Acta Mechanica 116, 229–233 (1996)

ACTA MECHANICA

Note

A note on the flow over a stretching sheet

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(Received October 17, 1994; revised January 25, 1995)

Summary. This study deals with the viscous incompressible flow over a stretching sheet. The velocity of the sheet is a quadratic polynomial of the distance from the slit and the sheet is subjected to a linear mass flux. A closed form solution is obtained under some restrictions on the linear mass flux. Stream line patterns are plotted and the effect of mass flux on the flow is also studied.

1 Introduction

The fluid dynamics due to a viscous flow over a stretching sheet is important in polymer industry (Char and Chen [4]). McCormack and Crane [1] gave a similar solution in a closed analytic form for the two dimensional stretching of a flat surface with a velocity proportional to the distance from the slit. This solution is extended by Gupta and Gupta [2] with the sheet being subjected to a constant mass flux. Wang [3] studied the flow caused by the stretching of a flat surface in two lateral directions. Gupta and Gupta [2] pointed out that a stretching sheet may not always conform to the linear speed assumed by them. Hence, in the present note, apart from the linear speed a quadratic term is assumed for the stretching sheet which is also subjected to a linear mass flux apart from the constant mass flux. A closed form solution is obtained which reduces to the published results when the linear mass flux part and the quadratic speed part is not considered.

2 Analysis

Consider a flat sheet issuing from a long, thin slit at $x = 0$ and $y = 0$, stretched in the x direction (Fig. 1). The particle velocities on the surface are assumed to be

$$\text{on } y = 0, \quad u = \beta x + \alpha x^2, \quad v = v_c + \delta x \tag{1}$$

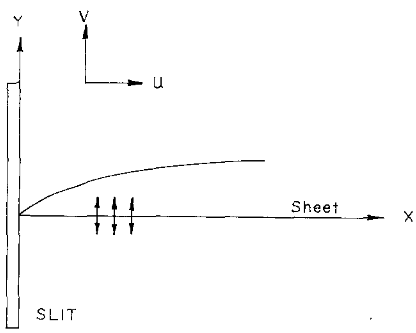

Fig. 1. Configuration of stretching sheet

where (u, v) are the velocity components in the (x, y) directions, and β, α, v_c, δ are constants. The fluid has no lateral motion as $y \to \infty$,

$$u = 0 \quad \text{for} \quad y \to \infty . \tag{2}$$

Assuming the boundary layer approximations, the equations of continuity and momentum in the usual notation are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}, \tag{4}$$

respectively, where ν is the kinematic viscosity.

Defining the stream function as

$$\psi = \sqrt{\beta \nu}\, x f(\eta) - \frac{\delta}{2} x^2 f'(\eta) \tag{5}$$

where

$$\eta = y \sqrt{\beta/\nu} \tag{6}$$

we get u, v as

$$u = \psi_y = \beta x f'(\eta) - \frac{\delta}{2} \sqrt{\beta/\nu}\, x^2 f''(\eta), \tag{7}$$

$$v = -\psi_x = -\sqrt{\beta \nu} f(\eta) + \delta x f'(\eta). \tag{8}$$

Now Eq. (3) is satisfied while Eq. (4) yields, on equating the coefficients of x, x^2 and x^3,

$$f'^2 - ff'' = f''', \tag{9}$$

$$f'f'' - ff''' = f^{iv}, \tag{10}$$

$$f''^2 - f'f''' = 0. \tag{11}$$

Equations (1) and (2) take the form

$$f'(0) = 1, \quad f'(\infty) = f''(\infty) = 0, \tag{12}$$

$$f''(0) = -2 \frac{\alpha}{\delta} \sqrt{\nu/\beta}, \tag{13}$$

$$f(0) = -\frac{v_c}{\sqrt{\beta \nu}}. \tag{14}$$

Equation (10) is redundant as it can be got by differentiation of Eq. (9).

Equation (11) can be integrated as

$$(f''/f')' = 0 \Rightarrow f'' + sf' = 0.$$

Hence,

$$f = a + be^{-s\eta}$$

where a, b, s are arbitrary constants.

Substitution of this solution in Eq. (9) yields $a = s$. Also, with the aid of Eqs. (12)–(14), we get

$$f = s - \frac{e^{-s\eta}}{s} \quad (s > 0), \tag{15}$$

$$\frac{\alpha}{\delta} = \frac{s}{2}\sqrt{\beta/\nu}, \tag{16}$$

$$\frac{1}{s} - s = \frac{v_c}{\sqrt{\beta\nu}}. \tag{17}$$

It is clear from Eq. (17) that $0 < s < 1$ corresponds to injection ($v_c > 0$) and $s > 1$ corresponds to suction ($v_c < 0$).

Hence Eq. (15) is the solution of Eqs. (1)–(4) if s is given by Eq. (17), and α, δ are related by Eq. (16).

Using the dimensionless variables

$$\psi^* = \frac{\psi}{\nu}, \quad \xi = x\sqrt{\beta/\nu}, \quad b = \frac{\delta}{2\beta} \tag{18}$$

Eq. (5) becomes

$$\psi^* = \xi f(\eta) - b\xi^2 f'(\eta). \tag{19}$$

Now, the stream lines $\psi^* = C$ (a constant) are given by

$$\eta = \frac{1}{s}\log\left[\frac{\xi\left(\frac{1}{s} + b\xi\right)}{s\xi - C}\right]. \tag{20}$$

3 Discussion

The problem admits a closed form solution only for both effects, i.e., if the quadratic part in stretching velocity and the linear part in mass flux are simultaneously present (Eq. (16)) or simultaneously absent. This may be because of the simplicity in the form of the solution assumed.

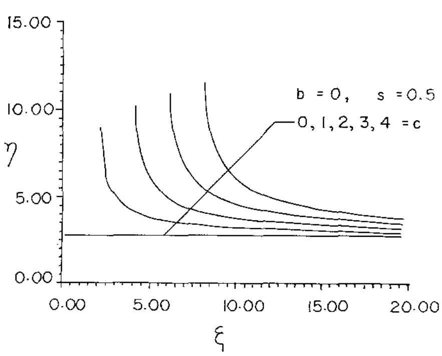

Fig. 2

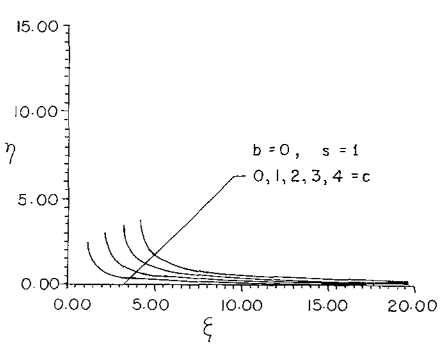

Fig. 3

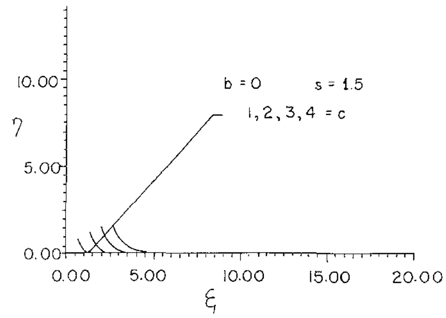

Fig. 4

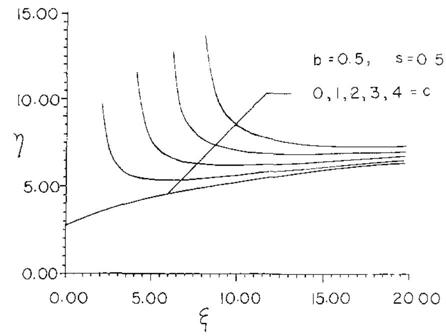

Fig. 5

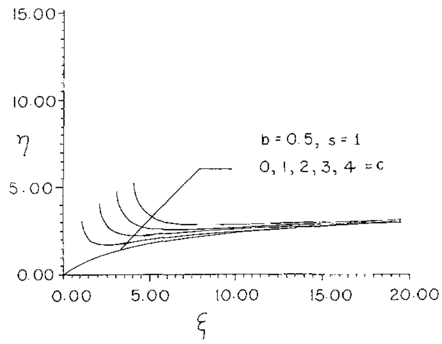

Fig. 6

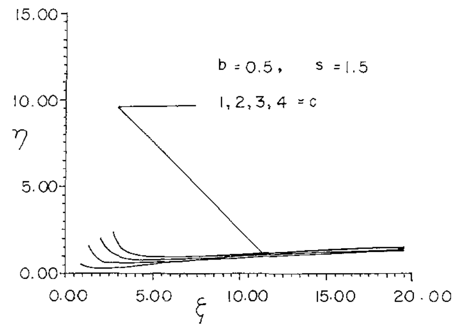

Fig. 7

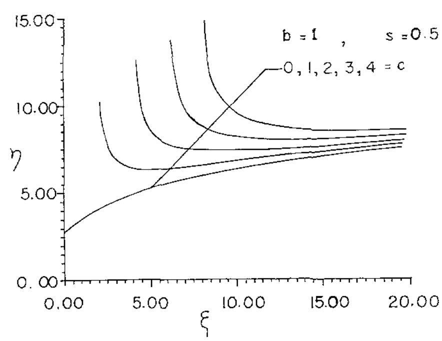

Fig. 8

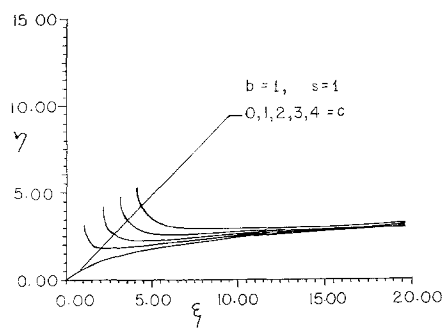

Fig. 9

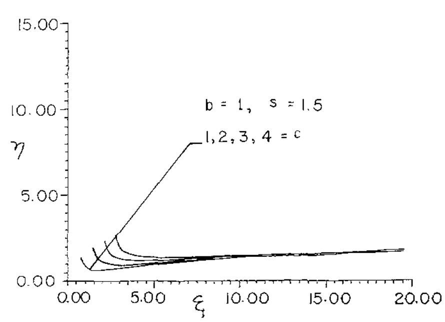

Fig. 10

To understand the effects of the mass flux the stream lines are shown in Figs. 2 – 10 for different values of the parameters s, b, and C. Decrease in s, i.e., increase in v_c will blow up the stream lines as seen from Figs. 2 – 4. This may be because of the external mass flux input. The stream lines fall very steeply near the stretching sheet (Figs. 8, 6, and 4) by increasing s and decreasing b, i.e., for increasing β. Increasing b, i.e., increasing δ or α will cause the stream lines to have positive slopes away from the slit unlike stream lines with negative slopes for the case $b = 0$, because of this, fluid near the origin will drift towards the origin (Figs. 5 – 10).

Acknowledgement

The authors are thankful to Prof. Dr. Dr. J. Zierep for his valuable suggestions. Also, Dr. V. Kumaran is grateful to the Council of Scientific and Industrial Research, New Delhi, India, for the financial support.

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