating flows. It is found, that $C_f(Re_x)^{1/2}$ decreases when α increases, but $Nu_x(Re_x)^{-1/2}$ increases. This is due to the fact, that

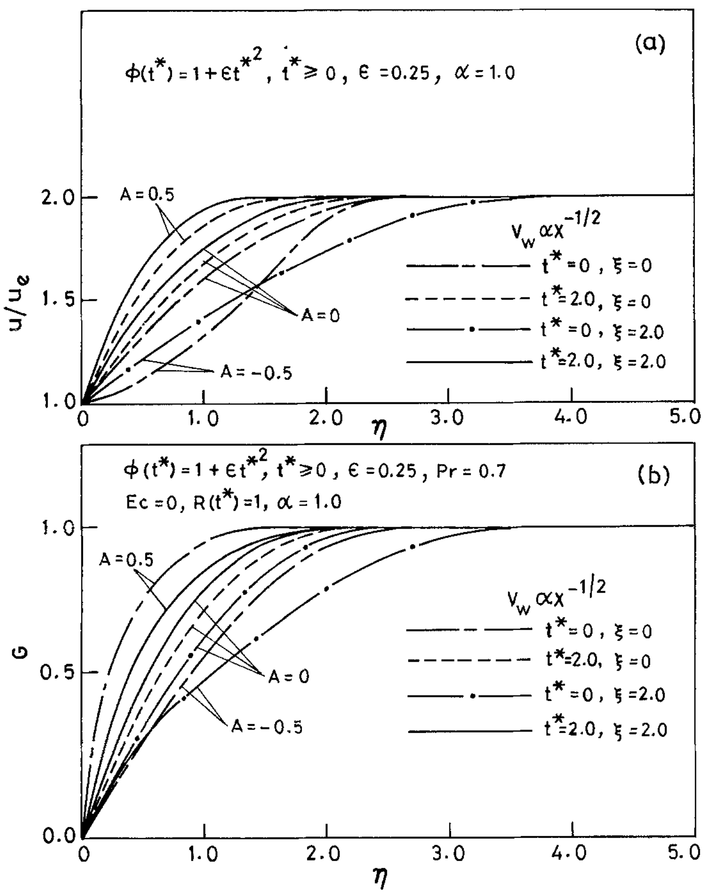

Fig. 7. a Velocity profiles **b** Temperature profiles (accelerating free stream)

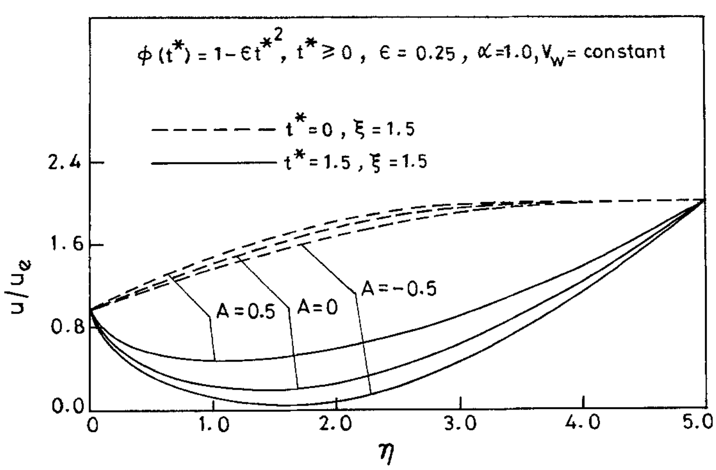

Fig. 8. Velocity field in the decelerating flow

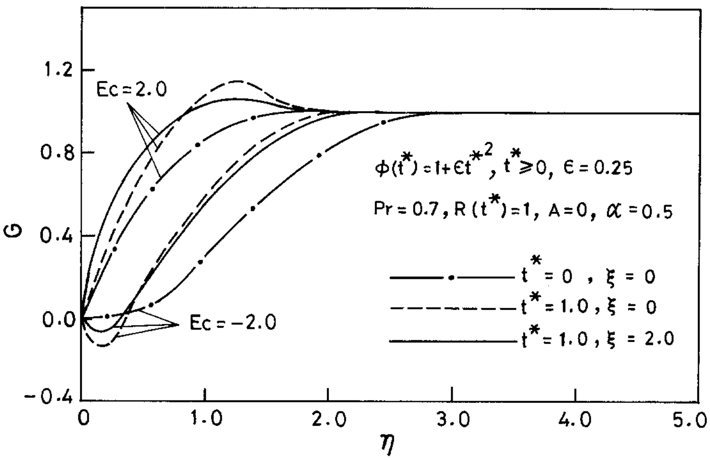

Fig. 9. Influence of viscous dissipation on the temperature field

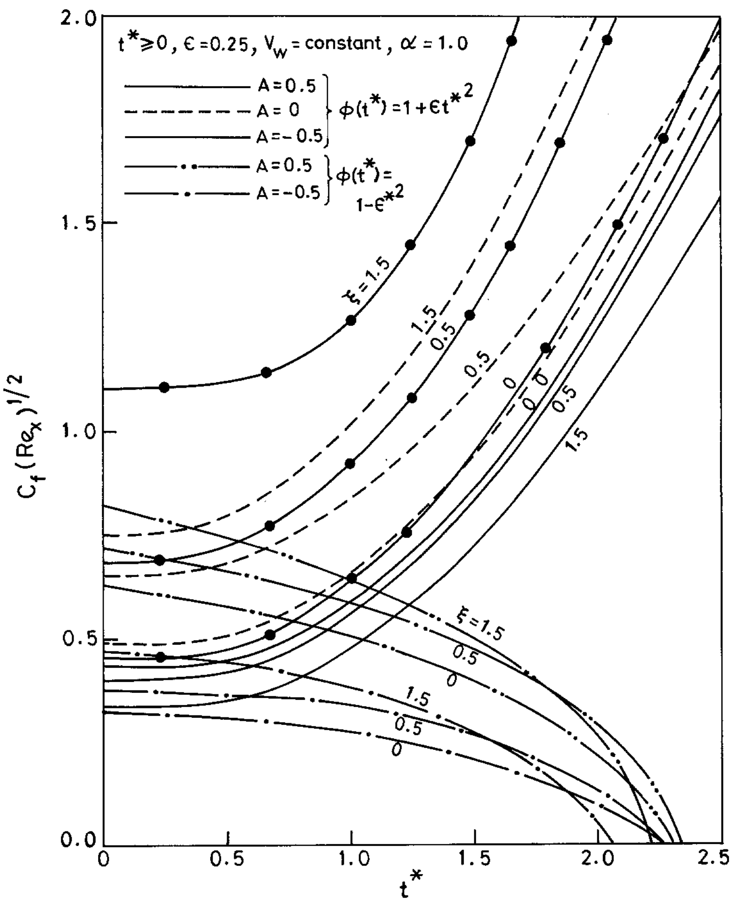

Fig. 10. Skin friction coefficient results

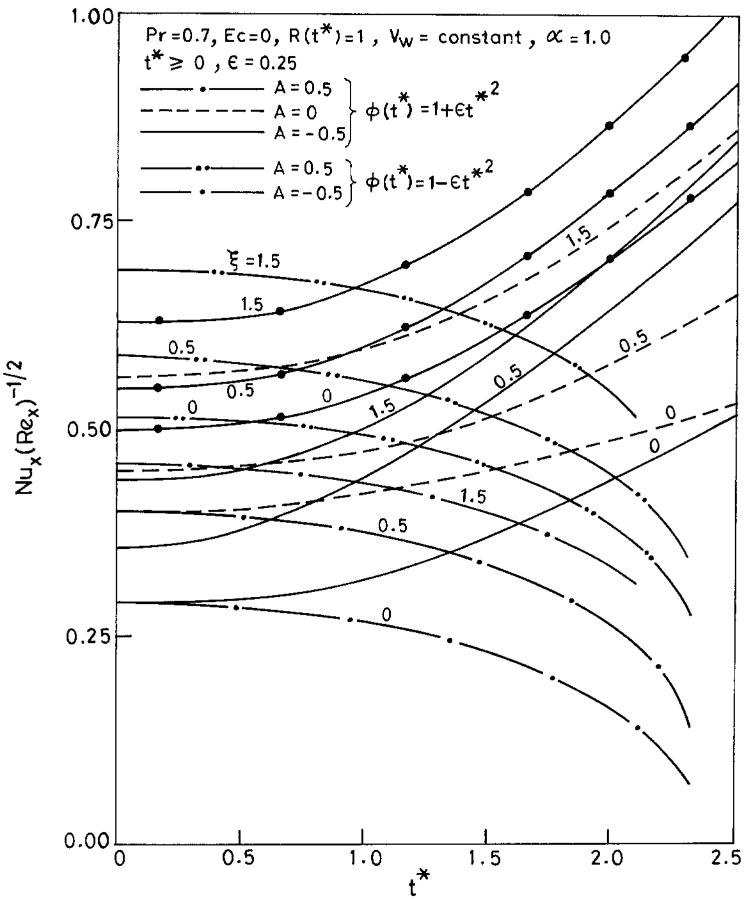

Fig. 11. Heat-transfer coefficient results

when $\alpha \to 2$ ($u_w \to u_e$), the fluid tends to be inviscid. This causes a considerable reduction in skin friction. In fact, $C_f(\mathrm{Re}_x)^{1/2}$ vanishes when $\alpha = 2$. On the other hand, the difference between the surface temperature and the free stream temperature ($T_w - T_\infty$) increases. This results in an increase in heat-transfer.

The variation of the skin friction and the heat-transfer $[C_f(\mathrm{Re}_x)^{1/2}, \mathrm{Nu}_x(\mathrm{Re}_x)^{-1/2}]$ with time t^*, when $\phi(t^*) = 1 + \varepsilon t^{*2}$ ($\varepsilon > 0$), for several values of α and for two values of the curvature parameter ξ is shown in Fig. 4. It is observed that the increase in α results in the increase of $\mathrm{Nu}_x(\mathrm{Re}_x)^{-1/2}$, but $C_f(\mathrm{Re}_x)^{1/2}$ decreases. However, for a given α, with respect to time, both $C_f(\mathrm{Re}_x)^{1/2}$ and $\mathrm{Nu}_x(\mathrm{Re}_x)^{-1/2}$ increase considerably.

The skin friction values for upstream moving cylinders are interesting. It may be remarked here that Hussaini and Lakin [25] and Hussaini et al. [26] have studied the problem of boundary-layer flow on a flat plate ($\xi = 0$), which is moving with constant velocity α^* ($\alpha^* = -\alpha$, $\alpha > 0$), opposite in the direction to that of the free stream. It was found, in their work, that the solution exists only if α^* does not exceed a critical value α_c^*, where $\alpha_c^* = 0.3541$. Furthermore, the solutions of the problem are found to be nonunique in the region $\alpha_c^* < \alpha^* < 0$. For our case, being identical to that one of Hussaini and Lakin [25] for $\xi = t^* = 0$ and $F(\infty) = 1$, a similar phenomenon is observed in connection with the calculation of the skin friction. The present value of the skin friction (F_w') at $\alpha_c^* = 0.3541$ was found to be 0.2185, as compared to 0.2180, being

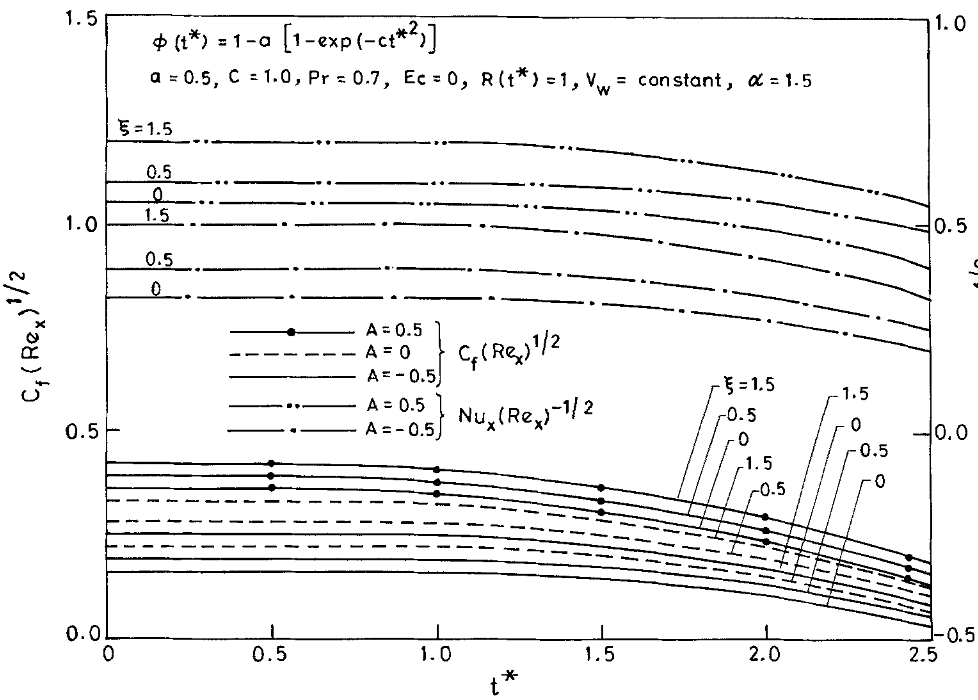

Fig. 12. Skin friction and heat-transfer for exponentially decelerating flow

reported in [25]. To illustrate the nature of the flow in this noteworthy region of $\alpha_c^* < \alpha^* < 0$, we have obtained results for different values of ξ and t^*. These are presented in Figs. 5 and 6. Figure 5 shows the representative values of F_w' along with the comparison of our steady state results for the flat plate with those of [25]. Dual values of F_w' for all values of ξ and t^* are easily seen in this figure, confirming the existence and non-uniqueness of solutions of our boundary-layer problem for $\alpha_c^* < \alpha^* < 0$. The relevant velocity and temperature fields have been studied in the range of α^* under consideration, corresponding to two values of skin friction (see Fig. 5). The appropriate velocity and temperature profiles have been presented in Fig. 6. The profiles, corresponding to the upper branch of surface shear stress (I), increase monotonically across the boundary-layer to reach their free stream values. On the other hand, profiles corresponding to the lower branch of surface shear stress (II), reveal the thickening of the boundary-layer.

The velocity profiles u/u_e $(= F/2)$ and temperature profiles G for an accelerating free stream with $\phi(t^*) = 1 + \varepsilon t^{*2}$, $\varepsilon > 0$, are presented in Fig. 7. These profiles approach the free stream values rapidly as the time t^* increases. The suction $(A > 0)$ makes the profiles to become steep whereas injection $(A < 0)$ does the opposite. The steepness in these profiles is due to the reduction of the boundary-layer thickness, caused by mass removal (suction).

The velocity profiles for a decelerating free stream $\phi(t^*) = 1 - \varepsilon t^{*2}$, $\varepsilon > 0$, are shown in Fig. 8. These profiles are strongly time dependent. Further, the profiles exhibit a declining nature in the vicinity of the surface. The reason for such a behaviour in these profiles is due to the deceleration of the flow. The velocity gradients near the surface become less than those of the cylinder velocity before reaching their free stream conditions. However, in the steady flow they simply approach to free stream values, since the effect of unsteadiness is absent.

The effect of the viscous dissipation on the temperature profiles is displayed in Fig. 9 where the surface temperature is constant (i.e., $R(t^*) = 1$). If Ec > 0, $t^* > 0$, the temperature profiles exhibit an overshoot and the magnitude of the overshoot decreases with an increase of t^*. But when Ec $= -2.0$ and $t^* \geq 1$, the profiles show $G < 0$ near the surface. This indicates that due to viscous dissipation, the fluid near the surface heats up and its temperature becomes larger than that of the cylinder, although originally the surface of the cylinder was at higher temperature. Thus the cylinder is heated instead of being cooled, because the 'heat cushion', provided by the frictional heat, prevents cooling. However, when $t^* = 0$ and Ec $= -2.0$ no such phenomenon is observed and the heat is transferred from the cylinder to the fluid.

The variation of $C_f(\mathrm{Re}_x)^{1/2}$ and $\mathrm{Nu}_x(\mathrm{Re}_x)^{-1/2}$ with time t^* and for several values of the transverse curvature and mass transfer (ξ, A), is shown in Figs. 10–12. Figures 10 and 11 show, respectively, the results of $C_f(\mathrm{Re}_x)^{1/2}$ and $\mathrm{Nu}_x(\mathrm{Re}_x)^{-1/2}$, both for accelerating flow as well as decelerating flow while Fig. 12 gives the results for exponentially decelerating flow with $\phi(t^*) = 1 - a(1 - \exp(-ct^{*2})), a > 0, c > 0$. In the accelerating flow it is observed (Fig. 10), that for a given ξ and t^* suction increases both: $C_f(\mathrm{Re}_x)^{1/2}$ and $\mathrm{Nu}_x(\mathrm{Re}_x)^{-1/2}$, whereas the effect of injection is just the opposite. Moreover, $C_f(\mathrm{Re}_x)^{1/2}$ increases rapidly as compared to $\mathrm{Nu}_x(\mathrm{Re}_x)^{-1/2}$ with the increase of t^*. The reverse trend is observed in the case of decelerating flow (Fig. 11). The growth of these coefficients in the accelerating flow is due to the fact that we have taken $\phi(t^*) = 1 + \varepsilon t^{*2}$ $(\varepsilon > 0)$, which corresponds to the accelerating velocity distribution of the free stream. On the other hand, the velocity distribution $\phi(t^*) = 1 - \varepsilon t^{*2}$, $(\varepsilon > 0)$ corresponds to

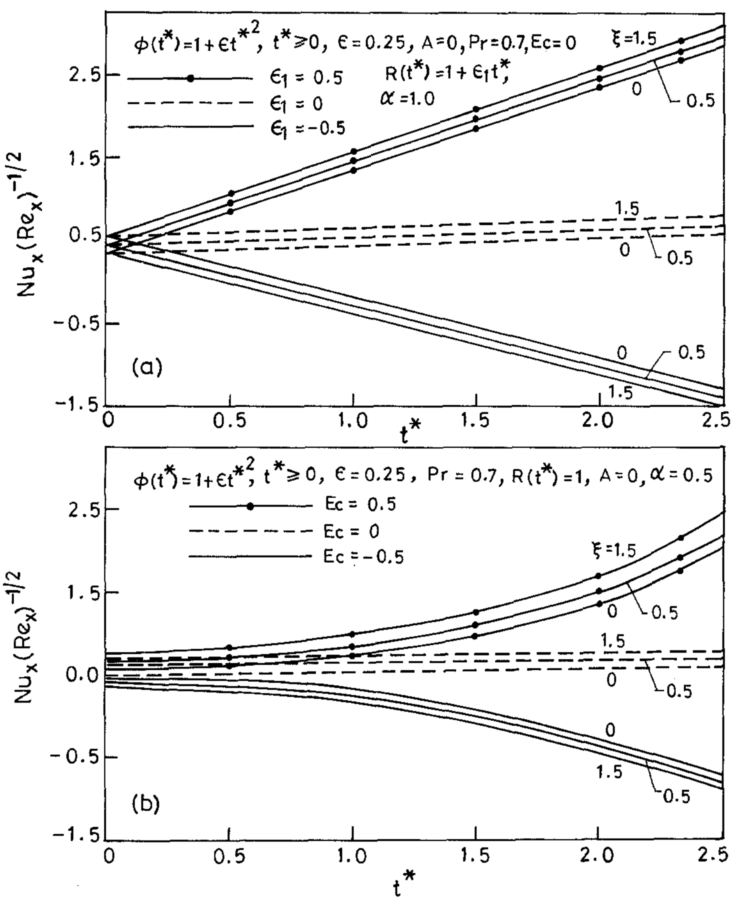

Fig. 13. **a** Surface temperature variation effects on heat transfer **b** Viscous dissipation effects on heat-transfer

a decelerating flow of the free stream, wherein these physical quantities decrease with time. Further it is observed that for a given value of A, in the case of an unsteady accelerating flow, $C_f(\mathrm{Re}_x)^{1/2}$ is more than twice that of $C_f(\mathrm{Re}_x)^{1/2}$ in the steady flow. But the rate of heat-transfer in both cases remains to be the same. Indeed, for a fixed value of A, the percentage increase in $C_f(\mathrm{Re}_x)^{1/2}$ for an increase in ξ from 0 to 1.5, is 41% for a steadily and 93% for an unsteadily accelerating flow. Also, for a given value of A and for a fixed value of ξ, the percentage increase in $C_f(\mathrm{Re}_x)^{1/2}$ for an increase in t^* from 0 to 2.5 is quite high, but in the case of $\mathrm{Nu}_x(\mathrm{Re}_x)^{-1/2}$, the percentage of increase is about 13%. On the other hand and in the case of decelerating flow, due to the increase in both the momentum and the thermal boundary-layer thicknesses, both $C_f(\mathrm{Re}_x)^{1/2}$ and $\mathrm{Nu}_x(\mathrm{Re}_x)^{-1/2}$ decrease with increasing time. And it is observed that for a fixed value of ξ and A, the percentage decrease in $C_f(\mathrm{Re}_x)^{1/2}$ for an increase in t^* from 0 to 2.0 is 50%, whereas in the case of $\mathrm{Nu}_x(\mathrm{Re}_x)^{-1/2}$ it is about 12%. The results for exponentially decelerating flow (Fig. 12) show that $C_f(\mathrm{Re}_x)^{1/2}$ as well as $\mathrm{Nu}_x(\mathrm{Re}_x)^{-1/2}$ decrease with time t^*. Further it is found that the skin friction and the heat-transfer values reach their new steady state values after $t = t_1{}^*$. The behaviour of $C_f(\mathrm{Re}_x)^{1/2}$, $\mathrm{Nu}_x(\mathrm{Re}_x)^{-1/2}$ as well as the velocity and the temperature profiles, mentioned above for different unsteady velocity distributions, is qualitatively the same, whenever v_w is constant or v_w varies as $x^{-1/2}$. Only a few selected results have been presented here, for the sake of brevity.

The effects of the variation of surface temperature and viscous dissipation on $\mathrm{Nu}_x(\mathrm{Re}_x)^{-1/2}$ are shown in Fig. 13. It is observed in Fig. 13 a, that, when the surface temperature increases, the heat-transfer is from the cylinder to the fluid and increases with time t^*. But when the surface temperature decreases with time, initially $\mathrm{Nu}_x(\mathrm{Re}_x)^{-1/2} \geqq 0$ (i.e., for $t^* \leqq 0.4$) but it decreases for $t^* > 0.4$. Further, it can be seen easily in Fig. 13 b that $\mathrm{Nu}_x(\mathrm{Re}_x)^{-1/2} \gtrless 0$ according to $\mathrm{Ec} \gtrless 0$. This implies that heat is transferred from the cylinder to the fluid if $\mathrm{Ec} > 0$ and from the fluid to the cylinder if $\mathrm{Ec} < 0$.

4 Conclusions

The skin friction and heat-transfer results are significantly affected by the time-dependent free stream velocity distributions as well as by the cylinder velocity. This increase is due to transverse curvature and suction. The effect of injection is found to do just the opposite. In the case of a downstream moving cylinder, the skin friction decreases with the increase of the cylinder velocity but the heat-transfer increases. Skin friction solutions for the upstream moving cylinder exist only for a small range of α and further these are found to be nonunique in this region. The viscous dissipation and the variation of the surface temperature have a pronounced effect on the heat-transfer. The temperature field is strongly influenced by the viscous dissipation.

References

[1] Sakiadis, B. C.: Boundary-layer behavior on continuous solid surfaces. I. Boundary-layer equations for two-dimensional and axisymmetric flow. Am. Inst. Chem. Eng. J. **7**, 26–28 (1961); II. The boundary-layer on a continuous flat surface. Am. Inst. Chem. Eng. J. **7**, 221–225 (1961).

[2] Crane, L. J.: Flow past a stretching plate. Z. Angew. Math. Phys. **21**, 645–647 (1970).

[3] Gupta, P. S., Gupta, A. S.: Heat and mass transfer on a stretching sheet with suction and blowing. Canad. J. Chem. Engng. **55**, 744–746 (1977).

[4] Carragher, P., Crane, L. J.: Heat-transfer on a continuous stretching sheet. Z. Angew. Math. Mech. **62**, 564–565 (1982).
[5] Danberg, J. E., Fansler, K. S.: A nonsimilar moving wall boundary-layer problem. Quart. Appl. Math. **34**, 305–309 (1976).
[6] Vleggaar, J.: Laminar boundary-layer behaviour on continuous accelerating surfaces. Chem. Engng. Sci. **32**, 1517–1525 (1977).
[7] Abdelhafez, T. A.: Skin friction and heat-transfer on a continuous flat surface moving in a parallel free stream. Int. J. Heat Mass Transfer **28**, 1234–1237 (1985).
[8] Chakrabarti, A., Gupta, A. S.: Hydromagnetic flow and heat-transfer over a stretching sheet. Quart. Appl. Math. **37**, 73–78 (1979).
[9] Chappidi, P. R., Gunnerson, F. S.: Analysis of heat and momentum transport along a moving surface. Int. J. Heat Mass Transfer **32**, 1383–1386 (1989).
[10] Vajravelu, K.: Hydromagnetic flow and heat-transfer over a continuous moving, porous, flat surface. Acta Mech. **64**, 179–185 (1986).
[11] Dutta, B. K.: Heat-transfer from a stretching sheet with uniform suction and blowing. Acta Mech. **78**, 255–262 (1989).
[12] Takhar, H. S., Kumari, M., Nath, G.: Boundary-layer forced convection flow over a moving wall with a magnetic field. Mech. Res. Commun. **16**, 257–263 (1989).
[13] Lee, S. L., Tsai, J. S.: Cooling of a continuous moving sheet of finite thickness in the presence of natural convection. Int. J. Heat Mass Transfer **33**, 457–464 (1990).
[14] Sakiadis, B. C.: Boundary-layer behavior on continuous solid surfaces. III. The boundary-layer on a continuous cylindrical surface. Am. Inst. Chem. Eng. J. **7**, 467–472 (1961).
[15] Bourne, D. E., Elliston, D. G.: Heat-transfer through the axially symmetric boundary-layer on a moving circular fibre. Int. J. Heat Mass Transfer **13**, 583–593 (1970).
[16] Karnis, J., Pechoc, V.: The thermal laminar boundary-layer on a continuous cylinder. Int. J. Heat Mass Transfer **21**, 43–47 (1978).
[17] Choi, I. G.: The effect of variable properties of air on the boundary-layer for a moving continuous cylinder. Int. J. Heat Mass Transfer **25**, 597–602 (1982).
[18] Pop, I., Kumari, M., Nath, G.: Non-Newtonian boundary-layers on a moving cylinder. Int. J. Engng. Sci. **28**, 303–312 (1990).
[19] Keller, H. B., Cebeci, T.: Accurate numerical methods in boundary-layer flows. In: Two-dimensional laminar flows (Holt, M., ed.), pp. 92–100. Second Int. Conf. on Numerical Methods in Fluid Dynamics. New York: Springer 1971.
[20] Sparrow, E. M., Yu, H. S.: Local nonsimilarity thermal boundary-layer solutions. Trans. A.S.M.E., J. Heat Transfer **93**, 328–334 (1971).
[21] Wanous, D. J., Sparrow, E. M.: Longitudinal flow over a circular cylinder with surface mass transfer. A.I.A.A. J. **3**, 147–149 (1965).
[22] Sparrow, E. M., Quack, H., Boerner, C. J.: Local nonsimilarity boundary-layer solutions. A.I.A.A. J. **8**, 1936–1942 (1970).
[23] Inouye, K., Tate, A.: Finite-difference version of quasilinearization applied to boundary-layer equations. A.I.A.A. J. **12**, 558–560 (1974).
[24] Keller, H. B., Cebeci, T.: Accurate numerical methods for boundary-layer flows. II. Two-dimensional turbulent flows. A.I.A.A. J. **10**, 1193–1199 (1972).
[25] Hussaini, M. Y., Lakin, W. D.: Existence and non-uniqueness of similarity solutions of a boundary-layer problem. Quart. J. Mech. Appl. Math. **39**, 15–24 (1986).
[26] Hussaini, M. Y., Lakin, W. D., Nachman, A.: On similarity solutions of a boundary-layer problem with an upstream moving wall. SIAM J. Appl. Math. **47**, 699–709 (1987).

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