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MHD flow between two parallel plates with heat transfer

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Summary. In the present paper, the steady flow of an electrically conducting, viscous, incompressible fluid bounded by two parallel infinite insulated horizontal plates and the heat transfer through it are studied. The upper plate is given a constant velocity while the lower plate is kept stationary. The viscosity of the fluid is assumed to vary with temperature. The effect of an external uniform magnetic field as well as the action of an inflow perpendicular to the plates together with the influence of the pressure gradient on the flow and temperature distributions are reported. A numerical solution for the governing non-linear ordinary differential equations is developed.

1 Introduction

The hydrodynamic channel-flow is a classical problem for which exact solutions can be obtained [1]. The introduction of electromagnetic phenomena to the fluid dynamics problem establishes new physical phenomena. Exact solutions for the flow and temperature fields of simplified problems can be found in many references [2]–[6]. The governing equations for laminar flow in a channel are the Navier-Stokes and the energy equations. When viscosity is assumed to be temperature dependent, these equations are mutually coupled and become nonlinear. So far there are only few studies of channel-flow problems which include the effects of viscosity variation or viscous dissipation. The hydrodynamic problem is solved by assuming certain property laws [7], [8], or by means of a linear perturbation theory to account for variable viscosity effects and to avoid the coupling of the governing equations [9].

In the present work, the steady flow of an electrically conducting, viscous, incompressible fluid between two parallel infinite insulated horizontal plates under the influence of an external uniform magnetic field directed perpendicular to the plates is studied. The viscosity of the fluid is assumed to vary exponentially with the temperature, and the two plates are kept at different constant temperatures. The fluid motion is also subject to a uniform suction and injection at the upper and lower plates respectively. The upper plate is given a horizontal velocity, and a pressure gradient is applied in the horizontal direction. The flow and temperature distributions are governed by the coupled set of the continuity, momentum transfer, and energy equations. The resulting non-linear ordinary differential equations are solved numerically using the finite difference approximations. The effects of the external magnetic field, pressure gradient, inflow, and the temperature dependent viscosity on both the flow and temperature distributions are studied.

2 Basic equations

A Cartesian frame of axes is located with its origin on the lower plate, its x-axis along the main flow. The y-axis is perpendicular to the two plates. Since the two plates are infinitely extended in the x and z directions, the problem is essentially one dimensional and the x-component fluid velocity u and temperature T are functions of y only. The no-slip condition implies that at the upper plate the x-component velocity of the fluid is equal to the velocity of the plate, and at the lower plate is equal to zero. The vertical component of the fluid velocity is always constant and equal to the suction velocity. The value of the uniform magnetic field, which is directed along the y-axis, is assumed to be unaltered by making the necessary assumptions that guarantee the neglection of the induced electric and magnetic fields [2]. The electromagnetic effect is to restrain the motion of the flow by imposing a force which is proportional to the velocity. The flow is also subjected to a pressure gradient in the x-direction. At the upper and lower plates, the fluid temperatures are equal to the temperatures of the plates.

The continuity, momentum transfer, and energy equations when applied to the fluid give [2], [3]

$$\varrho v_W \frac{du}{dy} = -\frac{dp}{dx} + \frac{d}{dy}\left(\mu \frac{du}{dy}\right) - \sigma {B_0}^2 u \tag{1.1}$$

$$\varrho c v_W \frac{dT}{dy} = K \frac{d^2T}{dy^2} + \mu \left(\frac{du}{dy}\right)^2 + \sigma {B_0}^2 u^2 \tag{1.2}$$

where ϱ is the fluid density, c the specific heat of the fluid, σ the electric conductivity of the fluid, B_0 the magnetic flux density, K the thermal conductivity of the fluid, v_W the suction velocity, and μ the fluid viscosity, defined as $\mu = \mu_0 \mu(T)$, $\mu_0 = \varrho v_0$.
The boundary conditions are

$$u = 0 \quad \text{and} \quad T = T_1 \quad \text{at} \quad y = 0$$

$$u = u_1 \quad \text{and} \quad T = T_2 \quad \text{at} \quad y = h.$$

We replace the variables by the following dimensionless variables:

$$y^* = y/h, \quad u^* = uh/v_0, \quad T^* = (T - T_1)/(T_2 - T_1), \quad u_1{}^* = u_1 h/v_0 = \mathrm{Re}$$

$$\mathrm{Pr} = \mu c/K, \quad \mathrm{Ec} = {v_0}^2/(h^2 c(T_2 - T_1)), \quad \mathrm{Ha} = \mathrm{B_0 h}\sqrt{\sigma/\mu}, \quad G = -dp/dx, \quad \$ = v_W h/v_0.$$

Equations (1) become, in terms of these dimensionless variables (star dropped),

$$\$ \frac{du}{dy} = G + \mu(T) \frac{d^2u}{dy^2} + \frac{d\mu(T)}{dy}\frac{du}{dy} - \mathrm{Ha}^2 \tag{2.1}$$

$$\$ \frac{dT}{dy} = \frac{1}{\mathrm{Pr}} \frac{d^2T}{dy^2} + \mathrm{Ec}\,\mu(T) \left(\frac{du}{dy}\right)^2 + \mathrm{Ec}\,\mathrm{Ha}^2\, u^2 \tag{2.2}$$

where Re, Pr, Ec, and Ha are the Reynolds-, Prandtl-, Eckert-, and Hartmann numbers respectively. G and $\$$ are the pressure gradient factor and the suction parameter, respectively. The imposed boundary conditions are

$$u = 0 \quad \text{and} \quad T = 0 \quad \text{at} \quad y = 0 \tag{3.1}$$

$$u = \mathrm{Re} \quad \text{and} \quad T = 1 \quad \text{at} \quad y = 1. \tag{3.2}$$

By assuming the viscosity to vary exponentially with the temperature the function $\mu(T)$ takes the form [10]

$$\mu(T) = \exp(-aT) \tag{4}$$

where "a" is defined as a viscosity parameter $= \ln(\mu_1/\mu_2)$, and where μ_1 and μ_2 are two values for the coefficient of viscosity evaluated at temperatures T_1 and T_2, respectively.

Equations (2) represent a system of coupled and non-linear equations which needs to be solved numerically.

3 Numerical solution

The system of non-linear ordinary differential equations (2) is solved under the boundary conditions (3) by a two-point finite difference technique [11]. We first write this system as a set of first order equations by introducing the new dependent variables W and H defined by $W = du/dy$ and $H = dT/dy$. In terms of the new variables and using Eq. (4), Eqs. (2) take the form

$$\$\, W = G + \exp(-aT)\frac{dW}{dy} - a\exp(-aT)\,HW - \mathrm{Ha}^2\, u,$$

$$\$\, H = \frac{1}{\mathrm{Pr}}\frac{dH}{dy} + \mathrm{Ec}\exp(-aT)\,W^2 + \mathrm{Ec}\,\mathrm{Ha}^2\,\mathrm{u}^2,$$

with the boundary conditions (3).

The non-linear differential equations are solved iteratively. The computational domain is divided in the y direction into N intervals. Finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximations. The resulting difference equations are solved numerically using Thomas' algorithm [11].

4 Results and discussion

Calculations have been carried out to study the flow between two infinite parallel plates. Velocity and temperature distributions have been represented under variable conditions, namely the magnetic field, pressure gradient, inflow normal to the plates, and temperature dependent viscosity. In all the foregoing calculations, the nondimensional velocity of the upper plate u_1 is taken to be unity which corresponds to a unit Reynolds number.

Figure 1 shows the velocity distribution u against the vertical distance y for different values of the pressure factor G ranging from -5 to $+5$ in the case when both the magnetic field and the suction are turned off and the viscosity is temperature independent. For $G = 0$, the velocity distribution is linear as expected. For $G = -5$, reversed flow is marked for circulation depth equal to 0.22.

Figure 2 represents the velocity distribution for different values of the Hartmann number ranging from 0 to 10 at constant pressure gradient ($G = -5$). The condition of zero magnetic field is released, while both the suction parameter $\$$ and the viscosity parameter "a" are still set equal to zero. The figure indicates that the magnetic field has a marked effect on restraining the reversed flow and limiting the flow recirculation depth.

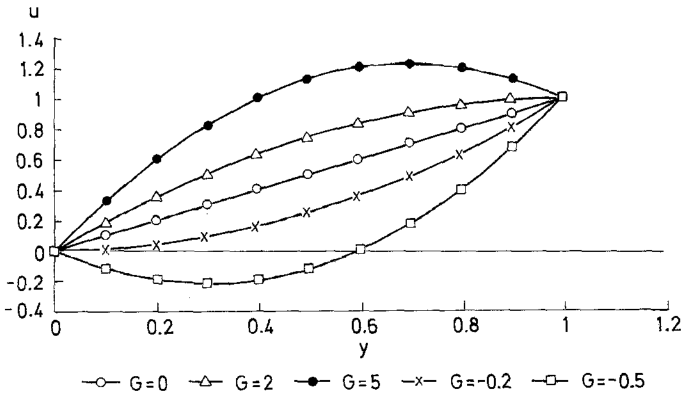

Fig. 1. The influence of pressure gradient on a velocity distribution with constant viscosity (Ha = 0, $\$ = 0$)

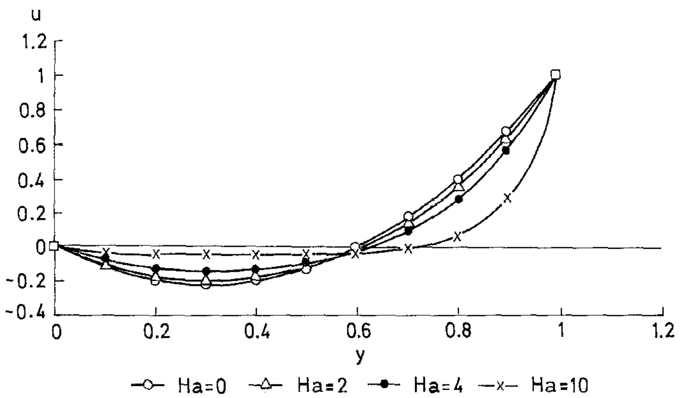

Fig. 2. The influence of the magnetic field on the velocity distribution ($G = -5$, $\$ = 0$)

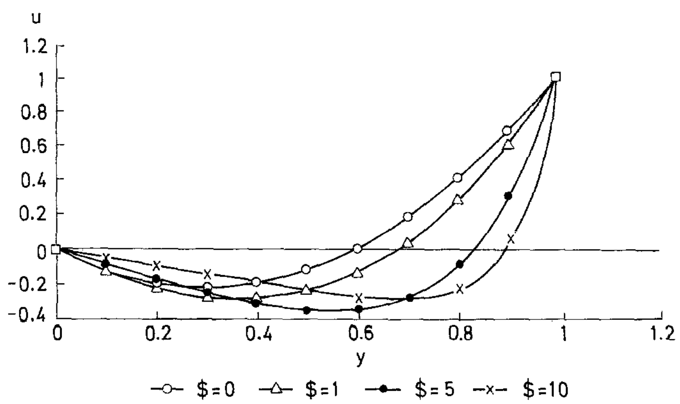

Fig. 3. The influence of the blowing ($v_W > 0$) on the velocity distribution ($G = -5$, Ha = 0)

Figure 3 shows the influence of suction inflow on the velocity distribution with $G = -5$, $Ha = 0$, and $a = 0$. Increasing the inflow parameter $\$$ up till 5 increases the reversed flow depth, but for the case $\$ = 10$ a reduction in the recirculation flow depth is indicated in the figure.

Figure 4 shows the effect of viscosity on the velocity distribution for zero values of the pressure gradient factor G, Hartmann number Ha, and suction parameter $\$$. Changing the viscosity parameter from 0 to 2 has a marked effect on the velocity profile. Increasing the viscosity parameter reduces the mass flow rate represented by the area between the velocity curves and the y-axis. In some fluids, the viscosity increases with temperature [10], that is, the viscosity

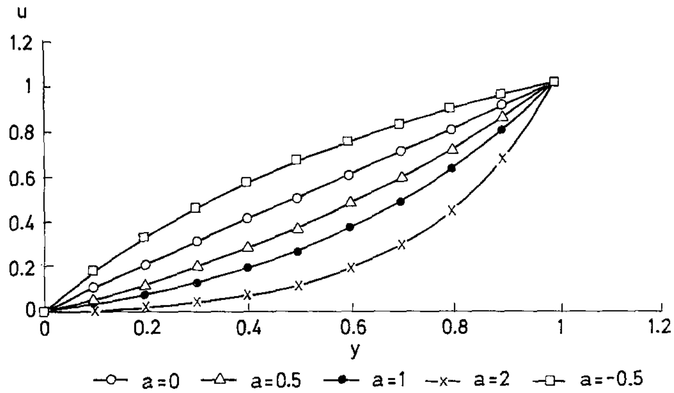

Fig. 4. The influence of viscosity on the velocity distribution ($G = 0$, $\mathrm{Ha} = 0$, $\mathcal{S} = 0$)

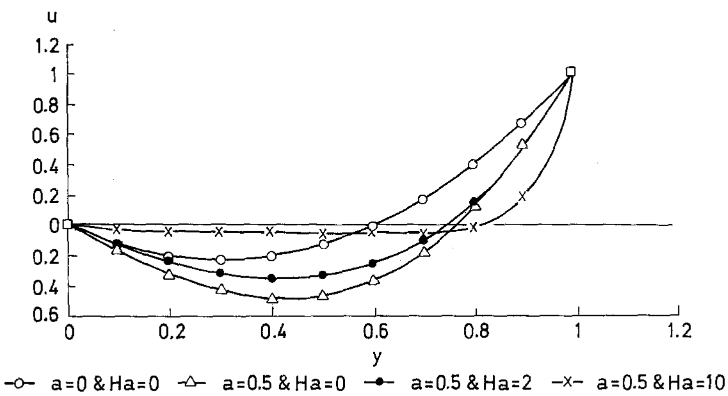

Fig. 5. The influence of the magnetic field on the velocity distribution with variable viscosity ($G = -5$, $\mathcal{S} = 0$)

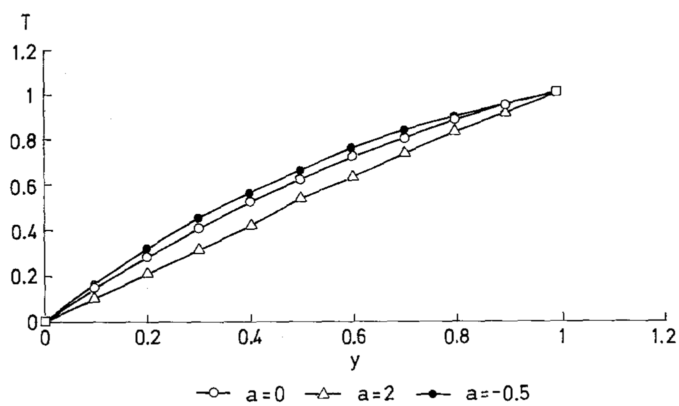

Fig. 6. The influence of viscosity on the temperature distribution ($G = 0$, $\mathrm{Ha} = 0$, $\mathcal{S} = 0$, $\mathrm{Ec} = 1$, $\mathrm{Pr} = 1$)

parameter has a negative value. This case is examined and the results shown in the figure indicate that the mass flow rate increases as the viscosity parameter changes from 0 to -0.5.

Figure 5 represents the influence of Hartmann number with variable viscosity for the pressure gradient factor $G = -5$ and the viscosity parameter $a = 0.5$ while the suction parameter $\mathcal{S} = 0$. It can be seen that increasing the Hartmann number reduces the reversed flow depth. Comparison has been made for the case of $Ha = 0$ between the cases with $a = 0$ and $a = 0.5$. The figure shows that increasing the viscosity parameter "a" increases the recirculation flow depth.

Figure 6 shows the effect of the viscosity parameter on the temperature distribution for $G = 0$, $Ha = 0$, and $\mathcal{S} = 0$. It is indicated from the figure that increasing the viscosity parameter "a"

decreases the temperature. This reduction in the temperature results from the reduction in the viscosity and consequently the viscous dissipation term. For large values of the viscosity parameter "a", $a = 2$, the viscous dissipation term vanishes and the temperature changes linearly with the distance y. On the other hand, when the viscosity parameter is negative, $a = -0.5$, the temperature increases as a result of increasing the viscous dissipation effect.

5 Conclusions

A numerical solution of the momentum transfer and energy equations has been developed using the finite difference method for the flow between two parallel infinite plates. The flow of the electrically conducting, incompressible fluid has been considered to be laminar and fully developed. The velocity and temperature distributions have been presented under the effect of an external uniform magnetic field, pressure gradient, inflow normal to the plates, and temperature dependent viscosity. A comparison of the velocity and temperature distributions has been made for the cases of constant and variable viscosity.

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