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The effects of in-plane core stiffness on the wrinkling behavior of thick sandwiches

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Summary. This contribution presents a refined analytical solution for the wrinkling of sandwich plates with isotropic face layers and thick orthotropic cores, taking into account *in-plane deformations of the core*. A single explicit equation for the critical wrinkling load in an asymptotic sense in derived. The results have been verified extensively by a numerical model [1] and show that, when dealing with highly orthotropic cores (e.g., honeycombs), the wrinkling loads and deformation patterns can strongly depend on the in-plane stiffness of the core. This new theoretical finding which is of considerable practical importance is the main motivation for this paper. Classical wrinkling formulae [2], [3], [4] can lead to significant errors when used in connection with highly orthotropic cores.

1 Introduction

Wrinkling (i.e., short wavelength buckling of the face layers) is a common local stability problem of sandwich plates and shells under compressive or bending loads, leading to a loss in stiffness of the structure. The problem is characterized by the interaction between the sandwich core and the face layer, which is bonded to the core. As a result, the critical force leading to wrinkling of the face layers is a function of the stiffness parameters of the face layer and the core, the geometry of the problem and the bonding and loading conditions.

Classical papers dealing with analytical calculations of critical wrinkling loads (see e.g., [5], [6], [7], [8]) are based on simplifying assumptions regarding the influence of the in plane stiffness of the core. More recent papers dealing with wrinkling concentrate on numerical solutions (see e.g., [9], [10], [11]) or on post-buckling and interaction of global and local buckling (see e.g., [12], [13]).

In the present paper a rather fundamental problem is treated, namely the analytical consideration of the in-plane core stiffness on the bifurcation behavior in terms of wrinkling of thick sandwiches.

Plantema [2] showed, that for core materials with the shear moduli $G^c_{xz} = G^c_{yz}$ and isotropic face layers, the critical wrinkling load under biaxial compressive load is solely determined by the major compressive membrane force in the face layer. Therefore, many of the different analytical models that have been set up to describe this problem use the equation for the isotropic plate on an elastic foundation specialized for *unidirectional loading* (in x-direction, see Fig. 1):

$$K^f \frac{\partial^4 w^f}{\partial x^4} + P^f_x \frac{\partial^2 w^f}{\partial x^2} + \hat{q} = 0\,, \tag{1}$$

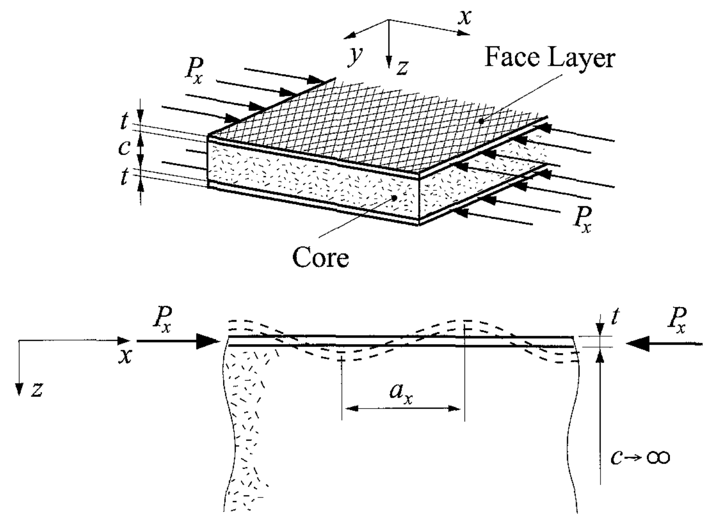

Fig. 1. Uniaxial wrinkling, 3D and 2D model

where K^f denotes the plate bending stiffness and P_x^f the compressive force per unit width of the face layer. It is generally assumed here, that the whole in-plane load is carried by the face layers.

The aim of the analytical approaches to this problem is the determination of $\hat{q}$, the reaction pressure of the core with respect to the plate (i.e., the foundation stiffness of the core multiplied by the deflection w). The two main approaches use either the differential equation or the energy method to derive $\hat{q}$. However, in-plane action of the core has been disregarded up to now.

The wrinkling pattern (in z-direction) of the unidirectionally loaded face layer, and therefore also for the core surface which is perfectly bonded to it, is a sinus pattern,

$$w^f = w^c_{z=0} = w^* \sin\left(\frac{\pi x}{a_x}\right) , \tag{2}$$

where the superscripts f and c denote face layer and core, respectively, and a_x is the half-wavelength of the deformation pattern. The main two closed form solutions in use are the Winkler model and the model for (infinitely) thick cores, both of which have been compared and discussed closer (see e.g., [7]) and are used in the standard sandwich literature (see e.g., [2], [14], [3]).

1.1 Winkler foundation model

In the Winkler foundation model the continuous elastic medium of the core is modeled as a set of parallel closely set elastic springs (Winkler foundation). This model takes only the transverse core stiffness E_z^c into account. Neglecting the shear stiffness seems to lead only to useful results if the core thickness is relatively low. The face layers are assumed to undergo symmetrical buckling, therefore the mid surface of the sandwich remains undeformed during buckling. Using this boundary condition, $\hat{q}$ can be calculated (compare e.g., [14], [7]):

$$\hat{q} = w^f k^W = w^f E_z^c \frac{2}{c} , \tag{3}$$

where c denotes the core thickness. Inserting this result into Eq. (1), P_x^f is obtained as a function of the wavelength a_x. The minimization of P_x^f with respect to the wavelength a_x (for nontrivial solutions of w^f) leads to the critical compressive force per unit width in the face layer for undirectional loading P^W_{crit} :

$$P^W_{crit} = 2\sqrt{k^W K^f} = 2t\sqrt{\frac{E^f E^c_z t}{6c\left(1-(\nu^f)^2\right)}}\,. \tag{4}$$

1.2 Model for thick cores

In the case of the thick cores it can be assumed that the two face layers do not influence each other. Taking only global uniaxial compression into account, the core is considered as orthotropic plane stress problem (in x-z direction, see Fig. 1). The face layer, which is orders of magnitude stiffer than the core, is generally assumed to lead to $\varepsilon^c_{x,z=0} = 0$ in the core-face layer interface.

In the differential equation approach (see e.g., [4]), this boundary condition is simplified by taking the core stiffness in loading direction E^c_x to be infinity. This leads to $\varepsilon^c_x = 0$ in the whole core. On the other hand, the core stiffness in loading direction can be very small in reality (e.g., honeycomb cores). Therefore, the assumption of an infinitely high stiffness seems to be questionable. However, this simplification appears to be based on the assumption, that the influence of in-plane core action can generally be neglected in this problem.

The majority of authors ([6], [7], [2], [3] etc ...) used an energy approach to this problem, and assumed the in-plane displacements u (in x-direction) to be zero. This is essentially the same assumption as mentioned above. The results are also practically the same as in the differential equation approach cited above.

Using these simplifications, $\hat{q}$ can be calculated in an asymptotic sense, i.e., no interaction between upper and lower face:

$$\hat{q} = w^f \frac{\pi}{a_x} k^{old} = w^f \frac{\pi}{a_x} \sqrt{E^c_z G^c_{xz}}\,, \tag{5}$$

with k^{old} being the effective core stiffness corresponding to this "old" model for thick cores. Equation (5) leads (in analogy to Sect. 1.1) to the critical force per unit width per face layer P^{old}_{crit}:

$$P^{old}_{crit} = 1.89\sqrt[3]{K^f E^c_z G^c_{xz}} \quad \overset{\nu_f=0.3}{\Longrightarrow} \quad 0.85t\sqrt[3]{E^f E^c_z G^c_{xz}}\,. \tag{6}$$

From experimental investigations on sandwiches with *isotropic* cores by [6] the following formula is recommended for design:

$$P^{old}_{crit,pr} = 0.5t\sqrt[3]{E^f E^c G^c}\,. \tag{7}$$

2 Refined analysis for thick orthotropic cores

The simplifications that have been made to gain the results above are rather far reaching. Therefore, a refined theory is derived. In order to calculate the core reaction $\hat{q}$, the differential equation for Airy's stress function F of the orthotropic elastic plane stress problem for unit width (see e.g., [4]) is used to model a section of the core in the loading plane (see Fig. 1).

$$\frac{1}{D^c_z}\frac{\partial^4 F}{\partial x^4} + \frac{2}{D^c_{xz}}\frac{\partial^4 F}{\partial x^2 \partial z^2} + \frac{1}{D^c_x}\frac{\partial^4 F}{\partial z^4} = 0\,, \tag{8}$$

$$\sigma^c_x = \frac{\partial^2 F}{\partial z^2}\,, \qquad \sigma^c_z = \frac{\partial^2 F}{\partial x^2}\,, \qquad \sigma^c_{xz} = \frac{\partial^2 F}{\partial x \partial z}\,, \tag{9}$$

with the membrane stiffness

$$D^c_x = E^c_x\,, \qquad D^c_{xz} = 2E^c_x G^c_{xz}/(E^c_x - 2\nu^c_{xz} G^c_{xz})\,, \qquad D^c_z = E^c_z\,. \tag{10}$$

The stresses and strains perpendicular to the x-z plane are neglected (as in the other models described above), but no further restrictions, apart from the necessary and sensible boundary conditions below, are imposed on the core deformation.

The displacement field at $z = 0$ of the core in z-direction is assumed to follow Eq. (2). The stresses are assumed to vanish within the infinitely thick core as z approaches infinity. The governing Eq. (8) requires this decrease to be of an exponential form. To represent this, the following definition of the function F is used:

$$F = C\left(\frac{a_x}{\pi}\right)^2 \exp\left(\frac{\mu\pi z}{a_x}\sqrt[4]{\frac{D^c_x}{C^c_z}}\right)\sin\left(\frac{\pi x}{a_x}\right). \tag{11}$$

Inserting Eq. (11) into Eq. (8) yields the solutions for μ

$$\mu_{1,3} = \pm\sqrt{\xi + \sqrt{\xi^2 - 1}}\,, \qquad \mu_{2,4} = \pm\sqrt{\xi - \sqrt{\xi^2 - 1}}\,, \qquad \xi = \frac{\sqrt{D^c_x D^c_z}}{D^c_{xz}}\,. \tag{12}$$

Now F and, therefore, the stresses are defined except for the constants C, i.e., C_1 to C_4, which are determined by the boundary conditions. The positive solutions for μ, i.e., μ_1 and μ_2 have to disappear to allow an exponential decrease of the stresses.

Therefore the corresponding constants C_1 and C_2 are set to zero. The stresses in the plane can now be written as:

$$\sigma^c_x = \left(C_3\mu_1^2 \exp\left(\frac{-\mu_1\pi z}{a_x}\sqrt[4]{\frac{D^c_x}{D^c_z}}\right) + C_4\mu_2^2 \exp\left(\frac{-\mu_2\pi z}{a_x}\sqrt[4]{\frac{D^c_x}{D^c_z}}\right)\right)\sqrt{\frac{D^c_x}{D^c_z}}\sin\left(\frac{\pi x}{a_x}\right), \tag{13}$$

$$\sigma^c_z = -\left(C_3 \exp\left(\frac{-\mu_1\pi z}{a_x}\sqrt[4]{\frac{D^c_x}{D^c_z}}\right) + C_4 \exp\left(\frac{-\mu_2\pi z}{a_x}\sqrt[4]{\frac{D^c_x}{D^c_z}}\right)\right)\sin\left(\frac{\pi x}{a_x}\right), \tag{14}$$

$$\sigma^c_{xz} = -\left(C_3\mu_1 \exp\left(\frac{-\mu_1\pi z}{a_x}\sqrt[4]{\frac{D^c_x}{D^c_z}}\right) + C_4\mu_2 \exp\left(\frac{-\mu_2\pi z}{a_x}\sqrt[4]{\frac{D^c_x}{D^c_z}}\right)\right)\sqrt[4]{\frac{D^c_x}{D^c_z}}\cos\left(\frac{\pi x}{a_x}\right). \tag{15}$$

The remaining two constants C_3, C_4 can be calculated using the following two boundary conditions. In an approximation, the strains in the core at the interface ($z = 0$) in x-direction are set to zero. This common assumption (see e.g., [2], [14], [1], [3]) is especially meaningful if one deals with rather stiff face layers on cores which show low in-plane stiffness. Such combinations are considered here. This leads to the boundary condition:

$$\varepsilon^c_{x,z=0} = \frac{1}{E^c_x}(\sigma^c_{x,z=0} - \nu^c_{xz}\sigma^c_{z,z=0}) = 0\,. \tag{16}$$

The second boundary condition is, that the stress at the interface in z-direction is distributed as a sine wave with a certain amplitude $\sigma^c_{z,0}$, corresponding to the assumed deformation given in Eq. (2):

$$\sigma^c_{z,z=0} = \sigma^c_{z,0}\sin\left(\frac{\pi x}{a_x}\right) = -\hat{q}\,. \tag{17}$$

Using these two boundary conditions in Eqs. (13), (14) gives the following values for the constants C_3 and C_4, which can be split up into the boundary stress amplitude $\sigma^c_{z,0}$ and dimensionless factors X_3 and X_4:

$$C_3 = \sigma^c_{z,0} X_3 = \sigma^c_{z,0} \frac{\mu_2^2 + \nu^c_{xz}\sqrt{\frac{D^c_z}{D^c_x}}}{\mu_1^2 - \mu_2^2}\,, \qquad C_4 = -\sigma^c_{z,0} X_4 = -\sigma_{z,0} \frac{\mu_1^2 + \nu^c_{xz}\sqrt{\frac{D^c_z}{D^c_x}}}{\mu_1^2 - \mu_2^2}\,. \tag{18}$$

Using Hooke's Law, the strains in the core in z-direction can now be calculated:

$$\varepsilon^c_z = \frac{1}{E^c_z}(\sigma^c_z - \nu^c_{zx}\sigma^c_x)\,. \tag{19}$$

To obtain the deformation w (in z-direction) of the core at the interface to the face layer, the strain ε^c_z has to be integrated over the infinitely thick core:

$$w^c_{z=0} = \int_\infty^0 \varepsilon^c_z dz = -\sigma^c_{z,0}\left[\left(\frac{X_4}{\mu_2} - \frac{X_3}{\mu_1}\right) + \nu^c_{zx}(X_4\mu_2 - X_3\mu_1)\sqrt{\frac{E^c_x}{E^c_z}}\right]$$
$$\times \frac{1}{E^c_z}\frac{a_x}{\pi}\sqrt[4]{\frac{E^c_z}{E^c_x}}\sin\left(\frac{\pi x}{a_x}\right)\,. \tag{20}$$

Inserting Eqs. (20) and (2) into (17) gives the core reaction pressure $\hat{q}$, and thus (in analogy to Eq. (5)) the effective stiffness in this new model k^{thick} can be determined.

$$\hat{q} = w_f \frac{\pi}{a_x} k^{thick} \Rightarrow k^{thick} = \left[\left(\frac{X_4}{\mu_2} - \frac{X_3}{\mu_1}\right) + \nu^c_{zx}(X_4\mu_2 - X_3\mu_1)\sqrt{\frac{E^c_x}{E^c_z}}\right]^{-1} E^c_z\sqrt[4]{\frac{E^c_x}{E^c_z}}\,. \tag{21}$$

Inserting Eq. (21) into Eq. (1), P^f_x can be calculated as a function of the wavelength a_x. To obtain the critical wrinkling force per unit length P^{thick}_{crit}, P^f_x is minimized analytically with respect to the wavelength a_x leading to

$$a_{x,crit} = \sqrt[3]{2K^f \frac{\pi^3}{k^{thick}}}\,, \tag{22}$$

$$P^{thick}_{crit} = \left(\sqrt[3]{2} + \sqrt[3]{\frac{1}{4}}\right)\sqrt[3]{K^f (k^{thick})^2} \overset{\nu_f=0.3}{\Longrightarrow} 0.85t\sqrt[3]{E^f (k^{thick})^2}\,. \tag{23}$$

These results are similar to the old model (see Eq. (6)) with respect to the influence of the face layer, but the core stiffness enters the results in a very different way.

3 Results and numerical validation

3.1 Numerical reference model

To validate the results obtained by the theory derived above, the semi-analytical-numerical approach in [1] was used. This approach models the core as a complete three-dimensional orthotropic continuum and biaxial loading of the sandwich as well as finite core thickness can be considered. In order to calculate the critical wrinkling load a numerical minimization scheme has to be used, which makes that model effectively much more complicated than the theory derived in Sect. 2. However, that model is able to calculate symmetrical as well as anti-metrical wrinkling loads, the lower of which is considered to be decisive. That approach was extended in [11] to take orthotropic face layers into account. Moreover, the numerical model accounts for the three axial stress state in the core, whereas the analytical theory derived above is only capable of predicting the critical wrinkling load in an asymptotic sense for thick cores and a plane stress state.

Very recently, another numerical model has been published by [9], who used mechanics of incremental deformation and modeled the infinitely thick core as well as the face layers as fully 2D orthotropic elastic plane strain ploblems. He concentrated on the influence of face layer orthotropy, rather than core orthotropy, and deals only with very stiff cores that are not likely to be used in classical sandwich construction. However, where applicable the new analytical model and Hwang's numerical results show good agreement.

3.2 *Parametric study*

An extensive parameter study has been performed to evaluate the influence of the core material parameters on the critical wrinkling load in the new analytical as well as in the numerical reference model. In the case of sufficiently thick cores, the agreement between the results obtained from both approaches was excellent. The analytical model leads to marginally lower critical loads due to the assumption of plane stress in the core which neglects the stiffening due to transverse effects. In the case of isotropic thick cores, the old theory (see Eq. (6)) gives a proper estimate of the critical wrinkling load. However, in the case of highly orthotropic cores, such as e.g., honeycomb cores, it is shown, that the old theory is not sufficient. This study shows, that the main influence parameters for the uniaxial critical wrinkling load of sandwich panels with thick highly orthotropic cores are:

- Thickness of the face layer t
- Young's modulus of the face layer E^f
- Poisson ratio of the face layer ν^f
- Young's modulus of the core in transverse direction E_z^c
- Young's modulus of the core in longitudinal direction E_x^c
- Transverse shear modulus of the core G_{xz}^c.

All of these main parameters are accounted for in the new analytical model derived above, whereas the design formula used up to now (Eq. (6)) neglects the influence of E_x^c which must not be neglected if $E_x^c \ll E_z^c$.

3.2.1 Influence of longitudinal core stiffness E_x^c

In Fig. 2 the critical wrinkling force in the face leayers is plotted over the ratio of E_x^c/E_z^c on a logarithmic scale for a sandwich plate made of commercial NOMEX honeycomb (with respect to E_z^c, G_{xz}^c) and aluminum face layers. For the numerical reference model [1], the ratio c/t is taken to be 5000 which is a fictious number just to ensure that the two face layers do not influence each other.

The symmetrical P_{crit}^{sym} and anti-metrical P_{crit}^{ant} critical wrinkling forces per face layer per unit width calculated by the numerical reference model match exactly due to the sufficiently thick core. The critical wrinkling force calculated by the analytical model P_{crit}^{thick} is a little bit

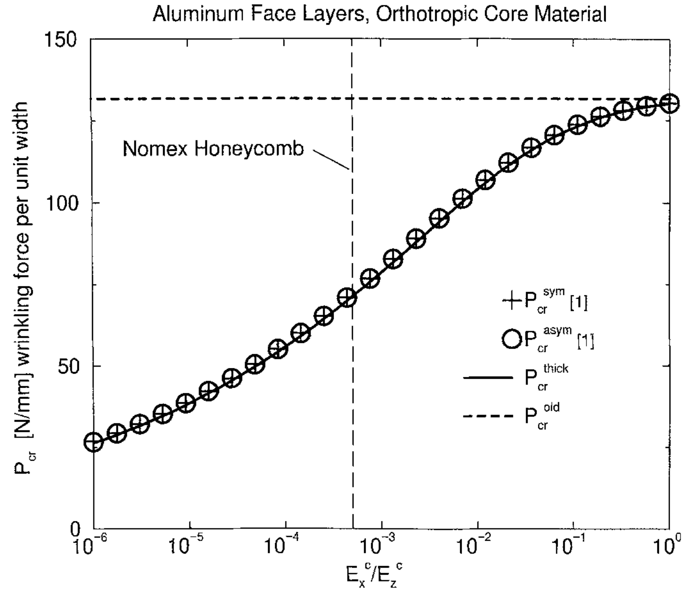

Fig. 2. Critical wrinkling force for infinitely thick cores

lower, as mentioned above, but still it represents an excellent estimate. The old analytical model for thick cores is denoted by P^{old}_{crit} and does not account for the parameter E^c_x. In the case of real honeycombs, the ratio of E^c_x/E^c_z is about 1/2000 (see e.g., [15]) which is marked by a vertical dashed line in the plot. Within this parameter range, the error of the old analytical model is definitely too large to be neglected. However, for the nearly isotropic case $\left(\log(E^c_x/E^c_z) \approx 0\right)$, the old analytical model matches very well.

It can clearly be seen, that the wrinkling stress decreases as E^c_x gets smaller. This effect has only very recently been reported for the first time by [9] who used a numerical model, but no explanation was given. He concluded that E^c_x has an influence only if the ratio of face layer stiffness to transverse core stiffness is not too high, which practically excluded the usual application of structural sandwiches from this effect. However, apart from the NOMEX honeycomb shown in Fig. 2, numerous other honeycomb materials have been investigated, and although the stiffness ratio mentioned above has been well over thousand, it was always obvious, that the influence of E^c_x must not be disregarded. This discrepancy to Hwang's considerations can be explained by the limited parameter range Hwang investigated, which did not include the high grade of orthotropy that is found in the homogenized honeycomb material values.

To gain a proper explanation for the decrease of the critical wrinkling force, the core deformation patterns should be considered. Due to the boundary conditions, the core deformation pattern has to be a sine wave on the surface and has to decrease to an undeformed state within an infinitely thick core. The old analytical theory assumes, that the only existing strains are shear and transverse strains. Therefore, the deformation pattern of the core is forced to look like Fig. 3. The new approach also accounts for strains in the longitudinal, i.e., x-direction. For the case of very small ratios of E^c_x/E^c_z the deformation pattern is shown in Fig. 4. The strains in longitudinal direction must obviously not be neglected. In the case of $E^c_x \rightarrow 0$ it is even possible to get a deformation pattern without any shear strains. So, from the viewpoint of deformation kinematics, the longitudinal strains are able to substitute the shear strains in the core for the given boundary conditions. Whether the deformation pattern

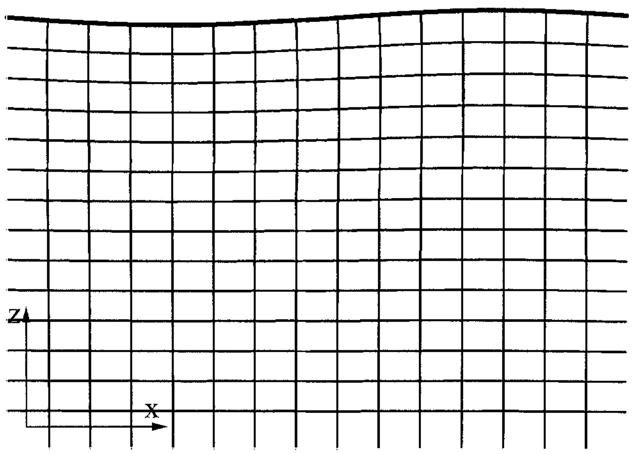

Fig. 3. Core deformation pattern for infinitely thick cores, old model (Sect. 1.2)

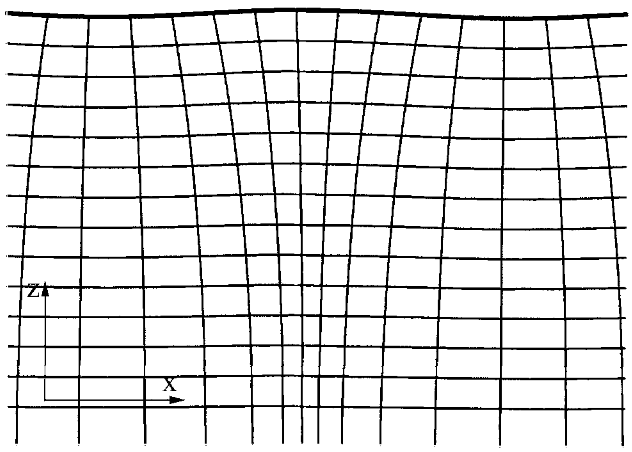

Fig. 4. Core deformation pattern for infinitely thick cores, new model (Sect. 2) at $E_x^c/E_z^c = 10^{-4}$

exhibits predominantly shear or longitudinal strains (apart from the transverse strains) is dependent on the energy stored in these two modes and, therefore, on the moduli G_{xz}^c and E_x^c. In the case of isotropy, shear strains are dominant.

It is also worth noting that the region of the core which exhibits high deformations is much thicker for small ratios of E_x^c/E_z^c than for high ones. This finding is in accordance with [16] who observed, that a higher grade of material anisotropy leads to a longer St. Venant decay length. The effects of this can be seen in the following section.

3.2.2 Cores of finite thickness

Considering cores of a given finite thickness, the effect of the longitudinal stiffness on the wrinkling load can be seen in Fig. 5, which is based on an c/t ratio of 100. The notation is equal to Fig. 2, and additionally P_{crit}^W (Winkler foundation, see Eq. (4)) is also displayed. For slightly orthotropic cores it can be seen, that the symmetrical P_{crit}^{sym} and anti-metrical P_{crit}^{ant} critical wrinkling force calculated by the numerical model match well, which shows that, for the chosen c/t ratio there is hardly an interaction between the two face layers and an infinitely

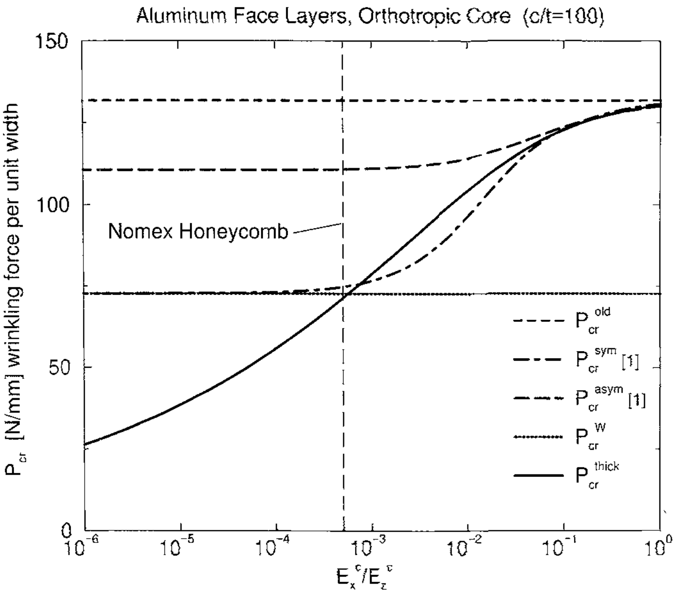

Fig. 5. Critical wrinkling force for cores of finite thickness

thick core can be assumed. Therefore, the analytical calculation derived within this work (P_{crit}^{thick}) gives a good approximation of the critical wrinkling force.

For a higher grade of orthotropy of the core material, the decrease of E_x^c/E_z^c leads at some point to an interaction between the face layers, and, therefore, the symmetrical P_{crit}^{sym} and anti-metrical P_{crit}^{ant} critical wrinkling forces calculated by the numerical model do not match anymore. In this parameter region the assumption of an infinitely thick core is not appropriate anymore and the analytical result (P_{crit}^{thick}) starts to differ from the numerical results. It is noteworthy that this effect is also observed at very high ratios of c/t (e.g., 100 in the case of Fig. 5) for realistic material values of honeycomb core materials.

For further decrease of the E_x^c/E_z^c ratio, the numerically calculated critical wrinkling stress approaches the Winkler solution. The Winkler solution itself considers only the transverse strains, and no longitudinal or shear strains. At the low ratios of E_x^c/E_z^c, where the numerical result and the Winkler solution coincide, there are no shear strains present anymore and the longitudinal strains contain practically no energy due to the low longitudinal stiffness of the core, however a strong interaction between the two face layers exists. Therefore, The Winkler solution gives proper critical wrinkling stresses also for thick cores, if the core material exhibits a sufficiently low ratio of E_x^c/E_z^c. The new solution presented here, i.e., P_{crit}^{thick}, fails for too small c/t ratios, because it does not take the face layer interaction into account.

It is obvious, that the effects described above get more pronounced for higher ratios of G_{xz}^c/E_z^c.

4 Conclusions

The newly derived analytical model leads to a single explicit equation for the critical wrinkling load and is, therefore, efficient and easy to apply in the case of sufficiently thick cores. It is the only closed form solution which takes the effect of a finite, nonzero longitudinal core stiffness into account.

The influence of the longitudinal core stiffness on the critical wrinkling load has been evaluated in a newly derived analytical and a numerical approach. Both approaches agree, that the influence must not be neglected in the case of a low ratio of longitudinal stiffness to transverse stiffness of the core, as it happens for example in the case of honeycomb cores.

For making decisions whether or not the core can be characterized as being thick, it is not sufficient to consider the c/t ratio (as in [2], [3], [17], [1], [4] etc ...), the ratios E_x^c/E_z^c and G_{xz}^c/E_z^c have also to be considered. It is therefore shown, that it poses a complex problem to determine whether a sandwich can be considered to have a thick core or not, as far as the question of coupling between the face layers is concerned.

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