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Heat and mass transfer effects on flow past an impulsively started vertical plate

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Summary. An exact solution to the problem of flow past an impulsively started infinite vertical plate in the presence of uniform heat and mass flux at the plate is presented by the Laplace-transform technique. The velocity, the temperature and the concentration profiles are shown graphically. The rate of heat transfer, the skin-friction, and the Sherwood number are also shown on graphs. The effect of different parameters like Grashof number, mass Grashof number, Prandtl number, and Schmidt number are discussed.

List of symbols

C'	species concentration near the plate
C'_∞	species concentration in the fluid far away from the plate
C	dimensionless concentration
C_p	specific heat at constant pressure
D	mass diffusion coefficient
g	acceleration due to gravity
Gr	thermal Grashof number
Gc	mass Grashof number
j''	mass flux per unit area at the plate
K	thermal conductivity of the fluid
Nu	Nusselt number
Pr	Prandtl number
q	heat flux per unit area at the plate
Sc	Schmidt number
t'	time
t	dimensionless time
T'	temperature of the fluid near the plate
T'_∞	temperature of the fluid far away from the plate
T'_w	temperature of the plate
u'	velocity of the fluid in the x'-direction
u_0	velocity of the plate
u	dimensionless velocity
x'	coordinate axis along the plate
y'	coordinate axis normal to the plate
y	dimensionless coordinate axis normal to the plate
β	volumetric coefficient of thermal expansion
β^*	volumetric coefficient of expansion with concentration
μ	coefficient of viscosity

ν kinematic viscosity
ϱ density
τ' skin-friction
τ dimensionless skin-friction
θ dimensionless temperature
$erfc$ complementary error function
η similarity parameter

1 Introduction

In many manufacturing processes such as hot rolling, hot extrusion, wire drawing, continuous casting and fibre drawing, heat transfer occurs between a moving material and the ambient medium. In most cases, the moving material is hotter than the surroundings, and the heat transfer to the ambient occurs at the surface of the moving material. In the case of wire drawing and continuous casting processes, the material is cooled by passing it through a colder ambient medium like water or, in some cases, just quiescent ambient air.

The flow of an incompressible viscous fluid past an impulsively started infinite horizontal plate, in its own plane, was first studied by Stokes [1]. It is also known as Rayleigh's problem in the literature. Following Stokes' analysis, Soundalgekar [2] first presented an exact solution to the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate. The solution was derived using the Laplace-transform technique, and the effects of heating or cooling of the plate on the flow-field were discussed through Grashof number. The fluid considered in this study was pure air or water. However, in nature, the availability of pure air or water is very difficult. Air and water are contaminated with impurities like CO_2, NH_3, O_2, etc. or salts in water. The presence of such impurities is studied in the literature by considering it as a foreign mass. Soundalgekar [3] has studied mass transfer effects on flow past an impulsively started infinite vertical plate. The solution to this problem governed by coupled linear differential equations was derived by using the Laplace-transform technique. The study of convection with heat and mass transfer is very useful in fields such as chemistry, agriculture and oceanography. A few representative fields of interest in which combined heat and mass transfer play an important role are the design of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees, damage of crops due to freezing, and pollution of the environment. This technique is used in the cooling processes of plastic sheets, polymer fibres, glass materials, and in drying processes of paper. Das et al. [4] considered the mass transfer effects on flow past an impulsively started infinite isothermal vertical plate with constant mass flux. The solutions are derived by the Laplace-transform technique. In this study, the plate temperature and rate of concentration are assumed to be constant.

In the present analysis, it is proposed to study the effects of simultaneous constant heat and mass flux on the flow past an impulsively started infinite vertical plate by the Laplace-transform technique. In this case, the effect of various chemical impurities has been discussed. Such a study is found useful in chemical processing industries such as food processing, crystal growing and polymer production.

In Sect. 2, the problem is posed mathematically, and the solutions are derived by the usual Laplace-transform technique for the velocity, temperature and concentration profiles. They are shown graphically, followed by a discussion. In Sect. 3, the conclusions are given.

2 Mathematical analysis

Here, the flow of a viscous incompressible fluid past an impulsively started infinite vertical plate is considered. The x'-axis is taken along the plate in the vertically upward direction, and the y'-axis is taken normal to the plate. At time $t' \leq 0$, the plate and the fluid are at the same temperature and concentration in a stationary condition. At time $t' > 0$, the plate is given an impulsive motion in the vertical direction with constant velocity u_0, and at the plate constant heat and mass flux are imposed. Then under usual Boussinesq's approximation, the unsteady flow past an infinite vertical plate is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2}, \tag{1}$$

$$\varrho C_p \frac{\partial T'}{\partial t'} = K \frac{\partial^2 T'}{\partial y'^2}, \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2}, \tag{3}$$

with the following initial and boundary conditions:

$$\begin{aligned}
&t' \leq 0 \qquad u' = 0\,, \qquad T' = T'_\infty\,, \qquad C' = C'_\infty \qquad \text{for all} \qquad y' \leq 0\,,\\
&t' > 0: \qquad u' = u_0\,, \qquad \frac{\partial T'}{\partial y'} = -\frac{q}{K}\,, \qquad \frac{\partial C'}{\partial y'} = -\frac{j''}{D} \qquad \text{at} \qquad y' = 0\,,\\
&u' = 0\,, \qquad T' \to T'_\infty\,, \qquad C' \to C'_\infty \qquad \text{as} \qquad y' \to \infty\,.
\end{aligned} \tag{4}$$

On introducing the following nondimensional quantities:

$$\begin{aligned}
&U = \frac{u'}{u_0}\,, \qquad t' = \frac{{u_0}^2 t'}{\nu}\,, \qquad y = \frac{y' u_0}{\nu}\,, \qquad \theta = \frac{T' - T'_\infty}{\left(\dfrac{q\nu}{K u_0}\right)}\,, \qquad C = \frac{C' - C'_\infty}{\left(\dfrac{j''\nu}{D u_0}\right)}\,,\\
&\mathrm{Gr} = \frac{\nu g \beta \left(\dfrac{q\nu}{K u_0}\right)}{{u_0}^3}\,, \qquad \mathrm{Gc} = \frac{\nu g \beta^* \left(\dfrac{j''\nu}{D u_0}\right)}{{u_0}^3}\,, \qquad \mathrm{Pr} = \frac{\mu C_p}{K}\,, \qquad \mathrm{Sc} = \frac{\nu}{D}
\end{aligned} \tag{5}$$

Eqs. (1) – (4) lead to

$$\frac{\partial u}{\partial t} = \mathrm{Gr}\,\theta + \mathrm{Gc}\,C + \frac{\partial^2 u}{\partial y^2}, \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\mathrm{Pr}} \frac{\partial^2 \theta}{\partial y^2}, \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{\mathrm{Sc}} \frac{\partial^2 C}{\partial y^2}. \tag{8}$$

The corresponding initial and boundary conditions in dimensionless form are:

$$\begin{aligned}
&t \leq 0: \qquad u = 0\,, \qquad \theta = 0\,, \qquad C = 0\,, \qquad \text{for all} \qquad y \leq 0\,,\\
&t > 0: \qquad u = 1\,, \qquad \frac{\partial \theta}{\partial y} = -1\,, \qquad \frac{\partial C}{\partial y} = -1 \qquad \text{at} \qquad y = 0\,,\\
&u = 0\,, \qquad \theta \to 0\,, \qquad C \to 0 \qquad \text{as} \qquad y \to \infty\,.
\end{aligned} \tag{9}$$

These Eqs. (6)–(8), subject to the boundary conditions (9), are solved by the usual Laplace-transform technique, and the solutions are derived as follows:

Case I: $\mathrm{Sc} \neq 1$

$$\theta = 2\sqrt{t}\left[\frac{\exp(-\eta^2 \mathrm{Pr})}{\sqrt{\pi}\sqrt{\mathrm{Pr}}} - \eta\, erfc(\eta\sqrt{\mathrm{Pr}})\right], \tag{10}$$

$$\begin{aligned} u = erfc(\eta) &+ \frac{\mathrm{Gr}\, t\sqrt{t}}{3(\mathrm{Pr}-1)\sqrt{\mathrm{Pr}}}\left[\frac{4}{\sqrt{\pi}}(1+\eta^2)\exp(-\eta^2) - \frac{4}{\sqrt{\pi}}(1+\eta^2\,\mathrm{Pr})\exp(-\eta^2\,\mathrm{Pr})\right.\\ &\left. + \eta\sqrt{\mathrm{Pr}}(6+4\eta^2\,\mathrm{Pr})\, erfc(\eta\sqrt{\mathrm{Pr}}) - \eta(6+4\eta^2)\, erfc(\eta)\right]\\ &+ \frac{\mathrm{Gc}\, t\sqrt{t}}{3(\mathrm{Sc}-1)\sqrt{\mathrm{Sc}}}\left[\frac{4}{\sqrt{\pi}}(1+\eta^2)\exp(-\eta^2) - \frac{4}{\sqrt{\pi}}(1+\eta^2\,\mathrm{Sc})\exp(-\eta^2\,\mathrm{Sc})\right.\\ &\left. + \eta\sqrt{\mathrm{Sc}}(6+4\eta^2\,\mathrm{Sc})\, erfc(\eta\sqrt{\mathrm{Sc}}) - \eta(6+4\eta^2)\, erfc(\eta)\right] \end{aligned} \tag{11}$$

$$C = 2\sqrt{t}\left[\frac{1}{\sqrt{\pi}\sqrt{\mathrm{Sc}}}\exp(-\eta^2\,\mathrm{Sc}) - \eta\, erfc(\eta\sqrt{\mathrm{Sc}})\right], \tag{12}$$

where $\eta = y/2\sqrt{t}$.

Case II: $\mathrm{Sc} = 1$

$$C = 2\sqrt{t}\left[\frac{1}{\sqrt{t}}\exp(-\eta^2) - \eta\, erfc(\eta)\right], \tag{13}$$

$$\begin{aligned} u = erfc(\eta) &+ \frac{\mathrm{Gr}\, t\sqrt{t}}{3(\mathrm{Pr}-1)\sqrt{\mathrm{Pr}}}\left[\frac{4}{\sqrt{\pi}}(1+\eta^2)\exp(-\eta^2) - \frac{4}{\sqrt{\pi}}(1+\eta^2\,\mathrm{Pr})\exp(-\eta^2\,\mathrm{Pr})\right.\\ &\left. + \eta\sqrt{\mathrm{Pr}}(6+4\eta^2\,\mathrm{Pr})\, erfc(\eta\sqrt{\mathrm{Pr}}) - \eta(6+4\eta^2)\, erfc(\eta)\right]. \end{aligned} \tag{14}$$

In order to get physical insight into the problem, the numerical values of u, θ and C are evaluated for different values of the Prandtl number, the Grashof number and the mass Grashof number. The numerical values of the Schmidt number Sc are chosen such that they represent a reality in case of air ($\mathrm{Pr} = 0.71$), see Gebhardt and Pera [5].

The velocity profiles for air are shown in Fig. 1 for different values of Sc and $\mathrm{Gr} = 2.0$, $\mathrm{Gc} = 5.0$ and $t = 0.2$. It is observed that an increase in the Schmidt number Sc leads to a fall in the velocity of air. However, at very small values of the Schmidt number Sc we observe that the velocity tends to overshoot near the plate. Physically, it can be interpreted as follows: For a given Pr, as Sc decreases, the relative thickness of the concentration boundary layer increases, and the contribution from mass diffusion-induced buoyancy becomes more significant, and hence the velocity increases near the plate or there is an overshoot in the velocity at small values of the Schmidt number.

In Fig. 2, the velocity profiles are shown for different values of the Grashof number and the mass Grashof number, and we conclude that there is a rise in velocity, when Gr or Gc increases. Physically, this is possible because as the Grashof number increases, the contribution from the buoyancy near the plate becomes significant, and hence a short rise in the velocity near the plate is observed.

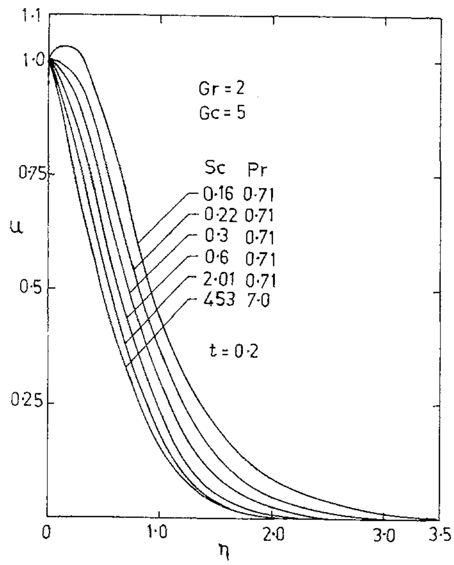

Fig. 1. Velocity profiles for different Sc

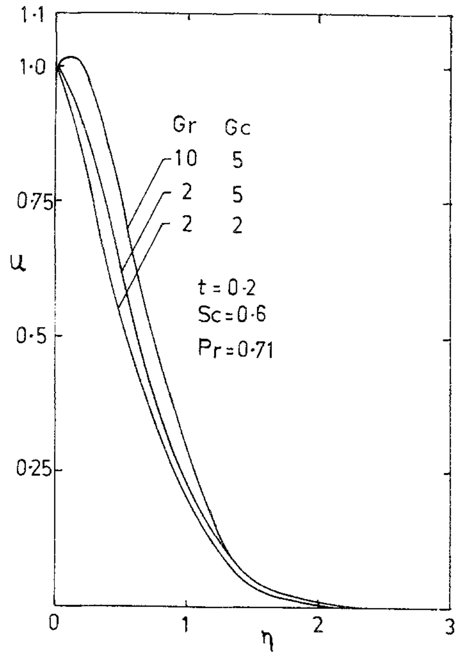

Fig. 2. Velocity profiles for different Gr and Gc

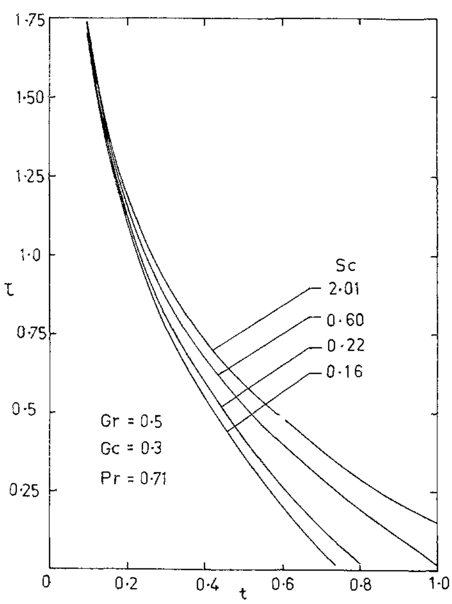

Fig. 3. Skin-friction for different Sc

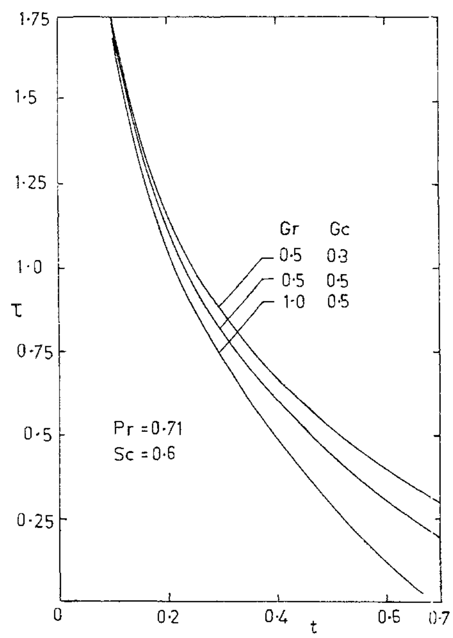

Fig. 4. Skin-friction for different Gr and Gc

From the velocity field, the skin-friction is studied and is given by

$$\tau' = -\mu \left(\frac{\partial u'}{\partial y'}\right)_{y'=0}, \tag{15}$$

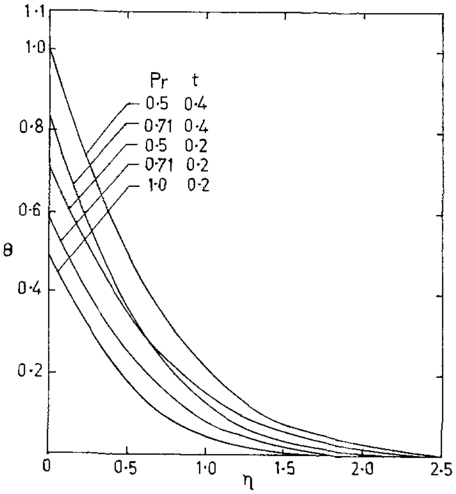

Fig. 5. Temperature profiles

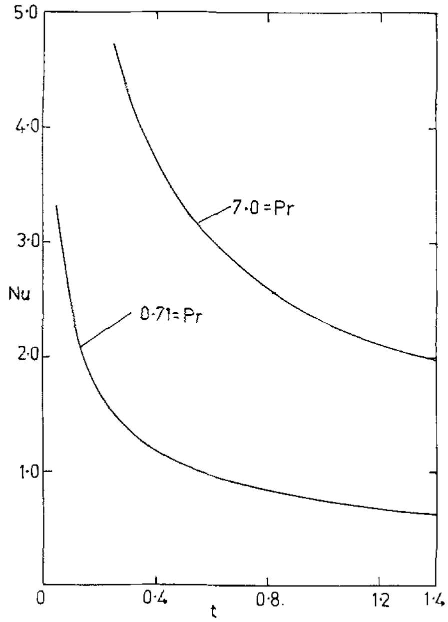

Fig. 6. Nusselt number

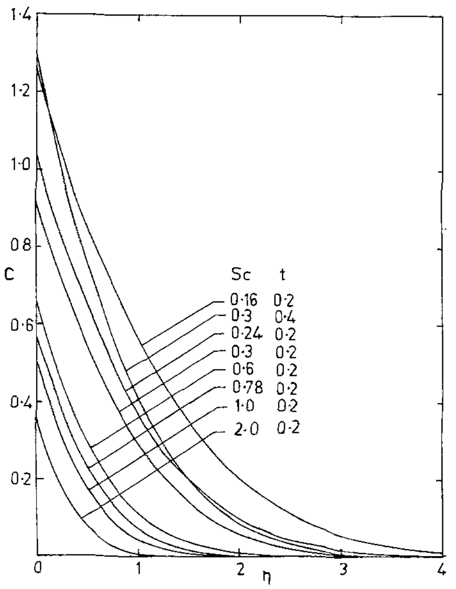

Fig. 7. Concentration profiles

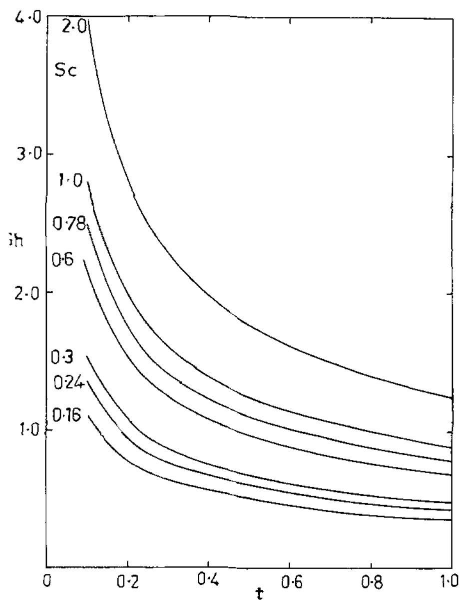

Fig. 8. Sherwood number

and in view of (5), Eq. (15) reduces to

$$\tau = \frac{\tau'}{\varrho {u_0}^2} = -\left(\frac{du}{dy}\right)_{y=0}. \tag{16}$$

The numerical values of τ are computed, and these are plotted in Figs. 3 and 4. From Fig. 3, it is observed that an increase in the Schmidt number leads to an increase in the value of the skin-friction. The skin-friction for different values of the Grashof number and the mass Grashof number are shown in Fig. 4. It is observed that an increase in the Grashof number or mass Grashof number leads to a fall in the skin-friction when water vapour ($\mathrm{Sc} = 0.6$) is present in air.

The temperature profiles are calculated from Eq. (10), and these are shown in Fig. 5 for air ($\mathrm{Pr} = 0.71$) and water ($\mathrm{Pr} = 7.0$), and it is clear that there is a fall in temperature with increasing the Prandtl number. From the temperature field, the rate of heat transfer is studied and is given in nondimensional form as

$$\mathrm{Nu} = -\frac{1}{\theta(0)}\left(\frac{d\theta}{dy}\right)_{y=0} = \frac{1}{\theta(0)}\,, \tag{17}$$

and the numerical values of the Nusselt number are plotted in Fig. 6. It is observed that the Nusselt number increases with increasing the Prandtl number.

The numerical values of the concentration profiles are computed from Eqs. (12) and (13), and these values are plotted in Fig. 7 for different values of the Schmidt number. It shows that an increase in the Schmidt number leads to a decrease in the concentration, but concentration increases with time. From the concentration field, the rate of concentration transfer is analyzed, which is expressed in terms of the Sherwood number as

$$\mathrm{Sh} = -\frac{1}{C(0)}\left(\frac{dC}{dy}\right)_{y=0} = \frac{1}{C(0)}\,. \tag{18}$$

Using boundary condition (9), the numerical values of the Sherwood number are calculated, and these values are plotted in Fig. 8. It is observed that the Sherwood number increases with increasing the Schmidt number.

3 Conclusions

An exact solution has been carried out for the flow past an impulsively started infinite vertical plate with simultaneous heat and mass flux. The dimensionless governing equations are solved using the usual Laplace-transform technique. The study concludes the following results:

(i) The velocity decreases with increasing the Schmidt number and increases with increasing the Grashof number or mass Grashof number.
(ii) The temperature increases with decreasing the Prandtl number.
(iii) An increase of the Schmidt number, the Grashof number or the mass Grashof number leads to a decrease of the skin-friction.
(iv) The rate of heat transfer increases with increasing the Prandtl number.
(v) An increase in the Schmidt number leads to a fall in the concentration and a rise in the Sherwood number.

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