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Unsteady MHD-boundary-layer of a source and vortex flow adjacent to a stationary surface

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Summary. The steady laminar incompressible flow of an electrically conducting fluid over an infinite permeable disk in the presence of an axial magnetic field has been investigated, and a self-similar solution of the boundary-layer equations is obtained numerically. For large values of the suction parameter, a closed form solution is obtained. Also, an asymptotic solution is found for large values of the independent variable. The surface-shear stresses in the radial and tangential directions and the surface heat transfer strongly depend on the suction parameter, the ratio of the source and vortex flow and the magnetic field except the surface heat transfer which weakly depends on the magnetic field. The similarity solution of the boundary-layer equations exists only when a certain minimum suction or magnetic field is applied. The results of the analytical solution are in good agreement with those of the numerical solution for the suction parameter $f_w \geq 3$.

1 Introduction

Rotating flows have been extensively studied due to their applications in meterology, in geophysics and cosmological fluid dynamics, in gaseous core nuclear reactors, and in vortex power generators etc. When a rotating flow interacts with a stationary surface, a complicated three-dimensional flow occurs which is found frequently both in external and internal flows. The rotating flow of a viscous incompressible fluid over an infinite stationary disk was first studied by Bödewadt [1] who considered the rigid-body rotation of the fluid. Smith and Colton [2] extended the above problem to include the effect of mass-transfer. The effect of the magnetic field on the rotating flow under boundary-layer approximations was considered by King and Lewellen [3], King [4], and Stewartson and Troesch [5]. They [3]–[5] investigated the generalized vortex flow $v_e \propto r^n, -1 \leq n \leq 1$, where v_e is the tangential velocity at the edge of the boundary-layer, r is the radial distance ($n = 1$ corresponds to a solid body rotation case, and $n = -1$ represents the potential vortex case), and they have shown that no similarity solution of the potential vortex case exists unless a certain minimum magnetic field is applied. The potential vortex flow over a stationary surface in the absence of the magnetic field and suction was investigated by Kidd and Farris [6] who found that there is no similarity solution to the boundary-layer equations, and the solution to the Navier-Stokes equations exists only up to a certain (small) value of the Reynolds number. Nanbu [7] studied the effect of large suction on the potential vortex flow over a stationary disk under the boundary-layer approximations and obtained the solution analytically. The interaction of a potential vortex with a source flow of equal strength on an infinite stationary disk was considered by Cham [8] and Hoffman [9] who obtained the solution of the Navier-Stokes equations only up to the Reynolds number $\mathrm{Re} = 16$. They also found that the similarity solution does exist for the boundary-layer equations.

In this paper, we have investigated the steady laminar boundary-layer over an infinite stationary disk due to the source and vortex flow of unequal strength in the presence of a magnetic field and (or) suction. The inclusion of a magnetic field and (or) suction enables us to obtain the similarity solution of the boundary-layer equations which does not exist in their absence. The fluid is assumed to be viscous, incompressible and electrically conducting. The ordinary differential equations governing the flow and heat transfer are solved numerically using a shooting method [10]. For large suction an analytical solution is obtained using a perturbation technique. Also, an asymptotic solution is found for a large value of the independent variable η (i.e., as $\eta \to \infty$).

2 Problem formulation

Let us consider a steady laminar boundary-layer flow of an electrically conducting fluid over an infinite stationary disk caused by the interaction of a potential vortex with a source flow. The physical model and the coordinate system are shown in Fig. 1. The magnetic field B is applied in the z-direction (i.e., the axial direction). The magnetic Reynolds number $Rm\ (= \mu_0 \sigma V L)$ is small, where μ_0 is the magnetic permeability, σ is the electrical conductivity, and V and L are the characteristic velocity and length, respectively. Under this condition it is possible to neglect the induced magnetic field in comparison to the applied magnetic field. The electrical current flowing in the fluid will give rise to an induced magnetic field if the fluid were an insulator. Here we have taken the fluid to be electrically conducting. The effects of viscous dissipation, Ohmic heating and Hall currents are neglected. The wall and the free stream temperatures are kept constant. The flow is assumed to be axisymmetric. Under the above assumptions the boundary-layer equations governing the flow and heat transfer over an infinite stationary disk are given by [3]–[5], [7], [11],

$$u_r + u/r + w_z = 0\,, \tag{1}$$

$$uu_r + wu_z - v^2/r = -\varrho^{-1} p_r + \nu u_{zz} - \sigma B^2 u/\varrho\,, \tag{2}$$

$$uv_r + wv_z + uv/r = \nu v_{zz} - \sigma B^2 (v - v_e)/\varrho\,, \tag{3}$$

$$uT_x + wT_z = \alpha T_{zz}\,. \tag{4}$$

The boundary conditions are given by

$$\begin{aligned} &u(r,0) = v(r,0) = 0\,, \qquad w(r,0) = w_o\,, \qquad T(r,0) = T_w\,, \\ &u(r,\infty) = u_e(r)\,, \qquad v(r,\infty) = v_e(r)\,, \qquad T(r,\infty) = T_\infty\,. \end{aligned} \tag{5}$$

The velocity components at the edge of the boundary-layer, u_e and v_e, are given by

$$u_e = a/r\,, \qquad v_e = b/r\,, \qquad a > 0\,, b > 0\,. \tag{6}$$

Hence, from Eqs. (2) and (6) we obtain

$$-\varrho^{-1} Pr = u_e (u_e)_r - {v_e}^2/r + \sigma B^2 u_e/\varrho = -(a^2 + b^2)/r^3 + \sigma B^2 a/(\varrho r)\,. \tag{7}$$

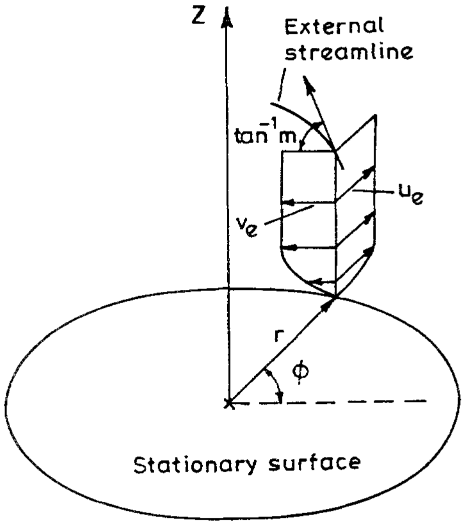

Fig. 1. Physical model and coordinate system

Here r, ϕ, z are the cylindrical polar coordinates; u, v and w are the velocity components along r, ϕ and z directions, respectively; B is the magnetic field; T is the temperature; ϱ and ν are the density and kinematic viscosity, respectively; p is the static pressure; α is the thermal diffusivity; a and b are constants; the subscripts r and z denote the derivatives with respect to r and z, respectively; and the subscripts e, w and ∞ denote the conditions at the edge of the boundary-layer, on the wall and in the free stream, respectively.

It may be noted that the flow considered here is fully viscous which can be divided into two regions. The main flow consists of a region in which axial gradients are negligible in comparison with the radial gradients. Hence, the flow can be considered independent of z. The other region is the boundary-layer region near the surface where the axial shear is much larger than the radial shear. These two regions should be matched to obtain the whole flow field.

For rotating flows over a stationary disk, the radial velocity is directed towards the disk, and the angular momentum is created at a large radius and transported to a small radius.

It is possible to reduce the partial differential equations (1)–(4) to a system of ordinary differential equations by using the following transformations:

$$\eta = (b/\nu)^{1/2}(z/r)\,, \qquad u = (a/r)\, f'(\eta)\,, \qquad v = (b/r)\, g(\eta)\,,$$
$$w = (az/r^2)\, f'(\eta) - (a/r)\,(\nu/b)^{1/2} f(\eta)\,, \qquad \theta(\eta) = (T - T_w)/(T_\infty - T_w)\,,$$
$$S = \sigma B^2 r/\varrho \nu_e\,, \qquad f_w = -(w_0/u_e)\,(b/r)^{1/2}\,, \qquad Pr = \nu/\alpha\,, m = a/b > 0\,. \tag{8}$$

Consequently, Eq. (1) is identically satisfied, and Eqs. (2)–(4) reduce to

$$f''' + m(f'' + f'^2) + (g^2/m) - (m^2 + 1)/m - S(f' - 1) = 0\,, \tag{9}$$
$$g'' + mfg' - S(g - 1) = 0\,, \tag{10}$$
$$\theta'' + Pr\, mf\theta' = 0\,. \tag{11}$$

The boundary conditions (5) reduce to

$$f = f_w\,, \qquad f' = g = \theta = 0 \quad \text{at} \quad \eta = 0\,,$$
$$f' = g = \theta = 1 \quad \text{as} \quad \eta \to \infty\,. \tag{12}$$

Here η is the transformed variable; f' and g are the dimensionless velocity components in the radial and tangential directions, respectively; S is the dimensionless magnetic parameter; θ is the dimensionless temperature; Pr is the Prandtl number; and a prime denotes a derivative with respect to η. The suction parameter $f_w (f_w > 0)$ is a constant if w_0 is selected in such a manner that (w_0/u_e) is a constant. The ratio m is a dimensionless constant and denotes the relative magnitude of the source and the vortex flow; $m > 1$ implies that the source flow dominates over the vortex flow, and for $m < 1$ it is the other way around. Also, for the magnetic parameter S to be of $0(1)$ while the magnetic Reynolds number $Rm \ll 1$, it is necessary that the ratio $\left(B^2/(\varrho\mu_0\nu_e{}^2)\right) \gg 1$.

3 Analytical solution

For a large suction rate ($f_w \geq 3$), it is possible to obtain analytical solutions of Eqs. (9)–(11) under the boundary conditions (12) using a perturbation technique when the parameters S and m are of order one. For large suction f_w, the boundary-layer becomes very thin, and there is a large change in the dependent variables across the boundary-layer. Hence, it is convenient to stretch the coordinates by using the following transformations:

$$\xi = f_w \eta, \qquad f(\eta) = f_w F(\xi), \qquad g(\eta) = G(\xi), \qquad \theta(\eta) = \phi(\xi), \qquad \varepsilon = f_w^{-2}. \tag{13}$$

Substituting relations (13) into Eqs. (9)–(12), we obtain

$$F''' + m(FF'' + F'^2) - \varepsilon S F' + \varepsilon^2 (G^2/m) - \varepsilon^2 b_1 = 0, \tag{14}$$

$$G'' + mFG' - \varepsilon S(G-1) = 0, \tag{15}$$

$$\phi'' + Pr\, mF\phi' = 0, \tag{16}$$

$$F = 1, \qquad F' = G = \phi = 0 \quad \text{at} \quad \xi = 0; \quad F' = \varepsilon, \quad G = \phi = 1 \quad \text{as} \quad \eta \to \infty. \tag{17}$$

Here $b_1 = (m^2 + 1 - mS)/m$ is a constant, and a prime denotes a derivative with respect to ξ. For large values of f_w the perturbation quantity ε is small. Hence, F, G and ϕ can be expanded in terms of ε as follows:

$$F = F_0 + \varepsilon F_1 + \varepsilon^2 F_2 + \dots,$$

$$G = G_0 + \varepsilon G_1 + \varepsilon^2 G_2 + \dots,$$

$$\phi = \phi_0 + \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \dots \tag{18}$$

Substituting relations (18) into Eqs. (14)–(17) and equating the coefficients of various powers of ε, we get the following system of equations, along with boundary conditions:

Zero-th order equations

$$F_0''' + m\left(F_0 F_0'' + (F_0')^2\right) = 0, \tag{19.1}$$

$$G_0'' + mF_0 G_0' = 0, \tag{19.2}$$

$$\phi_0'' + Pr\, mF_0\phi_0' = 0, \tag{19.3}$$

$$F_0 = 1, \qquad F_0' = G_0 = \phi_0 = 0 \quad \text{at} \quad \xi = 0,$$

$$F_0' = 0, \qquad G_0 = \phi_0 = 1 \quad \text{as} \quad \xi \to \infty. \tag{19.4}$$

First-order equations

$$F_1''' + m(F_0F_1'' + 2F_0'F_1' + F_0''F_1 - SF_0') = 0\,, \tag{20.1}$$

$$G_1'' + m(F_0G_1' + G_0'F_1) - S(G_0 - 1) = 0\,, \tag{20.2}$$

$$\phi_1'' + Pr\,m(F_0\phi_1' + \phi_0'F_1) = 0\,, \tag{20.3}$$

$$F_1 = F_1' = G_1 = \phi_1 = 0 \quad \text{at} \quad \xi = 0\,,$$

$$F_1' = 1\,, \qquad G_1 = \phi_1 = 0 \quad \text{as} \quad \xi \to \infty\,. \tag{20.4}$$

Second-order equations

$$F_2''' + m(F_0F_2'' + 2F_0'F_2' + F_0''F_2' + F_1F_1'' + F_1'^2) - SF_1' + G_0^2/m - b_1 = 0\,, \tag{21.1}$$

$$G_2'' + m(F_0G_2' + F_1G_1' + F_2G_0') - SG_1 = 0\,, \tag{21.2}$$

$$\phi_2'' + Pr\,m(F_0\phi_2' + F_1\phi_1' + F_2\phi_0') = 0\,, \tag{21.3}$$

$$F_2 = F_2' = G_2 = \phi_2 = 0 \quad \text{at} \quad \xi = 0\,; \quad F_2' = G_2 = \phi_2 = 0 \quad \text{as} \quad \xi \to \infty\,. \tag{21.4}$$

The solutions of the previous system of equations are given by

$$F_0 = 1\,, \tag{22.1}$$

$$G_0 = 1 - \exp(-m\xi)\,, \tag{22.2}$$

$$\phi_0 = 1 - \exp(-Pr\,m\xi)\,, \tag{22.3}$$

$$F_1 = \xi - m^{-1}[1 - \exp(-m\xi)]\,, \tag{23.1}$$

$$G_1 = [m\xi^2/2 + S\xi/m + 1/(2m)]\,\exp(-m\xi) - (2m)^{-1}\exp(-2m\xi)\,, \tag{23.2}$$

$$\begin{aligned}\phi_1 = {} & [m^{-1}(Pr+1)^{-1}Pr^2 - (Pr-1)\,\xi + Pr\,m\,\xi^2/2]\,\exp(-Pr\,m\,\xi) \\ & - m^{-1}(Pr+1)^{-1}Pr^2\,\exp[-m(1+Pr)\,\xi]\,,\end{aligned} \tag{23.3}$$

$$\begin{aligned}F_2 = {} & [2^{-1}m^{-4}(3 - 2mS) + m^{-3}(m^2 + 2 - mS)\,\xi - \xi^2/2]\,\exp(-m\xi) \\ & + [(2m^2+1)\,\exp(-2m\xi) - (2m^2 - 4mS + 7)]/(4m^4)\,,\end{aligned} \tag{24.1}$$

$$\begin{aligned}G_2 = {} & [-m^2\xi^4/8 - (2m+S)\,\xi^3/6 + \{2m^3 + (2S-7)\,m^2 - 2mS - 2S^2\}\,\xi^2/(4m^2) \\ & + \{2m^3 - 2(3-S)\,m^2 - 5Sm - 2S^2\}\,\xi/(2m^3) \\ & + (3m^2 - 18Sm + 34 + 3S)/(24m^4)]\,\exp(-m\xi) \\ & + [(m^2 + 6Sm - S - 11)/(8m^4) + (m^2 - 2mS + 4)\,\xi/(4m^3) + \xi^2/4]\,\exp(-2m\xi) \\ & - (24m^4)^{-1}\,(6m^2+1)\,\exp(-3m\xi)\,,\end{aligned} \tag{24.2}$$

$$\begin{aligned}\phi_2 = {} & [a_1\xi - a_2\xi^2/2 + 2^{-1}(1-m)\,Pr\,\xi^3 - 2^{-3}(Pr\,m)^2\,\xi^4 + a_6]\,\exp(-Pr\,m\,\xi) \\ & + [a_3m^{-1} + a_4(m\xi+1)/m^2 + (Pr/2)\,(m^2\xi^2 + 2m\xi + 2)/m^3]\,\exp[-(Pr+1)\,m\xi] \\ & - a_5(2m)^{-1}\,\exp[-(Pr+2)\,m\xi]\,,\end{aligned} \tag{24.3}$$

where

$$a_1 = [12m^2(1-m) - Pr\,(6m^2 - 4mS + 7)]\,(4Pr\,m^4)\,,$$

$$a_2 = (Pr + 3m - 3)/m\,,$$

$$a_3 = (Pr+1)^{-3} m^{-4} [4m^2 Pr^4 + (12m^2 + 2mS - 3) Pr^3 + (8m^2 + 6mS - 10)\, Pr^2 - (2m^2 - 4mS + 7)\, Pr - 2m^2] ,$$

$$a_4 = Pr\,(2m^2\, Pr + mS - 2)/m^3 (Pr+1) ,$$

$$a_5 = Pr\,(4m^2\, Pr + 2m^2 + 1)/4m^4 (Pr+2) ,$$

$$a_6 = m^{-1} a_3 - m^{-2} a_4 - m^{-3}\, Pr + (2m)^{-1} a_5 . \tag{24.4}$$

The surface shear stresses in the radial and the tangential directions are given by

$$f''(0) = f_w^3 [F_0''(0) + f_w^{-2} F_1''(0) + f_w^{-4} F_2''(0)] = f_w m + f_w^{-1} (2Sm - 2m^2 - 3)/2m^2 , \tag{25.1}$$

$$g'(0) = f_w [G_0'(0) + f_w^{-2} G_1'(0) + f_w^{-4} G_2'(0)] = f_w m + f_w^{-1} (2S + m)/(2m) + f_w^{-3} [12m^3 + (24S - 69)\, m^2 - 66Sm + 11 + 3S - 24S^2]/24m^3) , \tag{25.2}$$

$$\begin{aligned}\theta'(0) &= f_w [\phi_0'(0) + f_w^{-2} \phi_1'(0) + f_w^{-4} \phi_2'(0)] \\ &= Pr\, m f_w + f_w^{-1} (Pr+1)^{-1} + f_w^{-3} [Pr+1)^{-1} (Pr^4 - Pr^3 + Pr^2 - Pr - 2) \\ &\quad - (4m^4)^{-1} (2m^2 - 4Sm + 7) - Pr\,(Pr+1)^{-2} m^{-4} (2Pr\, m^2 + Sm - 2) \\ &\quad + m^{-2} (Pr+1)^{-1} (Pr+2)^{-1} \{ (Pr/m^2)\, (2m^2 - 8Sm + 13) \\ &\quad - (Pr/2m)^2\, (2m^2 + 4Sm - 7) - (Pr+1)^{-1}\, (Pr+2)\, (Pr^3 + Pr^2 - 1) \}] .\end{aligned} \tag{25.3}$$

It is evident from (25.3) that the surface heat transfer $\theta'(0)$ is weakly dependent on the magnetic parameter S. The surface shear stresses in the radial and the tangential directions $(f''(0), g'(0))$ and the surface heat transfer $(\theta'(0))$ are found to be in good agreement with the corresponding numerical results when $f_w \geq 3$, and S and m are of order one.

4 Asymptotic solution

Here we consider the asymptotic behaviour of the solutions of Eqs. (9)–(11) under the boundary conditions (12) for large η (i.e., as $\eta \to \infty$). As $\eta \to \infty$, f', g and θ tend to 1. Also,

$$\eta \to f + \int_0^\eta (1 - f')\, d\eta . \tag{26}$$

Since the above integral is the displacement thickness of the boundary-layer, it must be finite and is usually small. This implies that $f \to \eta$ for large η. Hence, we set for large η

$$f(\eta) = \eta - f_1(\eta) , \qquad g(\eta) = 1 - g_1(\eta) , \qquad \theta(\eta) = 1 - \theta_1(\eta) , \tag{27}$$

where f_1, g_1 and θ_1 are small and their squares and products can be neglected. Using relations (27) in Eqs. (9)–(11) and linearizing, we obtain

$$f_1''' + m\eta f_1'' + (2m - S)\, f_1' = -2g_1/m , \tag{28}$$

$$g_1'' + m\eta g_1' - S g_1 = 0 , \tag{29}$$

$$\theta_1'' + Pr\, m\eta \theta_1' = 0 . \tag{30}$$

The boundary conditions are given by

$$f_1' = g_1 = \theta_1 \to 0 \quad \text{as} \quad \eta \to \infty . \tag{31}$$

We first solve Eq. (29), and the solution is given in terms of parabolic cylinder functions [12]

$$g_1 = \exp(-m\eta^2/4)\,[C_1 D_1(m^{-1/2}\eta) + C_2 D_2(im^{-1/2}\eta)]\,, \tag{32}$$

where

$$D_1(m^{1/2}\eta) = \exp(-m\eta^2/4)\,(m^{1/2}\eta)^{-(S+m)/m}\left[1 - \frac{(S+m)(S+2m)}{2m^2(m\eta^2)} + 0(m^2\eta^{-4})\right], \tag{33.1}$$

$$D_2(im^{1/2}\eta) = \exp(m\eta^2/4)\,(m^{1/2}\eta)^{S/m}\left[1 + \frac{S(S-m)}{2m^2(m\eta^2)} + 0(m^2\eta^{-4})\right], \tag{33.2}$$

and C_1 and C_2 are arbitrary constants. In view of the condition $g_1 \to 0$ as $\eta \to \infty$ given in (31), the divergent term D_2 must be omitted. Hence the constant C_2 in relations (32) must be zero. The solution of Eqs. (30) under the boundary conditions (31) is given by

$$\theta_1 = -C_3(Pr\,m\eta)^{-1}\exp(-Pr\,m\eta^2/2)\,[1 - (Pr\,m\eta^2)^{-1} + \ldots]\,, \tag{34}$$

where C_3 is an arbitrary constant.

Using relations (34) in Eq. (28) and then solving under the boundary conditions (31), we get

$$f_1{}' = C_4\,\exp(-m\eta^2/2)\,(m^{1/2}\eta)^{-(S+m)/m}\left[1 - \frac{(S+m)(S+2m)}{2m^2(m^{1/2}\eta)^2} + 0(m^2\eta^{-4})\right],$$

$$-(C_1/m^2)\,\exp(-m\eta^2/2)\,(m^{1/2}\eta)^{-(S+m)/m)}\left[1 - \frac{(S+m)(S+2m)}{3m^2(m^{1/2}\eta)^2} + 0(m^2\eta^{-4})\right], \tag{35}$$

where C_4 is an arbitrary constant. It is evident from the above equations that $f_1{}', g_1$ and θ_1 tend to zero exponentially as $\eta \to \infty$. Hence, f', g and θ tend to 1 exponentially as $\eta \to \infty$ (see [27]). Also f', g and θ are monotonic increasing functions of η in the range $0 \le \eta \le \infty$.

Since $f \to \eta, f' \to 1$ as $\eta \to \infty$, the axial velocity w (see [8]) for large η is given by

$$w = (az/r^2) - (a/r)\,(\nu/b)^{1/2}\eta\,, \tag{36.1}$$

and for small η it is given by

$$w = (az/r^2)\,f''(0)\,\eta - (a/r)\,(\nu/b)^{1/2} f''(0)\,\eta^2/2\,. \tag{36.2}$$

Existence of the similarity solution

It is possible to show that Eq. (9) under the boundary conditions (12) does not admit the similarity solution in the absence of suction f_w and the magnetic field S, but the similarity solution exists either in the presence of suction or the magnetic field or both. Integrating Eq. (9) twice with respect to η from $\eta = 0$ to $\eta = \eta_\infty$ (where η_∞ is the edge of the boundary-layer) and using the boundary conditions in (12), we obtain

$$m(f(\infty) - \eta_\infty)^2/2 = mf_w{}^2/2 + S\int_0^{\eta_\infty} d\eta \int_0^{\eta_\infty}(1-f')\,d\eta - 1 - m^{-1}\int_0^{\eta_\infty} d\eta\int_0^{\eta_\infty}(1-g^2)\,d\eta\,. \tag{37}$$

In the previous section, it was mentioned that f' and g tend to 1 exponentially as $\eta \to \infty$, and they are monotonically increasing functions. Hence for all η in $0 \le \eta \le \eta_\infty, 0 \le f' \le 1$, $0 \le g \le 1, , g \ge g^2$. Also f_w, η_∞, S and m are positive constants. For $f_w = 0$ (without suction) and $S = 0$ (without magnetic field), the left-hand side of (37) is positive $(f(\infty) \neq \eta_\infty)$, but the right-hand side is negative which is inconsistent. This implies that no solution exists for $f_w = S = 0$. When $f_w \geq f_w^*$ or $S \geq S^*$, the right-hand side is also positive, and relation (37) becomes consistent. Hence for $f_w \geq f_w^*$ or $S \geq S^*$, the similarity solution exists. The numerical results for $m = 1$ show that for $S = 0, f_w^* = 1.851$ and for $f_w = 0, S^* = 1.652$, respectively.

5 Results and discussion

Equations (9)–(11) under the boundary conditions (12) have been solved numerically using a shooting method [10]. We have examined the effect of the grid size $\Delta\eta$ and η_∞ on the solution, and the results presented here are independent of $\Delta\eta$ and η_∞ at least up to the 4th decimal place.

The results for the surface shear stresses $(f''(0), g'(0))$ when $S = 0$ (no magnetic field) and $u_e = 0$ (no source flow) are compared with those of Nanbu [7] who considered the potential vortex flow in the presence of large suction. The results are found to be in very good agreement (the maximum difference is less than 0.5 per cent for $f_w \geq 3$). Hence, for the sake of brevity the comparison is not shown here. Also, the surface shear stresses $(f''(0), g'(0))$ corresponding to the potential vortex flow ($u_e = 0$) for $S = 1, 2, 10$ are compared with those of King and Lewellen [3], and these are found to be in good agreement. This comparison is given in Table 1.

The variation of the surface shear stresses in the radial and tangential directions $(f''(0), g'(0))$ and the surface heat transfer $(\theta'(0))$ with the suction parameter f_w for several values of the magnetic parameter S is given in Figs. 2–4. These figures also show the results of the analytical solutions (Eqs. (25.1–25.3)). It can be seen that the analytical results are in good agreement with the numerical results for $f_w \geq 3$. The interesting result is that the solution exists beyond a certain value of the suction parameter f_w or the magnetic parameter S. For $S = 0$, the solution exists for $f_w \geq 1.851$, and for $S = 1$, it exists for $f_w \geq 0.4501$. For $S \geq 1.652$, the solution exists for all values of the suction parameter. Below these values the surface shear stress in the radial direction, $f''(0) < 0$, but the boundary-layer equations are not valid in such a situation. The effect of the suction parameter ($f_w > 0$) on the surface shear stresses and heat transfer $(f''(0), g'(0), \theta'(0))$ is more pronounced than that of the magnetic parameter S. For $S = 1, f''(0), g'(0)$ and $\theta'(0)$ increase by about 447%, 148% and 200%, respectively, as f_w increases from 1.5 to 5, but for $f_w = 3$ they increase by about 66%, 45%, 9%, respectively, when S increases from zero to 5. The increase in $f''(0), g'(0), \theta'(0)$ is caused by the reduction of both the momentum and thermal boundary-layer thicknesses. Also, the reason for the weak dependence of the heat transfer $(\theta'(0))$ on S is that the energy equation does not contain the magnetic field explicitly.

Table 1. Comparison of the surface shear stresses in the radial and tangential directions $(f''(0), g'(0))$ for $f_w = 0, m = 1, u_e = 0$

	Present results		King and Lewellen [3]	
S	$-f''(0)$	$g'(0)$	$-f''(0)$	$g'(0)$
1	0.6615	0.9682	0.655	0.959
2	0.4776	1.4113	0.473	1.397
10	0.2521	3.1541	0.250	3.131

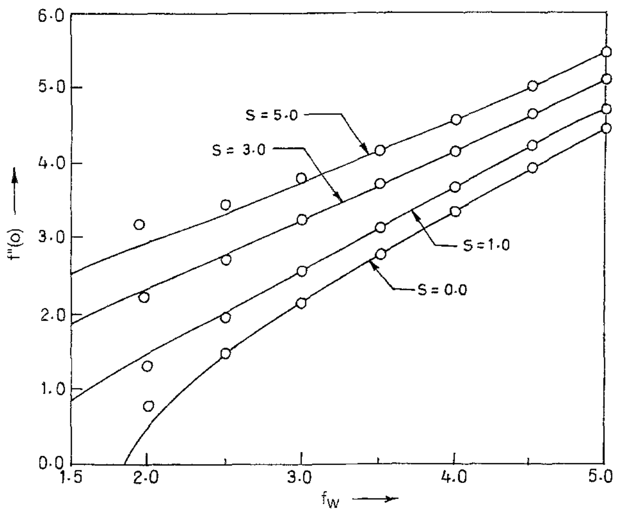

Fig. 2. Variation of the surface shear stress in the radial direction $(f''(0))$ with f_w and $S = 0, 1, 3, 5$, $m = 1$. o = analytical solution (Eq. (25.1))

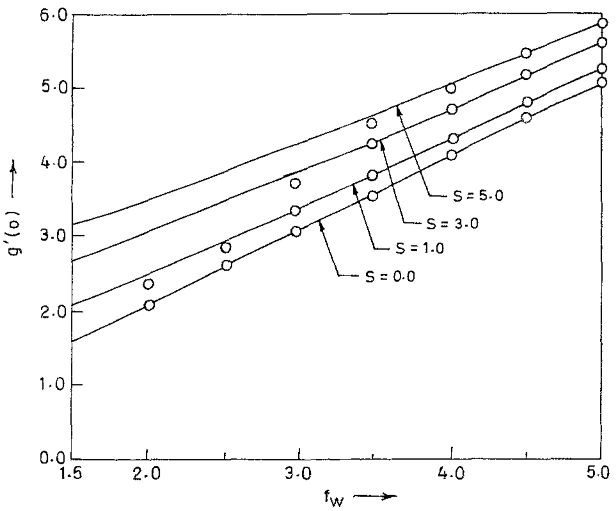

Fig. 3. Variation of the surface shear stress in the tangential direction $(f''(0))$ with f_w and $S = 0, 1, 3, 5$, $m = 1$. o = analytical solution (Eq. (25.2))

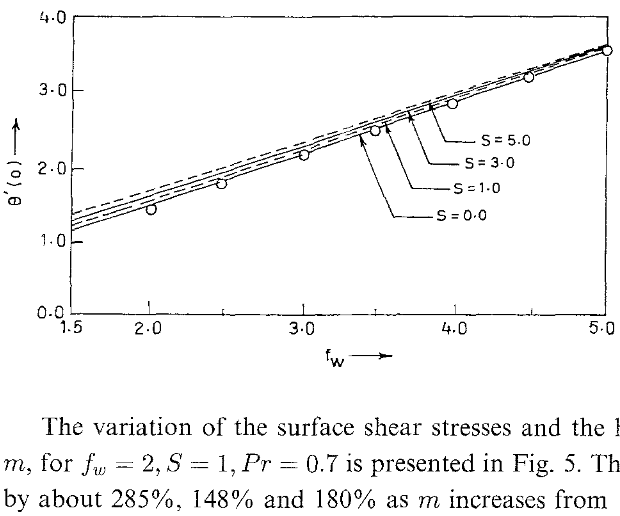

Fig. 4. Variation of the surface-heat transfer $(\theta'(0))$ with f_w for $Pr = 0.7$, $m = 1$. o = analytical solution (Eq. (25.3))

The variation of the surface shear stresses and the heat transfer $(f''(0), g'(0), \theta'(0))$ with m, for $f_w = 2, S = 1, Pr = 0.7$ is presented in Fig. 5. The functions $f''(0), g'(0), \theta'(0)$ increase by about 285%, 148% and 180% as m increases from 1 to 3. The strong dependence of the skin friction and heat-transfer parameters on m is due to the fact that m occurs explicitly in

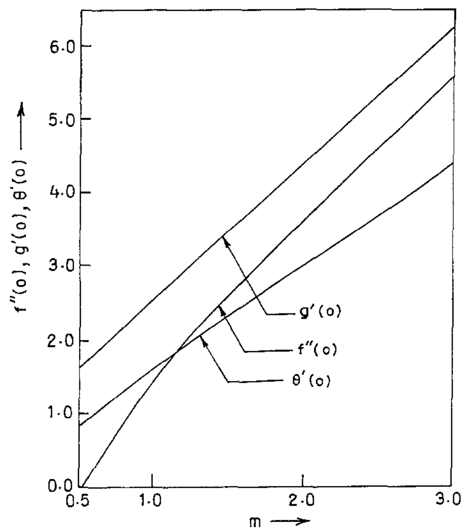

Fig. 5. Variation of the surface shear stresses and heat transfer $\left(f''(0), g'(0), \theta'(0)\right)$ with m for $f_w = 2$, $S = 1$, $Pr = 0.7$, $m = 1$

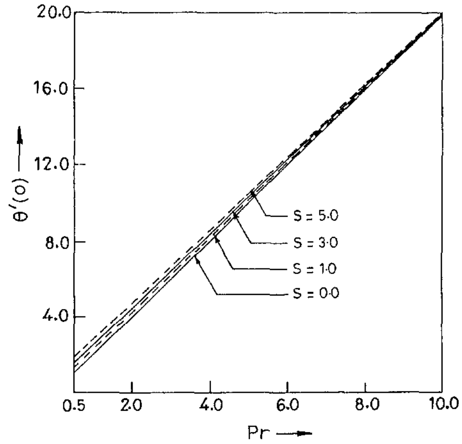

Fig. 6. Variation of the surface heat transfer $\left(\theta'(0)\right)$ with Pr for $m = 1$, $f_w = 2$

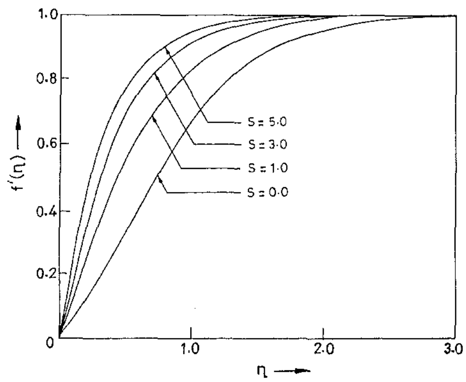

Fig. 7. Velocity profiles in the radial direction $\left(f'(\eta)\right)$ for $m = 1$, $f_w = 2$

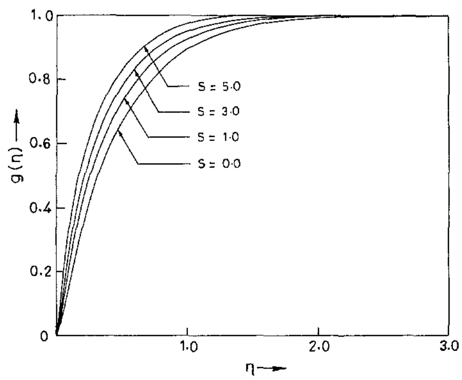

Fig. 8. Velocity profiles in the tangential direction $\left(g(\eta)\right)$ for $m = 1$, $f_w = 2$

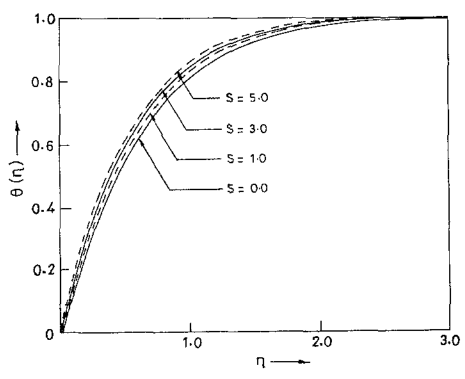

Fig. 9. Temperature profiles $\left(\theta(\eta)\right)$ for $m = 1$, $f_w = 2$, $Pr = 0.7$

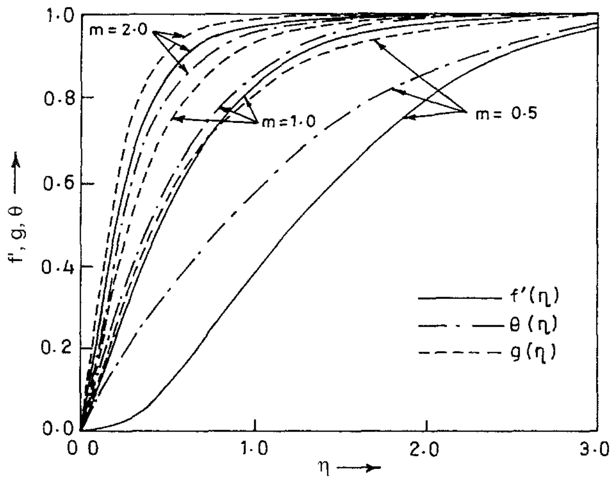

Fig. 10. Velocity and temperature profiles $(f'(\eta)), g(\eta), \theta(\eta)$ for $f_w = 2,\ S = 1,\ Pr = 0.7$

the momentum and the energy equations. Also, the solution exists for $m \geq 0.5375$, when $f_w = 2, S = 1$.

The effect of the Prandtl number Pr on the surface heat transfer $\theta'(0)$ for $f_w = 2, m = 1$ is displayed in Fig. 6. The surface shear stresses $(f''(0), g'(0))$ are not affected by the Prandtl number. The surface heat transfer increases significantly with Pr due to the reduction in the thermal boundary-layer thickness. As mentioned earlier, the heat transfer changes little with the change in the magnetic parameter S. For $m = S = 1, f_w = 2, \theta'(0)$ increases by about 16 times its value as Pr increases from 0.5 to 10. Since Pr occurs in the energy equation, it strongly affects the heat transfer.

The effect of the magnetic parameter S on the velocity and temperature profiles $((f', g, \theta)$ for $f_w = 2, m = 1, Pr = 0.7$ is presented in Figs. 7–9. The velocity and temperature profiles (f', g, θ) increase with S due to the enhanced Lorentz force. However, the effect of S on the temperature profiles is small.

The effect of m on the velocity and temperature profiles $((f', g, \theta)$ for $S = 1,\ f_w = 2,\ Pr = 0.7$ is shown in Fig. 10. The velocity and temperature profiles increase with m due to the reduction in the momentum and thermal boundary-layers.

6 Conclusions

The surface shear stresses in the radial and tangential directions and the surface heat transfer increase with suction, the magnetic field and the ratio of the source and vortex flow, and they strongly depend on these parameters, except the heat transfer which is weakly dependent on the magnetic field. Also, the surface heat transfer significantly increases with the Prandtl number. The solution exists beyond a certain value of the suction parameter or the magnetic parameter. The analytical results are found to be in good agreement with the numerical results for $f_w \geq 3$.

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