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Free Convection in the Laminar Boundary Layer Flow of a Thermomicropolar Fluid past a Vertical Flat Plate with Suction/Injection

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With 4 Figures

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Summary

The effect of suction/injection in the laminar free convection flow of a thermomicropolar fluid past a nonuniformly heated vertical flat plate has been considered. The conditions under which similarity exists have been examined. The resulting system of nonlinear ordinary differential equations has been solved numerically after transforming the infinite domain of boundary layer coordinate into a finite domain. The effects of variation of the boundary condition parameter and suction/injection parameter on the velocity, microrotation and temperature fields and the heat transfer coefficient have been studied graphically. The skin-friction parameter and the gradient of microrotation on the wall have been tabulated. It is found that there is significant increase in velocity, skin-friction and the heat transfer coefficient with the decreasing concentration of microelements.

1. Introduction

In the recent years, the study of free convection phenomenon has been the object of extensive research. The intensity of research in this field is due to enhanced concerns in science and technology about buoyancy-induced motions in the atmosphere, in bodies of water and in quasi-solid bodies such as earth. Heat transfer effects under the conditions of free convection are now dominant in many engineering applications such as rocket nozzels, cooling of nuclear reactors, high speed aircrafts and their atmospheric reentry, high sinks in turbine blades, chemical devices and process equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees, damage of crops due to freezing and pollution of the environment and so on. Hence a thorough investigation and knowledge of heat transfer process must be acquired in order to be able to design heat exchangers, bearing etc., so that no over heating or damage is caused to the components.

In case of heated vertical plate the heat transferred from the surface caused a decrease in liquid density and a subsequent upward flow due to buoyancy force. Mathur [1] has studied the free convection flow of an elastico-viscous fluid past a non-uniformly heated vertical plate. An excellent survey of this

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problem for Newtonian and non-Newtonian fluids has been given in [1]. Although a number of studies on the laminar free convection flow and heat transfer of Newtonian and non-Newtonian fliuds have been reported in literature, these do not give satisfactory results if the fluid is a mixture of heterogeneous means such as liquid crystals, ferro liquid, liquid with polymer additives, which is more realistic and important from technological point of view. For the realistic description of the flow of fluids such as fluids with polymeric additives etc., the classical continuum mechanics cannot be used. To overcome this, Eringen [2] has introduced the theory of thermomicropolar fluids. The theory of thermomicropolar fluids is an extension of the theory of micropolar fluids [3]. Physically, this theory may be considered to describe the flow behaviour of colloidal fluids, polymeric fluids, real fluids with suspensions and possibly animal blood. This theory is also capable of explaining the experimentally observed phenomenon of skin-friction reduction near a rigid body [4], [5] in fluids containing extremely small amount of polymeric additives when compared with the skin-friction in the same fluids without additives. Latto and Shen [6] studied the effects of injecting dilute aqueous polymer solutions into a turbulent boundary layer formed on a flat plate. They found that polymer concentration and injection velocity reduce the frictional drag. The phenomenon of drag reduction has been discussed in great detail in [7]. This phenomenon cannot be explained on the basis of classical continuum mechanics. Recently, Říha [8] has applied this theory for the adequate representation of fluid suspensions of rigid particles in a Newtonian fluid. The most complete statement of carried out investigation in this region and possible application of this theory has been reported in [9].

For micropolar fluids, Balaram and Sastry [10] have studied the convective heat transfer in a vertical channel. Sastry and Maiti [11] have studied the combined convective heat transfer in a micropolar fluid flowing in an annulus of two vertical pipes. Recently, Jena and Mathur [12] have studied the natural convection in the laminar boundary layer flow of a thermomicropolar fluid past a nonuniformly heated vertical flat plate.

The present work concerns with the free convection flow of a thermomicropolar fluid past a nonuniformly heated vertical flat plate in the presence of suction or injection. In Section 2, we have given the formulation of the problem. In Section 3, the conditions under which similarity exists have been examined. The resulting system of nonlinear ordinary differential equations have been solved numerically in a finite domain by 'shooting method'. The transformation which carries infinite domain to finite domain is given by Sills [13]. In Section 4, we have presented the results graphically for velocity, microrotation, temperature and the heat transfer coefficient. Numerical values for skin-friction parameter and the gradient of microrotation on the wall have been tabulated for different values of the boundary condition parameter and suction/injection parameter.

2. Formulation of the Problem

We choose a two-dimensional cartesian co-ordinate system (x, y) in which x is measured along the vertical plate and y is normal to the plate. The equations governing the steady laminar flow of a thermomicropolar fluid in this co-ordinate system are:

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

Momentum:

$$\varrho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = (\mu_v + k_v) \frac{\partial^2 u}{\partial y^2} + k_v \frac{\partial \nu}{\partial y} + \varrho g \beta (T - T_\infty) \tag{2.2}$$

Moment of Momentum:

$$\varrho j \left(u \frac{\partial \nu}{\partial x} + v \frac{\partial \nu}{\partial y}\right) = \gamma_v \frac{\partial^2 \nu}{\partial y^2} - k_v \left(\frac{\partial u}{\partial y} + 2\nu\right) \tag{2.3}$$

Energy:

$$\varrho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = K_c \frac{\partial^2 T}{\partial y^2} + \alpha^* \left(\frac{\partial T}{\partial x} \frac{\partial x}{\partial y} - \frac{\partial T}{\partial y} \frac{\partial \nu}{\partial x}\right), \tag{2.4}$$

where

u, v = components of velocity along and normal to the vertical flat plate
ν = component of microrotation whose direction of rotation is in the $(x - y)$ plane
ϱ, T = density and temperature of the fluid
μ_v, k_v, γ_v = viscosity, vortex viscosity and spin-gradient viscosity
j, K_c, α^* = micro-inertia density, thermal conductivity and micropolar heat conduction coefficient
g, β, C_p = acceleration due to gravity, coefficient of expansion and specific heat of the fluid at constant pressure.

The details of the derivation of the boundary layer Eqs. (2.1)—(2.4) are available in [14], [15].

The heat transfer due to free convection results in low velocities and is normally associated with large temperature differences. Since velocities are small, the viscous dissipation terms in the energy Eq. (2.4) have been neglected. Mathur et al. [15] have also shown recently that the viscous dissipation has very little effect on the temperature field and the rate of heat transfer for the flow of an incompressible thermomicropolar fluid past a circular cylinder placed in such a way that its axis is normal to the oncoming free stream.

Eringen [2] has discussed in detail the inequalities to be satisfied by the various material parameter involved in the theory of thermomicropolar fluids. These inequalities, which arise from the thermodynamic restrictions, are

$$\begin{gathered} k_v \geqq 0, \quad 2\mu_v + k_v \geqq 0, \quad \beta_v + \gamma_v \geqq 0, \quad K_c \geqq 0, \\ (\alpha^* - \beta^* T^{-1})^2 \leqq 2K_c T^{-1}(\gamma_v - \beta_v), \quad j \geqq 0, \end{gathered} \tag{2.5}$$

where β_v is the gradient of viscosity and β^* is the micropolar heat conduction.

Wall boundary conditions

Velocity field

$$u(x, 0) = 0, \qquad v(x, 0) = v_w(x), \tag{2.6}$$

where $v_w(x)$ is the suction or injection velocity. ($v_w < 0$ refers to suction and $v_w > 0$ refers to injection.)

Microrotation field

$$\nu(x, 0) = \bar{s}\left(\nu + \frac{1}{2}\frac{\partial u}{\partial y}\right)_{y=0} \tag{2.7}$$

where $\bar{s}$ is a parameter such that $0 \leqq \bar{s} \leqq \infty$. This condition relates the microrotation to the antisymmetric part of the stress. When $\bar{s} = 0$ and $\bar{s} \to \infty$, (2.7) reduce, respectively, to no relative spin and no antisymmetric part of the stress on the boundary.

Temperature field

$$T(x, 0) = T_w(x) \tag{2.8}$$

where $T_w(x)$ is the variable temperature of the wall.

At the edge of the boundary layer, we must have

$$y \to \infty: \quad u \to 0, \quad \nu \to 0, \quad T \to T_\infty. \tag{2.9}$$

T_∞ is the constant temperature of the fluid outside the boundary layer.

3. Method of Solution

A search for similar solution reveals that the similarity solutions for the governing system of Eqs. (2.1)—(2.4) and the boundary conditions (2.6)—(2.9) exist only under the condition that the suction or injection velocity $v_w(x)$ is constant. The appropriate similarity transformation is

$$\left.\begin{aligned} &\psi = Axf(\eta), \quad \nu = Cxg(\eta), \quad \eta = By, \\ &T_w - T_\infty = Nx, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \end{aligned}\right\} \tag{3.1}$$

where A, B, C, N are constants and $\psi(x, y)$ is a stream function with $u = \dfrac{\partial \psi}{\partial y}$ and $v = -\dfrac{\partial \psi}{\partial x}$. From Eq. (3.1), it is evident that the constants A, B, C and N have, respectively, the dimensions of velocity, the reciprocal of length, the reciprocal of the product of length and time, and of the ratio (temperature/length).

Making use of dimensional analysis, we obtain

$$\left.\begin{aligned} &A = [\bar{\alpha}^2 Ng\beta]^{1/4}, \quad B = [(Ng\beta)/\bar{\alpha}^2]^{1/4}, \\ &C = [(Ng\beta)^3/\bar{\alpha}^2]^{1/4}, \quad \bar{\alpha} = \frac{K_c}{\varrho C_p}, \\ &\Pr = \frac{\mu_v C_p}{K_c}, \quad N_1 = \frac{k_v}{\mu_v}, \quad N_2 = \frac{\varrho j (Ng\beta)^{1/2}}{\mu_v}, \\ &N_3 = \frac{\gamma_v \varrho (Ng\beta)^{1/2}}{\mu_v^2}, \quad N_4 = \frac{\varrho \beta_v (Ng\beta)^{1/2}}{\mu_v^2}, \\ &N_5 = \frac{\alpha^* (Ng\beta)^{1/2}}{K_c}, \quad N_6 = \frac{\beta^* (Ng\beta)^{1/2}}{\mu_v C_p T_\infty}, \end{aligned}\right\} \tag{3.2}$$

where Pr is the Prandtl number and $\bar{\alpha}$ is the thermal diffusivity.

Substituting (3.1) into (2.1)—(2.9) and making use of (3.2) we obtain the equations

$$f'^2 - ff'' = \Pr\,(1 + N_1)\, f''' + \Pr N_1 g' + \theta \tag{3.3}$$

$$N_2(f'g - fg') = \Pr N_3 g'' - N_1(2g + f'') \tag{3.4}$$

$$f'\theta - f\theta' = \theta'' + N_5(\theta g' - \theta' g) \tag{3.5}$$

with the boundary conditions

$$\begin{aligned} &\eta = 0: \quad f = -\frac{v_w}{A} = C_0\ (\text{constant}), \qquad f' = 0, \\ &\qquad g = -\frac{s}{2} f'', \qquad \theta = 1, \\ &\eta \to \infty: \quad f' \to 0, \qquad g \to 0, \qquad \theta \to 0, \end{aligned} \tag{3.6}$$

where $s = \bar{s}/(\bar{s} - 1)$ and $0 \leqq s \leqq 1$. Here dash denotes differentiation with respect to η.

In Eqs. (3.3)—(3.6), the dimensionless parameters N_1, N_2, N_3 and N_5, respectively, characterize the vortex viscosity, microinertia density, spin-gradient viscosity and the micropolar heat conduction. The parameters N_4 and N_6 will appear in the expressions for couple stress components and the rate of heat transfer. In terms of these parameters, the inequalities (2.5) become

$$\begin{aligned} &N_1 \geqq 0, \qquad N_1 + 2 \geqq 0, \qquad N_3 + N_4 \geqq 0, \qquad \Pr \geqq 0, \\ &(N_5 - \Pr N_6 \Phi^{-1})^2 \leqq 2\Phi^{-1}(N_3 - N_4)\,\frac{\Pr E}{R}, \qquad N_2 \geqq 0, \end{aligned} \tag{3.7}$$

where $\Phi = (T/T_\infty)$ (dimensionless temperature), $E = \dfrac{A^2}{C_p T_\infty}$ (like Eckert number) and $R = \dfrac{\varrho A}{B\mu_v}$ (like Raynolds number).

Transformation to Finite Domain

For computational convenience, we transform the system of simultaneous ordinary differential Eqs. (3.3)—(3.5) to a new independent variable wherein the infinite domain of η is replaced by a finite domain. Employing the transformation

$$\xi = 1 - e^{-\lambda\eta}, \tag{3.8}$$

where λ is a constant that can be used as a scaling factor to provide an optimum distribution at nodal points across the boundary layer, and letting

$$z = \lambda(1 - \xi), \tag{3.9}$$

we arrive at the following set of equations

$$\begin{aligned}\Pr(1+N_1)\,z^3f''' + z^2f''[f - 3\lambda \Pr(1+N_1)]\\ + zf'[\lambda^2 \Pr(1+N_1) - \lambda f] - z^2f'^2 + \Pr N_1 z g' + \theta = 0,\end{aligned} \tag{3.10}$$

$$\begin{aligned}\Pr N_3 z^2 g'' + zg'(N_2 f - \lambda \Pr N_3) - g(2N_1 + zN_2 f')\\ - z^2 N_1 f'' + \lambda z N_1 f' = 0,\end{aligned} \tag{3.11}$$

$$z\theta'' + \theta'(f - N_5 g - \lambda) + \theta(N_5 g' - f') = 0. \tag{3.12}$$

In view of (3.8) and (3.9), the boundary conditions become

$$\left.\begin{aligned}\xi = 0:\quad f = C_0, \quad f' = 0, \quad g = -\frac{\lambda^2 s}{2} f'', \quad \theta = 1,\\ \xi = 1:\quad f' = 0, \quad g = 0, \quad \theta = 0.\end{aligned}\right\} \tag{3.13}$$

In the above equation, dash denotes differentiation with respect to ξ.

We have solved the Eqs. (3.10)–(3.12) with the boundary conditions (3.13) numerically by 'shooting method' employing Taylor series at an interval of $\Delta\xi = 0.05$. We have carried out the numerical computation work on a DEC-10 computer. In the present investigation, we are primarily interested in studying the effect of suction/injection together with the variation of boundary condition parameter 's'. The effect of N_1, N_2, N_3, N_5 and Pr on the velocity, microrotation and temperature fields and skin-friction, rate of heat transfer in the absence of suction/injection has already been studied in detail in [12]. In our computations, the following values have been chosen for the parameters:

$$N_1 = 0.1, \quad N_2 = 0.002, \quad N_3 = 0.02, \quad N_5 = 1.0, \quad \lambda = 1.0, \quad \Pr = 9.0.$$

These values satisfy the restrictions given by the inequalities in Eq. (3.7). The parameters C_0 and 's' have been assigned various values in the numerical calculations.

Ahmadi [16] and Tözeren and Skalak [17] have stated that the parameter N_1 depends on the shape and the concentration of the microelements. For a given shape of the microelements, N_1 directly gives a measure of concentration of microelements. The parameters N_2 and N_3 can be thought of as fluid properties depending on the relative size of microstructure in relation to a geometrical length.

Skin-Friction and Wall Couple Stress

The skin-friction coefficient C_f and wall couple stress coefficient M_w, in terms of the non-dimensional quantities are

$$C_f = \frac{(t_{yx})_{y=0}}{\varrho A^2} = \Pr[\lambda^2(1+N_1)\,f''(0) + N_1 g(0)]\,\bar{x},$$

$$M_w = \frac{B(m_{yz})_{y=0}}{\varrho A^2} = \Pr\left[\lambda \Pr N_3 \bar{x} g'(0) + \frac{RN_5}{E}\,\theta(0)\right],$$

where $\bar{x} = Bx$.

Heat Transfer Coefficient

The non-dimensional heat transfer coefficient called Nusselt number $N(\bar{x})$ is defined as

$$N(\bar{x}) = \left(\frac{q}{BK_c T_\infty}\right),$$

where q is the heat flux at the wall and defined as

$$q = \left(K_c \frac{\partial T}{\partial y} + \beta^* \frac{\partial v}{\partial x}\right)_{y=0}$$

In terms of non-dimensional variables, we have

$$N^*(\bar{x}) = \frac{N(\bar{x})}{(N/BT_\infty)} = \frac{N(\bar{x})\, BT_\infty}{N} = \lambda \bar{x} \theta'(0) + \Lambda N_6 \Pr g(0),$$

where $\Lambda = \dfrac{BT_\infty}{N}$ is the dimensionless ratio fo free stream temperature to characteristic wall temperature.

4. Results and Discussion

In order to study the effect of suction or injection, we have chosen the following values of the suction or injection parameter C_0:

$$-0.5, \quad -0.25, \quad 0.0, \quad 1.0, \quad 1.5.$$

The positive values of C_0 correspond to suction and negative values correspond to injection at the wall. It is found from our calculations that the curves representing velocity, microrotation, temperature and the heat transfer coefficient differ in magnitude only for $C_0 = -0.5$ and $C_0 = -0.25$ and also for $C_0 = 1.0$ and $C_0 = 1.5$. Due to this, we have plotted these curves in Figs. 1, 2, 3 and 4 only for $C_0 = -0.5$ and $C_0 = 1.0$.

In order to study the effect of variation of 's' (the parameter appearing in the boundary condition of microrotation) on velocity, microrotation and temperature fields, we have chosen the following values of 's':

$$0.0, \quad 0.5, \quad 1.0.$$

The value of $s = 0.0$ correspond to the no spin condition and $s = 1.0$ correspond to the vanishing of the antisymmetric part of the stress on the wall. It is to be noted that large values of 's' correspond to low concentration and small values to high concentration of microelements.

Velocity field

The effects of variation of 's' and C_0 on velocity profiles have been plotted in Fig. 1. For a given C_0, it can be seen that velocity increases with the increase of 's'. This means that fluid velocity is more in the case of weak concentration

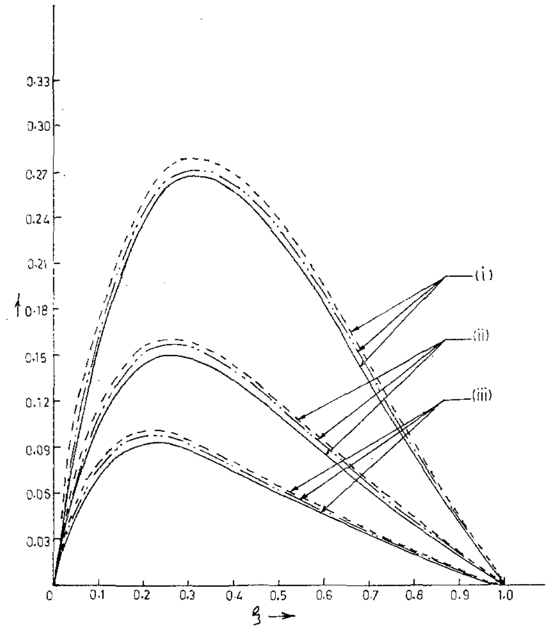

Fig. 1. Effect of variation of s and C_0 on the velocity profiles for $N_1 = 0.1$, $N_2 = 0.002$, $N_3 = 0.02$, $N_5 = 1.0$ and $\mathrm{Pr} = 9.0$. *i* $C_0 = 1.0$, *ii* $C_0 = 0.0$, *iii* $C_0 = -0.5$; ——— $s = 0.0$, —·—·— $s = 0.5$, ----- $s = 1.0$

of microelements. Further, we can see that the velocity increases with suction and decreases with injection and this is true for all values of 's'. The same type of results were obtained in [14].

Microrotation Field

In Fig. 2, we have plotted the microrotation profiles showing the effect of variation of 's' and C_0. For a given C_0, it is found that the microrotation near the surface decreases with decrease in concentration, i.e., with increase of 's'. The nature of the microrotation profiles for various values of 's' is the same as obtained in [18]. Due to the change in concentration the microrotation profile near the wall decreases with suction and increases with injection and this is true for all values of 's'. This is in qualitative agreement with the results obtained in [14] but the flow pattern in our case is slightly different from that of [14].

Temperature Field

In Fig. 3, we have plotted the temperature profiles showing the effect of variation of 's' and C_0. For a given C_0, it can be seen that the effect of variation of 's' on temperature is rather insignificant. It can also be seen that the temperature decreases with suction and increases with injection and this is true for all values of 's'. This type of behaviour of temperature is in agreement with the results obtained in [14]. Similar results have also been obtained in [19] for Newtonian fluids.

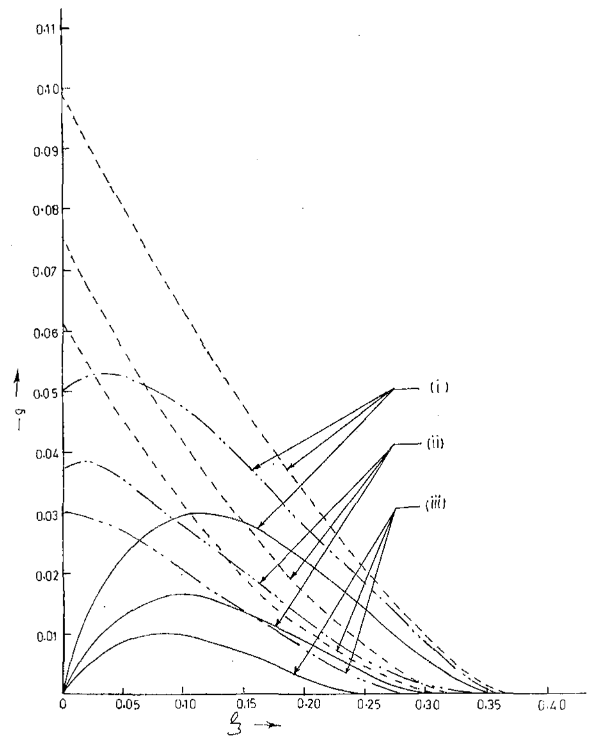

Fig. 2. Effect of variation of s and C_0 on the microrotation profiles for $N_1 = 0.1$, $N_2 = 0.002$, $N_3 = 0.02$, $N_5 = 1.0$ and Pr $= 9.0$. *i* $C_0 = 1.0$, *ii* $C_0 = 0.0$, *iii* $C_0 = -0.5$; ——— $s = 0.0$, —·—·—·— $s = 0.5$, - - - - - $s = 1.0$

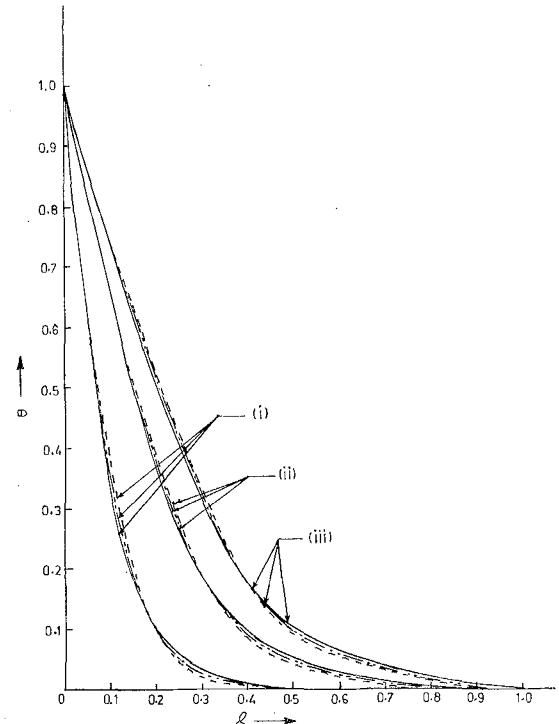

Fig. 3. Effect of variation of s and C_0 on the temperature profiles for $N_1 = 0.1$, $N_2 = 0.002$, $N_3 = 0.02$, $N_5 = 1.0$ and Pr $= 9.0$. *i* $C_0 = 1.0$, *ii* $C_0 = 0.0$, *iii* $C_0 = -0.5$; ——— $s = 0.0$, —·—·—·— $s = 0.5$, - - - - - $s = 1.0$

Table 1. *Effect of variation of 's' and C_0 on the skin-friction parameter and the gradient of microrotation on the wall for* $N_1 = 0.1$, $N_2 = 0.002$, $N_3 = 0.02$, $N_5 = 1.0$ *and* $\Pr = 9.0$

C_0	$f''(0)$			$g'(0)$		
	$s = 0.0$	$s = 0.5$	$s = 1.0$	$s = 0.0$	$s = 0.5$	$s = 1.0$
−0.5	0.1150	0.1175	0.1201	−0.0313	0.0003	0.0330
−0.25	0.1322	0.1350	0.1401	−0.0379	−0.0020	0.0345
0.0	0.1482	0.1541	0.1572	−0.0450	−0.0042	0.0362
1.0	0.1919	0.1966	0.2035	−0.0682	−0.0169	0.0364
1.5	0.2022	0.2068	0.2102	−0.0737	−0.0212	0.0368

In Table 1, we have tabulated the skin-friction parameter $f''(0)$ and the gradient of microrotation $g'(0)$ on the wall for various values of the boundary condition parameter 's' and suction/injection parameter C_0. It can be seen that with the decrease in concentration, the skin-friction and the gradient of microrotation increase for a given C_0. It can also be seen from Table 1 that skin-friction parameter $f''(0)$ increases with suction and decreases with injection for all values of 's'. Similar effects of suction and injection on the skin-friction parameter have been obtained in [20] for Newtonian fluids. Using the values of $f''(0)$ and $g'(0)$ from Table 1, C_f and M_w can be computed for the prescribed values of N_1, N_3, N_5, Pr, R, E and λ.

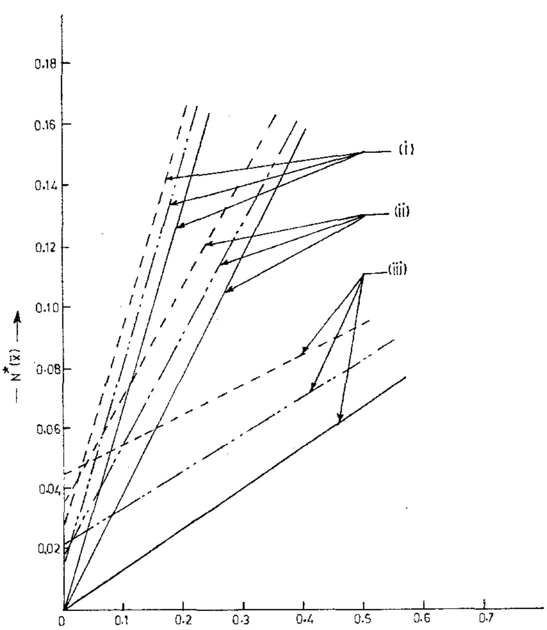

Fig. 4. Effect of variation of s and C_0 on the rate of heat transfer for $N_1 = 0.1$, $N_2 = 0.002$, $N_3 = 0.02$, $N_5 = 1.0$, $N_6 = 0.05$, $\Pr = 9.0$ and $\Lambda = 1.0$. *i* $C_0 = 1.0$, *ii* $c_0 = 0.0$, *iii* $C_0 = -0.5$; ——— $s = 0.0$, —·—·— $s = 0.5$, ----- $s = 1.0$

Fig. 4 shows the effect of variation of 's' and C_0 on the dimensionless heat transfer coefficient $[-N^*(\bar{x})]$. It is found that for high concentration the heat transfer coefficient $[-N^*(\bar{x})]$ increases with suction and decreases with injection and it is also true for low concentration except on the wall. Similar effects of suction and injection on the rate of heat transfer for Newtonian fluids and micropolar fluids have been obtained respectively in [19]—[21] and [14].

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