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Forced convection over a continuous sheet with suction or injection moving in a flowing fluid

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Summary. The analysis of forced convection flow and heat transfer about a flat sheet with suction or injection continuously moving in a quiescent or flowing fluid has been carried out. This kind of problem finds applications in a variety of manufacturing processes such as hot rolling, extrusion of plastic sheets, continuous casting, and cooling of a metallic plate in a cooling bath. The governing differential equations are reduced to nonlinear ordinary differential equations by similarity transformations. These equations are solved numerically based on a finite difference algorithm. Representative velocity and temperature profiles within the boundary layer are presented at selected values of free stream velocity and injection parameter. The friction factor and Nusselt number are illustrated for a wide range of governing parameters. For the same injection parameter, Prandtl number, and normalized velocity difference $|U_w - U_\infty|$, higher values of the Nusselt number and friction factor result from $U_w > U_\infty$ than from $U_w < U_\infty$. Also, an increase in the velocity ratio U_∞/U_w results in an increase in the heat transfer rate, but a decrease in the friction factor. Furthermore, the heat transfer is enhanced due to increasing the values of the free stream velocity, the injection parameter, and Prandtl number; while it is reduced due to increasing the velocity difference.

Nomenclature

C_f	friction factor, $\tau_w/(\varrho u_r^2/2)$
F	dimensionless stream function
F_w	injection parameter
h	local heat transfer coefficient
k	thermal conductivity
Nu	Nusselt number, hx/k
Pr	Prandtl number, ν/α
q_w	wall heat flux
Re	Reynolds number, $u_r x/\nu$
T	temperature
T_w	temperature at the fluid-sheet interface
T_∞	free stream temperature
U_w	normalized velocity of the sheet, u_w/u_r
U_∞	normalized free stream velocity, u_∞/u_r
u	fluid velocity component in x-direction
u_r	reference velocity, Eq. (8)
u_w	velocity of the continuous sheet
u_∞	free stream velocity
v	fluid velocity component in y-direction
v_w	fluid velocity component in y-direction at the fluid-sheet interface
x	streamwise coordinate
y	cross-stream coordinate

Greek symbols

α	thermal diffusivity
δ	boundary-layer thickness
η	dimensionless cross-stream coordinate
θ	dimensionless temperature
ν	kinematic viscosity
τ_w	wall shear stress
ψ	stream function

1 Introduction

From a technological point of view the study of heat transfer from a continuously moving surface is of considerable practical interest. Such systems are used in a wide variety of manufacturing processes such as hot rolling, glass fiber drawing, crystal growing, plastic extrusion, continuous casting, and wire drawing processes. Sakiadis [1]–[3] initiated the theoretical analyses for the momentum transfer occurring in the boundary layers adjacent to continuous surfaces moving steadily through an otherwise quiescent fluid environment. In his pioneering papers, solutions to this different class of boundary-layer problems are substantially different from those of boundary-layer flow over stationary surfaces. Later an experimental and analytical treatment was made by Tsou et al. [4] to show that such a flow is a physically realizable one by confirming the data from Sakiadis. Thereafter, many related analytical solutions have been obtained for different aspects of this class of boundary-layer problems. Such extensions include consideration of mass transfer, nonuniform surface velocity and temperature, application to non-Newtonian fluids, and other physical effects such as magnetic fields, mixed convection, and chemical reaction.

In this regard, Fox et al. [5] considered the effects of suction and injection on heat and mass transfer coefficients of a continuous flat surface moving with a constant velocity through a fluid medium at rest for certain values of the Prandtl and Schmidt numbers. Soundalgekar and Murty [6] studied the effects of power-law surface temperature variation on the heat transfer from a continuous moving sheet with uniform surface motion. Grubka and Bobba [7] examined the effects of variable surface temperature on a linear stretching surface. Murty and Sarma [8] carried out an analysis of flow past a continuous moving plate with prescribed surface heat flux. Through a scale analysis approach Jacobi [9] developed a convenient correlation, with applicability for all Prandtl numbers, for heat transfer from an isothermal surface continuously moving through a quiescent ambient fluid. An analysis has been carried out by Bianchi and Viskanta [10] to predict thermal transport occurring in the laminar boundary layer on a continuous surface that moves in the opposite direction of the free stream. The influence of a transverse magnetic field on the thermal boundary layer over a linearly stretching sheet has been examined by Chiam [11]. The flow of a viscous ferrofluid over a stretching sheet in the presence of a magnetic dipole has been considered by Andersson and Valnes [12] to explore numerically the effects of the magneto-thermomechanical interaction on skin friction and heat transfer. Recently, Chen [13] has investigated the heat transfer characteristics of a mixed convection flow over a non-isothermal, continuously stretching sheet that moves vertically upward or downward through an otherwise quiescent ambient fluid. Also, similarity solutions to the problem of mixed convective heat and mass transfer over a horizontal continuous plate have been reported by Fan et al. [14].

Most of the previous studies dealt with the problem of a continuously moving plate in a otherwise quiescent surrounding fluid medium, where the free stream velocity is zero. However, in many practical situations such as hot rolling, extrusion, and drawing, the continuous surface is in motion relative to a moving surrounding fluid. For example, a hot sheet issuing from a die or slot is cooled in a parallel free stream. In this paper we shall investigate the forced convective heat transfer from of a continuous surface with suction or injection moving through a parallel free stream. Based on similarity transformations the governing differential equations are reduced to nonlinear ordinary differential equations. Numerical solutions are generated by employing an efficient finite-difference method. Results for the velocity and temperature profiles, the wall shear stress, and he heat transfer rate are presented for various governing parameters such as the injection parameter, the normalized velocity difference, the free stream velocity, and the Prandtl number.

2 Mathematical formulations

Consider a continuous flat surface of temperature T_w issuing from a slot at a constant velocity u_w and being cooled by a forced convective flow with a free-stream velocity u_∞. The plate surface and the surrounding fluid are assumed to move in the same direction. The positive x-coordinate is measured along the moving surface with the slot as the origin, and the positive y-coordinate is measured normal to the sheet in the outward direction toward the fluid, as shown schematically in Fig. 1. By applying the boundary-layer approximation the governing equations for steady flow on a continuous surface with suction or injection, in usual notation, may be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \tag{1}$$

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial v}{\partial y} = \nu\,\frac{\partial^2 u}{\partial y^2}\,, \tag{2}$$

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \alpha\,\frac{\partial^2 T}{\partial y^2}\,, \tag{3}$$

subject to the following boundary conditions:

$$u = u_w\,, \qquad v = v_w(x)\,, \qquad T = T_w \quad \text{at} \quad y = 0 \tag{4}$$

$$u \to u_\infty\,, \qquad T \to T_\infty \quad \text{as} \quad y \to \infty\,. \tag{5}$$

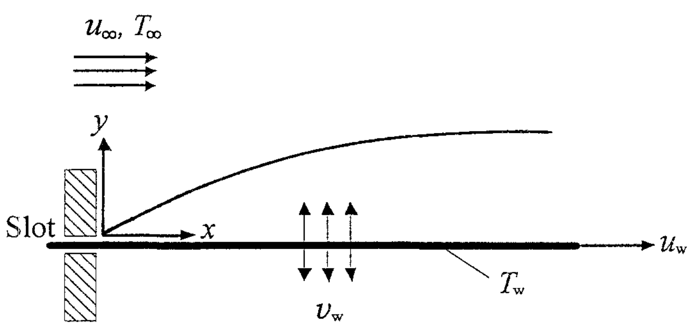

Fig. 1. Schematic representation and coordinate system

Let δ be the scale of the boundary-layer thickness and introduce the stream function ψ as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ such that the continuity equation is automatically satisfied. Then an order-of-magnitude scaling analysis shows

$$\delta \sim O(x/\mathrm{Re}^{1/2}), \tag{6}$$

$$\psi \sim O(\nu\,\mathrm{Re}^{1/2}), \tag{7}$$

where $\mathrm{Re} = u_r x/\nu$ is the Reynolds number with the reference velocity u_r defined by

$$u_r = \begin{cases} u_w & \text{if} \quad u_w > u_\infty \\ u_\infty & \phantom{\text{if}} \quad u_w < u_\infty \end{cases}. \tag{8}$$

Equations (6) and (7) suggest the following nondimensional variables:

$$\eta = \frac{y}{x}\,\mathrm{Re}^{1/2}, \tag{9}$$

$$F = \frac{\psi}{\nu\,\mathrm{Re}^{1/2}}, \tag{10}$$

and the nondimensional temperature can be defined as

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}. \tag{11}$$

In terms of these new variables, the velocity components are given by

$$u = u_r F', \tag{12}$$

$$v = -\frac{\nu}{x}\,\mathrm{Re}^{1/2}\left(\frac{1}{2}\,F + x\,\frac{\partial F}{\partial x} - \frac{1}{2}\,\eta F'\right), \tag{13}$$

and the boundary layer equations can be transformed into local similarity equations

$$F''' + \frac{1}{2}\,FF'' = 0, \tag{14}$$

$$\frac{1}{\mathrm{Pr}}\,\theta'' + \frac{1}{2}\,F\theta' = 0, \tag{15}$$

where $\mathrm{Pr} = \nu/\alpha$ is the Prandtl number of the fluid and the primes indicate differentiation with respect to η. The boundary conditions (4) and (5) become

$$F'(0) = U_w, \qquad F(0) = F_w, \qquad \theta(0) = 1, \tag{16}$$

$$F'(\infty) = U_\infty, \qquad \theta(\infty) = 0, \tag{17}$$

where $U_w = u_w/u_r$, $U_\infty = u_\infty/u_r$, and $F_w = -2v_w(x)\,(x/\nu u_r)^{1/2}$ is the injection parameter. It should be noted that positive and negative F_w indicate suction and injection, respectively.

The primary physical quantities of interest are the velocity distribution $u/u_r = F'(\eta)$, the temperature distribution $(T - T_\infty)/(T_w - T_\infty) = \theta(\eta)$, the friction factor C_f and the Nusselt number Nu. The last two quantities are defined, respectively, as

$$C_f = \frac{\tau_w}{\varrho {u_r}^2/2}, \qquad \mathrm{Nu} = \frac{q_w}{T_w - T_\infty}\,\frac{x}{k}. \tag{18}$$

With the aid of Eqs. (9)–(11), along with the definition of the wall shear stress $\tau_w = \mu(\partial u/\partial y)_{y=0}$ and the use of Fourier's law $q_w = -k(\partial T/\partial y)_{y=0}$, it follows that

$$C_f \, \mathrm{Re}^{1/2} = 2F''(0)\,, \tag{19}$$

$$\mathrm{Nu}/\mathrm{Re}^{1/2} = -\theta'(0)\,. \tag{20}$$

3 Numerical method

The transformed system of governing equations and boundary conditions (14)–(17) was solved by an implicit finite difference method as described in [13]. First we write the transformed differential equations and boundary conditions in terms of a first-order system, which is then converted into a set of finite-difference equations using central differences. Next, the nonlinear algebraic equations are linearized using the method of quasi-linearization, and the resulting linear system is then solved by the method of block-elimination factorization. For the sake of brevity, the details of the solution procedure are not presented here. It is worth noting that a step size of $\eta = 0.01$ is satisfactory in obtaining sufficient accuracy within a tolerance less than 10^{-5} in nearly all cases. To validate the present analysis, comparison is made with Fox et al. [5] in Table 1 for the wall shear stress, in terms of $F'(0)$, at selected values of the injection parameter F_w. Table 2 presents a comparison with Jacobi [9] for the Nusselt number for different values of the Prandtl number. It can be seen that the present results agree very well with the previously published data. Other comparisons for the heat transfer results, which are not presented here to conserve space, to those of Tsou et al. [4], and Soundalgekar and Murty [6] are also made. Again, a good agreement has been found. The following section presents the effects of various governing parameters on the flow and heat transfer characteristics of the continuously moving sheet.

Table 1. Comparison of $F''(0)$ for $U_w = 1, U_\infty = 0$, and for various values of F_w

F_w	Fox et al. [5]	Present results
0.0	–0.4437	–0.4438
0.1	–0.4738	–0.4737
0.5	–0.6040	–0.6030
1.0	–0.7864	–0.7864

Table 2. Comparison of $\mathrm{Nu}/\mathrm{Re}^{1/2}$ for $U_w = 1, U_\infty = 0$, and for various values of Pr

Pr	Jacobi [9]	Present results
0.7	0.3492	0.34925
1	0.4438	0.44375
7	1.387	1.3870
10	1.679	1.6802
100	5.448	5.5442

4 Results and discussion

The dimensionless velocity profiles for $U_w = 1, F_w = 0$, and for different values of U_∞ are shown in Fig. 2. It is obvious from this figure that the velocity within the boundary layer increases as the free stream velocity increases, associated with a decrease in the velocity gradient at the wall, and hence being expected to yield a decrease in the friction factor. The effects of injection parameter F_w on the velocity profiles are presented in Fig. 3 for $U_w = 1$ and $U_\infty = 0.5$. As might be expected, injection ($F_w < 0$) broadens the velocity distribution and thickens the boundary-layer thickness, while suction ($F_w > 0$) thins it. Also, the wall shear stress would be increased with the application of suction whereas injection tends to decrease the wall shear stress. This can be readily understood from the fact that the wall velocity gradient is increased with increasing the value of F_w.

Results for typical temperature profiles are illustrated in Figs. 4 and 5 for Pr = 0.7 (such as air) and for various values of free stream velocity and injection parameter. Figure 4 exhibits the dimensionless temperature distribution within the boundary layer at selected values of U_∞ for the case when $U_w = 1$ and $F_w = 0$. The thermal boundary-layer thickness is more reduced, together with a larger wall temperature gradient, when the free stream velocity becomes larger. Therefore, increasing the strength of the free stream velocity is expected to enhance the

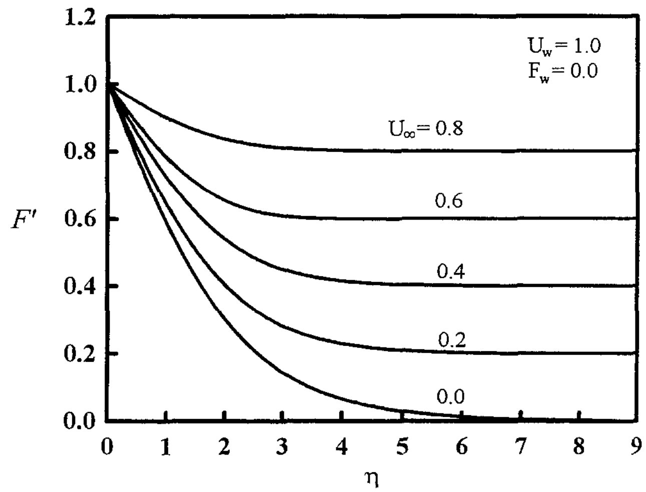

Fig. 2. Velocity distribution for different values of U_∞

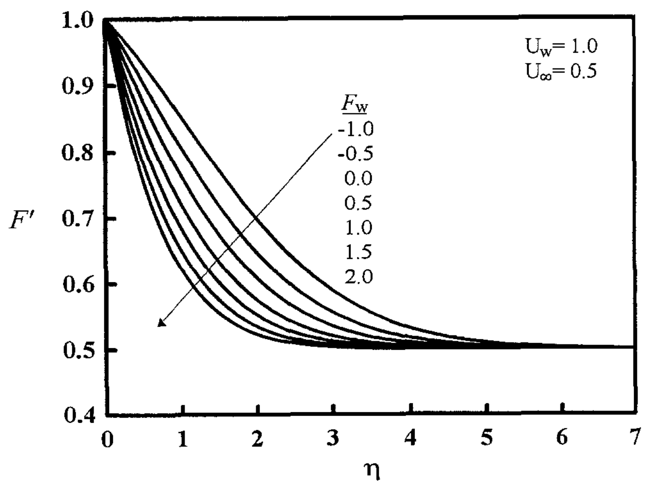

Fig. 3. Velocity distribution for different values of F_w

surface heat transfer rate. Figure 5 aims to explore the effects of the injection parameter on the temperature distribution. The effect of injection is found to broaden the temperature distribution, decrease the wall temperature gradient, and hence reduce the heat transfer rate. On the other hand, the thermal boundary layer becomes thinner and the wall temperature gradient becomes larger when suction is applied.

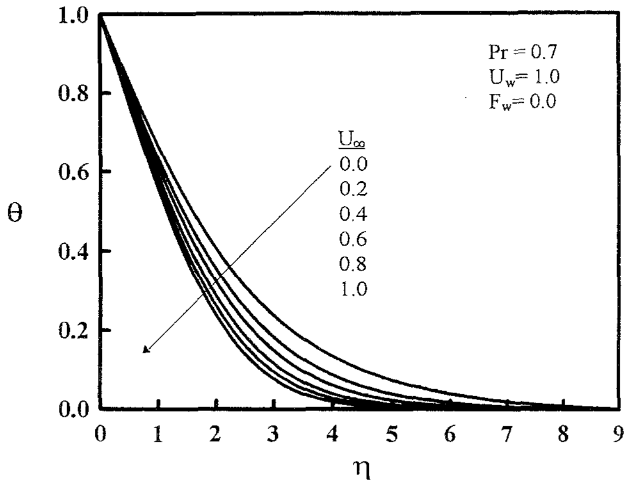

Fig. 4. Temperature distribution for Pr = 0.7 and for different values of U_∞

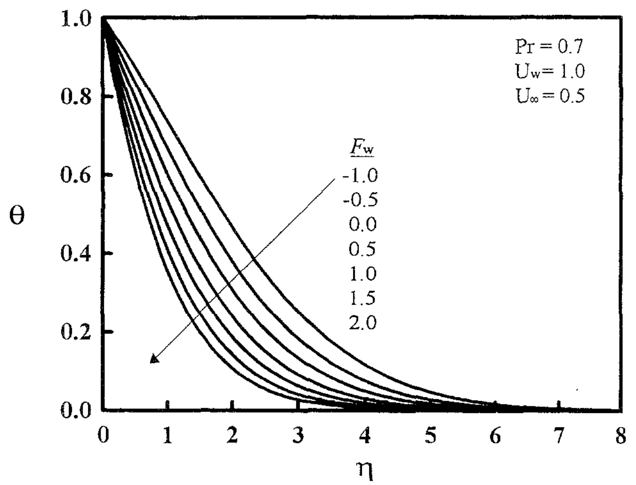

Fig. 5. Temperature distribution for Pr = 0.7 and for different values of F_w

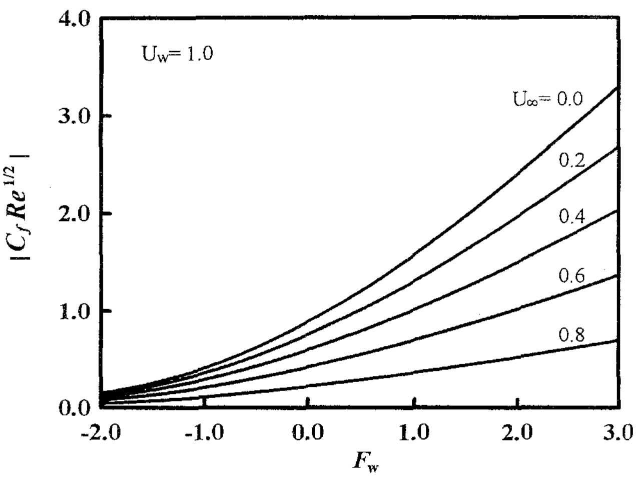

Fig. 6. Friction factor vs. F_w for $U_w = 1.0$ and for different values of U_∞

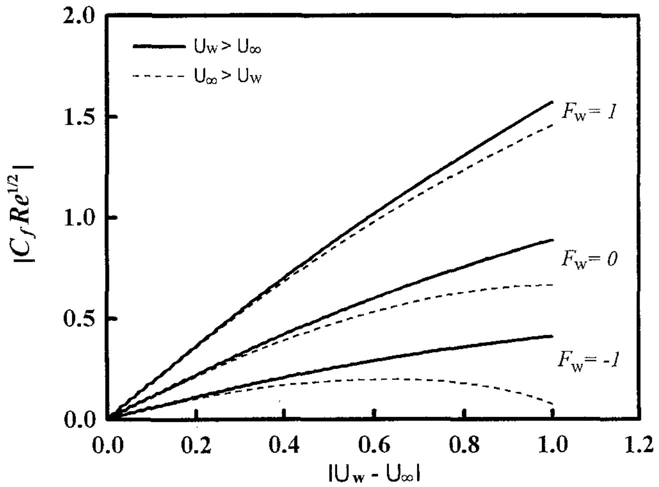

Fig. 7. Friction factor vs. the normalized velocity difference at selected values of F_w

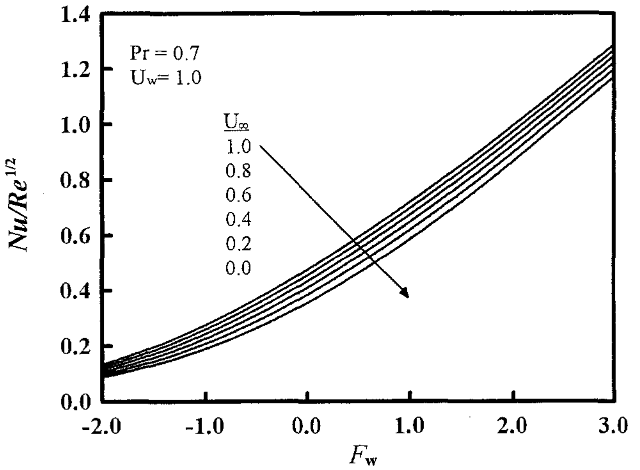

Fig. 8. Nusselt number vs. F_w for $\mathrm{Pr} = 0.7$ and for different values of U_∞

Figure 6 presents the friction factor as a function of the injection parameter F_w for $U_w = 1$ and for various values of U_∞. It can be seen that to increase the values of free stream velocity U_∞ or the velocity ratio U_∞/U_w is to decrease the friction factor. This observation is consistent with the results for velocity profiles shown in Fig. 2, where a decrease in the wall velocity gradient is caused by increasing the free stream velocity. Figure 7 illustrates the results for the friction factor against the normalized velocity difference $|U_w - U_\infty|$ at selected values of F_w. It is noted that for a given value of F_w the solutions for all possible combinations of U_w and U_∞ lie on two curves by introducing this method of normalization. The solid curves refer to the case where the velocity of the continuous sheet is larger than the free stream velocity while the dashed lines stand for the opposite case. In general, the friction factor increases with increasing the velocity difference except for the case where $F_w = -1$ and $U_\infty > U_w$. In that exceptional case the friction factor at first increases with increasing the normalized velocity difference, reaching a maximum value, and then diminishes with further increase in $|U_w - U_\infty|$. In addition, Fig. 7 reveals that the friction factor depends not only on the normalized velocity difference, but also on which moves faster, the sheet or the free stream. It was found that for a specified value of F_w the friction factor is larger for $U_w > U_\infty$ than for $U_w < U_\infty$.

Figure 8 represents the results for the heat transfer rate plotted as the Nusselt number $\mathrm{Nu}/\mathrm{Re}^{1/2}$ versus the injection parameter F_w for $\mathrm{Pr} = 0.7, U_w = 1$, and for several values of

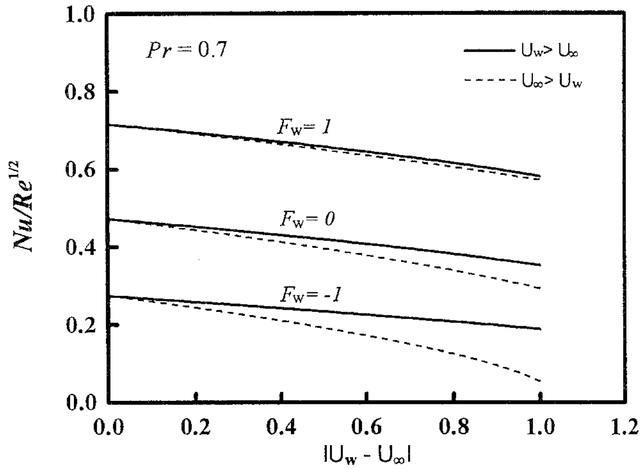

Fig. 9. Nusselt number vs. the normalized velocity difference for $\mathrm{Pr} = 0.7$ and for selected values of F_w

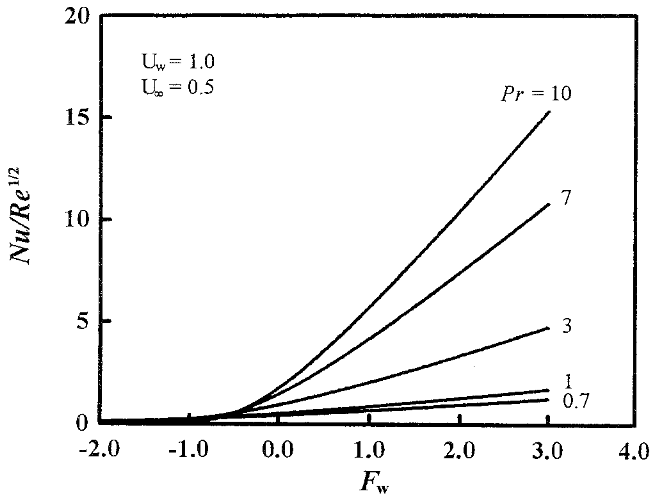

Fig. 10. Nusselt number vs. F_w for different values of Pr

U_∞. A slight increase in the heat transfer rate is observed as the free stream velocity is increased. This is clear from the fact that a large stream velocity improves the convection and thus enhances the heat transfer. Moreover, as compared to the case without suction or blowing ($F_w = 0$), the Nusselt number is increased for the suction case ($F_w > 0$) while it is decreased for the blowing case ($F_w < 0$). Thus, suction can be used as a means to get better cooling of the continuous sheet. It should be noted here that the results for the Nusselt number depicted in Fig. 8 are conformable to the results for the temperature profiles shown in Figs. 4 and 5, where larger values of U_∞ and F_w give rise to larger wall temperature gradients. Representative results for the Nusselt number versus the normalized velocity difference $|U_w - U_\infty|$ are displayed in Fig. 9 at selected values of F_w. The Nusselt number decreases monotonously with an increase in the normalized velocity difference. It is also noted that the heat transfer rate depends not only on the normalized velocity difference $|U_w - U_\infty|$ but also on whether $U_w > U_\infty$ or $U_w < U_\infty$. As the normalized velocity difference increases, the Nusselt number drops more rapidly for the case when $U_w < U_\infty$ (dashed lines) than for the case when $U_w > U_\infty$ (solid lines). This trend becomes more prominent when injection is activated (e.g., $F_w = -1$), as compared to the case when $F_w = 0$. The opposite trend is recognized when suction is used (e.g., $F_w = 1$). Figure 10 indicates the results for the heat transfer coefficient $\mathrm{Nu}/\mathrm{Re}^{1/2}$ as a function of F_w for $U_w = 1, U_\infty = 0.5$, and for several values of the Prandtl number. Higher values of $Nu/Re^{1/2}$ are predicted for larger values of the Prandtl number. This behavior can be explained from the fact that a fluid having a larger Prandtl number possesses a larger heat capacity and hence enhances the heat transfer. Therefore, the choice of fluids with larger Prandtl numbers will perform more efficiently the cooling of the heated continuous sheet.

5 Conclusions

In this work the problem of forced convection adjacent to a continuous flat sheet moving in a parallel free stream has been investigated numerically. Results for the velocity and temperature profiles, the friction factor, and Nusselt number are presented for a wide range of governing parameters. Effects of the free stream velocity, the normalized velocity difference, and the injection parameter on the flow and heat transfer characteristics are examined. It can be summarized that a higher Nusselt number can be obtained by applying higher values of the free stream velocity, the injection parameter, and Prandtl number of the fluid. Also, a greater heat transfer rate can be obtained from $U_w > U_\infty$ than from $U_w < U_\infty$ for the same normalized velocity difference $|U_w - U_\infty|$. The free stream velocity, choice of the fluid, and velocity difference between the ambient fluid and the surface may affect the thermal behavior of the continuous sheet in materials processing, and thus determine the quality of the final products. Therefore, the results obtained here to explore the basic thermal behavior of a continuous surface might be useful in achieving the design for relevant manufacturing processes.

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