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Asymptotic behaviour of the loss probability of the $M/G/1/K$ and $G/M/1/K$ queues

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This paper investigates the asymptotic behaviour of the loss probability of the $M/G/1/K$ and $G/M/1/K$ queues as the buffer size increases. It is shown that the loss probability approaches its limiting value, which depends on the offered load, with an exponential decay in essentially all cases. The value of the decay rate can be easily computed from the main queue parameters. Moreover, the close relation existing between the loss behaviour of the two examined queueing systems is highlighted and a duality concept is introduced. Finally some numerical examples are given to illustrate on the usefulness of the asymptotic approximation.

Keywords: Loss probability; asymptotic behaviour; exponential decay; duality.

1. Introduction

A great interest has recently arisen in the study of finite buffer queues (e.g. in the communication field, in conjunction with the proposal of the ATM technique). In particular, the analysis of the loss probability is becoming a central issue in the applications of queueing theory.

On the other hand, the behaviour of even simple loss systems has always turned out to be much harder to determine than the solution of the corresponding infinite buffer systems. In particular, no simple closed form solutions are known to the author, except for the $M/M/1/K$ queue; quite cumbersome closed formulae have been derived in Neuts [5] for the $M/PH/1/K$ and $PH/M/1/K$ queues, but they lend themselves to little more than numerical computation, especially for large dimensions of the state space of the PH distribution. Even the simple $M/G/1/K$ and $G/M/1/K$ queues have not yielded other than a numerical (albeit quite straightforward) solution, for example by using the *Imbedded Markov Chain* (IMC) approach.

Although in these cases there is no conceptual difficulty in solving for the loss probability, yet some even approximate characterization of the behaviour of such a probability as a function of the buffer size would be desirable, both in order to gain more insight into the system performance and because of possible numerical instability arising in the solution of large linear equation systems, required by large buffer sizes.

This paper addresses the problem of finding the asymptotic behaviour of the loss probability of the $M/G/1/K$ and $G/M/1/K$ systems, as the buffer size tends to infinity.

The main results obtained are that: (i) the loss probability for the two examined systems exhibits an exponential decay, i.e. it tends to its limiting value at an exponential rate, except for a single particular case, in which the decay is linear, (ii) the decay rate can be easily computed from the main queue parameters.

The numerical examples reported later show that, in all interesting cases, taking only the leading term of the asymptotic expansion of the loss probability results in negligible inaccuracies. This assures that the approximate formulae can be powerful tools for the analysis of the loss properties of a finite buffer queue of the types considered in this paper. Moreover, the asymptotic formulae can be straightforwardly inverted, thus allowing a simple and effective buffer dimensioning to be done.

Finally, there exists an intimate relationship between the whole *Queue Length Probability Distribution* (QLPD) the $M/G/1/K$ queue and the QLPD of its dual system, that is a suitably defined as the $G/M/1/K$ queue, in which the roles of the arrival and service processes of the original queue are interchanged. This duality property is exploited in the proof of the asymptotic behaviour of the loss probability.

As for the organization of the paper, in section 2 the main notations and general definitions used in the paper are introduced. Moreover, a duality definition and the relevant property are given. Section 3 deals with the characterization of the asymptotic behaviour of the loss probability. Finally, section 4 is devoted to some numerical examples.

2. General definitions and duality property

Let us now introduce some notation. Only steady-state probability distributions will be considered: for the two examined queues (and indeed for any finite buffer queue) such probability distributions always exist, whatever the value of the offered load is. The solution of both queues will be obtained via IMCs, choosing respectively the departure epochs and the arrival epochs for the $M/G/1/K$ system and the $G/M/1/K$ system.

The Laplace–Stieltjes transform of the cumulative distribution function of the interarrival time and the mean arrival rate will be denoted by $F^*(s)$ and λ, respectively. As for the service time, the corresponding quantities will be denoted by $H^*(s)$ and μ.

Further, we define the following items:

K buffer size; obviously, up to $K+1$ customers are allowed into the queue;

A_o mean offered traffic, equal to λ/μ;

$\psi(z)$ *Probability Generating Function* (PGF) of the random variable representing the number of arrivals within a service time in the $M/G/1$ queue;

R least modulus root of the equation $z=\psi(z)$, apart from the trivial root $z=1$;

$\phi(z)$ PGF of the random variable representing the number of service completions within an interarrival time in the $G/M/1$ queue;

σ least modulus root of the equation $z=\phi(z)$, apart from the trivial root $z=1$;

$\pi_i(K)$ ith element of the QLPD at the imbedded time points for the $M/G/1/K$ queue, $i=0, 1,\dots, K$;

$\pi_i(\infty)$ same as above for the infinite buffer $M/G/1$ queue, $i \geqslant 0$;

$\tilde{\pi}_i(K)$ ith element of the QLPD at imbedded time points for the $G/M/1/K$ queue, $i=0, 1,\dots, K+1$;

$\tilde{\pi}_i(\infty)$ same as above for the infinite buffer $G/M/1$ queue, $i \geqslant 0$;

c_i ratio of $\pi_i(K)$ to $\pi_0(K)$, for $i \geqslant 0$;

$C(z)$ generating function of the sequence $\{c_i\}$;

d_i sum of the first $i+1$ elements of the sequence $\{c_j\}: d_i = \Sigma_{j=0}^{i} c_j$, $i \geqslant 0$;

$D(z)$ generating function of the sequence $\{d_i\}$;

$\Pi(K)$ loss probability of the $M/G/1/K$ queue;

$\tilde{\Pi}(K)$ loss probability of the $G/M/1/K$ queue.

According to the definitions above, it can be immediately deduced that $\psi(z) = H^*(\lambda - \lambda z)$ and $\phi(z) = F^*(\mu - \mu z)$. Moreover, in Baiocchi [1] it is shown that:

(i) the equation $z=\psi(z)$ has no roots with modulus less than or equal to $\max\{1, R\}$, apart from 1 and R themselves; a perfectly analogous result holds for the equation $z=\phi(z)$, replacing R with σ;

(ii) if ρ_ψ denotes the radius of convergence of $\psi(z)$, then $A_o<1 \Rightarrow R \in (1, \rho_\psi)$, $A_o = 1 \Rightarrow R=1$ and $A_o>1 \Rightarrow R \in (0, 1)$;

(iii) if ρ_ϕ denotes the radius of convergence of $\phi(z)$, then $A_o<1 \Rightarrow \sigma \in (0, 1)$, $A_o=1 \Rightarrow \sigma=1$ and $A_o>1 \Rightarrow \sigma \in (1, \rho_\phi)$.

Finally, the duality property of the $M/G/1/K$ and $G/M/1/K$ queues is introduced.

If the states of the IMC of the $M/G/1/K$ queue are renumbered in reversed order, so that the state 0 becomes the Kth one, it can be recognized that the one-step transition probability matrix of this new Markov chain can be obtained from the original matrix by simply reverting all the columns and rows. The matrix we get has just the same structure and the same entries as the one-step transition probability matrix of the Markov chain imbedded at the arrival time points in a $G/M/1/K-1$ system, in which the role of the arrival and service processes have been interchanged with respect to the original $M/G/1/K$ queue. The $G/M/1/K-1$ queue so obtained will be referred to as dual of the $M/G/1/K$ queue.

Analogously, the dual queueing system of a $G/M/1/K$ queue is defined as a $M/G/1/K+1$ queue, in which the role of the arrival and service processes have been interchanged and such that the ith state of the Markov chain imbedded at the departure epochs in the dual queue corresponds to the $(K+1-i)$th state of the IMC of the original queue, $i=0, 1, \dots, K+1$.

In the following, a subscript d will distinguish the variables referring to the dual system of a given one. Then, the definition of duality entails that:

$$\begin{cases} \pi_i(K) = \tilde{\pi}_{d,K-i}(K-1) & i=0,\dots,K, \\ \tilde{\pi}_i(K) = \pi_{d,K+1-i}(K+1) & i=0,\dots,K+1. \end{cases} \tag{1}$$

Such properties will be exploited in the next section.

3. Asymptotic behaviour of the loss probability

In what follows a property of the power series will be used: let z_0 be a pole of multiplicity $m+1$ of a function $f(z)$, that has no singularities with modulus equal to or less than z_0 and let a_n, $n \geqslant 0$, be the sequence of coefficients of the McLaurin expansion of $f(z)$. Then in Baiocchi [1] it is shown that

$$\lim_{n\to\infty} a_n \frac{z_0^n}{\binom{n+m}{n}} = \lim_{z\to z_0} \left(1-\frac{z}{z_0}\right)^{m+1} f(z). \tag{2}$$

The following two theorems summarize the fundamental results of this paper.

THEOREM 1

Let us consider the $M/G/1/K$ queue, as the buffer size K increases: then the loss probability approaches its limiting value $\Pi(\infty) = \max\{0, 1-1/A_o\}$ according to the following expressions:

$$\Pi(K) - \Pi(\infty) = \begin{cases} \dfrac{(1-A_o)^2}{\psi'(R)-1} R^{-K} + o(R^{-K}), & A_o < 1, \\ \dfrac{1-\psi'(R)}{A_o^2} R^K + o(R^K), & A_o > 1, \\ \dfrac{\psi''(1)}{2} \dfrac{1}{K+2} + o\left(\dfrac{1}{K}\right), & A_o = 1, \end{cases}$$

as $K \to \infty$.

Proof

The loss probability can be expressed in terms of the probability of finding the queue empty at a generic time point, *Pr*{*empty*}, by means of a simple flow conservation argument, equating the mean flow into the system, i.e. $\lambda[1-\Pi(K)]$, and the mean flow out, i.e. $\mu[1-Pr\{empty\}]$. Since, according to Gross and Harris [2], *Pr*{*empty*} is given by $\pi_0(K)/(A_0+\pi_0(K))$, it follows that:

$$\Pi(K)=1-\frac{1}{\pi_0(K)+A_o}. \tag{3}$$

From this and the identity $\max\{0, 1-A_o\}=1-A_o+A_o\Pi(\infty)$, it can be deduced that, for any value of A_o,

$$\pi_0(K)-\max\{0, 1-A_o\}=\frac{\Pi(K)-\Pi(\infty)}{[1-\Pi(K)][1-\Pi(\infty)]}$$

and consequently:

$$\lim_{K\to\infty}\frac{\Pi(K)-\Pi(\infty)}{\pi_0(K)-\max\{0, 1-A_o\}}=[1-\Pi(\infty)]^2. \tag{4}$$

It now remains to characterize the asymptotic behaviour of $\pi_0(K)-\max\{0, 1-A_0\}$, as $K\to\infty$. To this end, we recall a well known result, stated for example in Gross and Harris [2], i.e. that the ratios $c_i=\pi_i(K)/\pi_0(K)$, $i=0, 1,\dots, K$, do not depend upon K, for an $M/G/1/K$ queue. The coefficients c_i can be generated according to the following set of linear equations

$$\alpha_i c_0+\sum_{j=1}^{i+1}\alpha_{i+1-j}c_j=c_i, \quad i\geqslant 0, \tag{5}$$

where α_i is the probability of i arrivals within a service time, $i\geqslant 0$, and $c_0=1$. From eq. (5), it can be easily derived that $C(z)=(1-z)\psi(z)/(\psi(z)-z)$. The radius of convergence of $C(z)$ is just equal to R. Moreover, by the definition of the sequence $\{d_i\}$, it follows that $D(z)=\psi(z)/(\psi(z)-z)$, whose radius of convergence equals $\min\{1, R\}$. Finally, from the normalization condition of the QLPD, it follows that $\pi_0(K)d_K=1$.

It is now convenient to consider three different cases, according to the value of A_o.

(i) $A_o<1$. Since in this case $R>1$ and therefore $C(z)$ is analytic for $z=1$, $\lim_{K\to\infty}d_K=C(1)=1/(1-A_o)$. Then, from $\Pi(\infty)=0$, $\pi_0(K)=1/d_K$ and eq. (4) it follows that

$$\lim_{K\to\infty}\frac{\Pi(K)}{\frac{1}{1-A_o}-d_K}=(1-A_o)^2. \tag{6}$$

Recalling the definition of R, it can be readily verified that the generating function $\Delta(z)$ of the sequence $\delta_K = 1/(1-A_o) - d_K$ has a simple pole in $z = R$ and no singularities with modulus equal to or less than R. Then, applying the result of eq. (2) with $z_0 = R$, $f(z) = \Delta(z)$ and $m = 1$, it can be obtained that $\lim_{K\to\infty} \delta_K R^K = 1/(\psi'(R) - 1)$. Recalling the definition of δ_K and taking into account eq. (6), it finally follows that

$$\lim_{K\to\infty} \Pi(K) R^K = \frac{(1-A_o)^2}{\psi'(R)-1}. \tag{7}$$

(ii) $A_o > 1$. In this case $R < 1$ and it can be easily seen that $D(z)$ has a simple pole in $z = R$ and no singularities with modulus equal to or less than R. Then, applying eq. (2) with $z_0 = R$, $f(z) = D(z)$ and $m = 1$, we obtain that $\lim_{K\to\infty} d_K R^K = 1/(1 - \psi'(r))$. Therefore, the equality $\pi_0(K) = 1/d_K$ and eq. (4) imply that

$$\lim_{K\to\infty} [\Pi(K) - \Pi(\infty)] R^{-K} = \frac{1-\psi'(R)}{A_o^2}. \tag{8}$$

(iii) $A_o = 1$. In this case $R = 1$ and $D(z)$ has a double pole in $z = 1$ and no singularities with modulus equal to or less than 1. Then, applying eq. (2) with $z_0 = 1$, $f(z) = D(z)$ and $m = 2$, we obtain that $\lim_{K\to\infty} d_K/(K+1) = 2/\psi''(1)$ and therefore the equality $\pi_0(K) = 1/d_K$ and eq. (5) yield:

$$\lim_{K\to\infty} (K+1)\Pi(K) = \frac{\psi''(1)}{2}. \tag{9}$$

We have thus completed the proof of theorem 1. □

It is to be noted that, in the proof of the case (iii) above, eq. (9) would hold as well, even if $K+1$ were replaced by $K+x$, for any real x. The choice made in the formulation of theorem 1 has the only purpose to be consistent with the expression of the loss probability of the $M/M/1/K$ queue, which reduces just to $1/(K+2)$ for $A_o = 1$. The same observation applies to the case of the $G/M/1/K$ queue.

THEOREM 2

Let us consider the $G/M/1/K$ queue, as the buffer size K increases: then the loss probability approaches its limiting value $\tilde{\Pi}(\infty) = \max\{0, 1 - 1/A_o\}$ according to the following expressions:

$$\tilde{\Pi}(K) - \tilde{\Pi}(\infty) = \begin{cases} [1-\phi'(\sigma)]\sigma^{K+1} + o(\sigma^{K+1}), & A_o < 1, \\ \dfrac{(1-A_o^{-1})^2}{\phi'(\sigma)-1}\sigma^{-(K+1)} + o(\sigma^{-(K+1)}), & A_o > 1, \\ \dfrac{\phi''(1)}{2}\dfrac{1}{K+2} + o\left(\dfrac{1}{K}\right), & A_o = 1, \end{cases}$$

as $K \to \infty$.

Proof

Since the imbedded time points are the arrival epochs, the loss probability, i.e. the probability that a customer finds no room in the system upon his arrival, is simply given by $\tilde{\pi}_{K+1}(K)$.

Let us consider the $M/G/1/K+1$ queue which is the dual of the given $G/M/1/K$. Since $\tilde{\Pi}(K) = \tilde{\pi}_{K+1}(K) = \pi_{d0}(K+1)$ and $A_{do} = 1/A_o$, it follows that $\tilde{\Pi}(K) - \tilde{\Pi}(\infty) = \pi_{d0}(K+1) - \max\{0, 1 - A_{do}\}$ and therefore, using eq. (4), it can be found that

$$\lim_{K\to\infty} \frac{\tilde{\Pi}(K) - \tilde{\Pi}(\infty)}{\Pi_d(K+1) - \Pi_d(\infty)} = \frac{1}{A_o^2\left[1 - \tilde{\Pi}(\infty)\right]^2}. \tag{10}$$

Moreover, on account of the duality, $\psi_d(z)$ reduces to $\phi(z)$ and therefore $R_d = \sigma$. Then, for $A_o < 1$, since $\tilde{\Pi}(\infty) = 0$, from eqs. (8) and (10) it follows that

$$\lim_{K\to\infty} \tilde{\Pi}(K)\sigma^{-(K+1)} = 1 - \phi'(\sigma). \tag{11}$$

For $A_o > 1$, using the eqs. (7) and (10), it can be deduced that

$$\lim_{K\to\infty} \left[\tilde{\Pi}(K) - \tilde{\Pi}(\infty)\right]\sigma^{K+1} = \frac{\left(1 - A_o^{-1}\right)^2}{\phi'(\sigma) - 1}. \tag{12}$$

Finally, for $A_o = 1$, taking into account the observation at the end of theorem 1, eqs. (9) and (10) yield the equality:

$$\lim_{K\to\infty} (K+2)\tilde{\Pi}(K) = \frac{\phi''(1)}{2}. \tag{13}$$

This completes the proof of theorem 2. □

The above theorems completely characterize the asymptotic behaviour of the loss probability, so that it can be stated in general that the asymptotic expression of the loss probability is the sum of its limiting value and of an "error" term that dies out at an exponential rate. This is true unless $A_o = 1$; in this particular case, the decay behaviour of the loss probability is linear.

It is to be stressed that the results obtained in theorem 1 depend solely on: (i) the validity of eq. (3); (ii) the particular structure of the one-step transition probability matrix of the Markov chain imbedded at the departure epochs of the $M/G/1/K$ queue. Moreover, the results of theorem 2 depend only on theorem 1 and on the possibility of establishing a duality principle such that eq. (1) holds. As a consequence, extension to other queueing models of the previous results can be envisaged.

For example, the above reasoning could be applied to the $Geo/G/1/K$ queue, defined as in Louvion, Boyer and Gravey [4], that is, the discrete-time version of the $M/G/1/K$ queue. In fact, the whole proof of theorem 1 could be repeated, as points (i) and (ii) above are satisfied for the $Geo/G/1/K$ queue. Moreover, if the time axis is divided into fixed size units (slots), p is the probability of an arrival in a slot and $G(z)$ is the PGF of the number of slots required for servicing a customer, then $\psi(z) = G(1-p+pz)$ and $A_o = pG'(1)$.

Moreover, since a duality principle can be introduced also between the $Geo/G/1/K$ and the $G/Geo/1/K$ queues, so that eq. (1) still holds, theorem 2 also could be extended to the discrete-time case.

As a final remark, it can be easily shown that, for $A_o < 1$, the tail of the infinite buffer QLPD tends to 0 according to an exponential decay, at the same rate as the loss probability for both $M/G/1/K$ and the $G/M/1/K$ queues.

As for the $G/M/1/K$ queue, such a result is a trivial consequence of eq. (11) and of the fact that $\tilde{\pi}_i(\infty) = (1-\sigma)\sigma^i$, $i \geqslant 0$ (see Kleinrock [3]). Instead, for the $M/G/1/K$ queue, it can be deduced form eqs. (4) and (7), observing that $\pi_i(K)/\pi_0(K) = c_i = \pi_i(\infty)/\pi_0(\infty)$, $i = 0, 1, \ldots, K$, and that therefore the equality $\pi_0(\infty) = 1 - A_o$ and the normalization condition of the QLPD of the $M/G/1/K$ queue imply that $\pi_0(K) = (1-A_o)/\Sigma_{i=0}^{K}\pi_i(\infty)$.

4. Numerical examples and results

The aim of this section is to assess the practical impact of the theory discussed in the previous section. Through the evaluation of some numerical example, it is shown that the loss probability can be adequately approximated by the simple exponential or linear asymptotic expansion given in theorems 1 and 2. The errors implied by such an asymptotic approximation are negligible, at least in the range of values of the loss probability that may be of practical interest.

In the following, the subscript "*asy*" will denote the leading term of the asymptotic expansion of the loss probability.

M/M/1/K queue. For this case, all of the theorems stated above hold.

The explicit expression of the loss probability can be straightforwardly computed for this case. Moreover, in this case $\psi(z) = [1 + A_o(1-z)]^{-1}$ and $R = 1/A_o$, while $\phi(z) = [1 + (1-z)/A_o]^{-1}$ and $\sigma = A_o$. The asymptotic approximation can be derived from both the theorems on the $M/G/1/K$ and $G/M/1/K$ queues and also by inspection from the expression of $\Pi(K)$, yielding:

$$\Pi_{asy}(K) - \Pi(\infty) = \begin{cases} (1-A_o)A_o^{K+1}, & A_o < 1, \\ \dfrac{1}{K+2}, & A_o = 1, \\ \left(1 - \dfrac{1}{A_o}\right)\left(\dfrac{1}{A_o}\right)^{K+2}, & A_o > 1. \end{cases} \tag{14}$$

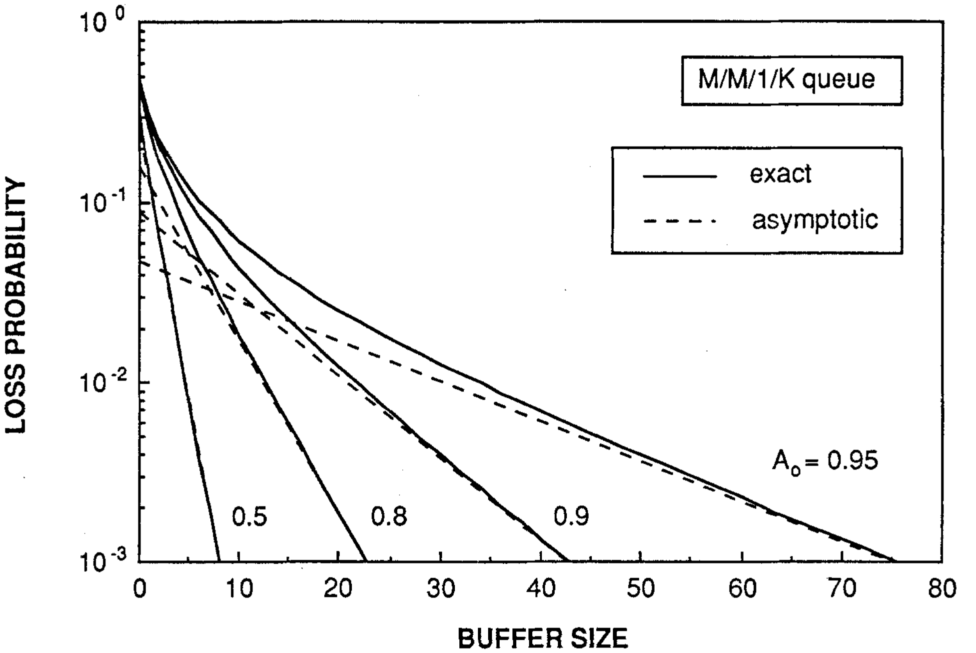

Fig. 1. $M/M/1/K$ queue: loss probability vs. the buffer size for various values of A_o; comparison between exact and asymptotic results.

Figure 1 shows the loss probability vs. the buffer size for various values of the mean offered load, both according to the exact solution and to the asymptotic approximation. The errors implied by the use of the simple approximate formulae of eq. (14) are null for the particular case $A_o = 1$, while for all the other cases they are negligible, except for low buffer sizes and values of A_o very close to 1.

However, it is to be noted that in the range of acceptable values of the loss probability for a practical system, the asymptotic approximation is in excellent agreement with the exact curves.

M/D/1/K queue. In this case $\psi(z) = e^{A_o(z-1)}$ and, applying the results of theorem 1, we obtain:

$$\Pi_{asy}(K) - \Pi(\infty) = \begin{cases} \dfrac{(1-A_o)^2}{A_o R - 1} R^{-K}, & A_o < 1, \\ \dfrac{1}{2(K+2)}, & A_o = 1, \\ \dfrac{1 - A_o R}{A_o^2} R^K, & A_o > 1, \end{cases} \tag{15}$$

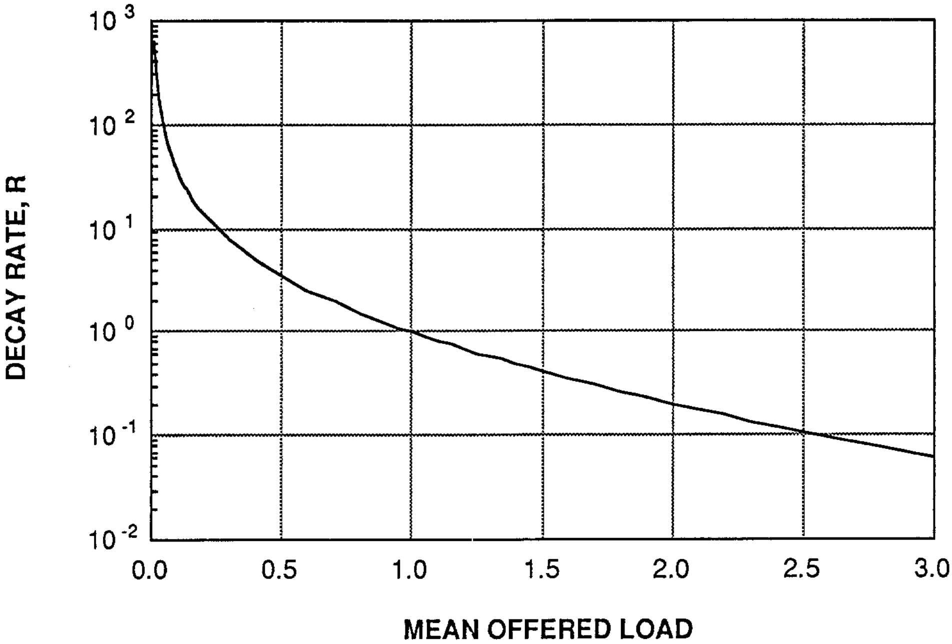

Fig. 2. Decay rate of the loss probability of the $M/D/1/K$ queue, i.e. the least modulus root of the equation $z = \exp[A_o(z-1)]$, apart from the trivial root $z=1$.

wherein R is the least modulus root of the equation $z = e^{A_o(z-1)}$, apart from the trivial root $z=1$. In fig. 2 such a root is plotted against the mean offered load.

In fig. 3 the exact curve of the loss probability is compared with the approximation resulting from eq. (15), whereas in fig. 4 the same comparison is made with respect only to the difference between the value of $\Pi(K)$ and its limiting value $1-1/A_o$. In both cases the asymptotic approximation can be used instead of the exact value with virtually no error for any practical value of the buffer size and of the loss probability.

D/M/1/K queue. In this case $\phi(z) = e^{(z-1)/A_o}$ and, applying the results of theorem 2, we obtain:

$$\tilde{\Pi}_{asy}(K) - \tilde{\Pi}(\infty) = \begin{cases} \dfrac{A_o - \sigma}{A_o}\sigma^{K+1}, & A_o < 1, \\ \dfrac{1}{2(K+2)}, & A_o = 1, \\ \dfrac{(1-A_o)^2}{A_o(\sigma - A_o)}\sigma^{-(K+1)}, & A_o > 1, \end{cases} \tag{16}$$

wherein σ is the least modulus root of the equation $z = e^{(z-1)/A_o}$, apart from the trivial root $z=1$. The behaviour of this root as a function of the mean

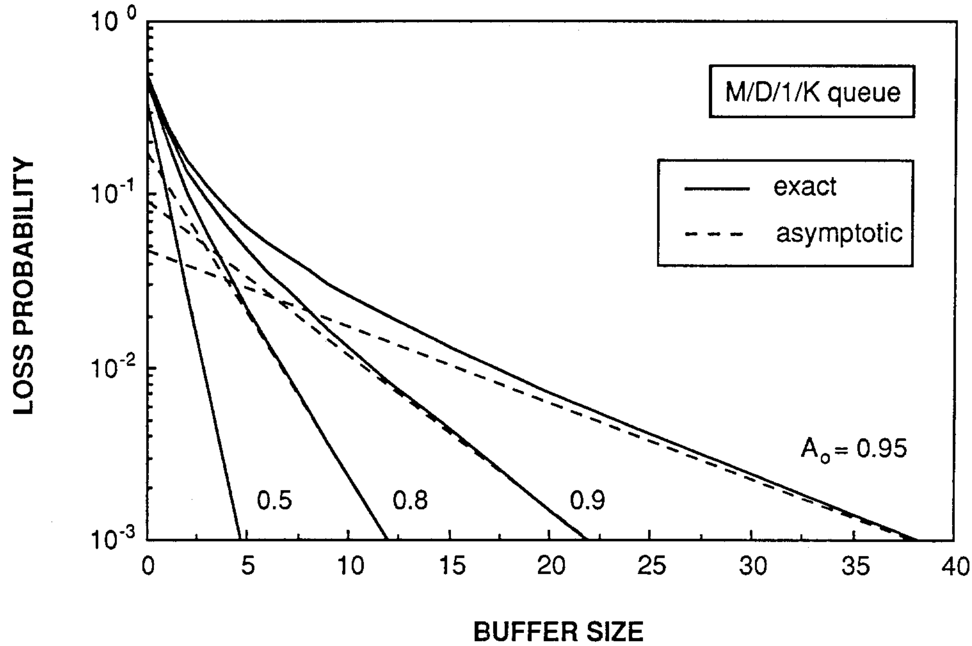

Fig. 3. $M/D/1/K$ queue: loss probability vs. the buffer size for various values of A_o; comparison between exact and asymptotic results.

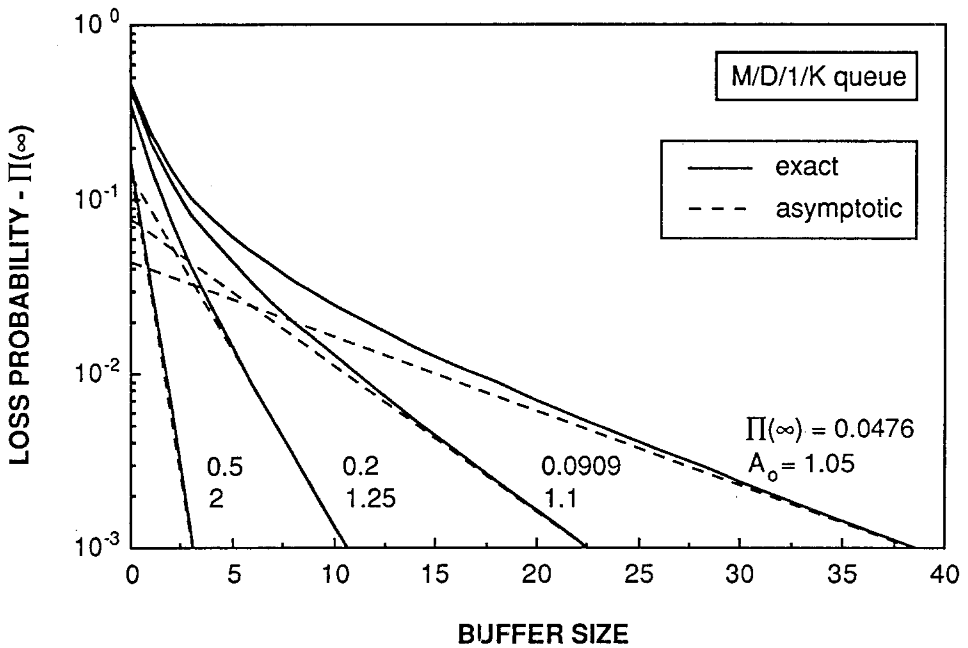

Fig. 4. $M/D/1/K$ queue: loss probability minus $\Pi(\infty)$ vs. the buffer size for various values of A_o; comparison between exact and asymptotic results.

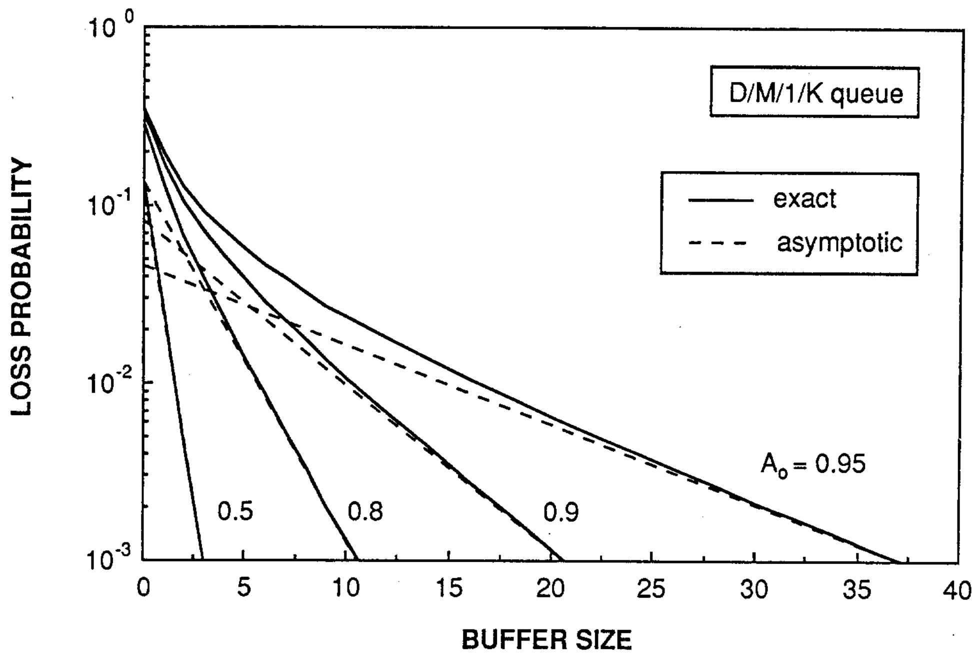

Fig. 5. $D/M/1/K$ queue: loss probability vs. the buffer size for various values of A_o; comparison between exact and asymptotic results.

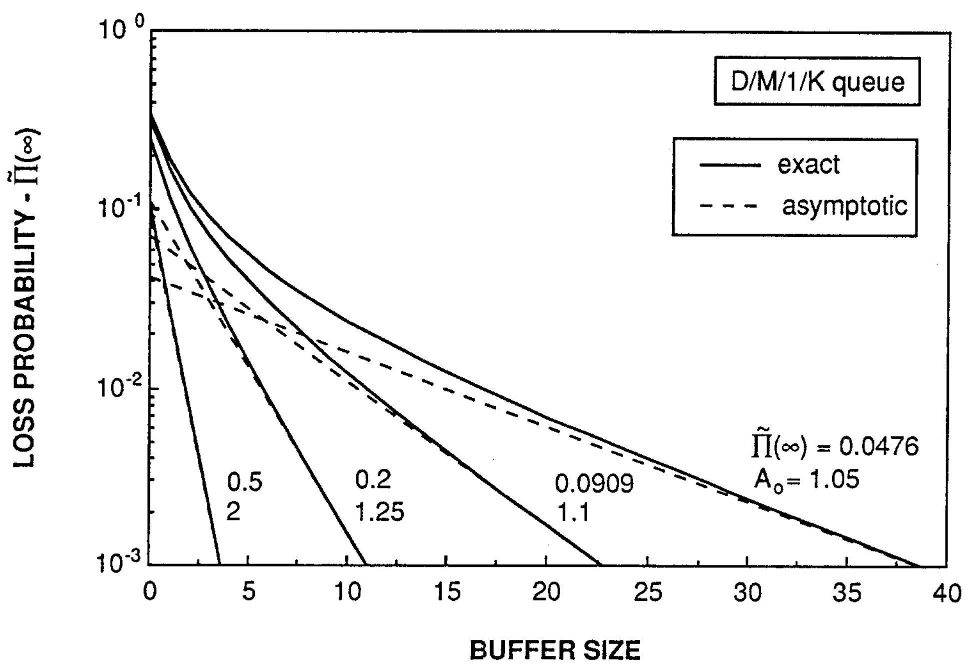

Fig. 6. $D/M/1/K$ queue: loss probability minus $\tilde{\Pi}(\infty)$ vs. the buffer size for various values of A_o; comparison between exact and asymptotic results.

offered load can be easily deduced from fig. 2, just replacing the values on the x-axis with their reciprocals.

Figures 5 and 6 show the comparison between the exact curve of the loss probability and the approximation resulting from eq. (16). As for the $M/G/1/K$ queue, in fig. 6 the comparison is carried out with respect only to the difference between the value of $\tilde{\Pi}(K)$ and its limiting value $1 - 1/A_o$.

Entirely similar comments apply to this case as for the previous one.

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