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Heat transfer in a second-order fluid over a continuous stretching surface

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Summary. The heat transfer characteristics of a second-order fluid over a continuous stretching surface with internal heat generation or absorption is analyzed. Two cases are studied, namely (i) the sheet with prescribed surface temperature (PST-case) and (ii) the sheet with prescribed wall heat flux (PHF-case). The solution and heat transfer characteristics are obtained in terms of Kummer's functions. For large values of Prandtl number a uniform approximation is given in terms of parabolic cylinder functions with a boundary layer of width $\sqrt{1/\mathrm{Pr}}$ in both the PST and PHF cases. It is also shown that no boundary layer type solution exists for small Prandtl number.

1 Introduction

In many engineering processes, boundary layer behavior occurs for a flow over a moving continuous solid surface. Manufacturing processes that involve extrusion of a material and heat-treated materials that travel between feed and wind-up rollers or on conveyer belts are examples that exhibit the characteristics of flow over a moving continuous surface. Sakiadis [1] initiated the study of these applications by considering the boundary layer flow over a continuous solid surface moving with constant speed. This flow is quite different than the boundary layer flow over a semi-infinite flat plate due to the entrainment of the ambient fluid. This problem was extended by Erickson et al. [2] to the case where the transverse velocity at the moving surface is nonzero with heat and mass transfer in the boundary layer accounted for.

These investigations address the problem of a polymer sheet extruded continuously from a dye. It is often implicitly assumed that the sheet is inextensible but it may be necessary to consider a stretching plastic sheet. This was noted by McCormack and Crane [3]. Danberg and Fansler [4] investigated the non-similar solution for the boundary layer flow past a wall that is stretched with a velocity proportional to the distance along the wall. Gupta and Gupta [5] analyzed the heat and mass transfer corresponding to the similarity solution for the boundary layer over a stretching sheet subject to suction or blowing. Recently, Chen and Char [6] investigated the effects of power law surface temperature and power law surface heat flux variation on the heat transfer characteristics of a continuous, linearly stretching sheet subject to suction or blowing.

All of the above investigators restricted their analysis to flows of a Newtonian fluid. However, recently non-Newtonian fluids have become of interest in industry. For example, Fox et al. [7] used both exact and approximate methods to study the laminar boundary layer on an inextensible continuous flat surface moving with a constant velocity in its own plane in a non-Newtonian fluid characterized by a power law model (Oswald-de Waele fluid). This power law model has some limitations as it does not exhibit any elastic properties such as normal stress differences in shear flow. In certain polymer processing applications, flow of a viscoelastic fluid over a stretching sheet is important. For this reason, Rajagopal et al. [8] studied the flow behavior of a viscoelastic fluid over a stretching sheet and gave an approximate solution for the flow. They considered an incompressible second-order fluid whose constitutive equation is based on the assumption of gradually fading memory suggested by Coleman and Noll [9] as

$$\boldsymbol{T} = -P\boldsymbol{I} + \mu \boldsymbol{A}_1 + \alpha_1 \boldsymbol{A}_2 + \alpha_2 \boldsymbol{A}_1^2, \tag{1}$$

where $\boldsymbol{T}$ is the stress tensor, P is the pressure, μ, α_1, α_2 are material constants with $\alpha_1 < 0$ and $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ defined as

$$\boldsymbol{A}_1 = (\operatorname{grad} \boldsymbol{v}) + (\operatorname{grad} \boldsymbol{v})^T, \tag{2}$$

$$\boldsymbol{A}_2 = \frac{d}{dt} \boldsymbol{A}_1 + \boldsymbol{A}_1 \cdot \operatorname{grad} \boldsymbol{v} + (\operatorname{grad} \boldsymbol{v})^T \cdot \boldsymbol{A}_1. \tag{3}$$

This model is applicable to some dilute polymer solutions (such as: (i) the 5.4 percent solution of polyisobutylene in cetane (see Markovitz and Coleman [10]); and (ii) the 0.83 percent solution of ammonium alginate in water (see Acrivos [11])) at low rates of shear. Recently, Troyt et al. [12] gave the exact solution to the problem of Rajagopal et al. [8].

The present authors motivated by the above analyses have studied heat transfer in a second-order fluid over a continuous stretching surface with power law surface temperature and power law surface heat flux including the effect of internal heat generation or absorption. A series solution to the energy equation in both cases is given in terms of Kummer functions. Several closed form analytical solutions are presented for special values of the parameters. Dilute polymer solutions like 0.83 percent ammonium alginate in water and 5.4 percent polyisobutylene in cetane have approximate Prandtl number of 440 and 3 respectively so the asymptotic cases of large and small Prandtl number are studied. (Also, the Prandtl number has different values at different temperatures and/or for different concentrations, for the same fluid. For further details, see Perry [14]). Further, the contributions of the elastic parameter m, the Prandtl number Pr, and heat source/sink parameter α to heat transfer characteristics are found to be quite significant.

2 Flow analysis

Consider the flow of a second-order fluid obeying (1) past a flat sheet coincident with the plane $y = 0$, and the flow confined to the region $y > 0$. Two equal and opposite forces are applied along the x-axis so that the wall is stretched but the origin stays fixed. The steady two-dimensional boundary layer equations for this fluid (see Beard and Walters [13] for details) in the usual notation are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{4}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \lambda \left[\frac{\partial}{\partial x} \left(u \frac{\partial^2 u}{\partial y^2} \right) + \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} + v \frac{\partial^3 u}{\partial y^3} \right] \tag{5}$$

where $\nu = \mu/\varrho$ and $\lambda = -\alpha_1/\varrho$. In deriving these equations it was assumed that the normal stress is of the same order of magnitude as the shear stress, in addition to the usual boundary layer approximations. Thus both ν and λ are $0(\delta^2)$, with δ the boundary layer thickness.

The appropriate boundary conditions for the problem are

$$\begin{aligned} &u = Bx, \quad v = 0 \quad \text{at} \quad y = 0, \quad B > 0, \\ &u \to 0 \quad \text{as} \quad y \to \infty. \end{aligned} \tag{6}$$

In this case, the flow is caused solely by the stretching of the sheet since the free stream velocity is zero. Equations (4) and (5) admit a similarity solution with

$$u = Bxf'(\eta), \qquad \tau = -(B\nu)^{1/2} f(\eta), \tag{7}$$

$$\eta = (B/\nu)^{1/2} y, \tag{8}$$

where a prime denotes differentiation with respect to η. Clearly u and v as defined above satisfy the continuity Eq. (4). Substituting (7) and (8) in (5) gives

$$(f')^2 - ff'' = f''' - \lambda_1[2f'f''' - (f'')^2 - ff^{\text{iv}}], \tag{9}$$

where $\lambda_1 = \lambda B/\nu$ is the elastic parameter. The boundary conditions (6) become

$$\begin{aligned} &f' = 1, \quad f = 0 \quad \text{at} \quad \eta = 0 \\ &f' \to 0 \quad \text{as} \quad \eta \to \infty. \end{aligned} \tag{10}$$

In 1987, Troy et al. [12] obtained the exact solution

$$f(\eta) = (1 - e^{-m\eta})/m, \qquad m = 1/\sqrt{1 - \lambda_1}, \tag{11}$$

for the differential equation (9) satisfying boundary conditions (10). This gives the velocity components

$$\begin{aligned} u &= Bxe^{-m\eta}, \\ v &= -(B\nu)^{1/2}(1 - e^{-m\eta})/m \end{aligned} \tag{12}$$

and the dimensionless shear stress at the wall is

$$\tau = (1 - \lambda_1) f''(0) = -(1 - \lambda_1)^{1/2}. \tag{13}$$

3 Heat transfer analysis

The governing boundary layer equation with internal heat generation or absorption is

$$\varrho C_P \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + Q(T - T_\infty). \tag{14}$$

The thermal boundary conditions depend on the type of heating process being considered. We consider two different heating processes, namely, (i) prescribed surface temperature and (ii) prescribed heat flux. The heat transfer analysis for these two cases is given in Sections 3.1 and 3.2.

3.1 Prescribed surface temperature (PST-case)

For this heating process, the boundary conditions are

$$\begin{aligned} &T = T_W[= T_\infty + A(x/l)^r] \quad \text{at} \quad y = 0, \\ &T \to T_\infty \quad \text{as} \quad y \to \infty, \end{aligned} \tag{15}$$

where l is the characteristic length and r is the temperature parameter. Defining the non-dimensional temperature as

$$\theta(\eta) = (T - T_\infty)/(T_W - T_\infty) \tag{16}$$

and using relations (7)–(8), then Eq. (14) and the boundary conditions (15) can be written as

$$\theta'' + \Pr f\theta' - (\Pr rf' - \alpha)\,\theta = 0, \tag{17}$$

$$\theta = 1 \quad \text{at} \quad \eta = 0, \qquad \theta \to 0 \quad \text{as} \quad \eta \to \infty, \tag{18}$$

where

$$\begin{aligned} &f(\eta) = (1 - e^{-m\eta})/m, \qquad f'(\eta) = e^{-m\eta} \\ &m = (1 - \lambda_1)^{-1/2}, \quad \text{elastic parameter} \\ &\Pr = \mu C_P/k, \quad \text{Prandtl number} \\ &\alpha = Q\nu/kB, \quad \text{heat source/sink parameter} \end{aligned} \tag{19}$$

and a prime denotes differentiation with respect to η.

Defining a new variable

$$\xi = \frac{-\Pr}{m^2}\, e^{-m\eta} \tag{20}$$

and substituting the solution f into Eq. (17), we get

$$\xi\theta'' + \left(1 - \frac{\Pr}{m^2} - \xi\right)\theta' + \left(r + \frac{\alpha}{m^2}\,\xi^{-1}\right)\theta = 0, \tag{21}$$

where a prime will now denote differentiation with respect to ξ. The boundary conditions are now

$$\theta\left(\frac{-\Pr}{m^2}\right) = 1, \qquad \theta(0^-) = 0, \tag{22}$$

where $\theta(0^-)$ denotes the left hand limit of θ at 0. The solution of Eq. (21) satisfying boundary conditions (22) is given in terms of Kummer's function (see [15]):

$$\theta(\xi) = \left(\frac{-m^2\xi}{\Pr}\right)^{(a+b)/2} \frac{M\left(\frac{1}{2}(a + b) - r,\, 1 + b;\, \xi\right)}{M\left(\frac{1}{2}(a + b) - r,\, 1 + b;\, -a\right)} \tag{23}$$

where $a = \Pr/m^2$ and $b = \dfrac{1}{m}\left(\dfrac{\Pr^2}{m^2} - 4\alpha\right)^{1/2}$ and

$$M(\alpha_1, \alpha_2; z) \equiv \sum_{n=0}^{\infty} \frac{(\alpha_1)_n Z^n}{(\alpha_2)_n n!}\,.$$

The solution (23) in terms of the η variable is

$$\theta(\eta) = e^{-(a+b)m\eta/2} \cdot \frac{M\left(\frac{1}{2}(a+b) - r,\ 1+b;\ -ae^{m\eta}\right)}{M\left(\frac{1}{2}(a+b) - r,\ 1+b;\ -a\right)}. \tag{24}$$

The nondimensional temperature gradient derived from (24) is

$$\theta'(0) = \frac{\frac{am}{2}(a+b-2r)(1+b)^{-1} M\left(\frac{1}{2}(a+b) - r + 1,\ 2+b;\ -a\right) - \frac{1}{2}(a+b)\, mM\left(\frac{1}{2}(a+b) - r,\ 1+b;\ -a\right)}{M\left(\frac{1}{2}(a+b) - r,\ 1+b;\ -a\right)} \tag{25}$$

and the local wall heat flux can be expressed as

$$q_W = -k\left(\frac{\partial T}{\partial y}\right)_W = -kA(B/\nu)^{1/2}(x/l)^r\,\theta'(0).$$

For several sets of values of a and b, closed form solutions can be given in terms of elementary functions and some of the interesting results are presented in Table 1. Also, the expressions in (24) and (25) are evaluated numerically and some of the qualitatively interesting results are presented in Figs. 1, 2 and 4.

3.2 Prescribed wall heat flux (PHF-case)

In this case the boundary conditions are

$$-k\frac{\partial T}{\partial y} = q_W = D(x/l)^s \quad \text{at} \quad y = 0,$$

and

$$T \to T_\infty \quad \text{as} \quad y \to \infty. \tag{26}$$

Defining

$$T - T_\infty = \frac{D(x/l)^s}{k}\left(\frac{\nu}{B}\right)^{1/2} g(\eta) \tag{27}$$

and substituting the relations (7) and (8) into (14) and (26), we get

$$g'' + \Pr fg' - (\Pr sf' - \alpha)\, g = 0, \tag{28}$$

$$g'(0) = -1, \qquad g(\infty) = 0, \tag{29}$$

where a prime denotes differentiation with respect to η, and all other parameters are as defined in Section 3.1.

Table 1a. Temperature expressions for various a and b

a $(= \mathrm{Pr}/m^2)$	b $\left(= \frac{1}{m}\left(\frac{\mathrm{Pr}^2}{m^2} - 4\alpha\right)^{1/2}\right)$	$\theta(\eta)$
$2r$	0	$e^{-mr\eta}$
$2r+1$	1	$e^{-mr\eta}\left(1 - \exp\left(-ae^{-m\eta}\right)\right)(1 - e^{-a})^{-1}$
a	$a - 2r$	$e^{mr\eta} \dfrac{\gamma(a - 2r, ae^{-a\eta})}{\gamma(a - 2r, a)}$
a	$a - 2r - 2$	$\exp\left(-m(a - r - 1)\eta - a(e^{-m\eta} - 1)\right)$
a	$a - 2r - 4$	$\left(1 + \dfrac{a(e^{-m\eta} - 1)}{2r + 3)}\right) \exp\left(-m(a - r - 2)\eta + a(1 - e^{-m\eta})\right)$

where γ is the incomplete gamma function

Table 1b. Temperature gradient expressions for various a and b

a	b	$\theta'(0)$
$2r$	0	$-mr$
$2r+1$	1	$\left(m(r - a)e^{-a} - mr\right)/(1 - e^{-a})$
a	$a - 2r$	$-mr - me^{-a}\dfrac{a^{a-2r}}{\gamma(a - 2r, a)}$
a	$a - 2r - 2$	$m(r + 1)$
a	$a - 2r - 4$	$m(r + 2) - \dfrac{(m + 1)a - 2r - 3}{2r + 3}$

Using transformation (20), we reduce Eq. (28) and the boundary conditions (29) to

$$\xi g'' + \left(1 - \frac{\mathrm{Pr}}{m^2} - \xi\right) g' + \left(s + \frac{\alpha}{m^2}\xi^{-1}\right) g = 0, \tag{30}$$

$$g'(-a) = -m/\mathrm{Pr}, \qquad g(0^-) = 0, \tag{31}$$

where prime now denotes differentiation with respect to ξ. The solution satisfying (30) and (31) is given by

$$g(\xi) = \frac{1}{m}\left[\frac{1}{2}(a + b) M\left(\frac{1}{2}(a + b) - s, 1 + b; -a\right)\right.$$
$$\left. - aM'\left(\frac{1}{2}(a + b) - s, 1 + b; -a\right)\right]^{-1} \left(\frac{-\xi}{a}\right)^{\frac{1}{2}(a+b)} M\left(\frac{1}{2}(a + b) - s, 1 + b; \xi\right) \tag{32}$$

where $M'(\alpha_1, \alpha_2; z) = \dfrac{\alpha_1}{\alpha_2} M(\alpha_1 + 1, \alpha_2 + 1; z)$.

Table 2. Temperature expressions for various a and b

a $(= \mathrm{Pr}/m^2)$	b $\left(= \frac{1}{m}\left(\frac{\mathrm{Pr}^2}{m^2} - 4\alpha\right)^{1/2}\right)$	$g(\eta)$
2	0	$\frac{2}{ma}\, e^{-m s \eta/2}$
$2s + 1$	1	$e^{-ms\eta}\left(1 - \exp\left(-ae^{-m\eta}\right)\right)\left(s + (a - s)\, e^{-a}\right)^{-1}$
a	$a - 2s$	$C_1 e^{-ms\eta}\gamma(a - 2s, ae^{-m\eta})$
a	$a - 2s - 2$	$\frac{-1}{m}(s + 1)^{-1}\exp\left(-m(a - s - 1)\,\eta - a(e^{-m\eta} - 1)\right)$
a	$a - 2s - 4$	$C_2\left(1 + \frac{a(e^{-m\eta} - 1)}{2s + 3}\right)\exp\left(-m(a - s - 2)\,\eta + a(1 - e^{-m\eta})\right)$

where

$$C_1 = \frac{1}{m}\left((a - s)\,\gamma(a - 2s, a) - \gamma(a - 2s + 1, a)\right)^{-1}$$

$$C_2 = \frac{1}{m}\left(\frac{a}{2s + 3} - s - 2\right)^{-1}$$

In terms of the variable η, the solution is

$$g(\eta) = \frac{1}{m}\left[\frac{1}{2}(a + b)\, M\left(\frac{1}{2}(a + b) - s, 1 + b; -a\right) - aM'\left(\frac{1}{2}(a + b) - s, 1 + b; -a\right)\right]^{-1} \times e^{-m(a+b)\eta/2} M\left(\frac{1}{2}(a + b) - s, 1 + b; -ae^{-m\eta}\right). \tag{33}$$

The wall temperature T_W is obtained from Eq. (27) as

$$T_W - T_\infty = \frac{D(x/l)^s}{k}(\nu/B)^{1/2}\, g(0). \tag{34}$$

As in Section 3.1, several closed form solutions are derived from (33) for special values of the parameters a and b. These are presented in Table 2. Also, numerical values of $g(0)$ for several values of the parameters m, α and Pr are calculated and presented in Fig. 3.

4 Asymptotic limit for large Prandtl number

In this Section, we derive the asymptotic results for large Prandtl number for the temperature functions $\theta(\eta)$ and $g(\eta)$, which arise in the PST and PHF cases respectively.

4.1 PST-case

In this case the boundary layer equation and boundary conditions are

$$\varepsilon\theta'' + f(\eta)\,\theta' - \left(rf'(\eta) - \varepsilon\alpha\right)\theta = 0, \tag{35}$$

$$\theta(0) = 1, \qquad \theta(\infty) = 0, \tag{36}$$

where $\varepsilon \equiv (\mathrm{Pr})^{-1}$ and a prime denotes differentiation with respect to η. The change of variable

$$\theta(\eta) = \exp\left(\frac{-1}{2\varepsilon} S(\eta)\right) \Phi(\eta), \qquad F(\eta) \equiv \int_0^\eta f(\eta')\, d\eta' \tag{37}$$

applied to (35) gives

$$\varepsilon^2\Phi'' - \left[\frac{1}{4} f(\eta)^2 + \left(r + \frac{1}{2}\right) \varepsilon f'(\eta) - \varepsilon^2\alpha\right] \Phi = 0. \tag{38}$$

Using WKB (Wentzel, Kramers and Brillouin) theory we obtain a uniform approximation in the limit of small ε (see [16]). The inner solution can be found by scaling η by $\sqrt{\varepsilon}$ in Eq. (38) which gives to lowest order

$$\frac{d^2\Phi}{d\tilde{\eta}^2} - \left[\frac{1}{4} \tilde{\eta}^2 - r + \frac{1}{2}\right] \Phi = 0 \quad \text{where} \quad \tilde{\eta} = \eta/\sqrt{\varepsilon}. \tag{39}$$

The inner solution is then proportional to $D_{-r-1}(\tilde{\eta})$. For $\eta \gg \sqrt{\varepsilon}$ the solution is found by looking for a solution of the form

$$\Phi(\eta) \sim \exp\left[\frac{1}{\varepsilon} \sum_{n=0}^{\infty} \varepsilon^n \beta_n(\eta)\right]. \tag{40}$$

Calculating $\beta_0(\eta)$ and $\beta_1(\eta)$ gives the approximation

$$\Phi(\eta) \sim C\big(f(\eta)\big)^{-r-1} \exp\left(\frac{-1}{2\varepsilon} F(\eta)\right) \tag{41}$$

with C determined by matching with the inner solution. The solution (39) and Eq. (41) can be combined in the uniform expansion

$$\theta(\eta) \sim \pi^{1/2}\, 2^{r+1}\Gamma\left(1 + \frac{r}{2}\right) \left(\frac{\sqrt{F(\eta)}}{f(\eta)}\right)^{r+1} D_{-r-1}\left(\sqrt{2F(\eta)/\varepsilon}\right) \exp\left(\frac{-1}{2\varepsilon} F(\eta)\right). \tag{42}$$

From (42) we observe that there is a boundary layer of width $\sqrt{\varepsilon}$.

4.2 PHF-case

The analysis in Section 4.1 can be applied to this case as well. The uniform asymptotic expression in this case is

$$g(\eta) \sim \pi^{-1/2} 2^{s+\frac{1}{2}} \varepsilon^{1/2} \Gamma\left(\frac{s+1}{2}\right) \left(\frac{\sqrt{F(\eta)}}{f(\eta)}\right)^{s+1} D_{-s-1}\left(\sqrt{2F(\eta)/\varepsilon}\right) \exp\left(\frac{-1}{2\varepsilon} F(\eta)\right). \tag{43}$$

As in the PST-case, the boundary layer is also $\sqrt{\varepsilon}$.

5 Asymptotic limit for small Prandtl number

As in Section 4, we would like to derive an asymptotic approximation for small Prandtl number for the temperature functions $\theta(\eta)$ and $g(\eta)$. However it is not possible to find matched asymptotic expansions in the usual sense. This is evident by considering the exact solution with Pr replaced by ε. The solution in both cases changes by 0(1) on a length scale of order $(\mathrm{Pr})^{-1}$, which is arbitrarily large for small Pr.

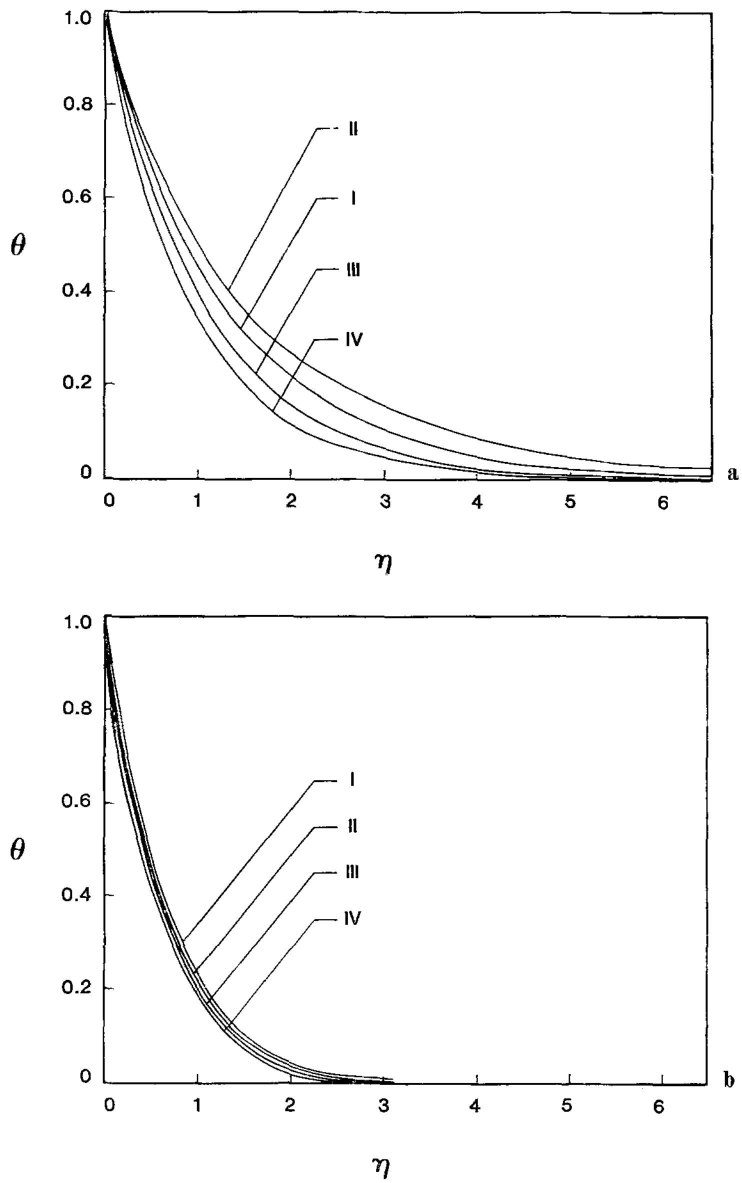

Fig. 1. Temperature profiles for $r = 1$ when **a** Pr $= 1$ and **b** Pr $= 2$

	I	II	III	IV
m	1	1.1	1.1	1.1
α	0.2	0.2	0.0	-0.2

5.1 PST-case

Letting $\varepsilon = \Pr$ in (17) gives

$$\theta'' + \varepsilon f(\eta)\, \theta' - \big(\varepsilon r f'(\eta) - \alpha\big)\, \theta = 0\,. \tag{44}$$

The solution satisfying (44) and boundary conditions

$$\theta(0) = 1\,, \qquad \theta(\infty) = 0$$

is

$$\theta(\eta) = e^{-\varepsilon\sigma\eta/2m} \frac{M\left(\frac{\varepsilon}{m^2}\sigma - r, 1 + \frac{\varepsilon}{m^2}(\sigma - 1); \frac{-\varepsilon}{m^2} e^{-m\eta}\right)}{M\left(\frac{\varepsilon}{m^2}\sigma - r, 1 + \frac{\varepsilon}{m^2}(\sigma - 1); \frac{-\varepsilon}{m^2}\right)} \tag{45}$$

where

$$\sigma = 1 + \left(1 - \frac{4\alpha m^2}{\varepsilon^2}\right)^{1/2}. \tag{46}$$

The term $e^{-\varepsilon\sigma\eta/2m}$ gives a slow exponential decay, which shows the impossibility of a boundary layer type solution.

5.2 *PHF-case*

Similarly $g(\eta)$ is given by

$$\begin{aligned} g(\eta) = \frac{1}{m} & \left[\frac{\varepsilon\sigma}{2m^2} M\left(\frac{\varepsilon}{m^2}\sigma - s, 1 + \frac{\varepsilon}{m^2}(\sigma - 1); \frac{-\varepsilon}{m^2}\right)\right. \\ & \left. - \frac{\varepsilon}{m^2} M\left(\frac{\varepsilon}{m^2}\sigma - s, 1 + \frac{\varepsilon}{m^2}(\sigma - 1); \frac{-\varepsilon}{m^2}\right)\right]^{-1} \\ & \times e^{-\varepsilon\sigma\eta/2m} M\left(\frac{\varepsilon}{m^2}\sigma - s, 1 + \frac{\varepsilon}{m^2}(\sigma - 1); \frac{-\varepsilon}{m^2} e^{-m\eta}\right) \end{aligned} \tag{47}$$

with σ as before in (46). Again, the solution decays slowly due to the $e^{-\varepsilon\sigma n/2m}$ term.

6 Discussion of results

In Fig. 1a, the temperature distribution $\theta(\eta)$ is plotted for Pr $= 1$ and several sets of values of m and α. Similar plots are given in Fig. 1b for Pr $= 2$. From these it is evident that the temperature increases with an increase in the viscoelastic parameter m. The temperature

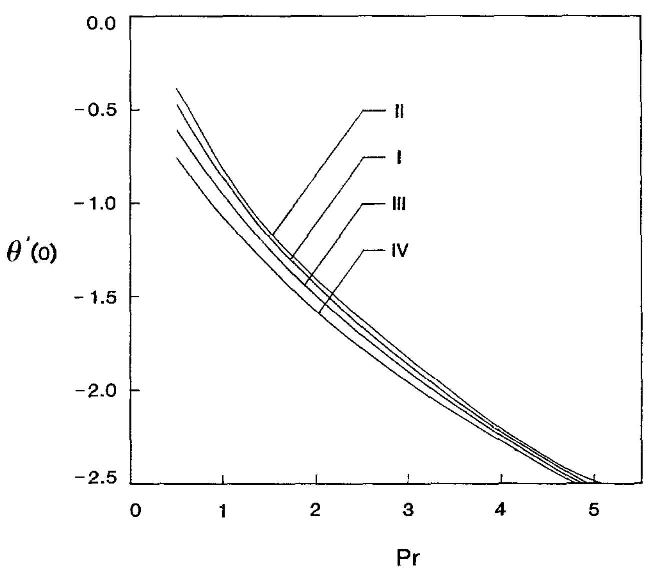

Fig. 2. Temperature gradient in the PST case for $r = 1$; curves as in Fig. 1

also increases with an increase in the value of the heat source/sink parameter α. Furthermore, an increase in the Prandtl number decreases the temperature at a given point in the fluid. This is consistent with the fact that the thermal boundary layer thickness decreases with increasing Prandtl number.

The wall temperature gradient $\theta'(0)$ as a function of Pr for several sets of values of m and α is shown in Fig. 2. For given values of m and α, the larger the Pr, the larger (in an absolute sense) the magnitude of the wall temperature gradient. Also, the wall temperature gradient increases in magnitude as m increases. This is true even for the heat source/sink parameter α.

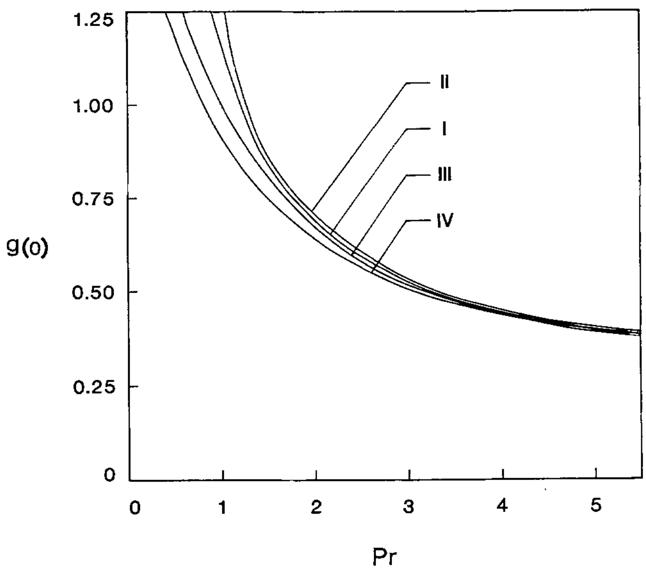

Fig. 3. Temperature at $\eta = 0$ in the PHF case for $s = 1$; curves as in Fig. 1

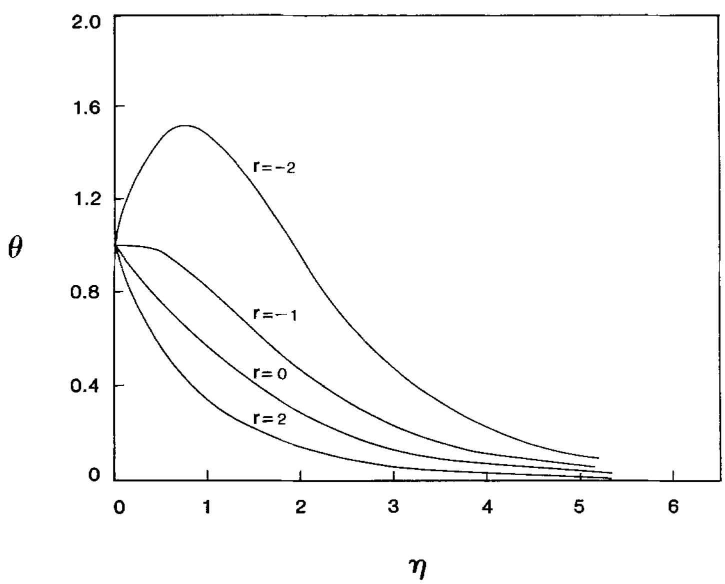

Fig. 4. Temperature in the PST case for $r = 2, 0, -1, -2$ at $m = 1.1$ and $\alpha = 0.2$

The behavior of the wall temperature $g(0)$ with changes in m, α and Pr is displayed in Fig. 3. From this figure it is seen that the wall temperature decreases rapidly for small values of Pr and then decreases slowly for further increases in the Prandtl number. Furthermore, the effects of m and α are to increase the wall temperature $g(0)$.

In Fig. 4 the effect of r, the temperature parameter, on the temperature distribution for fixed values of m, α and Pr is shown. When $r > 0$, heat flows from the stretching sheet to the ambient medium. The magnitude of the temperature gradient increases with r for this case. When $r < 0$, the temperature gradient is now positive and heat flows into the stretching sheet from the ambient medium.

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