Acta Mechanica 117, 101–113 (1996)

ACTA MECHANICA

Bond strength for adhesive-bonded single-lap joints

L. Tong, Sydney, New South Wales

(Received March 28, 1995)

Summary. Arbitrarily nonlinear stress-strain behaviour in both shear and peel for adhesive are utilised to formulate two coupled nonlinear governing equations for an adhesive-adherend sandwich of single-lap type. For a balanced adhesive-adherend sandwich, the two equations can be integrated, and simple formulas for bond strength are developed for characterising pure shear, peel and mixed failure in adhesive. These formulas define the bond strength in terms of the maximum strain energy density in the adhesive. It is shown that the product of the adhesive strain energy density and the adhesive thickness is equal to the energy release rate J of mode I, mode II and mixed fracture.

1 Introduction

Lap joint theories for adhesive-bonded single-lap joints have been developed to analyse the stresses in the adhesive and to predict the strength of the joints. Basing on the pioneer work by Goland and Reissner [1], many authors have made various assumptions regarding the behaviour of the adhesive and adherends to yield tractable differential equations, and have investigated the effects of various factors on the stresses in the adhesive layer and the joint strength [2]–[13]. These factors include adhesive plasticity [2], large deformation and rotation [3], [4], satisfaction of the stress free requirement at the adhesive end [5], spew fillet [6], bondline thickness [8], etc. It has been shown that, in addition to the large deformation, adhesive plasticity is another important factor and needs to be taken into account in order to appropriately predict the joint strength. While implement of nonlinear adhesive behaviour in the finite element model for the joint is routine, it is relatively difficult, if not impossible, to obtain analytical solutions for the joint when the nonlinear adhesive properties are considered. Hart-Smith [2] modelled the adhesive shear behaviour with an elastic-plastic model and assumed linear elastic behaviour for the peel in his analytical study, and obtained a simple formula for predicting the potential bond shear strength for adhesive-bonded single-lap joints. In his formula the potential bond shear strength is characterised in terms of the adhesive shear strain energy density. Due to the complexity involved when considering the nonlinear adhesive behaviour, there are few literature available on analytial solutions of the adhesive-bonded joints of single-lap type.

Strength of adhesive-bonded joint has also been predicted using fracture mechanics approach [9], [14], [15], [16]. Recently, Fernlund et al. [17], [18], [19] and Papini et al. [22] proposed an engineering approach to predict the fracture loads for adhesive joints. The approach based on the premise that the in-situ strength of the bondline can be characterised by a fracture envelope for a specific adhesive system. The fracture envelope determined the critical energy release rate as a function of the mode of loading using a single equal adherend beam specimen [20], [21]. They employed the concept of adhesive-adherend sandwich proposed by Bigwood and Crocombe [10] where the bonded overlap is isolated from the surrounding structures. Two methods for mode partitioning proposed by Sou and Hutchinson [23] and Williams [24] were utilised to calculate the mode ratio. It was shown that for the equal adherend single-lap joints, the average difference between measured and predicted fracture loads was 5% only [19].

In this study the arbitrarily nonlinear stress-strain curves in both shear and peel are used for the adhesive in formulation of the governing equations for both shear and peel strains in an adhesive-adherend sandwich [10]. It is assumed that the adhesive-adherend sandwich is subjected to a given load combination at both ends. For the equal adherend sandwich, the two nonlinear governing equations are decoupled and then integrated to develop simple formulas of bond strength for predicting pure shear, peel and mixed failure in adhesive. The relationship between the fracture load predicted using J integral and the present bond strength is also discussed.

2 Problem formulation

Consider an adhesive-adherend sandwich subjected to a combination of loading at the ends of the sandwich, as shown in Fig. 1. The sandwich consists of two adherends and a thin adhesive layer. The adherends are modelled as cylindrically bent plates and the adhesive as an interlayer capable of transmitting both peel (tensile) and shear forces. It is assumed that the adhesive exhibits a nonlinear stress-strain behaviour in both peel and shear while the adherends behaves elastically. For an infinitesimal element as shown in Fig. 2, we have the following fundamental governing equations:

For the adherend 1 and 2, the equilibrium equations are:

$$\frac{dN_1}{dx} + \tau = 0, \quad \frac{dQ_1}{dx} + \sigma = 0, \quad \frac{dM_1}{dx} - Q_1 + \frac{t_1}{2}\tau = 0 \tag{1a}$$

$$\frac{dN_2}{dx} - \tau = 0, \quad \frac{dQ_2}{dx} - \sigma = 0, \quad \frac{dM_2}{dx} - Q_2 + \frac{t_2}{2}\tau = 0 \tag{1b}$$

where N_i, Q_i and M_i $(i = 1, 2)$ are the longitudinal membrane forces, the transverse shear forces and the bending moments per unit width for the adherends.

The longitudinal membrane forces N_i $(i = 1, 2)$ and bending moments M_i $(i = 1, 2)$ for the adherend 1 and 2 can be expressed with the relevant longitudinal displacements u_i $(i = 1, 2)$ in the x direction and the deflection w_i $(i = 1, 2)$ as follows:

$$N_i = A_i \frac{du_i}{dx}, \qquad M_i = -D_i \frac{d^2 w_i}{dx^2} \qquad (i = 1, 2) \tag{2}$$

where $A_i = E_i t_i$ and $D_i = E_i t_i^3/12$ $(i = 1, 2)$ are the membrane and bending stiffness of the adherends.

For adhesive, the peel stress σ and the shear stress τ in the adhesive layer are assumed to take the following form:

$$\sigma = \sigma(\varepsilon), \qquad \tau = \tau(\gamma) \tag{3}$$

where $\sigma(\varepsilon)$ and $\tau(\gamma)$ are arbitrary functions of ε and γ, respectively, as shown in Fig. 3, and could be the shear stress-strain curve of the adhesive measured using the thick adherend lap shear test [25] or the napkin ring shear test [26], and the tensile stress-strain curve of the adhesive measured using the neat adhesive tensile test or butt joint test [9].

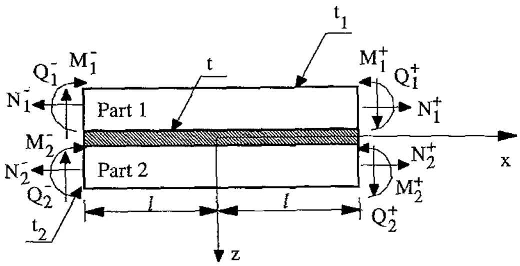

Fig. 1. Geometry and coordinates for an adhesive-adherend sandwich of single-lap type

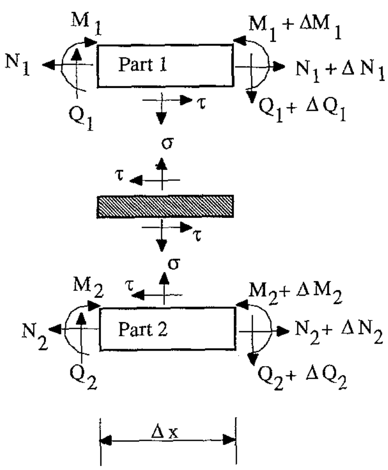

Fig. 2. Stresses and stress resultants of an element in the sandwich

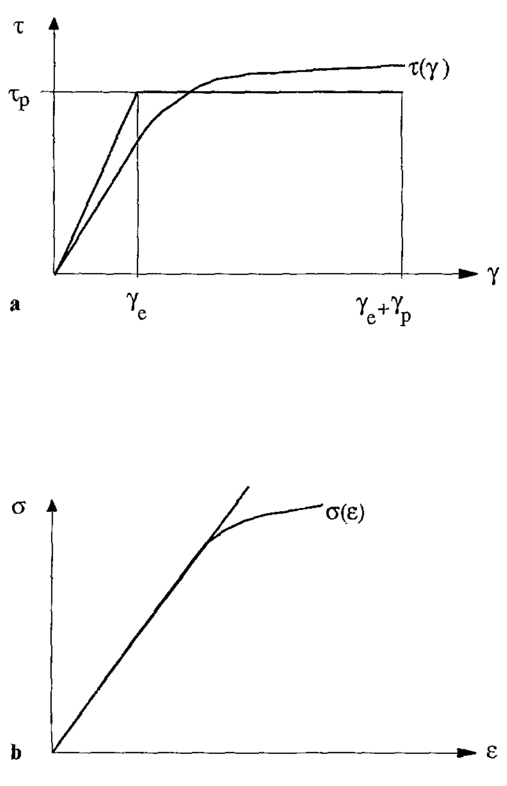

Fig. 3. Stress-strain curves for adhesive. **a** Shear stress-strain models, **b** peel stress-strain models

The peel and shear strains in the adhesive are assumed to be constant through the adhesive thickness and defined as [1], [2], [11]

$$\varepsilon = \frac{w_2 - w_1}{t}, \qquad \gamma = \frac{u_2 - u_1}{t} + \frac{1}{2t}\left(t_1 \frac{dw_1}{dx} + t_2 \frac{dw_2}{dx}\right). \tag{4}$$

The equations in Eqs. (1)−(4) are the fundamental governing equations for the adhesive-adherend sandwich. Differentiating γ three times and ε four times with respect to x and noting Eqs. (1)−(3), we can rewrite Eqs. (1)−(4) in terms of the shear and peel strains as follows:

$$\frac{d^3\gamma}{dx^3} - \alpha_1 \frac{d\tau}{dx} - \alpha_2 \sigma = 0, \tag{5}$$

$$\frac{d^4\varepsilon}{dx^4} + \alpha_3 \frac{d\tau}{dx} + \alpha_4 \sigma = 0, \tag{6}$$

where:

$$\alpha_1 = \frac{1}{t}\left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{t_1^2}{4D_1} + \frac{t_2^2}{4D_2}\right), \tag{7.1}$$

$$\alpha_2 = \alpha_3 = \frac{1}{t}\left(\frac{t_1}{2D_1} - \frac{t_2}{2D_2}\right), \tag{7.2}$$

$$\alpha_4 = \frac{1}{t}\left(\frac{1}{D_1} + \frac{1}{D_2}\right). \tag{7.3}$$

Equations (5) and (6) are two coupled second order differential equations of the shear strain γ and the peel strain ε, and do not permit a general closed-form solution when the adhesive is assumed to behave nonlinearly as defined in Eqs. (3). However, a general solution can be obtained when the adhesive exhibits linearly elastic behaviour.

When both adherends have the same thickness and material properties, the joint becomes a balanced one. In this case, $\alpha_2 = \alpha_3 = 0$, and Eqs. (5) and (6) are decoupled and can be simplified as

$$\frac{d^3\gamma}{dx^3} - \alpha_1 \frac{d\tau}{dx} = 0 \tag{8}$$

$$\frac{d^4\varepsilon}{dx^4} + \alpha_4 \sigma = 0 \tag{9}$$

where

$$\alpha_1 = \frac{2}{t}\left(\frac{1}{A_1} + \frac{t_1^2}{4D_1}\right), \qquad \alpha_4 = \frac{2}{tD_1}. \tag{10}$$

By assuming an ideal elastic-plastic stress-strain behaviour in shear (see Fig. 3a) and a linearly elastic stress-strain behaviour in peel for the adhesive, Hart-Smith [2] obtained an analytical solution for Eqs. (8) and (9). Once again because of the presence of the adhesive nonlinearity, Eqs. (8) and (9) do not permit a closed-form solution in general. However, similar to the case of double lap joints [28], simple explicit formulas can be obtained for defining the ultimate load or the load combination factor without completely solving Eqs. (8) and (9).

3 Bond shear strength for balanced joints

Bond shear strength is defined as the ultimate load or load combination factor when shear failure occurs in the adhesive layer. Using the maximum shear stress or strain criterion, shear failure is assumed to occur when the maximum shear stress or strain in the adhesive reaches its allowable.

To develop the expression of bond shear strength for a balanced sandwich, differentiating the shear strain γ defined in Eq. (4) with respect to x twice and noting Eqs. (1) to (3), we find

$$\frac{d^2\gamma}{dx^2} = \alpha_1\tau(\gamma) - \frac{t_1}{2tD_1}(Q_1 + Q_2). \tag{11}$$

Equation (11) is identical to Eq. (8) when differentiating it with respect to x once more and using the second equilibrium equations in the equilibrium equations given in Eqs. (1).

Multiplying $2\,(d\gamma/dx)$ on both sides of the Eq. (11) and rearranging the equation yields

$$d\left(\frac{d\gamma}{dx}\right)^2 = 2\alpha_1\tau(\gamma)\,d\gamma - \frac{t_1}{tD_1}(Q_1 + Q_2)\,d\gamma.$$

Integrating the above equation with respect to $(d\gamma/dx)^2$ for the first term and with respect to γ for others and noting that $Q_1 + Q_2$ is constant (It is assumed that there is no distributed lateral load acting on the top and bottom surfaces of the sandwich), we find the following approximate expression for the joint with sufficiently long overlap:

$$\left(\frac{d\gamma}{dx}\right)^2 = 2\alpha_1\int\limits_0^{\gamma}\tau(\gamma)\,d\gamma - \frac{t_1}{2tD_1}(Q_1 + Q_2)\,\gamma. \tag{12}$$

Equation (12) can be physically interpreted as: the slope of the shear strain distribution at any point in the adhesive is related to the shear strain energy density in the adhesive computed using the shear strain at that point and the work done by the transverse shear forces on the shear strain in the adhesive.

Using Eqs. (2) and (4), we can express the slope of the shear strain in terms of the longitudinal membrane forces and the bending moments in the adherends, and furthermore rewrite Eq. (12) as follows:

$$\left[\frac{N_2 - N_1}{A_1} - \frac{t_1(M_1 + M_2)}{2D_1}\right]^2 + \frac{t_1 t}{2D_1}(Q_1 + Q_2)\,\gamma = 4t\left(\frac{1}{A_1} + \frac{t_1{}^2}{4D_1}\right)\int\limits_0^{\gamma}\tau(\gamma)\,d\gamma. \tag{13}$$

The integration on the right hand side of the above equation is the area under the shear stress-strain curve or the shear strain energy density. Equation (13) reveals the relationship among the shear strain and the shear strain energy density in the adhesive, the longitudinal membrane forces, bending moments and the transverse shear forces in the two adherends at any point.

For an adhesive-adherend sandwich subjected to the combined loads as shown in Fig. 1, shear stress or strain always attains its maximum at one of the two ends of the model. Let us assume that, for all later discussion, shear stress or strain reaches maximum at the end of $x = -l$. When the loads acting on the adherends are determined from a global analysis without considering adhesive behaviour, the shear strain in the adhesive layer can be determined using Eq. (13). On the other hand, when shear failure occurs in the adhesive layer at the left end of the sandwich, the maximum shear strain reaches its allowable γ_{max}. Therefore by substituting the combined loads at the end of $x = -l$, we can determine the bond shear strength by

$$\frac{A_1}{16}\left[\frac{N_2^- - N_1^-}{A_1} - \frac{t_1(M_1^- + M_2^-)}{2D_1}\right]^2 + \frac{3t}{8t_1}(Q_1^- + Q_2^-)\,\gamma_{max} = t\int_0^{\gamma_{max}} \tau(\gamma)\,d\gamma \tag{14}$$

where the bond shear strength can be determined by solving Eq. (14) for the ultimate load or load combination factor. Evidently, the bond shear strength is expressed in terms of the maximum shear strain and the product of the adhesive thickness and the maximum shear strain energy density.

When the total transverse shear force is zero, namely, $Q_1^- + Q_2^- = 0$, Eq. (14) can be simplified as

$$\frac{A_1}{16}\left[\frac{N_2^- - N_1^-}{A_1} - \frac{t_1(M_1^- + M_2^-)}{2D_1}\right]^2 = t\int_0^{\gamma_{max}} \tau(\gamma)\,d\gamma. \tag{15}$$

In this case, bond shear strength can be determined in terms of the maximum shear strain energy density in the adhesive. Equation (15) can be used as an approximation of Eq. (14) for some lap joints where the transverse forces are less significant than the membrane forces and bending moments, for example, for these joints with relatively long unsupported adherends. Another point worth noting is that, when there are no transverse shear forces and bending moments acting at the ends of the model as shown in Fig. 1, Eq. (15) becomes identical to the formula given by Tong [30] for adhesive-bonded double-lap joints of balanced stiffness.

As an example, let us consider an End Notched Flexure (ENF) adhesive bonded specimen simply supported at both ends and subjected to an lateral vertical load P at the midpoint. The non-zero loads at the end of the adhesive layer are the bending moments and the transverse shear forces per unit width i.e., $M_1^- + M_2^- = Pa/2$, $Q_1^- + Q_2^- = P/2$, where a is the length of the debonded section. In this case, Eq. (14) becomes

$$\frac{3P^2a^2}{64D_1} + \frac{3tP}{16t_1}\gamma_{max} = t\int_0^{\gamma_{max}} \tau(\gamma)\,d\gamma.$$

If the shear strain γ_{max} on the left hand side of the above equation is approximated by $\gamma_{max} = P/2Gtk$, where G is the shear modulus and $k = 5/6$ is the shear correction factor, the above equation can be rewritten as

$$\frac{3P^2a^2}{64D_1}\left(1 + \frac{Et_1^2}{5Ga^2}\right) = t\int_0^{\gamma_{max}} \tau(\gamma)\,d\gamma.$$

It is noted that the terms on the left hand side of the above equation are identical to these in the formula of the strain energy release rate of mode II J_{IIC} given by Carlsson et al. [28]. For materials and geometries that result in neglecting Et_1^2/Ga^2, for example, when $a \gg t_1$, the above equation can be simplified as

$$t\int_0^{\gamma_{max}} \tau(\gamma)\,d\gamma = \frac{3P^2a^2}{64D_1} \tag{16}$$

which is identical to the formula of the strain energy release rate of mode II J_{IIC} given by Russell and Street [29], namely,

$$t \int_0^{\gamma_{\max}} \tau(\gamma)\, d\gamma = \frac{3P^2a^2}{64D_1} = J_{IIC}. \tag{17}$$

Hence it is revealed that for a given load P per unit width the energy release rate of mode II is the same as the product of the shear strain energy density and the adhesive thickness. Because both J_{IIC} and $t \int_0^{\gamma_{\max}} \tau(\gamma)\, d\gamma$ are intrinsic material properties, the terms on the left hand side of Eq. (14) can be used to calculate the Mode II component of the fracture energy for the adhesive-adherend sandwich subjected to a combined loading. This conclusion was found to be true for linear [27] and nonlinear [26] adhesive shear behaviour. It should be borne in mind that the above conclusion is applicable only when the adhesive layer is thin and the shear strain in the adhesive layer can be approximated as constant across the adhesive thickness.

4 Bond peel strength for balanced joints

Bond peel strength is defined as the ultimate load or load combination factor when the peel (or tensile) failure occurs in the adhesive layer. Using the maximum tensile stress or strain criterion, peel failure is assumed to occur when the maximum tensile stress or strain in the adhesive attains its allowable.

To characterise the bond peel failure for a balanced sandwich, let us multiply $d\varepsilon/dx$ on both sides in Eq. (9) and rewrite the equation as follows

$$d\left(\frac{d^3\varepsilon}{dx^3}\right)\frac{d\varepsilon}{dx} + \alpha_4\sigma(\varepsilon)\, d\varepsilon = 0.$$

Integration with respect to the state variables, namely, $d^3\varepsilon/dx^3$ for the first term and ε for the second term, yields

$$\frac{1}{2}\left(\frac{d^2\varepsilon}{dx^2}\right)^2 - \frac{d\varepsilon}{dx}\frac{d^3\varepsilon}{dx^3} = \alpha_4 \int_0^{\varepsilon} \sigma(\varepsilon)\, d\varepsilon.$$

By expressing $d^2\varepsilon/dx^2$ and $d^3\varepsilon/dx^3$ in terms of the bending moments and the transverse shear forces, we find

$$(M_1 - M_2)^2 - 2D_1t(Q_1 - Q_2)\frac{d\varepsilon}{dx} = 4D_1t \int_0^{\varepsilon} \sigma(\varepsilon)\, d\varepsilon. \tag{18}$$

Equation (18) established the relationship amongst the slope of the peel strain distribution and the peel strain energy density in the adhesive, and the bending moments and the transverse shear forces in the adherends. This relationship is valid for any point along the adhesive layer in the adhesive-adherend sandwich.

When the peel strain reaches its allowable $\varepsilon_{\max}$, peel failure occurs in the adhesive at one end of the adhesive-adherend sandwich. Assuming that failure occurs at the left end, the bond peel strength can be determined from the following equation

$$\frac{1}{4D_1}(M_1^- - M_2^-)^2 - \frac{t}{2}(Q_1^- - Q_2^-)\left.\frac{d\varepsilon}{dx}\right|^-_{\varepsilon=\varepsilon_{max}} = t\int_0^{\varepsilon_{max}} \sigma(\varepsilon)\, d\varepsilon. \tag{19}$$

When $Q_1^- - Q_2^- = 0$, Eq. (19) can be simplified as

$$\frac{1}{4D_1}(M_1^- - M_2^-)^2 = t\int_0^{\varepsilon_{max}} \sigma(\varepsilon)\, d\varepsilon. \tag{20}$$

In this case, bond peel strength can be characterised in terms of the maximum peel strain energy density in the adhesive. Equation (20) can be regarded as an approximation of Eq. (19) for some lap joints where the transverse shear forces is less significant than the bending moment components. For example, this approximation is applicable to those joints in which the unsupported parts are relatively large. Hart-Smith [2] and Oplinger [3] assumed $dw_1/dx = dw_2/dx$ (this assumption implies an approximation of $d\varepsilon/dx = 0$) at the ends when determining the bending moments acting on one adherend at the ends of the overlap for single-lap joints.

As an illustrative example, let us consider a Double Cantilever Beam (DCB) specimen. In this specimen, a predominantly mode I loading can be realised by applying the vertical loads at the ends of the two debonded adherends. As the loads are the same in magnitude and opposite in direction, the non-zero loads at the end of the adhesive layer are the bending moments and the transverse shear forces per unit width, e.g., $M_1^- = -M_2^- = Pa$, $Q_1^- = -Q_2^- = P$. Hence we have

$$\frac{P^2a^2}{D_1} - tP\left.\frac{d\varepsilon}{dx}\right|_{\varepsilon=\varepsilon_{max}} = t\int_0^{\varepsilon_{max}} \sigma(\varepsilon)\, d\varepsilon.$$

After imposing the built-in conditions [19] at the end of the adhesive layer, i.e., $dw_1/dx = dw_2/dx = 0$, because of symmetry about the adhesive centerline, we find

$$t\int_0^{\varepsilon_{max}} \sigma(\varepsilon)\, d\varepsilon = \frac{P^2a^2}{D_1}$$

which is identical to the formula for computing the critical energy release rate of mode I J_{IC}, namely,

$$t\int_0^{\varepsilon_{max}} \sigma(\varepsilon)\, d\varepsilon = \frac{P^2a^2}{D_1} = J_{IC}. \tag{21}$$

Equation (21) reveals that the energy release rate of mode I is the same as the product of the peel or tensile strain energy density and the adhesive thickness. Because both J_{IC} and $t\int_0^{\varepsilon_{max}} \sigma(\varepsilon)\, d\varepsilon$ are inherent material properties, the terms on the left hand side of Eq. (20) can be used to calculate the mode I component of the energy release rate for the sandwich shown in Fig. 1. Edde and

Verreman [27] considered the linear adhesive behaviours and obtained

$$G_{IC} = \frac{t\sigma^2_{max}}{2E_a}$$

where E_a is the Young's modulus of the adhesive, σ_{max} is the maximum tensile stress. Their result is a special case of the present one.

5 Bond strength for balanced joints

For an adhesive-adherend sandwich, both shear and peel strains usually exist simultaneously in the adhesive layer. Bond strength of the adhesive failure can be determined using one of the following two criteria:

Limit criterion: bond failure is assumed to occur when the less of the bond shear and peel strength is attained.

Interactive criterion: bond failure is assumed to occur when the maximum strain energy density in the adhesive attains its allowable W_c for a combination of shear strain γ_c and peel strain ε_c, namely,

$$\int_0^{\gamma_c} \tau(\gamma)\, d\gamma + \int_0^{\varepsilon_c} \sigma(\varepsilon)\, d\varepsilon = W_{IIC} + W_{IC} = W_c(\psi). \tag{22}$$

Where $W_c(\psi)$ is bond strength envelope defined by the critical strain energy density in the adhesive layer corresponding to the combination of the shear and peel strains. Similar to the mixed-mode fracture, the phase angle, which is a measure for the combination of shear strain γ_c and peel strain ε_c or the strain energy density ratio, is defined as

$$\psi = \arctan \sqrt{\frac{W_{IIC}}{W_{IC}}} = \arctan \sqrt{\frac{\int_0^{\gamma_c} \tau(\gamma)\, d\gamma}{\int_0^{\varepsilon_c} \sigma(\varepsilon)\, d\varepsilon}}, \tag{23}$$

where γ_c and ε_c are the critical shear and peel strains when the adhesive failure occurs because the maximum strain energy density in the adhesive reaches its allowable W_c.

Consider an arbitrary load combinations as shown in Fig. 1, failure occurs at the left end of the overlap when

$$\frac{A_1}{16}\left[\frac{N_2^- - N_1^-}{A_1} - \frac{t_1(M_1^- + M_2^-)}{2D_1}\right]^2 + \frac{3t}{8t_1}(Q_1^- + Q_2^-)\,\gamma_c + \frac{1}{4D_1}(M_1^- - M_2^-)^2 - \frac{t}{2}(Q_1^- - Q_2^-)\left.\frac{d\varepsilon}{dx}\right|^-_{\varepsilon=\varepsilon_c} = tW_c(\psi) \tag{24.1}$$

$$\psi = \arctan \sqrt{\frac{\frac{A_1}{16}\left[\frac{N_2^- - N_1^-}{A_1} - \frac{t_1(M_1^- + M_2^-)}{2D_1}\right]^2 + \frac{3t}{8t_1}(Q_1^- + Q_2^-)\,\gamma_c}{\frac{1}{4D_1}(M_1^- - M_2^-)^2 - \frac{t}{2}(Q_1^- - Q_2^-)\left.\frac{d\varepsilon}{dx}\right|^-_{\varepsilon=\varepsilon_c}}} \tag{24.2}$$

where $d\varepsilon/dx|_{\varepsilon=\varepsilon_c}$ is the slope of the peel strain distribution at the location where peel strain equals to its critical value.

When dropping off the contributing terms related to the transverse shear forces, the above equation can be simplified as

$$\frac{A_1}{16}\left[\frac{N_2^- - N_1^-}{A_1} - \frac{t_1(M_1^- + M_2^-)}{2D_1}\right]^2 + \frac{1}{4D_1}(M_1^- - M_2^-)^2 = tW_c(\psi), \tag{25.1}$$

$$\psi = \arctan\sqrt{\frac{A_1 D_1\left[\dfrac{N_2 - N_1}{A_1} - \dfrac{t_1(M_1 + M_2)}{2D_1}\right]^2}{4(M_1 - M_2)^2}}. \tag{25.2}$$

In the previous section, we have demonstrated the equivalence between the energy release rate of mode I or II and the product of the adhesive thickness and the strain energy density in peel or shear, respectively. In order to demonstrate the equivalence in the case of mixed loading, as an illustrative example, let us consider the double cantilever beam (DCB) specimen for which a mixed mode loading was realised using a novel load jig [20]. The load jig consists of a link-arm system. By altering the geometry of the link-arm system, the load jig provides a variety of ratios between the transverse shear forces, F_1 and F_2, acting on the upper and lower adherends of the specimen. The load jig is statically determinate, and the loads acting on the adherends at the end of the adhesive layer are given as:

$$Q_1^- = F_1, \quad Q_2^- = F_2, \quad M_1^- = F_1 a, \quad M_2^- = F_2 a$$

where a is the length of the debonded section in the DCB specimen. In this case, the strain energy density and the strain energy density ratio are given by

$$tW = \frac{a^2(7F_1^2 + 7F_2^2 - 2F_1F_2)}{16D_1} + \frac{3t}{8t_1}(F_1 + F_2)\,\gamma_c - \frac{t}{2}(F_1 - F_2)\left.\frac{d\varepsilon}{dx}\right|_{\varepsilon=\varepsilon_c} \tag{26.1}$$

$$\psi = \arctan\sqrt{\frac{\dfrac{3a^2}{16D_1}(F_1 + F_2)^2 + \dfrac{3t}{8t_1}(F_1 + F_2)\,\gamma_c}{\dfrac{a^2}{4D_1}(F_1 - F_2)^2 - \dfrac{t}{2}(F_1 - F_2)\left.\dfrac{d\varepsilon}{dx}\right|_{\varepsilon=\varepsilon_c}}}. \tag{26.2}$$

After neglecting the contributing terms related to the transverse shear forces for the case of $a \gg t_1$, the above equation can be rewritten as

$$tW = \frac{(F_1 a)^2}{2D_1}\left[1 + \left(\frac{F_2}{F_1}\right)^2 - \left(1 + \frac{F_2}{F_1}\right)^2\right], \tag{27.1}$$

$$\psi = \arctan\frac{\sqrt{3}\left(\dfrac{F_1}{F_2} + 1\right)}{2\left(\dfrac{F_1}{F_2} - 1\right)}. \tag{27.2}$$

The formulas in Eq. (27) for computing the strain energy density and determining the strain energy density ratio are the same as Eqs. (3) and (4) for calculating the mixed-mode energy release rate and the related phase angle given by Fernlund and Spelt [20]. It has been shown that (a) the product of the strain energy density and the adhesive thickness is equal to the mixed-mode energy release rate for an adhesive bonded specimen; (b) the strain energy density ratio is identical to the mode ratio of the mixed mode I and mode II fracture. It is thus demonstrated that there exists a equivalence between the fracture envelope $J_c(\psi)$ developed in [20] and the bond strength envelope $W_c(\psi)\ t$.

6 The relationship between bond strength and fracture load

In the previous section, it has been shown that the bond strength envelope $tW_c(\psi)$ is the same as the fracture envelope $J_c(\psi)$ that was generated using the DCB specimen subjected to a combined loadings [20]. In the following, let us consider these lap joints for which the contributing terms related to the transverse shear forces can be neglected, and discuss the relationship between the failure loads predicted by the present bond strength approach and the fracture mechanics method [17]–[22].

When neglecting the contributing terms related to the transverse shear forces, the product of the adhesive strain energy density and the adhesive thickness, for an adhesive-adherend sandwich subjected to a combined load as shown in Fig. 1, can be written as

$$tW = \frac{(N_2^- - N_1^-)^2}{16A_1} + \frac{7(M_1^-)^2 + 7(M_2^-)^2 - 2M_1^-M_2^-}{16D_1} - \frac{t_1(M_1^- + M_2^-)(N_2^- - N_1^-)}{16D_1} \tag{28}$$

where the components contributed from the shear and peel behaviour are given by

$$tW_I = \frac{1}{4D_1}(M_1^- - M_2^-)^2 \tag{29.1}$$

$$tW_{II} = \frac{A_1}{16}\left[\frac{N_2^- - N_1^-}{A_1} - \frac{t_1(M_1^- + M_2^-)}{2D_1}\right]^2. \tag{29.2}$$

Using J integral approach, Fernlund and Spelt [19] derived the following total energy release rate for a cracked adhesive-adherend sandwich undergoing large deformations. The path-independent J is given by

$$J = \frac{N_1^2}{2A_1} + \frac{M_1^2}{2D_1} + \frac{N_2^2}{2A_2} + \frac{M_2^2}{2D_2} - \frac{N_3^2}{2A_3} - \frac{M_3^2}{2D_3} \tag{30}$$

where N_i and M_i ($i = 1, 2, 3$) are the loads at the crack tip (see Fig. 4 in Fernlund and Spelt [19]). Using the following equilibrium conditions

$$N_3 = N_1 + N_2, \quad Q_3 = Q_1 + Q_2, \quad M_3 = M_1 + M_2 + \frac{t_1}{2}(N_1 - N_2)$$

we can rewrite Eq. (30) as follows

$$J = \frac{(N_2 - N_1)^2}{16A_1} + \frac{7M_1^2 + 7M_2^2 - 2M_1M_2}{16D_1} - \frac{t_1(M_1 + M_2)(N_2 - N_1)}{16D_1}. \tag{31}$$

Noting that the membrane forces and the bending moments N_i and M_i ($i = 1, 2$) equal to N_i^- and M_i^- ($i = 1, 2$), respectively, Eqs. (28) and (31) reveal that the energy release rate J is equal to the product of the strain energy density W and the adhesive thickness t, namely,

$$J = tW \tag{32}$$

and

$$J_I = tW_I, \quad J_{II} = tW_{II}. \tag{33}$$

These equations indicate that: for a balanced adhesive-adherend sandwich subjected to combined loads at the adhesive end, only the difference between the bending moments gives rise to a Mode I loading, and only the difference between the tensile forces and the summation of the bending moments result in a pure Mode II contribution. This is in agreement with the result given in [17].

Because the bond strength envelope is the same as the fracture envelope, it can be concluded that the bond strength predicted using the present analysis is identical to the fracture load predicted by Fernlund et al. [17]–[22] who showed there was less than 5% average difference between the test and the predicted fracture loads for equal adherend single-lap joints.

7 Conclusions

The adhesive-adherend sandwich model is used to predict bond strength for adhesive-bonded balanced joints in which adhesive exhibits arbitrarily nonlinear stress-strain behaviour in both shear and peel. The following conclusions are obtained: (1) when the adhesive in the joint is loaded in pure shear, the bond shear strength can be determined from the maximum shear strain energy density. The product of the adhesive thickness and the shear strain energy density is equal to the energy release rate of mode II. (2) When the adhesive in the joint is loaded in pure peel or tension, the bond peel strength can be determined from the maximum peel or tensile strain energy density. The product of the adhesive thickness and the peel or tensile strain energy density is equal to the energy release rate of mode I. (3) When the adhesive in the joint is loaded in both shear and peel (or tension), the bond strength can be determined from the maximum strain energy density and the phase angle of the shear to peel strain energy density ratio. The product of the adhesive thickness and the strain energy density is equal to the mixed-mode energy release rate of mode I and II, while the phase angle of the shear to peel strain energy density ratio is equal to that of mixed-mode fracture. (4) Bond shear strength, bond peel strength and bond strength are equal to the fracture loads of the equal adherend single-lap type joints subjected to pure mode II, pure mode I and mixed-mode loadings.

References

[1] Goland, M., Reissner, E.: The stresses in cemented joints. ASME J. Appl. Mech. **11,** A17–27 (1944).
[2] Hart-Smith, L. J.: Adhesive-bonded single lap joints. Langley Research Center, NASA CR-112236, Hampton, Virginia, 1973.
[3] Oplinger, D. W.: A layered beam theory for single lap joints. Army Materials Technology Laboratory Report MTL TR 91-23, 1991.
[4] Tsai, M. Y., Morton, J.: An evaluation of analytical and numerical solutions to the single-lap joints. Int. J. Solids Struct. **31,** 2537–2563 (1994).
[5] Chen, D., Cheng, S.: An analysis of adhesive-bonded single-lap joints. ASME J. Appl. Mech. **50,** 109–115 (1983).
[6] Crocombe, A. D., Adams, R. D.: Influence of the spew fillet and other parameters on the stress distribution in the single-lap joints. J. Adhesion **13,** 141–155 (1981).
[7] Harris, J. A., Adams, R. D.: Strength prediction of bonded single-lap joints by nonlinear finite element methods. Int. J. Adhesion Adhesives **4,** 65–78 (1984).

[8] Ojalvo, I. U., Eidinoff, H.: Bond thickness effects upon stresses in single-lap adhesive joints. AIAA J. **16,** 204–211 (1978).
[9] Adams, R. D.: Strength predictions for lap joints, especially with composite adherends. A review. J. Adhesion **30,** 219–242 (1989).
[10] Bigwood, D. A., Crocombe, A. D.: Elastic analysis and engineering design formulae for bonded joints. Int. J. Adhesion Adhesives **9,** 229–242 (1989).
[11] Carpenter, W. C.: A comparison of numerous lap joint theories for adhesively bonded joints. J. Adhesion **35,** 55–73 (1991).
[12] Crocombe, A. D.: Global yielding as a failure criterion for bonded joints. Int. J. Adhesion Adhesives **9,** 145–153 (1989).
[13] Adams, R. D., Wake, W. C.: Structural adhesive joints in engineering, London: Elsevier 1984.
[14] Hamaush, S. A., Ahmed, S. H.: Fracture energy release rate of adhesive joints. Int. J. Adhesion Adhesives **9,** 171–178 (1989).
[15] Ripling, E. J., Mostovoy, S., Corten, H.: Fracture mechanics: a tool for evaluating structural adhesives. J. Adhesion **3,** 107–123 (1971).
[16] Anderson, G. P., Brinton, S. H., Ninow, K. J., DeVries, K. L.: A fracture mechanics approach to predicting bond strength. In: Advances in adhesively bonded joints (Mall, S., Leichti, K. M., Vinson, J. R., eds.), pp. 93–101. New York: ASME 1988.
[17] Fernlund, G., Spelt, J. K.: Failure load prediction: I. analytical method. Int. J. Adhesion Adhesives **11,** 213–220 (1991).
[18] Fernlund, G., Spelt, J. K.: Failure load prediction: II. experimental results. Int. J. Adhesion Adhesives **11,** 221–227 (1991).
[19] Fernlund, G., Papini, M., McCammond, D., Spelt, J. K.: Fracture load predictions for adhesive joints. Comp. Sci. Techn. **51,** 587–600 (1994).
[20] Fernlund, G., Spelt, J. K.: Mixed mode energy release rates for adhesively bonded beam specimens. J. Comp. Tech. Res. **16,** 234–243 (1994).
[21] Fernlund, G., Spelt, J. K.: Mixed-mode fracture characterisation of adhesive joints. Comp. Sci. Techn. **50,** 441–449 (1994).
[22] Papini, G., Fernlund, G., Spelt, J. K.: The effect of geometry on the fracture of adhesive joints. Int. J. Adhesion Adhesives **14,** 5–13 (1994).
[23] Suo, Z., Hutchinson, J. W.: Steady-state cracking in brittle substrates beneath adherent film. Int. J. Solids Struct. **25,** 1337–1353 (1989).
[24] Williams, J. G.: On the calculation of energy release rates for cracked laminates. Int. J. Fracture **36,** 101–119 (1988).
[25] Krieger, R. B., Jr.: Stress analysis concepts for adhesive bonding of aircraft primary structures. In: Adhesively bonded joints: testing, analysis and design (Johnson, W. S., ed.), pp. 264–275. Philadelphia: American Society for Testing and Materials 1988.
[26] Chai, H.: Observation of deformation and damage at the tip of cracks in adhesive bonds loaded in shear and assessment of a criterion for fracture. Int. J. Fracture **60,** 311–326 (1993).
[27] Edde, F., Verreman, Y.: On the fracture parameters in a clamped cracked lap shear adhesive joint. Int. J. Adhesion Adhesives **12,** 43–48 (1992).
[28] Carlsson, L. A., Gillespie, J. W., Pipes, R. B.: On the analysis and design of the end notched flexture (ENF) specimen for mode II testing. J. Comp. Mat. **120,** 594–604 (1986).
[29] Russell, A. J., Street, K. N.: Factors affecting the interlaminar fracture energy of graphite/epoxy laminates, pp. 279. Proceedings of ICCM IV, Tokyo, 1982.
[30] Tong, L.: Bond shear strength for adhesive bonded double lap joints. Int. J. Solids Struct. **31,** 2919–2931 (1994).

Author's address: Dr. L. Tong, Department of Aeronautical Engineering, The University of Sydney. Sydney, NSW 2006 Australia