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Free convection heat and mass transfer along a vertical surface in a porous medium

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Summary. This study deals with free convection heat and mass transfer from a vertical plate embedded in a fluid saturated porous medium with constant wall temperature and concentration. The temperature and concentration variations across the boundary layer produce a buoyancy effect which gives rise to flow field. Integral method of Von-Karman type is applied to obtain the analytical solution of this fundamental problem.

1 Introduction

The pure convection heat transfer along a vertical plate embedded in a saturated porous medium has been dealt with at length in literature. However, the work on convection driven by two buoyancy effects due to variation of temperature and concentration across the boundary layer has received very little attention. From a fundamental perspective, Bejan and Khair [1] have used Darcy's law to study the features of free convection boundary layer flow driven by temperature and concentration gradients. The coupled heat and mass transfer due to buoyancy in saturated porous medium may be encountered in geophysical and geothermal applications where surface mass transfer on the bed rock is generated due to chemical reaction and in the underground disposal of nuclear wastes where the spread of radioactive materials may result from the failure of canisters. Recently, Lai and Kulacki [2] have re-examined the free convection boundary layer along a vertical wall with constant heat and mass flux including the effect of wall injection.

The aim of the present study is to apply the integral method to analyse free convection problem along a vertical wall in the presence of temperature and concentration gradients. The temperature and concentration profiles of fourth order are assumed and the expressions for the rates of heat and mass transfer cover all possible positive and negative ranges of buoyancy ratio, N and Lewis number, Le. It is observed that for $N > 0$, the rate of heat transfer decreases as Lewis number increases while for negative N situation is reversed. The rate of mass transfer increases with increasing Lewis number for all N.

2 Mathematical analysis

For heat and mass transfer driven only by buoyancy, the governing boundary layer equations for natural convection along a vertical wall embedded in a Darcian fluid are given by [1]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u = \frac{gK}{\nu}[\beta_T(T - T_\infty) + \beta_c(C - C_\infty)] \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2} \tag{4}$$

$$\varrho = \varrho_\infty[1 - \beta_T(T - T_\infty) - \beta_c(C - C_\infty)] \tag{5}$$

where β_T, β_c, ν, g, K, α, D are thermal expansion coefficient, concentration expansion coefficient, kinematic viscosity of the fluid, acceleration due to gravity, permeability of porous medium, thermal and concentration diffusivities of the porous medium respectively. T and C denote the temperature and concentration inside the boundary layer. The boundary conditions are:

$$\begin{aligned} &y = 0; \quad T = T_w, \quad C = C_w, \quad v = 0 \\ &y \to \infty; \quad T \to T_\infty, \quad C \to C_\infty, \quad u \to 0. \end{aligned} \tag{6}$$

3 Method of solution

Integrating Eqs. (3) and (4) with respect to y from 0 to ∞ and using Eq. (1), we get the following integral equations

$$\frac{\delta}{\delta x}\int_0^\infty (u \cdot \theta)\, dy = -\alpha \left.\frac{\delta\theta}{\delta y}\right|_{y=0}, \tag{7}$$

$$\frac{\delta}{\delta x}\int_0^\infty (u \cdot \phi)\, dy = -D \left.\frac{\delta\phi}{\delta y}\right|_{y=0}, \tag{8}$$

where $\theta = \dfrac{T - T_\infty}{T_w - T_\infty}$ and $\phi = \dfrac{C - C_\infty}{C_w - C_\infty}$.

Since in the free convection the sole driving force is represented by the buoyancy term caused due to the temperature and concentration differences, we treat the temperature, T and concentration, C as the primary variables. Using boundary conditions (6) and smoothness conditions:

$$\frac{\partial\theta}{\partial y} \to 0, \quad \frac{\partial\phi}{\partial y} \to 0, \quad \frac{\partial^2\theta}{\partial y^2} \to 0, \quad \frac{\partial^2\phi}{\partial y^2} \to 0 \quad \text{as} \quad y \to \infty \tag{9}$$

the temperature and concentration profiles become:

$$\theta = 1 - 2\eta + 2\eta^3 - \eta^4, \tag{10}$$

$$\phi = 1 - 2\eta_1 + 2\eta_1{}^3 - \eta_1{}^4, \tag{11}$$

where $\eta = y/\delta_t$, $\eta_1 = y/\delta_c$; δ_t, δ_c being the thermal and concentration boundary layer thicknesses to be determined with the help of Eqs. (7) and (8). These profiles satisfy the compatibility conditions

$$\frac{\partial^2\theta}{\partial y^2} = 0, \quad \frac{\partial^2\phi}{\partial y^2} = 0 \quad \text{at} \quad y = 0 \tag{12}$$

which are obtained by evaluating the equations (3) and (4) at $y = 0$. Using Eqs. (10), (11) and (2) in Eqs. (7) and (8) and integrating we obtain a set of two ordinary differential equations for unknown boundary layer thicknesses δ_t and δ_c, the solutions of which is easily obtained as

$$\delta_t = \delta_t^* \sqrt{\frac{\alpha v x}{K g \beta_T \theta_w}},$$

$$\delta_c = \delta_c^* \sqrt{\frac{\alpha v x}{K g \beta_T \theta_w}},$$

where the unknowns δ_t^* and δ_c^* are given by

$$\left(\frac{23}{126} + NH(\Delta)\right) \delta_t^{*2} = 4, \tag{13}$$

$$\left(\Delta H(\Delta) + \frac{23N}{126}\right) \delta_c^{*2} = \frac{4}{\mathrm{Le}}, \tag{14}$$

where $\Delta = \delta_t/\delta_c$, is the ratio of boundary layer thicknesses, $\mathrm{Le} = \alpha/D$, is Lewis number, $N = \beta_c \phi_w/(\beta_T \theta_w)$, is buoyancy ratio and

$$H(\Delta) = \frac{3}{10\Delta} - \frac{2}{15\Delta^2} + \frac{3}{140\Delta^4} - \frac{1}{180\Delta^5} \quad \text{for } \Delta > 1$$

$$= \frac{3}{10} - \frac{2}{15}\Delta + \frac{3}{140}\Delta^3 - \frac{1}{180}\Delta^4 \quad \text{for } \Delta < 1$$

4 Results and discussions

The physical characteristics of importance are rates of heat and mass transfer from the wall to the fluid. The heat transfer coefficient in terms of Nusselt number is obtained as [3]

$$\mathrm{Nu} = \frac{2}{\delta_t^*} \sqrt{\mathrm{Ra}}, \tag{15}$$

while the mass transfer coefficient in terms of Sherwood Number is

$$\mathrm{Sh} = \frac{2}{\delta_c^*} \sqrt{\mathrm{Ra}}, \tag{16}$$

where $\mathrm{Ra} = K\beta_T g \theta_w x/(\alpha v)$, is the modified Rayleigh number.

The variation of $\mathrm{Nu}/\sqrt{\mathrm{Ra}}$ and $\mathrm{Sh}/\sqrt{\mathrm{Ra}}$ with Lewis number and buoyancy ratio are compared with those obtained by Bejan and Khair [1] in Table 1.

As expected, it is found that the value of heat transfer for fixed N decreases monotonically as Lewis number increases while for Sherwood number situation is reversed. Nusselt number is plotted in Fig. 1 as a function of buoyancy ratio for various values of Lewis number. For $N > 0$ rate of heat transfer decreases as Lewis number increases while for $N < 0$, it increases with increasing Lewis number.

Table 1. Comparison of local Nusselt and Sherwood numbers

		$Nu/\sqrt{Ra}$		$Sh/\sqrt{Ra}$	
N	Le	Exact	Present	Exact	Present
4	1	.992	.955	.992	.955
	4	.798	.750	2.055	1.981
	10	.681	.634	3.290	3.168
	100	.521	.489	10.521	10.118
1	1	.628	.604	0.628	0.604
	4	.559	.532	1.358	1.311
	10	.521	.495	2.202	2.125
	100	.470	.449	7.139	6.884

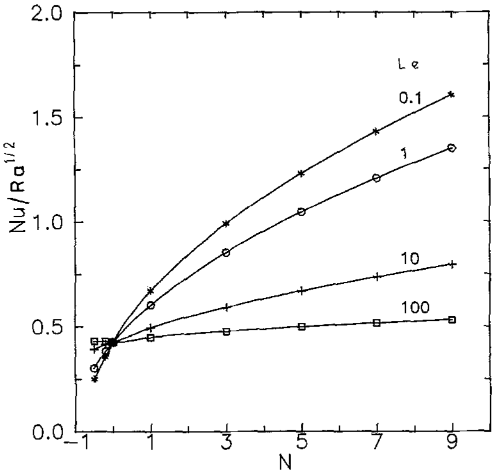

Fig. 1. Heat transfer coefficient as function of buoyancy ratio

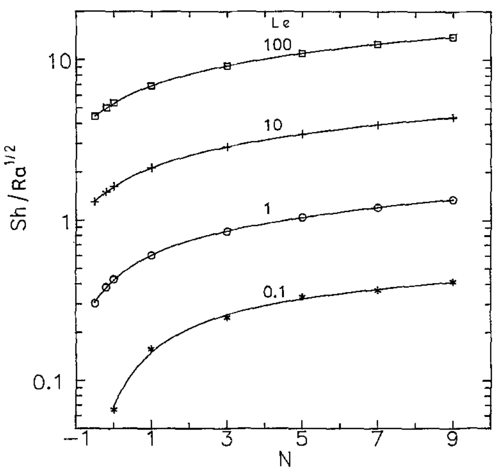

Fig. 2. Mass transfer coefficient as function of buoyancy ratio

The Sherwood number is plotted in Fig. 2 against buoyancy ratio for Le = 0.1, 1, 10 and 100. It is observed that the Lewis number has more pronounced effect on concentration field than it does on the temperature field. Figure 2 shows that the mass transfer coefficient is significantly increased when the Lewis number increases for all N. Boundary layer thickness ratio Δ defined by $\Delta = \delta_t/\delta_c$ is plotted in Fig. 3 in terms of its reciprocal and Lewis number, again an excellent

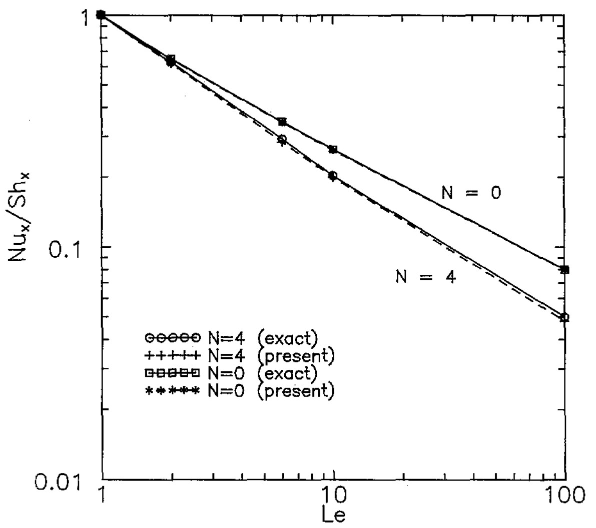

Fig. 3. Boundary layer thickness ratio

agreement is confirmed between the present approximate solution and exact solution of Bejan and Khair [1]. In the case of $N = 0$, the equation reduces to $\delta_t^* = \sqrt{(504/23)}$ which is the same as that obtained by Singh and Sharma [3]. It is also interesting to note that for $\Delta = 1$ and $\text{Le} = 1$; $H(\Delta) = 23/126$ and for this case the rates of heat and mass transfer reduce to

$$\frac{\text{Nu}}{\sqrt{\text{Ra}}} = \frac{\text{Sh}}{\sqrt{\text{Ra}}} = \sqrt{\frac{23(1+N)}{126}}$$

Conclusion

Heat and mass transfer coefficients obtained in the current study by the integral method agree very well with the prediction by similarity analysis [1], [2]. In the present approximate study, results are presented into a closed form analytical expressions which offers any practicing engineer a rapid way of obtaining physical characteristics of the problem for any arbitrary value of buoyancy ratio and Lewis number. The advantage of the present analysis is that the results are obtained with remarkable ease as compared to the task involved in numerical assault of the problem.

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