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Analysis and design of rate-based congestion control of high speed networks, I: stochastic fluid models, access regulation

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The paper gives models and analytic techniques for addressing critical issues of the Broadband Integrated Services Digital Network which will use the Asynchronous Transfer Mode. The traffic is expected to be highly bursty and variable at the source and consequently a key issue is admission control. We study a 4-parameter device called a regulator which acts as a policing device as well as a traffic shaper. The device is a generalized leaky bucket with a data buffer, a token buffer supplied by a constant-rate token stream, and a peak rate controller; the outputs of the device are streams of priority and marked cells. The composite system comprising of the source and the regulator is represented in a stochastic fluid model since fluid flow has been found to have properties well matched to the ATM environment, and the Markov Modulated Fluid Source allows bursty characteristics to be accurately modelled. A complete procedure based on spectral expansions for calculating the system's stationary state distribution is given. It is shown that with proper design the regulator effectively controls a three-way trade-off between throughput, delay and burstiness. Numerical results reveal that performance is sensitive to source characteristics such as the squared coefficient of variation of burst and silent periods. The second part of the paper characterizes the output of the regulator. The distributions of the time periods spent in the various states by the output process are calculated exactly. From this an approximate Markovian characterization is obtained. The output streams of priority and marked cells are coupled to capture their correlations. For the simple case of two-state on-off sources, the approximate Markovian characterization of the regulator's output rate processes is explicitly given and it is distinguished by the property that all moments are identical to those of the actual processes. With this characterization an original goal of analyzing a composite system of access regulation and statistical multiplexing is separated, decomposed and thereby made tractable.

Keywords: Markov Modulated Fluid Sources, bursty traffic, statistical multiplexing, loss priorities, Broadband ISDN, ATM.

1. Introduction

This paper is on models and analytic techniques for addressing several critical issues of the future Broadband Integrated Services Digital Network (B-ISDN) which will use the Asynchronous Transfer Mode (ATM). It is expected that the traffic on these networks will be characterized by high burstiness and high variability in the bit rates; this is clear from recent studies on video (Maglaris et al. [26], Kishino et al. [19]), packetized voice (Petr, DaSilva and Frost [34]) and facsimile (Chamzas and Duttweiler [5]). These characteristics are the driving force for admission control, access regulation and statistical multiplexing, all key elements of ATM networks and services (see the excellent papers by Coudreuse, Pays and Trouvat [7], and Eckberg, Luan and Lucantoni [13]); they are also at the core of this paper and its sequel. Admission control is a congestion avoidance procedure for deciding whether a call, with certain announced bandwidth, burstiness and other characteristics is to be carried or not, given certain information on the state of the network, such as the currently available bandwidth and buffers and the number of calls of various types in progress. The high level goal of the analyses of this and related papers is to facilitate admission control through, hopefully, simple tables, charts and guidelines. However, the concept of admission control of (admittedly) bursty sources raises fundamental questions of what constitutes the minimal description of burstiness which are consequential for network performance, which the user can provide and which the network can monitor. Such a universally accepted minimal descriptor does not exist today; in fact, one of the purposes of this paper is to demonstrate the sensitivity of network performance to some of the less visible features of burstiness in user-sources.

Assuming that a description has been agreed upon by the network and the user, and that a call has been accepted, it will remain for the network to enforce the "contract". Correspondingly, the user, who is aware of both the nature of the contract and the existence of the network enforcement, may want to optimize his overall cost and performance by shaping his traffic and trading-off delay, throughput and burstiness. A device which with appropriate parameter values may function both as network police and as traffic shaper is described below. Hence, it makes sense to have a pair of these devices with different parameters in a serial configuration. We call this device an "access regulator"; it has all the features of the usual leaky bucket policing device plus other elements (Turner [38], Sidi et al. [35], Eckberg, Luan and Lucantoni [12], Berger [2,3], Chuah and Cruz [4] and Monteiro, Gerla and Fratta [30]). A Markovian model of a bursty source in conjunction with a model of the access regulator, the analytic techniques for solving the equations describing the stationary state distribution of the composite model, calculating various performance features and trade-offs, and characterizing its output exactly and, also, approximately as another Markovian source are all subjects of this paper.

By the very statistical nature of burstiness, the estimation of the characteristics of a bursty source, such as mean rate, is subject to errors which increase as the estimation periods decrease. On the other hand, when the objective is enforcement and also because non-stationarity cannot be ruled out, estimation

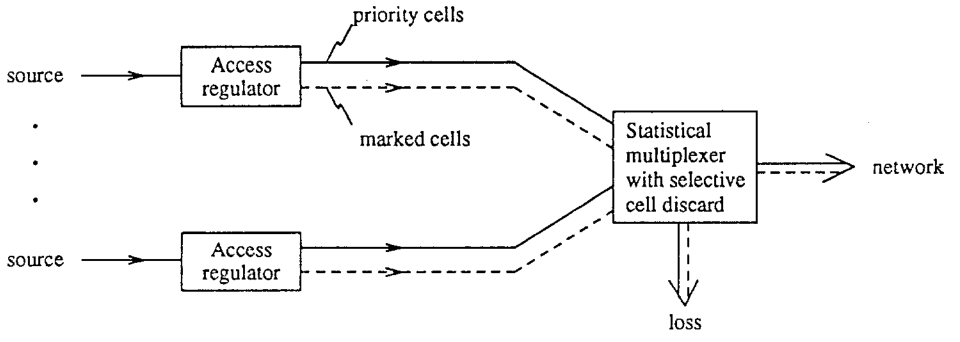

Fig. 1. Integrated system of access regulation and statistical multiplexing.

intervals cannot be long. This dilemma has several repercussions. First, for given contractual characteristics, the policing device has to have slack built into its design to minimize false detections of violations. However, this slack can be exploited by users which leads to inefficiencies. An alternative policy combines the minimum of slack with a "soft" violation tagging process. This is one reason that the concept of "loss priorities" is built into ATM concepts and standards (Coudreuse, Pays and Trouvat [7], Eckberg, Luan and Lucantoni [13], Woodruff et al. [41]). Cells which are judged to be in violation of contracts between network and user are "marked", typically carried and dropped only in the last resort. In the scheme that we analyze (and which has several tractable variants), see fig. 1, the access regulator does not drop cells and has two output cell streams, "priority" and "marked". The selective discarding of marked cells when the buffer content exceeds a pre-determined threshold is done at the statistical multiplexer. Although the entire system in fig. 1 is analyzed as an integrated system, for reasons of space we have deferred the analyses of the statistical multiplexer to this paper's sequel (Elwalid and Mitra [14]).

The access regulator studied in this paper is shown in fig. 2. It is a 4-parameter device $(r, B_T, B_D, \hat{v})$: r is the constant rate at which tokens arrive, B_T is the token buffer size, B_D is the data buffer size and $\hat{v}$ is the parameter which bounds the flow rate of priority cells leaving the regulator; an important parameter is $B \triangleq B_T + B_D$, the total buffer space in the device. The data buffer and the peak rate controller are not typical elements of the leaky bucket, although the data buffer is included in the configuration considered by Sidi et al. [35] and Berger [3].

The parameter r is naturally associated with the average source bit rate. The analysis here shows that in the rather general Markovian framework of this paper, the cell loss probability P_L depends on B_D and B_T only through their sum B. This has been known before (Berger [3]) in other contexts. Hence, prior to peak rate control, the throughput of priority cells depends on B_D and B_T only through B. Now r cannot be made too close to the average source cell rate without incurring the penalty of either high P_L or large B. Hence some slack in

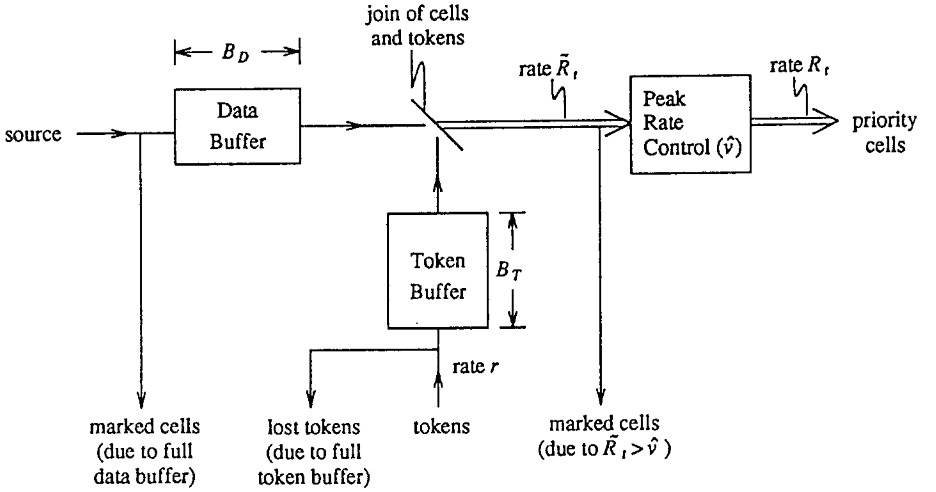

Fig. 2. The access regulator.

the form of $1 - \rho > 0$ must be introduced, where ρ is the ratio of the average source cell rate to r.

The partitioning of B into B_T and B_D determines the trade-off between the delay incurred by the priority cells and their burstiness, i.e. variability. For fixed B, small B_T gives large delays and smooth outgoing traffic. Our numerical studies show that this trade-off can be effectively controlled, but only if B_T is a small number of the order of the average amount of data generated by the source in a burst. We have observed that performance is insensitive to the partitioning for larger values of B_T. In summary, r, B_T and B_D control the three-way trade-off between throughput, delay and burstiness of the outgoing priority cell stream.

When the regulator is used as a policing device, the reasons for having the data buffer are not compelling. When the data buffer is non-existent or small, the issues are simpler: larger B_T slows down the reaction time in detecting violations, while decreasing the probability of false alarm wherein marking rates are higher than they should be. The throughput of unmarked cells leaving the data buffer is limited by r, and B_T limits the duration of periods during which the rate exceeds r. When the regulator is used as a traffic shaper, the role of the data buffer can be quite important. The user may benefit by paying the penalty of some delay to meet network specifications and thus obtain better guaranteed service from the network.

In this paper we have chosen to concentrate on the role of access regulation in ATM-BISDN networks. However, there is also an important role envisaged for access regulation in conjunction with feedback-based congestion control. This approach is of particular value in data networking where the delay requirements are not as stringent as in real-time services. An exposition of an approach which integrates access regulation with feedback-based congestion control is given by Mitra et al. [29].

The analyses of this paper and its sequel are based on stochastic fluid models (Anick, Mitra and Sondhi [1], Gaver and Lehoczky [16], Kosten [21,22], Weiss [39], Mitra [29], Stern and Elwalid [36], Coffman, Igelnik and Kogan [6]). The bursty sources are modelled as Markov Modulated Fluid Sources in which the state of a controlling continuous time Markov chain determines the rate of fluid generation. The particular class of two-state "on–off" sources have proven extremely useful and insightful. There are several fundamental reasons why fluid models are appropriate in the ATM environment: the small and uniform cell size (53 bytes) is, of course, important; the constant interarrival time between cells for several contiguous cells, at the time of generation, fits easily in the fluid framework and is difficult to handle in the queueing context; the numerical complexity of solving fluid models with finite buffers does not depend on buffer size, while with queueing models the complexity increases. The fluid approximation presumes a separation of time scales, i.e. the interarrival time of cells is small with respect to the time between changes in the rate, which is a feature of the high speed ATM environment. Hence, not surprisingly, several recent papers on packetized communications and ATM networks are based on stochastic fluid models (Tucker [37], Dittman and Jacobsen [9], Maglaris et al. [26], Li [25], Monteiro, Gerla and Fratta [30], Kobayashi [20], Norros et al. [33]). Several comparative evaluations of techniques for modelling and analyzing statistical multiplexing now exist (Daigle and Langford [8], Nagarajan, Kurose and Towsley [31], Kroner, Theimer and Briem [23]); these studies have found the approach in Anick, Mitra and Sondhi [1], which is based on stochastic fluid models, to be effective in its accuracy and capacity to solve large systems.

SUMMARY OF RESULTS

(i) The stationary state distribution of the system comprised of a Markov Modulated Fluid Source and the access regulator is obtained in section 2 from the spectral expansion of the solution of a system of ordinary differential equations and a set of natural boundary equations. Monotonicity and convexity properties are derived for on–off sources.

(ii) Section 3 presents numerical results on the effects of B on P_L as a function of ρ, on the dependence of the trade-off between delay and burstiness of the output rate $\tilde{R}_t$ (see fig. 2) on the partitioning of B into B_T and B_D, and on the effect of the squared coefficients of variation of the burst lengths and silent periods of on–off sources. Also, performance results from simulations of systems with ATM characteristics are compared with results from approximating fluid models and the agreement is found to be good.

(iii) Section 4 gives results on the distribution of the token and data buffers just after transition epochs of the source. These results are based on conditional PASTA and serve as tools for use later in the paper.

(iv) Section 5 gives results on the mean busy (i.e. non-empty) and blocking (i.e. full) periods of the buffer in a generic buffering system with a general Markov Modulated Fluid Source. These results are used later in the paper and also in the sequel.

(v) The rest of the paper (sections 6, 7 and 8) is devoted to the characterization of the output of the regulator. Section 6 gives the following mean values pertaining to the regulator: mean empty and busy periods of the token buffer, and the mean blocking periods of the token and data buffers. These results give the mean period during which the output rate R_t equals the token rate r.

(vi) Section 7 gives the exact distributions of the sojourn periods in states in which the output rates $\tilde{R}_t$ and R_t are greater than the token rate. (The distributions of periods in states where the output rates are less than the token rate are exponential.)

(vii) Section 8 gives an approximate characterization of the output streams, priority and marked, of the regulator as coupled Markov Modulated Fluid Sources, i.e. with a common controlling Markov chain and a pair of cell generation rates for each state. This is done for the case of $B_D = 0$ and on–off sources, although the results given earlier in the paper allow the procedure to be readily extended to general Markovian fluid sources. The Markovian characterization consists of approximating the distributions of sojourn times in certain states by exponentials with means which are correct and have been previously calculated. This approximation has the fortuitous property of yielding all moments for its rate processes which are identical to the corresponding moments of the actual process.

With this characterization we separate the problem of analyzing the composite of access regulation and statistical multiplexing, see fig. 1. While decompositions such as this have been proposed before, notably by 2-moment renewal-theoretic characterization (Kuehn [24], Whitt [40]), the innovation is in the Markovian and fluid features which experience has shown to be appropriate in the ATM-BISDN environment.

2. The access regulator

In this section we specify and then model a 4-parameter access regulator which polices and shapes the traffic from a source. The source, while typically single, may in fact be the multiplexed output of several sources. The burstiness of the source is modelled here (see fig. 3) as a Markov Modulated Fluid Source controlled by an N-state continuous time irreducible Markov chain (CTMC) with generator $\mathbf{M}$ and rates $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_N)$ where λ_i is the constant rate at which fluid is generated by the source when it is in state i. Special attention is given to "on–off sources", for which $N = 2$, state 1 is "off" and state 2 is "on".

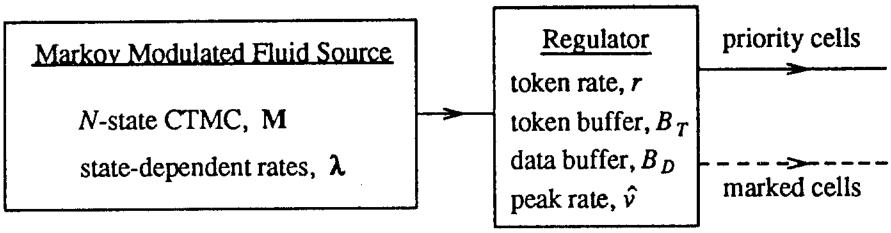

Fig. 3. The source and regulator investigated in section 2.

The specific function of the access regulator is to segregate the arriving cells into two streams, "priority" and "marked". Devices further downstream in the network, such as the statistical multiplexer, carry the marked cells only if residual resources are currently available. In this section we obtain the stationary state distribution of the composite of the source and the regulator.

The regulator is shown in fig. 3. An arriving cell which finds the data buffer full is marked and diverted from the rest of the regulator. Tokens arrive (as fluid) at the constant rate r. Arriving tokens which find the token buffer full are lost. Principles guiding further operations are (i) only paired cells and tokens move downstream, and (ii) the pairing is effected as soon as possible. A consequence of (ii) is that *both buffers cannot simultaneously be non-empty at any time*. The device described above is related to the leaky bucket which has several equivalent representations. Typically the data buffer and the peak rate control are not included; Sidi et al. [35] and Berger [3] include the data buffer. The operations described above are closely related to the "kanban" discipline used in manufacturing and production (Mitra and Mitrani [28]).

We let $\tilde{R}_t$ denote the rate at time t of fluid flow of unmarked cells which have been paired with tokens in the manner described above. There are only $(N+1)$ feasible values of $\tilde{R}_t$, namely, $\lambda_1, \ldots, \lambda_N$, r. This is because if the token buffer is non-empty then $\tilde{R}_t$ corresponds to the source rate, and if the token buffer is empty then $\tilde{R}_t = r$. A final step in the regulation is peak rate control which constrains the flow rate of priority cells leaving the regulator not to exceed $\hat{v}$ ($\hat{v} > r$). The excess over $\hat{v}$ of the rate of fluid flow is marked. Hence, the rate of priority cells leaving the regulator at time t, $R_t \in \{\lambda_1 \wedge \hat{v}, \ldots, \lambda_N \wedge \hat{v}, r\}$ where, in general, $x \wedge y = \min(x, y)$.

In an alternate configuration the peak rate controller is positioned before the pairing of cells and tokens. This does not result in any difference in the set of values of R_t, and if there is no data buffer, i.e. $B_D = 0$, then all differences vanish. However with $B_D > 0$, the present configuration shown in fig. 2 leads to a greater utilization of the data buffer.

A particularly interesting and viable special case of the regulator is when there is no data buffer, i.e. $B_D = 0$. All the important formulas derived in this paper apply to this special case. Note that when $B_D > 0$ there is the possibility of overtaking of marked cells over priority cells which necessitates resequencing at some later point. On the other hand, as we shall show, the presence of the data buffer gives an important additional degree of design flexibility. Moreover, with proper design the overtaking is small.

(1) STOCHASTIC FLUID MODEL OF THE REGULATOR

Let X_t and Y_t respectively denote the contents of the data buffer and the token buffer at time t $(0 \leqslant X_t \leqslant B_D;\ 0 \leqslant Y_t \leqslant B_T)$. As noted above, $X_t Y_t \equiv 0$. Let

$$W_t \triangleq X_t - Y_t + B_T, \tag{2.1}$$

so that $0 \leqslant W_t \leqslant B$. A little thought shows that knowledge of W_t gives X_t and Y_t. It is helpful to think of W_t as the "virtual buffer content". Let S_t denote the state of the source at time t and $\mathscr{S} \triangleq \{1, 2, \dots, N\}$ the state space. Let the state distribution of the source–regulator system in equilibrium be given thus

$$F_i(\xi) \triangleq \Pr[S = i,\ W \leqslant \xi] \quad (i \in \mathscr{S},\ 0 \leqslant \xi \leqslant B), \tag{2.2}$$

where, as in the rest of the paper, we drop the subscript t when specifying stationary distributions. A lexicographic arrangement gives the row vector $\boldsymbol{F}(\xi) = \{F_i(\xi) \mid i \in \mathscr{S}\}$. Let the scalar $F(\xi)$ be defined thus,

$$F(\xi) \triangleq \Pr(W \leqslant \xi) = \langle \boldsymbol{F}(\xi), \boldsymbol{1} \rangle, \tag{2.3}$$

where, in general, the vector product $\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \sum x_i y_i$, and $\boldsymbol{1} = (1, \dots, 1)$.

Following the procedure in, say, Anick, Mitra and Sondhi [1], it is straightforward to obtain the governing differential equations:

$$\frac{\mathrm{d}}{\mathrm{d}\xi} \boldsymbol{F}(\xi) \mathbf{D} = \boldsymbol{F}(\xi) \mathbf{M} \quad (0 < \xi < B) \tag{2.4}$$

where, recall from fig. 3, $\mathbf{M}$ is the generator of the controlling Markov chain of the source and $\mathbf{D} = \operatorname{diag}\{\lambda_1 - r, \dots, \lambda_N - r\}$. "$D$" is a mnemonic for "drift" and the ith element of $\mathbf{D}$, $\lambda_i - r$, is the drift or rate of change of the virtual buffer content W_t when the source state is i.

To complete the specification of the mathematical model, it is necessary to give boundary conditions to (2.4). See fig. 4 in this connection. It is helpful to define

$$\mathscr{S}_D \triangleq \{i \in \mathscr{S} \mid \lambda_i - r < 0\}, \quad \mathscr{S}_U = \{i \in \mathscr{S} \mid \lambda_i - r > 0\}, \tag{2.5}$$

i.e., $\mathscr{S}_D$ and $\mathscr{S}_U$ are the sets of source states giving downward and upward drifts, respectively, to the virtual buffer content. Assume for simplicity that no source rate exactly equals the token rate, i.e. $\lambda_i \neq r$; Mitra [27] has shown how exceptions may be handled. This assumption gives $\mathscr{S} = \mathscr{S}_D \cup \mathscr{S}_U$. Also let the stationary state distribution of the source be $\boldsymbol{w}$, i.e.,

$$\boldsymbol{w}\mathbf{M} = \boldsymbol{0}, \quad \langle \boldsymbol{w}, \boldsymbol{1} \rangle = 1. \tag{2.6}$$

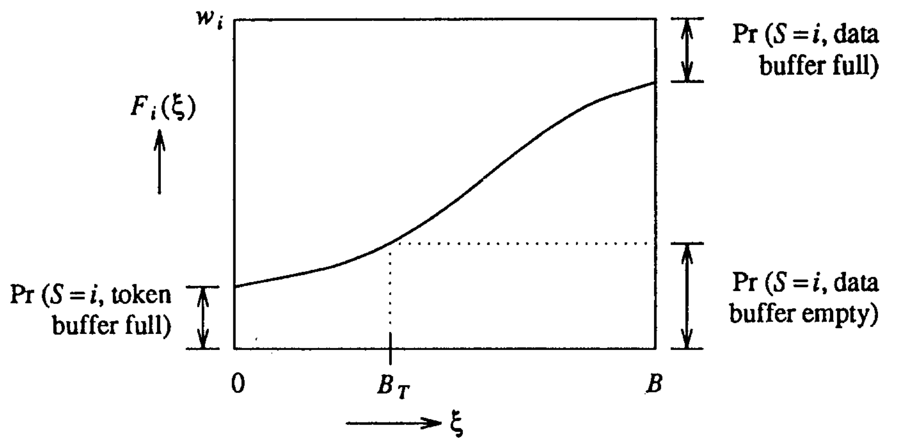

Fig. 4. Sketch of $F_i(\xi) = \Pr(S = i, W \leqslant \xi)$.

From fig. 4 note that

$$F_i(0+) = \Pr[S = i \text{ and token buffer full}], \tag{2.7,i}$$

$$w_i - F_i(B-) = \Pr[S = i \text{ and data buffer full}]. \tag{2.7,ii}$$

Hence, arguing as in Mitra [27], that if the source state $S_t \in \mathscr{S}_U$, then the token buffer cannot be full except at isolated instants of t, we obtain from (2.7,i),

$$F_i(0+) = 0 \quad (i \in \mathscr{S}_U). \tag{2.8,i}$$

Similarly, if the source state $S_t \in \mathscr{S}_D$ then the data buffer cannot be full except at isolated instants of t. Hence, from (2.7,ii),

$$F_i(B-) = w_i \quad (i \in \mathscr{S}_D). \tag{2.8,ii}$$

Since $\mathscr{S}_U \cup \mathscr{S}_D = \mathscr{S}$ we have in (2.8) exactly as many boundary conditions as the number of differential equations in (2.4), namely, N.

Now consider the solution of the system of differential equations in (2.4) in conjunction with the boundary conditions in (2.8). A particular method which has had considerable success in similar systems in the past is the method of spectral expansion (Anick, Mitra and Sondhi [1], Kosten [21,22], Mitra [27], Coffman, Igelnik and Kogan [6], Stern and Elwalid [36]). This method has two components, first, the calculation of eigenvalues and eigenvectors, and, second, the calculation of coefficients of the expansion from the boundary conditions. A great deal is known about the algebraic theory that in structured problems greatly simplify the first set of calculations (see for example Elwalid, Mitra and Stern [15]). We shall have to omit further discussions on these techniques.

The following is the spectral expansion of the system stationary state distribution

$$\boldsymbol{F}(\xi) = \sum_{j=1}^{N} a_j \boldsymbol{\phi}_j \exp(z_j \xi), \tag{2.9}$$

where $\{a_j\}$ is the set of coefficients and (z_j, ϕ_j) is an eigenvalue–eigenvector pair which satisfies the following equation

$$z_j \phi_j \mathbf{D} = \phi_j \mathbf{M} \quad (j = 1, 2, \ldots, N). \tag{2.10}$$

A particular pair, see (2.6), is $(0, w)$. Hence, from (2.9)

$$F(\xi) = a_1 w + \sum_{j=2}^{N} a_j \phi_j \exp(z_j \xi). \tag{2.11}$$

Substituting this form in (2.8) gives a linear system of N equations in the N unknowns $a_1, \ldots, a_N$; this system of equations is completely specified once the eigenvalues and eigenvectors have been computed. Typically, the linear equations have to be solved numerically.

A mathematical device which is quite often useful when the blocking probability is very small is to assume that $B = \infty$ and, by appropriately integrating the tail of the resulting distribution, to bound and estimate probabilities of interest in the real case of finite B. However, in the case of $B = \infty$ it is necessary for the existence of non-degenerate stationary distributions to have $\rho < 1$, where the traffic intensity

$$\rho \triangleq \frac{1}{r} \langle w, \boldsymbol{\lambda} \rangle. \tag{2.12}$$

In this case the boundary conditions are composed of the set of equations in (2.8,i) together with conditions that assert that if Re $(z_j) > 0$ then $a_j = 0$. It has been proved for a wide variety of conditions [27] that the number of such boundary conditions equals N.

We now give the distributions of X and Y, and some performance measures in terms of the expression in (2.11):

$$\Pr[\text{data buffer full}] = 1 - F(B), \tag{2.13,i}$$

$$\Pr[\text{token buffer full}] = F(0), \tag{2.13,ii}$$

$$\Pr[S = i, Y \leqslant y] = w_i - F_i(B_T - y) \quad (0 \leqslant y \leqslant B_T), \tag{2.13,iii}$$

$$\Pr[S = i, X \leqslant x] = F_i(x + B_T) \quad (0 \leqslant x \leqslant B_D). \tag{2.13,iv}$$

There are three interesting equivalent expressions for the stationary mean of $\tilde{R}_t$ which is the throughput of unmarked cells prior to peak rate control; the two given now are conservation equations for cells and tokens, respectively (the third expression is (2.27)). For the first,

$$\begin{aligned} \langle \tilde{R} \rangle &= \text{mean cell rate at source} - \text{mean cell loss rate due to full buffer} \\ &= \sum_{i=1}^{N} \lambda_i w_i - \sum_{i=1}^{N} (\lambda_i - r)\{w_i - F_i(B)\} \\ &= r + \langle F(B), \boldsymbol{\lambda} - r\mathbf{1} \rangle. \end{aligned} \tag{2.14,i}$$

The second expression is obtained analogously:

$$\langle \tilde{R} \rangle = r - \text{mean token loss rate due to full token buffer}$$
$$= r - \langle \boldsymbol{F}(0),\ r\boldsymbol{1} - \boldsymbol{\lambda} \rangle. \tag{2.14,ii}$$

The equivalence of all three expressions follows directly from the following identity (see Mitra [27]),

$$\langle \boldsymbol{F}(\xi),\ \boldsymbol{\lambda} - r\boldsymbol{1} \rangle \equiv \text{constant} \quad \forall \xi \quad (0 \leqslant \xi \leqslant B).$$

Once the throughput has been calculated, it is straightforward to obtain the cell loss probability, $P_L = 1 - \langle \tilde{R} \rangle / \langle \boldsymbol{w}, \boldsymbol{\lambda} \rangle$.

The distribution of delay seen by arriving cells, as given by the fluid model, is

$$\Pr(\text{delay} \leqslant t) = \frac{\langle \boldsymbol{F}(rt + B_T),\ \boldsymbol{\lambda} \rangle}{\langle \tilde{R} \rangle} \quad (t < B_D/r), \tag{2.15,i}$$

$$\Pr(\text{delay} = B_D/r) = \frac{r[1 - F(B)]}{\langle \tilde{R} \rangle}. \tag{2.15,ii}$$

The right hand expression in (2.15,i) is the fraction of the carried flow, i.e. throughput, which arrives at the data buffer when its content does not exceed (rt). Similarly, the right hand expression in (2.15,ii) is the fraction of the carried flow that arrives to a full data buffer.

Note that the procedure that we have given for calculating $F(\xi)$ depends on B_T and B_D only through their sum B (see (2.8,ii)). Hence notice from (2.14) that the throughputs $\langle \tilde{R} \rangle$ and $\langle R \rangle$ depend on B_T and B_D only through B. This has been observed independently by Berger [3] in a different framework.

(2) ON–OFF SOURCES WITH TWO STATES

This simple class of sources (Anick, Mitra and Sondhi [1]) is used throughout this paper to illuminate general results. For these sources the off and on periods are exponentially distributed with mean $1/\alpha$ and $1/\beta$, respectively, and while on the source transmit at rate λ. Hence,

$$\mathbf{M} = \begin{bmatrix} -\alpha & \alpha \\ \beta & -\beta \end{bmatrix}, \quad \boldsymbol{\lambda} = [0, \lambda], \tag{2.16}$$

which gives

$$\mathbf{D} = \operatorname{diag}(-r, \lambda - r), \quad \text{and} \quad \mathbf{w} = \left(\frac{\beta}{\alpha + \beta}, \frac{\alpha}{\alpha + \beta} \right). \tag{2.17}$$

We assume that $r < \lambda$ as otherwise no cells are marked. The two eigenvalues obtained by solving (2.10) are $z_1 = 0$ and z_2 which, in the special case of on–off sources, will be denoted simply by z,

$$z = -\frac{(\alpha + \beta)(1 - \rho)}{(\lambda - r)}, \tag{2.18}$$

where the traffic intensity,

$$\rho = \frac{\lambda}{r}\frac{\alpha}{\alpha+\beta}.$$

Hence, $\rho < 1$ implies $z < 0$. The eigenvector associated with z is

$$\phi = [(\lambda - r)/r, 1]. \tag{2.19}$$

The coefficients (a_1, a_2) in the special expansion (2.11) are

$$a_1 = -\frac{\alpha}{\alpha+\beta}\cdot\frac{1}{\Delta(B)}, \quad a_2 = \frac{1}{\Delta(B)}, \tag{2.20}$$

where, generally,

$$\Delta(\xi) \triangleq 1 - \frac{\alpha}{\beta}\frac{\lambda - r}{r}\exp(z\xi). \tag{2.21}$$

Hence, the complete system stationary state distribution is

$$F_1(\xi) = w_1\frac{\Delta(\xi)}{\Delta(B)}, \tag{2.22,i}$$

$$F_2(\xi) = w_2\frac{\{1-\exp(z\xi)\}}{\Delta(B)}. \tag{2.22,ii}$$

This gives

$$F(\xi) = \Pr[W \leqslant \xi] = \{1 - \rho\exp(z\xi)\}/\Delta(B) \quad (0 \leqslant \xi \leqslant B). \tag{2.23}$$

(3) OUTPUT RATE PROCESSES

Let $\tilde{R}_t$ be the rate of unmarked cells leaving the data buffer at time t, see fig. 2. $\tilde{R}_t$ is modulated by the process Ω_t with state space $= \{1, 2, \dots, N, N+1\}$. Let Ω_t be defined thus,

$$\begin{aligned} \Omega_t &= i \qquad \text{when } (S_t = i, Y_t > 0), \ (i = 1, 2, \dots, N), \\ &= N+1 \qquad \text{when } (Y_t = 0). \end{aligned} \tag{2.24,i}$$

When $\Omega_t = i$, $\tilde{R}_t = \tilde{v}_i$ $(1 \leqslant i \leqslant N+1)$ where,

$$\begin{aligned} \tilde{v}_i &= \lambda_i \quad (i = 1, 2, \dots, N), \\ &= r \quad (i = N+1). \end{aligned} \tag{2.24,ii}$$

For example, in the case of on–off sources, $(\tilde{v}_1, \tilde{v}_2, \tilde{v}_3) = (0, \lambda, r)$.

We let (Ω_t, R_t, v) be similarly identified with the output of the peak rate controller, which is also the output of priority cells of the regulator. The peak rate controller is modelled as a memoryless device:

$$R_t = \tilde{R}_t \wedge \tilde{v}. \tag{2.25}$$

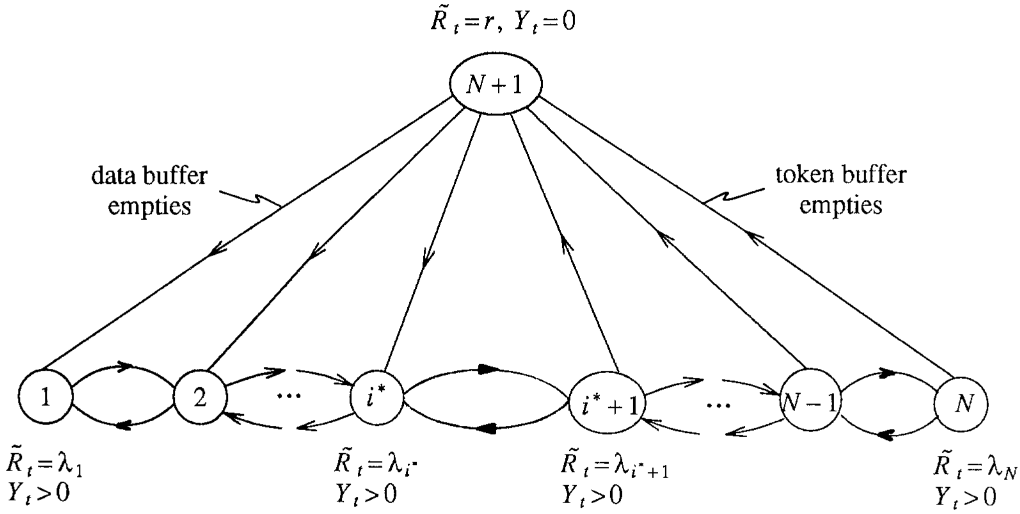

Fig. 5. Transition diagram of Ω_t. The rates $\tilde{R}_t$ are also shown. At source states $1, 2, \ldots, i^*$ data is generated at rates less than the token rate r, while the reverse is true for the remaining source states.

Figure 5 sketches the transitions of the process Ω_t for a typical source governed by a birth-and-death process. As indicated in the figure, transitions from $\tilde{R}_t = \lambda_i$ to $\tilde{R}_t = r$ cannot occur for $i \in \mathscr{S}_D$ and, similarly, transitions from $\tilde{R}_t = r$ to $\tilde{R}_t = \lambda_i$ cannot occur for $i \in \mathscr{S}_U$.

Note from (2.24) and (2.13), the stationary probabilities of Ω_t:

$$\Pr[\Omega = i] = \begin{cases} \Pr[S = i,\, Y > 0] = F_i(B_T) & (1 \leqslant i \leqslant N), \\ \Pr[Y = 0] = 1 - F(B_T) & (i = N+1). \end{cases} \tag{2.26}$$

The stationary mean and variance of the rate process $\tilde{R}_t$ are of considerable interest and readily obtained from (2.24) and (2.26). For the mean,

$$\langle \tilde{R} \rangle = \sum_{i=1}^{N+1} \tilde{v}_i \Pr[\Omega = i] = r\{1 - F(B_T)\} + \sum_{i=1}^{N} \lambda_i F_i(B_T). \tag{2.27}$$

Similarly for the second moment,

$$\langle \tilde{R}^2 \rangle = r^2\{1 - F(B_T)\} + \sum_{i=1}^{N} \lambda_i^2 F_i(B_T). \tag{2.28}$$

On introducing the spectral expansion of $F(\xi)$, see (2.11),

$$\langle \tilde{R}^2 \rangle = a_1 \langle R_s^2 \rangle + (1 - a_1) r^2 + \sum_{j=2}^{N} a_j \{\exp(z_j B_T)\} \left\{ \sum_{i=1}^{N} (\lambda_i^2 - r^2) \phi_{ji} \right\}, \tag{2.29}$$

where $\langle R_s^2 \rangle \triangleq \sum_{i=1}^{N} \lambda_i^2 w_i$ is the second moment of the source rates.

Notice that in contrast to the previously established result that the throughput of tokens, and hence $\langle \tilde{R} \rangle$, depends on B_T and B_D only through B, the second moment $\langle \tilde{R}^2 \rangle$ depends on both B_T and B_D. As the numerical results in the next section will show, this important dependence is pronounced.

(4) ON–OFF SOURCES (CONTINUED)

For these sources it follows from (2.22), (2.23), (2.27) and (2.28) that,

$$\langle \tilde{R} \rangle = r\left[1 - \frac{(1-\rho)}{\Delta(B)}\right], \tag{2.30}$$

$$\langle \tilde{R}^2 \rangle = r^2\left[1 - \frac{1}{\Delta(B)}\left\{\left(1 - \frac{\lambda\rho}{r}\right) + \rho\{\exp(zB_T)\}\left(\frac{\lambda}{r} - 1\right)\right\}\right], \tag{2.31}$$

where ρ, $\Delta(B)$ and z have been defined in section 2(2).

Let $C^2_{v,\tilde{R}}$ denote the squared coefficient of variation of $\tilde{R}_t$, i.e.

$$C^2_{v,\tilde{R}} = \frac{\langle \tilde{R}^2 \rangle}{\langle \tilde{R} \rangle^2} - 1. \tag{2.32}$$

It is easily verified that for fixed B,

$$\frac{\partial\left(C^2_{v,\tilde{R}}\right)}{\partial B_T} > 0 \qquad \forall \rho, \tag{2.33}$$

and

$$\frac{\partial^2\left(C^2_{v,\tilde{R}}\right)}{\partial B_T^2}\begin{cases} < 0 & \text{if } \rho < 1, \\ > 0 & \text{if } \rho > 1. \end{cases} \tag{2.34}$$

That is, with B held fixed, increasing B_T (and therefore decreasing B_D) increases $C^2_{v,\tilde{R}}$; also, $C^2_{v,\tilde{R}}$ is a concave function of B_T if $\rho < 1$ and convex if $\rho > 1$.

3. Numerical investigation and simulation results

In this section we study the performance of the regulator as measured by the cell loss probability (or equivalently the marking probability) P_L, the squared coefficient of variation of the output rate $C^2_{\tilde{R}}$, and the mean cell delay; we also investigate the trade-offs as functions of the regulator parameters B_T, B_D and r. In this investigation there is no peak rate control, i.e., the peak rate constraint is relaxed. The on–off source model is used and the effect of the mean length and variability of the on and off periods on the regulator performance are also reported. Finally, the accuracy of the fluid model and the analytical results are validated through computer simulation.

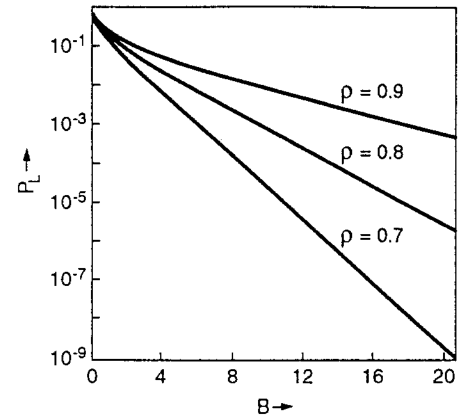

Fig. 6. Cell loss probability as a function of token rate r and total buffer space B. Source parameters: $\alpha = 0.35/0.65$, $\beta = 1$, $\lambda = 1$. Token rate $r = 0.5$, 0.44, 0.39 for $\rho = 0.7$, 0.8, 0.9 respectively.

In the exact source–regulator model, the source alternates between the on state and the off state. During the on state, whose mean duration is $1/\beta$ seconds, the source transmits cells (fixed length packets) at a constant (peak) rate of λ cells/second. The tokens arrive at the token bank at a constant rate of r tokens/sec. In all the examples considered here $1/\beta$ and $1/\alpha$ are set to 0.350 sec and 0.650 sec respectively and λ is set to 62.5 cells/sec. In the approximating fluid model we adapt the convention used by Anick, Mitra and Sondhi [1] and let the unit of time be the mean length of the on period and the unit of information be the average amount generated by the source during the on period. Thus, according to this convention, the source peak rate and mean rate are equal to 1 and 0.35 unit of information per unit of time respectively. Note that the unit of information is equivalent to 21.875 cells. These units are employed in figs. 6–10.

We first examine the effect of r and the total buffer size $B(=B_T+B_D)$ on the cell loss probability P_L. Figure 6 shows that as r approaches the source mean rate, i.e. as ρ approaches 1, the buffer size B required to achieve a desired value of P_L increases sharply. For P_L equal to 10^{-5}, for example, and as r decreases from 0.5 to 0.389, the required buffer size B increases from 10.8 to 17.8 units. This is an undesirable feature since the larger requirement of B translates in general into larger mean cell delay and less control on the variability of the output rate. However, as we will see below, the inherent flexibility in partitioning B into B_T and B_D provides a mean for achieving a reasonable compromise between mean delay and output rate variability while keeping the loss constant. We take note of an interesting phenomenon associated with fluid models. If in the model of the source the mean cycle time is reduced by a factor while keeping constant the ratio of the mean on and off periods, i.e. the jitteriness of the source is increased, then it is an easily verified

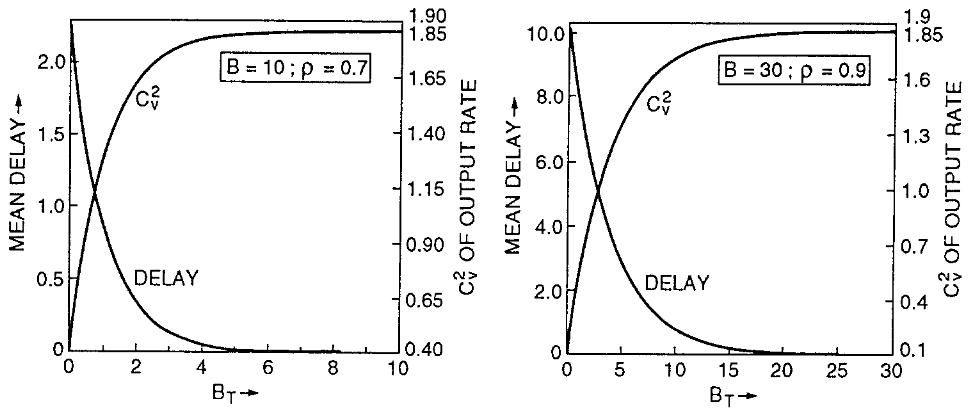

Fig. 7. The trade-off between delay and burstiness of output rate R_t controlled by the partitioning of B into B_T and B_D. (a) $B=10$, $P_L=2.26\times10^{-5}$, $r=0.5$; (b) $B=30$, $P_L=5.0\times10^{-5}$, $r=0.39$. Source parameters same as in fig. 6.

fact that in the units of our convention there is no change in the buffer requirements to achieve a given probability of loss. However, since the unit of information is equal to a number of cells which is smaller by the same factor, the buffer requirement in absolute terms is reduced by this factor. Note that as the mean cycle time approaches zero, all correlations in the source rate are removed, the rate approaches a constant value equal to the mean source rate and P_L approaches zero.

The trade-off between mean cell delay and the squared coefficient of variation of the output rate, $C^2_{v,\tilde{R}}$ is displayed in figs. 7(a) and (b) for r equal to 0.5 and 0.39 respectively. In fig. 7(a) the total buffer space B is set to 10 units of information (giving a loss probability of 2.26×10^{-5}) and the mean delay and $C^2_{v,\tilde{R}}$ are plotted as functions of B_T. As B_T increases from zero to B, the mean delay decreases from a maximum value of 2.165 units of time to zero, while $C^2_{v,\tilde{R}}$ increases from 0.65 to a maximum value approaching $C^2_{v,S}$, the squared coefficient of variation of the uncontrolled source rate. Similar qualitative results are also observed in fig. 7(b). Note the monotonicity and concavity of $C^2_{v,\tilde{R}}$. It is noteworthy that in order to achieve control on the variability, B_T should have a value of the order of one unit of information.

The dependence of the regulator performance on the variability of the on and off periods is investigated next. To generate an on period distribution with squared coefficient of variation larger than one, we have chosen the hyperexponential distribution with balanced means (see, for example, Kuehn [24]). The source model consists of three states: two on states during each of which the source rate is constant, and an off state. The sojourn time in each of the three states is exponentially distributed. The analysis of this model is a straight-forward generalization of the two state on–off source model. In particular the eigenvalues and eigenvectors are all real. As fig. 8 depicts, the loss probability is

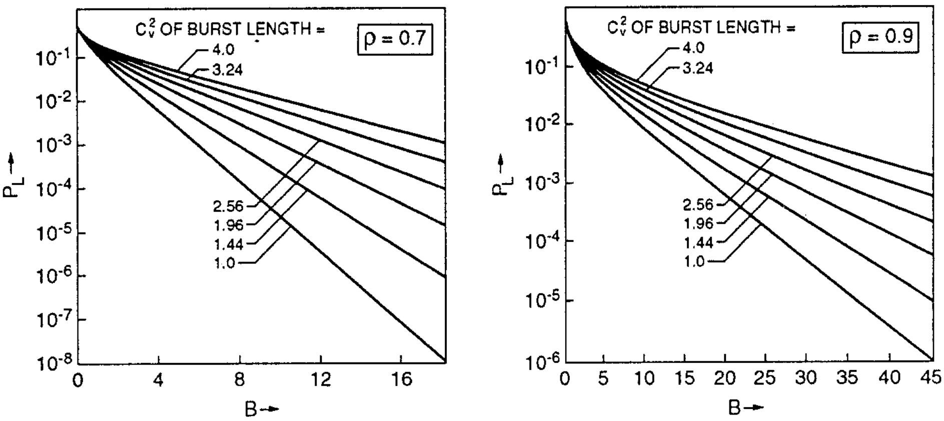

Fig. 8. The effect of the squared coefficient of variation of length of burst (i.e., on period) on cell loss probability. Mean burst length, mean silent period and source rate same as in fig. 6.

quite sensitive to the change in the squared coefficient of variation of the on period. For example, for ρ equal to 0.7, P_L increases by about two orders of magnitude as the squared coefficient of variations of the on period is increased from 1 to 1.44 while keeping B fixed at 18 units. In fig. 9 the loss probability is shown to depend in a similar manner on the coefficient of variation of the off period. Figure 10 elaborates on the theme of fig. 7. The tradeoff implicit in the data of fig. 7 is shown to depend on the squared coefficient of variation of the (hyperexponentially distributed) burst length. Observe that while a higher value of the latter parameter gives higher delay, the variability of the output rate process (as well as its mean) is lower. All of the data presented in figs. 8, 9 and 10 suggest that at least the second moment of the on and off periods needs to be specified as part of the declared parameters at the connection-setup phase in order to accurately predict the characteristics of the regulated traffic.

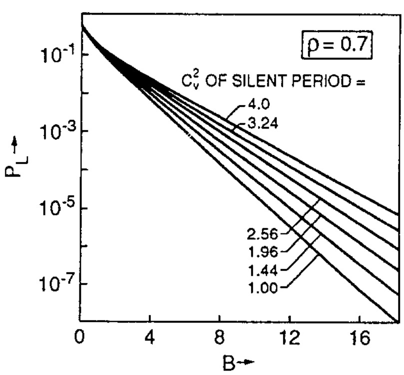

Fig. 9. The effect of the squared coefficient of variation of the silent (i.e., off) period on cell loss probability. Mean burst length, mean silent period and source rate same as in fig. 6.

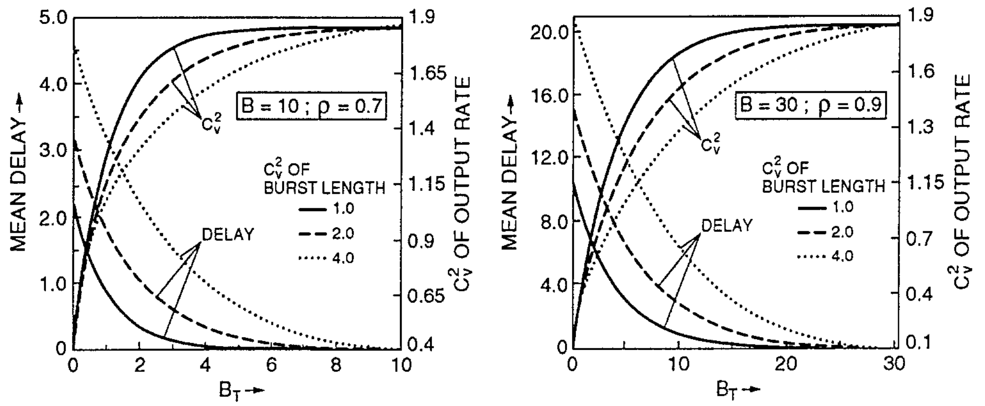

Fig. 10. The dependence of the trade-off between delay and burstiness of output rate on the squared coefficient of variation of the burst length. $r = 0.5$ and 0.39 in (a) and (b); mean burst length, mean silent period and source rate same as in fig. 6.

To validate the accuracy of the stochastic fluid model and its applicability to ATM networks, extensive simulation of the actual source-regulator model described above was performed. Table 1 provides a comparison between the analytical and simulation results for the mean data buffer, the mean token buffer, the throughput $\langle R \rangle$ and the standard deviation of the regulator output rate R_t. The token rate r is fixed at 0.4324 which corresponds to $\rho = 0.81$, and the total buffer space B is set to 44, 88 and 176 units. In each of the three cases, three different partitions of B into B_T and B_D are considered. We observe that there is excellent agreement between the analytical results based on the stochas-

Table 1
Results from simulation of source–regulator with ATM characteristics compared to analytic results from approximating stochastic fluid model. $1/\beta = 0.35$ sec, $1/\alpha = 0.65$ sec, $\lambda = 62.5$ cells/sec, $r = 27.03$ cell/sec; hence $\rho = 0.81$

B	B_T	Mean data buffer content		Mean token buffer content		Throughput, $\langle R \rangle$		Std. Dev (R)	
		Analysis	Simulation	Analysis	Simulation	Analysis	Simulation	Analysis	Simulation
44	11	8.08	8.72±0.91	4.21	4.72±0.26	20.16	20.36±1.83	19.37	20.14±2.68
44	22	3.68	3.97±0.48	10.82	11.20±0.84	20.16	20.21±0.64	23.62	23.31±0.38
44	33	1.13	1.25±0.05	19.27	19.24±0.46	20.16	20.20±0.52	26.43	25.81±0.38
88	22	11.25	12.08±0.40	8.90	9.12±0.45	21.38	21.53±0.07	21.58	21.74±0.51
88	44	4.33	4.73±0.40	23.97	23.86±0.45	21.38	21.55±0.06	25.90	25.50±0.51
88	66	1.07	1.18±0.17	42.72	42.32±0.57	21.38	21.59±0.14	28.16	27.24±1.38
176	44	10.24	11.52±0.94	22.10	21.99±0.59	21.82	22.07±0.16	24.92	24.38±1.83
176	88	2.81	3.18±0.55	58.67	58.01±0.21	21.82	22.08±0.15	28.29	27.44±1.41
176	132	0.50	0.44±0.19	100.36	99.23±0.31	21.82	22.05±0.13	29.39	28.57±1.43

tic fluid model and simulation results. The accuracy of the fluid model is due to the constancy of the cell length and of the cell arrival rate during bursts in the model being simulated, as well as on the fact that the cell interarrival time during the on period is much smaller than the duration of the on period, a condition which typically exists in high speed networks.

4. Buffer distributions just before and after source transitions

A high level goal of the rest of the paper is to characterize the output of the access regulator in sufficient detail for it to be a well-identified "source" for other devices, such as the statistical multiplexer, further downstream in the network. This characterization exploits knowledge of the stationary state distribution of the source–regulator system, $F(\xi)$, whose calculation was the subject of section 2. This section gives the virtual buffer content distribution just before and after epochs at which the source makes transitions. These results are used later in this paper and in the sequel. The results presented in this section make use of the conditional PASTA results in van Doorn and Regterschot [10] and van Doorn, Jagers and de Wit [11]. There are analogous results by Neuts [32] on the probability of states immediately before and after transitions of an $M/M/1$ queue in a Markovian environment.

Let E_n $(n = 1, 2, \ldots)$ denote the epochs at which the source process S_t $(t \geqslant 0)$ makes transitions and let $\chi_n = S_{E_n - 0}$. Let $q_1, \ldots, q_N$ denote the stationary state probabilities of the discrete-time Markov chain $\{\chi_n, n = 1, 2, \ldots\}$. It is known (see, for instance, Keilson [18] and van Doorn and Regterschot [10]) that

$$q_i = \frac{w_i \mu_i}{\langle \boldsymbol{w}, \boldsymbol{\mu} \rangle} \quad (1 \leqslant i \leqslant N), \tag{4.1}$$

where

$$\mu_i = -M_{ii} = \sum_{j \neq i} M_{ij} \tag{4.2}$$

and $\boldsymbol{w}$, see (2.6), is the stationary state distribution of the source. Let

$$G_i^-(n, \xi) \triangleq \Pr\left[\chi_n = i,\ W_{E_n - 0} \leqslant \xi\right] \quad (1 \leqslant i \leqslant N;\ 0 \leqslant \xi \leqslant B) \tag{4.3}$$

and

$$G_i^-(\xi) = \lim_{n \to \infty} G_i^-(n, \xi). \tag{4.4}$$

Similarly,

$$G_i^+(n, \xi) \triangleq \Pr\left[S_{E_n + 0} = i,\ W_{E_n + 0} \leqslant \xi\right] \tag{4.5}$$

and

$$G_i^+(\xi) = \lim_{n \to \infty} G_i^+(n, \xi). \tag{4.6}$$

An important bridge to the distribution $F(\xi)$ is provided by the following result from conditional PASTA (van Doorn and Regterschot [10]):

$$\frac{1}{q_i} G_i^-(\xi) = \frac{1}{w_i} F_i(\xi). \tag{4.7}$$

Our primary interest is in $\{G_i^+(\xi)\}$ which is related to $\{G_i^-(\xi)\}$ thus:

$$G_i^+(\xi) = \sum_{j=1}^{N} G_j^- T_{ji}, \quad (1 \leqslant i \leqslant N; 0 \leqslant \xi \leqslant B), \tag{4.8}$$

where $\mathbf{T}$ is the transition matrix of the aforementioned discrete-time Markov chain $\{\chi_n, n = 1, 2, \ldots\}$. Specifically,

$$T_{ij} = \delta_{ij} + \frac{1}{\mu_i} M_{ij}, \tag{4.9}$$

where δ_{ij} is Kronecker's delta function. It is straightforward to obtain (4.8).

The main result of this section is on the stationary distribution of the virtual buffer content just after a transitions epoch of the source.

PROPOSITION 4.1

$$H_i(\xi) \triangleq \lim_{n \to \infty} \Pr\left[W_{E_n+0} \leqslant \xi \mid S_{E_n+0} = i\right] = \frac{1}{w_i \mu_i} \sum_{j \in \mathscr{S} \setminus i} F_j(\xi) M_{ji}. \tag{4.10}$$

Proof

$$\begin{aligned} H_i(\xi) &= \frac{1}{q_i} G_i^+(\xi), && \text{from Bayes rule,} \\ &= \frac{1}{q_i} \sum_{j=1}^{N} G_j^-(\xi) T_{ji}, && \text{from (4.8),} \\ &= \sum_{j=1}^{N} \frac{q_j}{q_i} \frac{1}{w_j} F_j(\xi) T_{ji}, && \text{from (4.7),} \\ &= \frac{1}{w_i \mu_i} \sum_{j=1}^{N} \mu_j F_j(\xi) T_{ji}, && \text{from (4.1),} \end{aligned}$$

which gives (4.10) on substituting (4.9). □

The intuitive argument underlying the proposition is that the conditional distribution of the virtual buffer content just before a transition epoch of the source is, on account of the memory-less property of the exponentially distributed dwell times at the state, identical to the conditional virtual buffer content distribution at any point during the sojourn in the state before the transition.

The proposition allows us to calculate the token and data buffer distributions just after a source state transition from knowledge of $F(\xi)$. For instance, the conditional token buffer distribution

$$\lim_{n\to\infty} \Pr\left[Y_{E_n+0} \leqslant y \mid S_{E_n+0}=i\right] = \lim_{n\to\infty} \Pr\left[W_{E_n+0} > B_T - y \mid S_{E_n+0}=i\right]$$
$$= 1 - H_i(B_T - y) \quad (1 \leqslant i \leqslant N;\ 0 \leqslant y \leqslant B_T). \tag{4.11}$$

In particular, the stationary probability that the token buffer is empty just after the source makes a transition to state i is

$$\lim_{n\to\infty} \Pr\left[Y_{E_n+0} = 0 \mid S_{E_n+0}=i\right] = 1 - H_i(B_T). \tag{4.12}$$

On substituting the expression for $H_i(\cdot)$ given in proposition 4.1, the desired result is obtained.

In the special case of on–off sources, see section 2(2),

$$H_1(\xi) = \frac{1}{w_2} F_2(\xi), \quad H_2(\xi) = \frac{1}{w_1} F_1(\xi), \tag{4.13}$$

where w and $F(\xi)$ are given in (2.17) and (2.22). In particular, the stationary conditional probability that the token buffer content does not exceed y just after the source has made a transition to the "on" state, is

$$1 - F_1(B_T - y)/w_1, \tag{4.14}$$

and that the token buffer is empty is

$$1 - F_1(B_T)/w_1. \tag{4.15}$$

5. Mean busy and blocking periods of the buffer in a general storage system

The system considered in this section, see fig. 11, is a general data storage system and the analysis calculates the mean period that the buffer is non-empty,

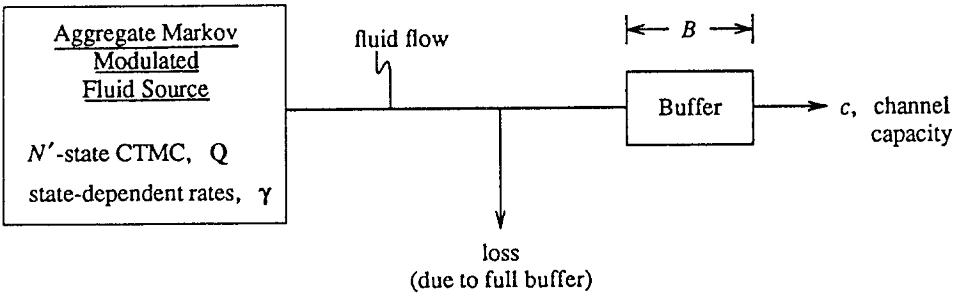

Fig. 11. A general storage system.

i.e. its busy period, and the complementary mean empty period. Also calculated are the mean blocking and non-blocking periods. The results obtained here are used later in connection with the regulator and, in the sequel, with the statistical multiplexer. The results are also of independent interest. Prior results on the mean blocking period for a particular system, packet voice, are given by Li [25], and Neuts [32] gives results on the characterization of the blocking and non-blocking periods by independent phase-type processes. For our purposes here all sources have been merged into one aggregate Markov Modulated Fluid Source with N' states and irreducible generator $\mathbf{Q}$. This source generates fluid at the constant rate of γ_σ when in state σ. The state space of the source is $\mathscr{S} = \{1, 2, \ldots, N'\}$; the source state and buffer content at time t are denoted by Σ_t and X_t respectively. The buffer capacity is B and the transmission capacity of the outgoing channel is c. The buffer capacity B may be infinite.

Let the stationary state distribution be denoted by $\boldsymbol{\pi}(x)$ where $\boldsymbol{\pi}(x) = \{\pi_\sigma(x) \mid \sigma \in \mathscr{S}\}$ and

$$\pi_\sigma(x) = \lim_{t \to \infty} \Pr(\Sigma_t = \sigma,\ X_t \leqslant x) \quad (0 \leqslant x \leqslant B). \tag{5.1}$$

The governing differential equations are easily obtained (compare with (2.4))

$$(\gamma_\sigma - c)\frac{\mathrm{d}}{\mathrm{d}x}\pi_\sigma(x) = \sum_{\sigma' \in \mathscr{S}} \pi_{\sigma'}(x) Q_{\sigma'\sigma} \quad (0 < x < B). \tag{5.2}$$

The term $(\gamma_\sigma - c)$ gives the drift rate of the buffer content when the source state is σ and the buffer is neither empty nor full. In this section we will not be concerned with any method for calculating $\boldsymbol{\pi}(x)$ and it is assumed to be known.

The source states are grouped in two sets depending upon whether a state gives a downward or a upward drift to the buffer content; the subscripts D and U serve as mnemonics.

$$\mathscr{S}_D \triangleq \{\sigma \in \mathscr{S} \mid \gamma_\sigma < c\}, \quad \mathscr{S}_U \triangleq \{\sigma \in \mathscr{S} \mid \gamma_\sigma > c\} \tag{5.3}$$

and it is assumed for simplicity that $\gamma_\sigma \neq c$, $\forall \sigma$. Let

$$\boldsymbol{\pi}_D(x) = \{\pi_\sigma(x) \mid \sigma \in \mathscr{S}_D\}, \quad \boldsymbol{\pi}_U(x) = \{\pi_\sigma(x) \mid \sigma \in \mathscr{S}_U\} \tag{5.4}$$

and let $\mathbf{Q}_{DD}$, $\mathbf{Q}_{DU}$, $\mathbf{Q}_{UD}$, $\mathbf{Q}_{UU}$ denote the submatrices obtained by partitioning $\mathbf{Q}$. Finally, let $\boldsymbol{p}$ denote the stationary probability vector associated with $\mathbf{Q}$, i.e., $\boldsymbol{p}\mathbf{Q} = \boldsymbol{0}$, and let $\boldsymbol{p}_D$ and $\boldsymbol{p}_U$ represent its partitions.

(1) EMPTY PERIODS OF THE BUFFER

For $\sigma \in \mathscr{S}_D$ and $\sigma' \in \mathscr{S}_D$, let $T_{\sigma\sigma'}$ denote the cumulative time spent by the source in state σ' during an empty period of the buffer, conditioned on the event that at the start of the period the source state is σ. It is known (Howard [17]) that the steady-state mean of $\{T_{\sigma\sigma'}\}$,

$$\langle \mathbf{T} \rangle = -\mathbf{Q}_{DD}^{-1}, \tag{5.5}$$

where the elements of $\mathbf{T}$ are $\{T_{\sigma\sigma'}\}$.

Let q_σ ($\sigma \in \mathscr{S}_D$) denote the stationary probability that the source state, $\Sigma_t = \sigma$ at the onset of the buffer's empty period. Also let $\boldsymbol{q} = \{q_\sigma \mid \sigma \in \mathscr{S}_D\}$ so that, in particular,

$$\langle \boldsymbol{q}, \boldsymbol{1} \rangle = 1. \tag{5.6}$$

It is a simple fact that

$$\boldsymbol{\pi}_D(0) = a\boldsymbol{q}\langle \mathrm{T} \rangle, \tag{5.7}$$

where a is some constant of proportionality. From (5.5), (5.6) and (5.7),

$$a = -\langle \boldsymbol{\pi}_D(0)\mathbf{Q}_{DD}, \boldsymbol{1} \rangle. \tag{5.8}$$

Hence, making use of (5.5) and (5.7),

$$\boldsymbol{q} = \frac{1}{\langle \boldsymbol{\pi}_D(0)\mathbf{Q}_{DD}, \boldsymbol{1} \rangle} \boldsymbol{\pi}_D(0)\mathbf{Q}_{DD}. \tag{5.9}$$

Now let t_σ denote the stationary expected cumulative time that $\Sigma_t = \sigma$ during the buffer's empty period, and $\boldsymbol{t} = \{t_\sigma \mid \sigma \in \mathscr{S}_D\}$. Obviously,

$$\boldsymbol{t} = \boldsymbol{q}\langle \mathrm{T} \rangle. \tag{5.10}$$

Hence from (5.5) and (5.9),

$$\boldsymbol{t} = \frac{-1}{\langle \boldsymbol{\pi}_D(0)\mathbf{Q}_{DD}, \boldsymbol{1} \rangle} \boldsymbol{\pi}_D(0). \tag{5.11}$$

It is almost always easier to compute the right hand quantity by noting that

$$\langle \boldsymbol{\pi}_D(0)\mathbf{Q}_{DD}, \boldsymbol{1} \rangle + \langle \boldsymbol{\pi}_D(0)\mathbf{Q}_{DU}, \mathbf{1} \rangle = 0. \tag{5.12}$$

Hence we obtain

PROPOSITION 5.1

t_σ, stationary mean cumulative time that $\Sigma_t = \sigma$

during empty period of buffer

$$= \frac{\pi_\sigma(0)}{\langle \boldsymbol{\pi}_D(0)\mathbf{Q}_{DU}, \boldsymbol{1} \rangle} \quad (\sigma \in \mathscr{S}_D) \tag{5.13}$$

$$\bar{t}, \text{ stationary mean empty period of buffer} = \frac{\pi(0)}{\langle \boldsymbol{\pi}_D(0)\mathbf{Q}_{DU}, \boldsymbol{1} \rangle}, \tag{5.14}$$

where $\pi(0)$, the stationary probability of empty buffer $= \Sigma_{\sigma \in \mathscr{S}_D} \pi_\sigma(0)$. □

In the special case that the Markov chain controlling the source is birth-and-death, $\mathbf{Q}$ is tridiagonal. In this case let σ^* denote the special state at the boundary of $\mathscr{S}_D$ and $\mathscr{S}_U$, that is, $\sigma^* \in \mathscr{S}_D$ and $(\sigma^*+1) \in \mathscr{S}_U$. Then,

$$t_\sigma = \frac{\pi_\sigma(0)}{\pi_{\sigma^*}(0)Q_{\sigma^*,\sigma^*+1}}, \quad \text{and} \quad \bar{t} = \frac{\pi(0)}{\pi_{\sigma^*}(0)Q_{\sigma^*,\sigma^*+1}}. \tag{5.15}$$

(2) MEAN BUSY PERIOD OF THE BUFFER

The buffer alternates between empty and busy (i.e. non-empty) periods. Let $\bar{b}$ denote mean busy period for the buffer in fig. 11. A simple renewal-theoretic argument gives

$$\pi(0) = \frac{\bar{t}}{\bar{t}+\bar{b}}, \tag{5.16}$$

and from (5.14),

$$\bar{b} = \frac{1-\pi(0)}{\langle \boldsymbol{\pi}_D(0)\mathbf{Q}_{DU}, \mathbf{1}\rangle}. \tag{5.17}$$

(3) MEAN BLOCKING AND NON-BLOCKING PERIODS OF THE BUFFER

By a symmetrical chain of reasoning it is straightforward to obtain an expression, analogous to (5.13), for the stationary mean blocking period of the buffer. Let l_σ denote the stationary mean cumulative time that $\Sigma_t = \sigma$ during a blocking period of the buffer, and $\bar{l}$ denote the stationary mean blocking period. We have

PROPOSITION 5.2

$$l_\sigma = \frac{p_\sigma - \pi_\sigma(B)}{\langle\{\boldsymbol{p}_U - \boldsymbol{\pi}_U(B)\}\mathbf{Q}_{UD}, \mathbf{1}\rangle}, \quad (\sigma \in \mathscr{S}_U) \tag{5.18}$$

and

$$\bar{l} = \frac{1-\pi(B)}{\langle\{\boldsymbol{p}_U - \boldsymbol{\pi}_U(B)\}\mathbf{Q}_{UD}, \mathbf{1}\rangle}, \tag{5.19}$$

where $\boldsymbol{p}$ is the stationary distribution of the source state and $\{1-\pi(B)\}$ is the stationary probability of the buffer being full, i.e. $\pi(B) = \sum_{\sigma \in \mathscr{S}} \pi_\sigma(B)$. □

Finally, let $\bar{g}$ denote the stationary mean period that the buffer is non-blocking. It is obtained from the relation

$$1-\pi(B) = \frac{\bar{l}}{\bar{l}+\bar{g}}, \tag{5.20}$$

since both sides of the expression give the fraction of time that the buffer is full.

6. Mean busy and blocking periods of the buffers in the regulator

We use the results of the preceding section to obtain explicit expressions for the mean empty and busy periods of the token buffer, as well as the mean blocking periods of the token and data buffers. The empty periods of the token buffer are of particular importance since during these periods the output rate process, $R_t = r$. The blocking periods of the data buffer are similarly important in characterizing the output rate process of marked packets, L_t.

As fig. 12 shows, the empty period of the token buffer is identical to the busy period of the data buffer, if the latter exists, and remains meaningful even in the absence of a data buffer. Let $\bar{t}_T$ and $\bar{b}_T$ respectively denote the mean empty and busy periods of the token buffer. Similarly $\bar{t}_D$ and $\bar{b}_D$ may be defined for the data buffer; clearly $\bar{t}_T = \bar{b}_D$ and $\bar{b}_T = \bar{t}_D$.

The results in this section on the calculation of $\bar{t}_T$ and $\bar{b}_T$ are in two parts. In the first part, section 6(1), we allow the data buffer, i.e. $B_D > 0$, but place restrictions on the source, while in the second part, section 6(2), the data buffer does not exist but the source is not restricted. The restriction on the source in the first part is that either $|\mathscr{S}_U| = 1$ or $|\mathscr{S}_D| = 1$, i.e., there is either a unique source state giving upward drift to the virtual buffer content, or a unique state giving downward drift. This restriction is satisfied, for instance, by on-off sources in which the on period is exponentially distributed while the off period is more generally distributed, say hyperexponentially. It is also satisfied if the distribution of the on period is more general while the off period is exponentially distributed. (Both these cases are represented in the numerical studies reported in section 3.)

In the second part, section 6(2), no such restriction is placed on the source model characterized by (M, λ); however, $B_D = 0$. Finally, in section 6(3) the mean blocking periods of the data and token buffers are calculated without restrictions on either the source or the data buffer.

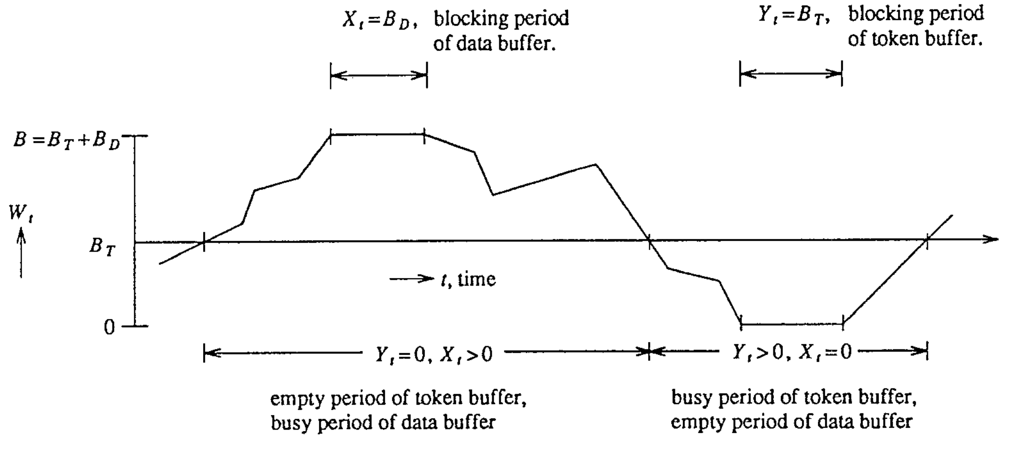

Fig. 12. Illustration of the busy, empty and blocking periods of the buffers in the access regulator.

(1) WITH DATA BUFFER

Our first observation is that

$$\Pr(W \leqslant B_T) = \frac{\bar{b}_T}{\bar{t}_T + \bar{b}_T}, \tag{6.1}$$

where, recall from section 2, the virtual buffer content, $W_t = X_t - Y_t + B_T$. The proof of (6.1) is based on a simple and familiar renewal-theoretic argument. By definition, see (2.3),

$$\Pr(W \leqslant B_T) = F(B_T). \tag{6.2}$$

Now suppose that $|\mathscr{S}_U| = 1$ and consider the calculation of $\bar{t}_T$. The key observation is that the empty period of the token buffer is identical in distribution to the busy period of a new construct, a data storage system as described in section 5 with a Markov Modulated Fluid Source characterized by $(\mathbf{M}, \boldsymbol{\lambda})$, channel capacity r and, importantly, a buffer of size B_D. This equivalence is critically dependent on the uniqueness of the source state at the commencement of the periods in the respective systems. It is for this reason that we need to assume $|\mathscr{S}_U| = 1$. Now (5.17) has given a formula for the mean busy period of the general data storage system. Translating (5.17) to the source-regulator system, we obtain

$$\bar{t}_T = \frac{1 - F(0; B_D)}{\langle \boldsymbol{F}_D(0; B_D)\mathbf{M}_{DU}, \boldsymbol{1} \rangle}, \tag{6.3}$$

in which we have introduced the general notation $\{F_i(\xi; B)\}$ to denote the solution, to be obtained as prescribed in section 2, for the equilibrium joint distribution for the system with a source characterized by $(\mathbf{M}, \boldsymbol{\lambda})$ and an access regulator with token rate r and total buffer space B. Hence, to obtain $\bar{t}_T$ from (6.3) we need to solve anew for the equilibrium joint distribution with total buffer space B_D.

Moreover, from (6.1) and (6.2), the busy period of the token buffer of regulator

$$\bar{b}_T = \bar{t}_T \frac{F(B_T; B)}{1 - F(B_T; B)}, \tag{6.4}$$

where $F(\xi; B) = \langle \boldsymbol{F}(\xi; B), \boldsymbol{1} \rangle$, and is therefore identical to $F(\xi)$ calculated in section 2. This completes the treatment for the case $|\mathscr{S}_U| = 1$.

Now suppose that $|\mathscr{S}_D| = 1$. The key observation in this case is that the busy period of the token buffer is identical in distribution to the non-blocking period of a new construct, a data storage system as described in section 5, with a Markov Modulated Fluid Source characterized by $(\mathbf{M}, \boldsymbol{\lambda})$, channel capacity r and, importantly, a buffer of size B_T. Once again the equivalence is critically dependent on the uniqueness of the source state at the commencement of the periods in the respective sytems, which follows from $|\mathscr{S}_D| = 1$. Translating (5.19) and (5.20) for the non-blocking period in the general system to the source–regulator system, gives

$$\bar{b}_T = \frac{F(B_T;\, B_T)}{\langle\{w_U - F_U(B_T;\, B_T)\}\mathbf{M}_{UD},\, \mathit{1}\rangle}. \tag{6.5}$$

Finally, $\bar{t}_T$ is obtained from the above and (6.4). This completes the treatment of the case $|\mathscr{S}_D| = 1$.

Now consider the special case of on–off sources with one on state and one off state, i.e. $|\mathscr{S}_U| = |\mathscr{S}_D| = 1$. From (6.3),

$$\bar{t}_T = \frac{1 - F(0;\, B_D)}{\alpha F_1(0;\, B_D)}, \tag{6.6}$$

which, together with (2.22) and (2.23), gives the following enlightening expression

$$\bar{t}_T = \frac{1}{\alpha}\frac{\rho}{1-\rho}\{1 - \exp(zB_D)\} + \frac{1}{\beta}\exp(zB_D), \tag{6.7}$$

where z and ρ have been given in (2.18). Finally, from (6.4) and (6.7),

$$\bar{b}_T = \frac{1 - \rho(\exp(zB_T))}{\alpha(1-\rho)\exp(zB_T)}. \tag{6.8}$$

The reader may verify that identical expressions for $\bar{t}_T$ and $\bar{b}_T$ are obtained from (6.5) and (6.4), and that in the limit as $B_D \to 0$ and $B_T \to 0$, the above expressions give $\bar{t}_T = 1/\beta$ and $\bar{b}_T = 1/\alpha$, respectively.

(2) WITHOUT DATA BUFFER

This case, where $B_D = 0$, is simpler, and expressions for $\bar{b}_T$ and $\bar{t}_T$ are obtained from a straightforward application of the results in the preceding section. This is because the empty period of the token buffer is identical in distribution to the blocking period of a new construct, a data storage system as described in section 5 with a Markov Modulated Fluid Source characterized by $(\mathbf{M}, \boldsymbol{\lambda})$ and, importantly, a buffer of size B_T. In a symmetrical manner, the busy period of the token buffer is now identical in distribution to the non-blocking period of the construct.

Hence by a simple translation of the result in (5.19) we obtain

$$\bar{t}_T = \frac{1 - F(B_T;\, B_T)}{\langle\{w_U - F_U(B_T;\, B_T)\}\mathbf{M}_{UD},\, \mathit{1}\rangle}, \tag{6.9}$$

and, from (5.20)

$$\bar{b}_T = \bar{t}_T\frac{F(B_T;\, B_T)}{1 - F(B_T;\, B_T)}. \tag{6.10}$$

(3) MEAN BLOCKING PERIODS OF THE TOKEN AND DATA BUFFERS

Let $\bar{l}_T$ and $\bar{l}_D$ respectively denote the mean blocking periods of the token and data buffers. Now $\bar{l}_T$ is also the mean empty period of a data storage system (as in section 5) with a source characterize by $(\mathbf{M}, \boldsymbol{\lambda})$ and a buffer of size B. Interpreting the result in (5.14) in the context of the regulator in section 2 gives

$$\bar{l}_T = \frac{F(0;\, B)}{\langle F_D(0;\, B)\mathbf{M}_{DU},\, \mathit{1}\rangle}. \tag{6.11}$$

Similarly, from (5.19),

$$\bar{l}_D = \frac{1 - F(B;\, B)}{\langle \{w_U - F_U(B;\, B)\}\mathbf{M}_{UD},\, \mathit{1}\rangle}. \tag{6.12}$$

The reader may verify that for two-state on–off sources, the above expressions reduce to $1/\alpha$ and $1/\beta$ respectively.

7. Characterization of the rate process R_t at output of regulator

In this section we proceed with the characterization of R_t, the rate of priority cells departing from the regulator. The main result of the section is the distribution of periods when $R_t = v_i$ or, equivalently, the sojourn time of the process Ω_t in state i ($i \in \mathscr{S}_U$). During any such period the output rate exceeds the token rate r. The calculation of these distribution makes use of the results in section 4.

Recall from the conventions established in section 2 that the state space of the process Ω_t is $\{1, 2, \ldots, N+1\}$; when $\Omega_t = i$, the output rate $R_t = v_i$. For $i \in \{1, 2, \ldots, N\}$ the rate v_i is simply the rate of the source in state i subject to peak rate control, i.e., $v_i = \lambda_i \wedge \hat{v}$, and, finally, $v_{N+1} = r$. See fig. 13.

Let $T_{n,i}$ denote the length of the nth sojourn of the process Ω_t in state i, i.e., during this sojourn $R_t = v_i$ ($n = 1, 2, \ldots$). Let $\Psi_{n,i}(t) = \Pr(T_{n,i} \leqslant t)$ and $\Psi_i(t) = \lim_{n\to\infty}\Psi_{n,i}(t)$. Hence $\Psi_i(t)$ denotes the steady state distribution of T_i,

$$\Psi_i(t) = \Pr(T_i \leqslant t) \quad (1 \leqslant i \leqslant N+1). \tag{7.1}$$

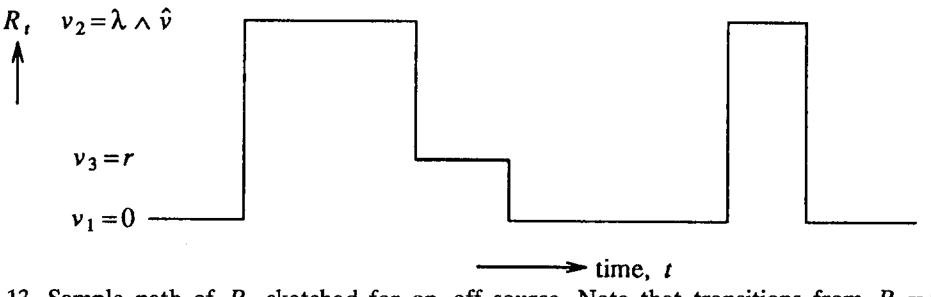

Fig. 13. Sample path of R_t sketched for on–off source. Note that transitions from $R_t = 0$ to $R_t = r$, as well as transitions from $R_t = r$ to $R_t = \lambda \wedge \hat{v}$ cannot occur.

Recall from (2.5) that $\mathscr{S}_D$ is the set of source states with data generation rates less than r. Clearly sojourn of the process Ω_t in state i, $i \in \mathscr{S}_D$, is entirely determined by the source's sojourn in the state. Hence in this case,

$$\Psi_i(t) = 1 - \exp(-\mu_i t) \quad (i \in \mathscr{S}_D), \tag{7.2}$$

where, see (4.2), $\mu_i = -M_{ii}$.

The preceding section has given the mean empty period of the token buffer, $\bar{t}_T$, see (6.5). This quantity is equal to the mean sojourn time of the output process in state $(N+1)$, $\langle T_{N+1} \rangle$. In this paper this is all the information we give on T_{N+1}.

The rest of this section is devoted to the calculation of $\Psi_i(t)$, $i \in \mathscr{S}_U$, the distribution of periods during which the output rate of the regulator exceeds the token rate r. We first obtain general expressions and this is followed by more explicit expressions for on–off sources.

(1) DISTRIBUTIONS OF PERIODS WHEN OUTPUT RATE EXCEEDS TOKEN RATE

The periods of interest here, T_i $(i \in \mathscr{S}_U)$, begin with those epochs of transitions of the source to state i for which the token buffer is not empty. This is because an output rate v_i, which exceeds the token rate r, can only be sustained while the token buffer is not empty. Consequently, also, the sojourn is terminated when either the token buffer becomes empty or the source makes a transition. Calculation of $\Psi_i(t)$ amounts to determining the outcome of a race between these two events. Specifically, $T_i = \min(T_i', T_i'')$ where T_i' is the exponentially distributed time to a transition of the source from state i to some other state, and T_i'' is the time for the token buffer to empty. During the sojourn of interest, the token buffer is depleted at a net (deterministic) rate of $(\lambda_i - r)$, and hence T_i'' is simply $Y_{E+0}/(\lambda_i - r)$, where Y_{E+0} is the token buffer content just after the source transition marking the start of the sojourn; as previously noted, the sojourn's start requires $Y_{E+0} > 0$. Observe that Y_{E+0} and T_i' (and therefore T_i' and T_i'') are mutually independent random variables.

We now make use of the expressions in section 4 giving the distribution of the token buffer content just after a source transition epoch. First note that for $i \in \mathscr{S}_U$,

$$\Pr(T_i'' \leqslant t) = 1 \quad \text{if } t > B_T/(\lambda_i - r). \tag{7.3}$$

For $t < B_T/(\lambda_i - r)$,

$$\begin{aligned}\Pr(T_i'' \leqslant t) &= \Pr(Y_{E+0} \leqslant t(\lambda_i - r) \mid S_{E+0} = i,\, Y_{E+0} > 0)\\ &= \frac{\Pr(0 < Y_{E+0} \leqslant t(\lambda_i - r) \mid S_{E+0} = i)}{\Pr(0 < Y_{E+0} \mid S_{E+0} = i)}\\ &= \left[H_i(B_T) - H_i\{B_T - t(\lambda_i - r)\}\right]/H_i(B_T),\end{aligned} \tag{7.4}$$

from (4.11) and (4.12). Hence from the expression in proposition 4.1,

$$\Pr(T_i'' \leqslant t) = 1 - \sum_{j \in \mathscr{S} \setminus i} F_j\{B_T - t(\lambda_i - r)\} M_{ji} / \sum_{j \in \mathscr{S} \setminus i} F_j(B_T) M_{ji}, \tag{7.5}$$

where $\{F_j(\xi)\}$, the stationary state distribution of the system composed of the source and regulator, has been calculated in section 2. Note that the distribution $\Pr(T_i'' \leqslant t)$ has value 0 at $t = 0$ and exhibits a jump at $t = B_T/(\lambda_i - r)$. Now

$$1 - \Psi_i(t) = \Pr(T_i > t) = \Pr(T_i' > t)\Pr(T_i'' > t),$$

and $\Pr(T_i' > t) = \exp(-\mu_i t)$ where $\mu_i = -M_{ii}$. Hence we have,

PROPOSITION 7.1

For $i \in \mathscr{S}_U$,

$$1 - \Psi_i(t) = \exp(-\mu_i t) \frac{\sum_{j \in \mathscr{S} \setminus i} F_j\{B_T - t(\lambda_i - r)\} M_{ji}}{\sum_{j \in \mathscr{S} \setminus i} F_j(B_T) M_{ji}} \quad \text{if } t < \frac{B_T}{(\lambda_i - r)},$$

$$= 0 \quad \text{if } t > \frac{B_T}{\lambda_i - r}. \quad \square \tag{7.6}$$

(2) ON–OFF SOURCES

For the special case of on–off sources, where only v_2 exceeds r, see fig. 13, we obtain on substituting $F_1(\cdot)$ given in (2.22),

$$1 - \Psi_2(t) = \Pr(T_2 > t) = \exp(-\beta t) \frac{\Delta\{B_T - t(\lambda - r)\}}{\Delta(B_T)}, \quad \text{if } t < B_T/(\lambda - r), \tag{7.7}$$

where $\Delta(\cdot)$ is defined in (2.21). Also, the mean period,

$$\langle T_2 \rangle = \frac{1}{\Delta(B_T)} \int_0^{B_r/(\lambda - r)} \exp(-\beta t) \Delta\{B_T - t(\lambda - r)\} \, dt$$

$$= \frac{1 - \exp(zB_T)}{\beta \Delta(B_T)}. \tag{7.8}$$

Note that for on–off sources $\Psi_i(t)$, $i \in \mathscr{S}_U$, does not depend on the size of the data buffer B_D.

8. Approximate Markovian characterization of output of regulator

This section gives an approximate Markovian characterization of the process Ω_t, see section 2(3). Thereby the priority and marked cell streams are characterized as coupled Markov Modulated Fluid Sources. The characterization allows us to separate the analysis of the access regulator from the rest of the network, including the statistical multiplexer, and thus achieve a decomposition which is essential for tractability of the entire integrated system. The characterization given here approximates the distributions of the sojourn time in the various states of the output rate processes by exponential distributions with means which are the previously calculated exact values.

In the characterization given here, a N_0-state continuous time Markov chain with generator **G** controls the fluid source. For each state i $(1 \leqslant i \leqslant N_0)$ there exists two rates, $v_i^{(1)}$ and $v_i^{(2)}$, corresponding to the flow of priority and marked cells, respectively. Thus the coupled Markov modulated fluid source is here characterized by $(\mathbf{G}; \boldsymbol{v}^{(1)}, \boldsymbol{v}^{(2)})$. Notice the fundamentally important feature of *coupling* between the two streams. Bursty sources are more likely to generate marked cells at a high rate precisely when they are generating priority cells at a high rate. This feature is not lost in our model.

The results presented here are for the case of on–off sources. Moreover we assume for simplicity that the data buffer is not present, i.e. $B_D = 0$. The reader will find that the expressions given earlier in this paper, together with the line of reasoning given below, allow such an approximate Markovian characterization to be obtained for general Markovian sources.

It will be shown below that the approximate Markovian characterization has the remarkable property that its rate processes have moments which exactly equal the moments of the actual rate processes.

Notice that all the transitions depicted in fig. 5 for two-state on–off sources are reflected in fig. 14. Also note that the flow rates of the priority cells (R_t) and marked cells (L_t) which are shown in the figure are exact in each of the three states depicted. The parameters θ and p are derived below; α and β are from

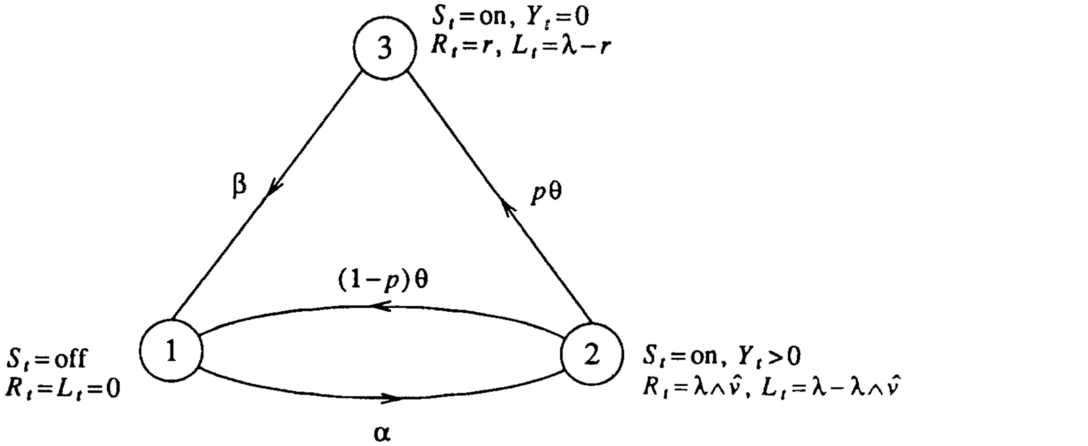

Fig. 14. The approximate Markovian characterization of the rate processes of priority (R_t) and marked (L_t) cells at output of regulator, for on-off sources.

the model of the on–off source, see (2.16). For the characterization in fig. 14 we have $N_0 = 3$,

$$\mathbf{G} = \begin{bmatrix} -\alpha & \alpha & 0 \\ (1-p)\theta & -\theta & p\theta \\ \beta & 0 & -\beta \end{bmatrix}, \quad \begin{aligned} v^{(1)} &= [0, \lambda \wedge \hat{v}, r], \\ v^{(2)} &= [0, \lambda - \lambda \wedge \hat{v}, \lambda - r]. \end{aligned} \tag{8.1}$$

Now consider the parameters θ and p. Section 7(2) has given the distribution $\Psi_2(\cdot)$ for T_2 which is the time during which S_t = on and $Y_t > 0$. We let $1/\theta$ denote the mean, i.e., from (7.8),

$$1/\theta = \langle T_2 \rangle = \frac{1 - \exp(zB_T)}{\beta \Delta(B_T)}, \tag{8.2}$$

where z and $\Delta(\cdot)$ are defined in (2.18) and (2.21). We also let p denote the probability that the token buffer empties before the source makes a transition from on to off, i.e.,

$$p = \Pr(T_2'' \leqslant T_2'). \tag{8.3}$$

From (7.7),

$$\Pr(T_2' > t) = \exp(-\beta t) \tag{8.4}$$

and

$$\begin{aligned} \Pr(T_2'' > t) &= \Delta\{B_T - t(\lambda - r)\}/\Delta(B_T) \quad &\text{if } t \leqslant B_T/(\lambda - r), \\ &= 0 &\text{if } t > B_T/(\lambda - r). \end{aligned} \tag{8.5}$$

Hence, from (8.3)–(8.5),

$$p = 1 - \frac{1 - \exp(zB_T)}{\Delta(B_T)}. \tag{8.6}$$

In summary, the approximate Markovian characterization is complete with θ and p specified in (8.2) and (8.6).

Now consider the equilibrium state distribution s of the characterization in fig. 14. Since $s\mathbf{G} = \mathit{0}$, we have

$$s = \frac{1}{c}\left[\frac{\theta}{\alpha}, 1, \frac{p\theta}{\beta}\right], \tag{8.7}$$

where $c = 1 + \theta/\alpha + p\theta/\beta$. On substituting the expression for θ and p in (8.2) and (8.6), we obtain

PROPOSITION 8.1

The equilibrium state probabilities of the approximate Markovian characterization in fig. 14 are

$$s_1 = \frac{\beta}{\alpha + \beta}; \quad s_2 = \frac{\alpha}{\alpha + \beta}\frac{1 - \exp(zB_T)}{\Delta(B_T)}; \quad s_3 = \frac{\alpha}{\beta}(1 - \rho)\frac{\exp(zB_T)}{\Delta(B_T)}. \quad \square \tag{8.8}$$

The following important observation on these formulas may be made: these probabilities are identical to their actual, natural counterparts whose exact values have been calculated in section 2. This observation is now summarized.

PROPOSITION 8.2

For $B_D = 0$ and two-state on–off sources, the analysis in section 2(2) has obtained

$$\Pr(R=0,\ L=0) = \Pr(S=\text{off}) = \beta/(\alpha+\beta), \tag{8.9,i}$$

$$\Pr(R=\lambda\wedge\hat{v},\ L=\lambda-\lambda\wedge\hat{v}) = \Pr(S=\text{on},\ Y>0) = F_2(B_T)$$
$$= \frac{\alpha}{\alpha+\beta}\,\frac{1-\exp(zB_T)}{\Delta(B_T)}, \tag{8.9,ii}$$

$$\Pr(R=r,\ L=\lambda-r) = \Pr(S=\text{on},\ Y=0) = w_2 - F_2(B_T)$$
$$= \frac{\alpha}{\beta}(1-\rho)\frac{\exp(zB_T)}{\Delta(B_T)}. \tag{8.9,iii}$$

Hence, comparison with (8.8) shows that the following correspondences exists between the above stationary probabilities for the output rates of the access regulator, and the stationary probabilities of the approximate Markovian characterization,

$$s_1 = \Pr(R=0,\ L=0);\quad s_2 = \Pr(R=\lambda\wedge\hat{v},\ L=\lambda-\lambda\wedge\hat{v});$$
$$s_3 = \Pr(R=r,\ L=\lambda-r).\quad \square \tag{8.10}$$

A corollary to the above correspondences is that *all* the stationary moments of the actual rate processes are *identical* to the corresponding moments of the approximate Markovian characterization. This is easily seen from the following expressions for $\langle R^m\rangle$ and $\langle L^m\rangle$, the mth stationary moments of the output rate processes R_t and L_t ($m = 1, 2, \ldots$):

$$\langle R^m\rangle = (\lambda\wedge\hat{v})^m \Pr(S=\text{on},\ Y>0) + r^m \Pr(S=\text{on},\ Y=0)$$
$$= (\lambda\wedge\hat{v})^m s_2 + r^m s_3. \tag{8.11}$$

$$\langle L^m\rangle = (\lambda-\lambda\wedge\hat{v})^m \Pr(S=\text{on},\ Y>0) + (\lambda-r)^m \Pr(S=\text{on},\ Y=0)$$
$$= (\lambda-\lambda\wedge\hat{v})^m s_2 + (\lambda-r)^m s_3. \tag{8.12}$$

In the sequel to this paper we study congestion control strategies based on loss priorities which are employed by network nodes. The analyses there rely on the results of this paper on the Markovian source approximation of the output of the regulator and the coupling between the priority and marked cells.

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