Acta Mechanica 40, 159–168 (1981)

ACTA MECHANICA

Application of the Method of Singular Integral Equations to Elasticity Problems with Concentrated Loads

By

N. I. Ioakimidis, Patras, Greece

With 1 Figure

(*Received July 16, 1980*)

Summary — Zusammenfassung

Application of the Method of Singular Integral Equations to Elasticity Problems with Concentrated Loads. The method of singular integral equations is a well-known method for solving plane and antiplane elasticity problems and efficient methods for the numerical solution of these equations have been developed. In this paper this method is used for problems where concentrated loads are applied on the boundary of the elastic medium. An application to a straight crack problem in plane isotropic elasticity is also made. Finally, the case of curvilinear crack problems with concentiated loads is considered. The results of this paper can further be applied to more complicated problems with concentrated loads.

Die Anwendung der Methode der singulären Integralgleichungen bei Elastizitätsproblemen mit konzentrierten Lasten. Die Methode der singulären Integralgleichungen ist eine sehr bekannte Methode für die Behandlung von ebenen und antiebenen Elastizitätsproblemen und es wurden erfolgreiche Methoden für die numerische Lösung dieser Gleichungen entwickelt. In dieser Arbeit wird die Methode für Probleme angewendet, bei welchen konzentrierte Lasten am Rande des elastischen Mediums aufgebracht werden. Eine Anwendung für ein Problem eines geradlinigen Risses bei ebener, isotroper Elastizität wird gezeigt. Abschließend wird auch der Fall von Problemen mit gekrümmten Rissen bei konzentrierten Lasten behandelt. Die Ergebnisse dieser Arbeit können ferner bei komplizierteren Problemen mit konzentrierten Lasten Anwendung finden.

1. Introduction

The method of solution of plane and antiplane isotropic and anisotropic elasticity problems by reducing them to singular integral equations (with Cauchy type kernels) has gained high popularity in recent years. Some of the relevant literature is contained in the review papers by Erdogan, Gupta and Cook [1] and Erdogan [2], [3], who developed also efficient methods for the numerical solution of singular integral equations. Considerable further progress on the topic is due to Ioakimidis [4], who considered particularly crack problems and developed new methods for the numerical solution of singular integral equations. The results of Ioakimidis can be found in a series of papers by him and Theocaris, some of which are mentioned in [5].

In spite of the fact that various elasticity problems have been treated by the method of singular integral equations, to this author's best knowledge no such

0001-5970/81/0040/0159/$02.00

problem with concentrated loads on the boundary of the elastic medium has been solved by this method. Only problems with jump discontinuities in loading were solved by the above method by Karihaloo (see, e.g., [6]), Theocaris, Chrysakis and Ioakimidis [7] and Ioakimidis [8]. It seems that the main reason for which problems involving concentrated loads have not been treated by the method of singular integral equations is the fact that in such problems the use of the Dirac δ-function, which is a generalized function, is necessary, contrary to what happens in problems not involving concentrated loads.

In this paper we will illustrate the application of the method of singular integral equations to elasticity problems with concentrated loads by using just the elementary properties of the δ-function and the available techniques for the formulation and numerical solution of singular integral equations. The problem of a periodic array of straight cracks with two pairs of opposite and symmetrically applied concentrated loads will be considered at first and some numerical results will be presented in the case of one pair of opposite concentrated loads. Furthermore, the problem of curvilinear cracks of arbitrary shape will be considered in some detail in the case of opposite concentrated loads along the crack edges. It is hoped that the illustration of the application of the method of singular integral equations in problems involving concentrated loads in this paper will permit its further wide use to more complicated or practical problems, contrary to what happened in the past when such problems were treated mainly by methods aiming to their closed-form solutions.

2. The Case of Straight Cracks

We consider the problem of a straight crack of length $2a$ inside an infinite isotropic elastic medium under plane strain or generalized plane stress conditions, loaded by a pressure distribution $f(x)$ acting along both crack edges. This problem is easily reducible to the following singular integral equation [1], [4]

$$\frac{1}{\pi}\int_{-1}^{1}\frac{w(t)\,g(t)}{t-x}\,dt = f(x), \qquad -1 < x < 1, \tag{1}$$

accompanied by the condition of single-valuedness of displacements

$$\int_{-1}^{1} w(t)\,g(t)\,dt = 0, \tag{2}$$

where

$$w(t) = (1-t^2)^{-1/2}. \tag{3}$$

Similar equations hold also true in the antiplane case.

Equations (1) and (2) possess the closed-form solution [10, p. 426]

$$g(t) = -\frac{1}{\pi}\int_{-1}^{1}\frac{f(\tau)}{w(\tau)\,(\tau-t)}\,d\tau, \tag{4}$$

which defines the unknown function $g(t)$ (proportional to the edge-dislocations density) along the crack. The reduced stress intensity factors (as if $a = 1$) $K(\pm 1)$ at the crack tips are given by [1], [4]

$$K(\pm 1) = \pm g(\pm 1). \tag{5}$$

The above theoretical results remain valid even when the compressive loading distribution $f(x)$ presents discontinuities or singularities of any form provided that the integral in Eq. (4) exists in the principal value sense. A pair of compressive concentrated loads P acting at the point ax_0 ($|x_0| < 1$) of the crack corresponds to the following form of $f(x)$

$$f(x) = P\delta(x - x_0), \tag{6}$$

where $\delta(x)$ is the well-known Dirac's delta-function defined by its properties [9]

$$\delta(x) = 0 \quad \text{if} \quad x \neq 0, \qquad \delta(x) = \infty \quad \text{if} \quad x = 0, \quad \int_{-\varepsilon}^{\varepsilon} \delta(x)\, dx = 1, \qquad \varepsilon > 0. \tag{7}$$

From Eqs. (7) it is clear that Eq. (6) gives

$$\int_{-1}^{1} f(x)\, dx = P \tag{8}$$

as expected.

Furthermore, for the loading distribution $f(x)$ defined by Eq. (6) we obtain from Eq. (4) for $g(t)$

$$g(t) = \frac{P(1 - x_0^2)^{1/2}}{\pi(t - x_0)} \tag{9}$$

as can easily be verified on the basis of Eqs. (7). Then we obtain from Eq. (5) for the dimensionless stress intensity factors $K(\pm 1)$

$$K(\pm 1) = \frac{P}{\pi} \left(\frac{1 \pm x_0}{1 \mp x_0} \right)^{1/2}. \tag{10}$$

This is a well-known result [11, p. 5.9]. As regards the complex potential $\Phi(z)$ of Muskhelishvili [12], it can be determined by [4]

$$\Phi(z) = -\frac{1}{2\pi} \int_{-1}^{1} \frac{w(t)\, g(t)}{t - z}\, dt, \tag{11}$$

which, because of Eq. (9), yields

$$\Phi(z) = \frac{P(1 - x_0^2)^{1/2}}{2\pi(z^2 - 1)^{1/2}\,(z - x_0)}. \tag{12}$$

This is also a well-known result [11, p. 5.9]. Of course, in the above equations all lengths have been reduced to the half crack length a. Finally, it can easily be seen from Eqs. (9) and (12) that

$$\Phi^{+}(t) - \Phi^{-}(t) = -iw(t)\, g(t) \tag{13}$$

and this is a verification of the correctness of these equations [4].

Moreover, if instead of one pair of concentrated loads P at the point ax_0, a series of such pairs of concentrated loads P_i $\big(i = 1(1)\, n\big)$ acts at the points ax_{0i} of the crack, then, obviously, Eq. (9) should be modified as

$$g(t) = \frac{1}{\pi} \sum_{i=1}^{n} \frac{P_i(1 - x_{0i}^2)^{1/2}}{t - x_{0i}}. \tag{14}$$

A similar modification should be made in this case to Eqs. (10) and (12).

Now we come to the case of a periodic array of cracks loaded by two pairs of concentrated loads P acting symmetrically on the cracks as shown in Fig. 1. The

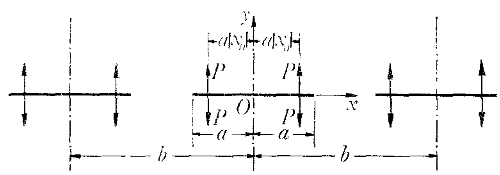

Fig. 1. A periodic array of cracks along a straight line loaded by two pairs of compressive concentrated loads acting symmetrically on the cracks

length of the cracks is equal to $2a$ and the period of the array is equal to b. The singular integral equation (1) takes in the case of a periodic array of cracks the form [4], [13]

$$\frac{1}{\pi} \int_{-1}^{1} w(t)\, k(t, x)\, g(t)\, dt = f(x), \qquad -1 < x < 1, \tag{15}$$

with

$$k(t, x) = \frac{\pi a}{b} \cot \frac{\pi a(t - x)}{b} \tag{16}$$

and remains accompanied by Eq. (2). The kernel $k(t, x)$ is sufficiently simple even in this case and Eq. (15) possesses also a closed-form solution. But here we wish to illustrate the numerical procedure for solving (15) by making use of the solution (9) for the case of a simple straight crack. Evidently, in more complicated cases equations of the form (15) do not have closed-form solutions and the application of a numerical technique becomes indispensable. In the case of Fig. 1 we will use only the closed-form expression for the dimensionless stress intensity factors [11, p. 7.7]

$$K(\pm 1) = \frac{2}{b} \left(\tan \frac{\pi a}{b} \Big/ \frac{\pi a}{b}\right)^{1/2} \cos \frac{\pi a x_0}{b} \left(\sin^2 \frac{\pi a}{b} - \sin^2 \frac{\pi a x_0}{b}\right)^{-1/2} \tag{17}$$

to check our numerical results.

For the numerical solution of Eq. (15) we will apply the Lobatto-Chebyshev direct quadrature method proposed by Ioakimidis [4] and further reported by Theocaris and Ioakimidis [14]. This method, although a little less accurate than the modified Gauss-Chebyshev direct quadrature method [15], presents the advantage that it permits the direct evaluation of the stress intensity factors from Eqs. (5) without the necessity of using interpolation formulas [15]. Moreover, since $f(x)$ is given by

$$f(x) = P[\delta(x - x_0) + \delta(x + x_0)] \tag{18}$$

for the array of cracks of Fig. 1 and, hence, it presents singularities along the integration interval $[-1, 1]$ of Eq. (15), the modification of the Lobatto-Chebyshev method of numerical solution of singular integral equations, originally proposed in [16] and further applied in [8] to crack problems with jump discontinuities in loading, must be used.

In accordance with this method, we replace Eq. (15) by the following equation

$$\frac{1}{\pi} \int_{-1}^{1} w(t)\, k(t, x)\, \tilde{g}(t)\, dt = F(x), \qquad -1 < x < 1, \tag{19}$$

where

$$F(x) = -\frac{1}{\pi} \int_{-1}^{1} w(t)\, [k(t, x) - 1/(t - x)]\, g_0(t)\, dt \tag{20}$$

with $g_0(t)$ being the closed-form solution of Eqs. (1) and (2) with $f(x)$ given by Eq. (18). Then $g(t)$ will be determined by [16, 8]

$$g(t) = g_0(t) + \tilde{g}(t). \tag{21}$$

From Eq. (20) it is clear that $F(x)$ is a continuous function along $(-1, 1)$, if Eq. (16) is also taken into account, and, hence, the Lobatto-Chebyshev method can be applied to Eqs. (19) and (2) in its original form [4], [14]. As regards $g_0(t)$, it is determined, on the basis of the previous developments, from Eq. (14) and is given by

$$g_0(t) = \frac{P}{\pi} (1 - x_0^2)^{1/2} \left(\frac{1}{t - x_0} + \frac{1}{t + x_0}\right). \tag{22}$$

Finally, since the kernel $[k(t, x) - 1/(t - x)]$ in Eq. (20) is a regular kernel, but $g_0(t)$ presents strong singularities at the points $t = \pm x_0$, the use of numerical integration rules for Cauchy type principal value integrals [4], [17] is necessary for the evaluation of $F(x)$ at the collocation points used for the numerical solution of Eqs. (19) and (2). Such rules for the weight function $w(t)$ (defined by Eq. (3)) are mentioned in [4], [17]. By using a sufficiently large number of nodes m in the quadrature rule used, it is possible to evaluate $F(x)$ from Eq. (20) up to the accuracy of the computer, practically exactly.

As a numerical application, we present in Table 1 the numerical results obtained by the above procedure for the dimensionless (with $a = 1$ and $P = 1$)

Table 1. *Convergence of the numerical results for the dimensionless stress intensity factors at the tips of the cracks (of length 2a) of a periodic array of cracks (of period b) along a straight line loaded by a pair of compressive concentrated loads (of intensity 2P) acting at the midpoints of the cracks*

n	$2a/b = 0.2$	$2a/b = 0.8$
2	0.6583528549	1.506154804
3	0.6582058845	1.357859822
4	0.6582043079	1.323329871
5	0.6582042990	1.317472268
6	0.6582042990	1.316567356
7	0.6582042990	1.316432040
8	0.6582042990	1.316412044
9	0.6582042990	1.316409104
10	0.6582042990	1.316408672
Theoretical values	0.6582042990	1.316408598

stress intensity factors $K(\pm 1)$ at the crack tips of the array of cracks of Fig. 1 in the special case when $x_0 = 0$, that is in the case when a pair of concentrated compressive loads of intensity $2P$ acts at the midpoints of the cracks. Both cases when $2a/b = 0.2$ and $2a/b = 0.8$ were considered and the number of nodes n in the Lobatto-Chebyshev method of numerical solution of Eqs. (19) and (2) took the values $n = 2(1)\,10$. Moreover, in the same table the theoretical values for these factors are presented as determined from Eq. (17). From the results of Table 1 the rapid covergence of the numerical results to their correct values, even in the case when $2a/b = 0.8$ (when the successive cracks lie too close to each other), is clear. This is a justification of the technique proposed in this section, which, probably, will find wide application in more complicated or more interesting problems involving concentrated loads.

Of course, not only crack problems are reducible to singular integral equations. For example, the problems of a finite or an infinite medium [18] or inclusion problems [19] in plane elasticity can be reduced to singular integral equations and the above technique, based on the use of the Dirac δ-function, to treat problems involving concentrated loads remains applicable. Yet, since most plane elasticity problems are reduced to complex singular integral equations, we will show how our method is applicable to this class of equations in the next section. This will be made without splitting the complex singular integral equation into two real singular integral equations. The case considered will be that of curvilinear crack problems [4], but the same complex singular integral equation holds also true for finite or infinite media if their boundaries are interpreted as cracks [4], [18].

3. The Case of Curvilinear Cracks

We consider now the problem of a smooth curvilinear crack L in the complex plane $z = x + iy$. We denote by τ, t the points of the crack and the corresponding values of the complex variable z. Assuming that the crack lies in an infinite plane isotropic elastic medium, we can reduce the problem to the following complex singular integral equation [4]

$$\frac{1}{\pi i}\int_L \left[\frac{1}{\tau - t} - \frac{dt}{\overline{dt}}\,\frac{\bar{\tau} - \bar{t}}{(\tau - t)^2}\right] \varphi(\tau)\, d\tau \\ -\frac{1}{\pi i}\int_L \left[\frac{1}{\bar{\tau} - \bar{t}} + \frac{dt}{\overline{dt}}\,\frac{1}{\tau - t}\right] \overline{\varphi(\tau)}\,\overline{d\tau} = \overline{2f(t)}, \qquad t \in L, \tag{23}$$

where $f(t)$ is a known function representing the loading on the crack edges (assumed for convenience the same on both these edges) and $\varphi(t)$ is the unknown function proportional to the edge-dislocations density along the crack. Moreover, a bar over a variable or a function denotes its complex conjugate and the quantities dt and $\overline{dt}$ are defined by: $dt = ds \exp(i\theta)$ and $\overline{dt} = ds \exp(-i\theta)$, where ds is an elementary arc along the crack and θ is the angle of the tangent of the crack (in the direction of increasing s) with respect to the positive $0x$-axis [4]. Of course, Eq. (23) is supplemented by the condition of single-valuedness of displacements

$$\int_L \varphi(\tau)\, d\tau = 0. \tag{24}$$

A singular integral equation similar in nature to Eq. (23) but of a more complicated form is valid in the case of a curvilinear crack inside an infinite anisotropic plane elastic medium [20].

Following the results of the previous section, we assume that a pair of concentrated loads acts at a point $t = t_0$ of the crack L, characterized by the value $s = s_0$ of the arc-length. Then we can assume that

$$f(t) = P\delta(s - s_0)\exp(-i\theta_0), \tag{25}$$

where θ_0 is the value of θ corresponding to the point t_0 of application of the pair of concentrated loads. This equation is analogous to Eq. (6) of the previous section. Of course, P may be a complex quantity (a compressive concentrated load together with a tangential concentrated load). Moreover, we can easily find from Eq. (25) that

$$\int_L f(t)\, dt = P \tag{26}$$

if Eqs. (7) are also taken into account.

Now, by taking into consideration the behavior of the kernels in Eq. (23) as $\tau \to t$, we can easily see that this equation is of the form

$$-\frac{2}{\pi i}\int_L \frac{\overline{\varphi(\tau)}}{\bar{\tau} - \bar{t}}\,\overline{d\tau} + \int_L \left[k(\tau, t)\,\varphi(\tau)\, d\tau + k^*(\tau, t)\,\overline{\varphi(\tau)}\,\overline{d\tau}\right] = \overline{2f(t)}, \qquad t \in L, \tag{27}$$

where $k(\tau, t)$ and $k^*(\tau, t)$ are regulars kernels. Hence, if we write $\varphi(t)$ as

$$\varphi(t) = \varphi_0(t) + \tilde{\varphi}(t), \tag{28}$$

following the developments of the previous section, we will have to determine $\varphi_0(t)$ from

$$\frac{1}{\pi i}\int_L \frac{\varphi_0(\tau)}{\tau - t}\, d\tau = f(t), \qquad t \in L, \tag{29}$$

as can easily be seen from Eq. (27), whereas $\tilde{\varphi}(t)$ should be determined from the solution of the following singular integral equation resulting from Eq. (27) on the basis of Eqs. (28) and (29)

$$\begin{aligned} -\frac{2}{\pi i}\int_L \frac{\overline{\tilde{\varphi}(\tau)}}{\bar{\tau} - \bar{t}}\, \overline{d\tau} &+ \int_L \left[k(\tau, t)\, \tilde{\varphi}(\tau)\, d\tau + k^*(\tau, t)\, \overline{\tilde{\varphi}(\tau)}\, \overline{d\tau}\right] \\ &= -\int_L \left[k(\tau, t)\, \varphi_0(\tau)\, d\tau + k^*(\tau, t)\, \overline{\varphi_0(\tau)}\, \overline{d\tau}\right], \qquad t \in L. \end{aligned} \tag{30}$$

Of course, the condition (24) remains valid and it is convenient to assume it valid for both functions $\varphi_0(t)$ and $\tilde{\varphi}(t)$.

As regards Eq. (30), it has now a regular right-hand side and can be solved by the direct quadrature methods of numerical solution of complex singular integral equations [4], [14], [15], [21] and, particularly, by the Lobatto-Chebyshev [4], [14], [21] or the modified Gauss-Chebyshev [15] methods. Of course, it is necessary, before applying these techniques, to find the closed-form solution of Eq. (29). If $t = \alpha$ and $t = \beta$ are the tips of the crack L, then we have [10, p. 426]

$$\varphi_0(t) = \frac{1}{\pi i X(t)}\int_L \frac{X(\tau)\, f(\tau)}{\tau - t}\, d\tau, \tag{31}$$

where

$$X(t) = i[(t - \alpha)\,(\beta - t)]^{1/2} \tag{32}$$

is the canonical function of Eq. (29) [10, p. 429]. Now, by inserting the expression (25) for $f(t)$ in Eq. (31), we find because of Eqs. (7)

$$\varphi_0(t) = \frac{iPX(t_0)}{\pi X(t)\,(t - t_0)}. \tag{33}$$

Moreover, it can be mentioned that the integral of the right-hand side of Eq. (30) can be evaluated, with $\varphi_0(t)$ given by (33), by using the Gauss-Chebyshev or the Lobatto-Chebyshev quadrature rules for Cauchy type principal value integrals [4], [17].

It can also be mentioned that Eq. (33) reduces to (9) in the case of a straight crack with $\alpha = -1$, $\beta = 1$, whence

$$X(t) = i(1 - t^2)^{1/2}, \tag{34}$$

if we take also into account Eq. (3), as well as the fact that

$$\varphi_0(t) = iw(t)\, g(t) \tag{35}$$

clear from a comparison of Eqs. (1) and (23). Next, the case of more than one pair of concentrated loads can be treated by the method of superposition as was made in the previous section. Similarly, in the case when we have not only concentrated loads acting along the crack edges but also a pressure distribution, we can also apply the principles of the above technique. This is similarly the case when we have also jump discontinuities in $f(t)$; Eq. (31) will remain valid but in some cases we will have to perform the integration by combining the closed-form formula (33) with numerical integration techniques for Cauchy type principal value integrals so that the part of the integral in Eq. (31) corresponding to the regular part of $f(t)$ can be evaluated. Of course, it is also possible to replace $f(t)$ in Eqs. (29) and (31) by its part $f_0(t)$ presenting strong or weaker singularities and further modify accordingly Eq. (30) by adding to its right-hand side the regular part

$$\bar{f}(t) = f(t) - f_0(t) \tag{36}$$

of $f(t)$. Several more analogous possibilities and generalizations are also possible but of a trivial character.

Acknowledgements

The research results presented in this paper belong to a research project supported by the National Hellenic Research Foundation. The financial support of this Foundation is most gratefully acknowledged.

The author wishes also to thank his colleagues Dr. P. Yannopoulos and Mr. M. Eftaksias for their kind help during the preparation of the numerical results reported in this paper.

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Prof. Dr. N. I. Ioakimidis
Assoc. Professor of Mathematics
School of Engineering
The University of Patras
P. O. Box 25B
Patras, Greece