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An experimental investigation into dynamic fracture: III. On steady-state crack propagation and crack branching

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Abstract

This is the third in a series of four papers in which problems of dynamic crack propagation are examined experimentally in large, thin sheets of Homalite-100 such that crack growth in an unbounded plate is simulated. In the first paper crack initiation resulting from stress wave loading to the crack tip as well as crack arrest were reported. It was found that for increasing rates of loading in the microsecond range the stress intensity required for initiation rises markedly. Crack arrest occurs abruptly without any deceleration phase at a stress intensity lower than that which causes initiation under quasi-static loading.

In the second paper we analyze the occurrence of micro cracks at the front of the running main crack which control the rate of crack growth. The micro cracks are recorded by real time photography. By the same means it is shown that these micro cracks grow and turn away smoothly from the direction of the main crack in the process of branching.

In the present paper we report results on crack propagation and branching. It is found that crack propagation occurs at a constant velocity although the stress intensity factor changes markedly. Furthermore, the velocity is determined by the stress wave induced intensity factor at initiation. The terminal velocity in Homalite-100 was found to be about half the Rayleigh wave speed (0.45 C_r). These observations are analyzed in terms of a microcrack model alluded to in the second paper of this series. A mechanism for crack branching is proposed which considers branching to be a natural evolution from a "cloud" of microcracks that accompany and lead the main crack. These results are believed to apply to quasi-brittle materials other than Homalite-100 and the reasons for this belief are discussed briefly in the first paper of this series.

In the final paper of the series the effect of stress waves impinging on the tip of a rapidly moving crack is examined. Waves affect the velocity and the direction of propagation as well as the process of crack branching.

1. Introduction

In the first paper of this series we motivated the experimental work reported in the four paper sequence [1–3] by an examination of the inadequacies in current theories of dynamic fracture. Although the material employed in our studies is the polymer Homalite-100, a polyester, we believe the results to be more generally applicable, at least with respect to the so-called "brittle" materials. Reasons for this belief are outlined in [1].

In order to better understand the physical processes that take place during dynamic fracture, it was necessary to study the fracture at the microscopic level and that investigation was discussed in the second paper of the series [2]. The ideas and observations of that presentation on the microstructural aspects of dynamic crack propagation form the basis for the discussion in this paper which addresses explanations of macroscopically observed crack propagation behavior. A unified view of the dynamic crack propagation process is proposed here that would at least qualitatively explain at this stage the observed crack propagation behavior, including specifically the lower limiting velocities as well as the phenomenon of crack branching. A quantitative evaluation of this model is beyond the present scope of this work.

In the following, we examine first the steady-state crack propagation behavior, considering both the constant velocity phase as well as the limiting velocity of crack propagation. In addressing next the problem of crack branching, we review branching mechanisms proposed in the literature and then propose a mechanism derived from our real-time photographic observations of branching at the microscopic level; these observations establish the "branching event" to be a continuous evolution from propagation along a plane.

2. Steady-state crack propagation

We first consider results of our experiments conducted with large sheets of Homalite-100. The requisite experimental arrangements have been discussed in [1] and in detail in [4]. For purpose of continuity of presentation, we state here that the thin test sheet specimens were semi-infinite crack configurations, with pressure applied to the crack surfaces via a trapezoidal pressure load history. The rate of loading in subsequent tests was progressively higher, ranging from $2.5 \cdot 10^4$ MPa/sec to $6.2 \cdot 10^5$ MPa/sec, and resulting in the crack being loaded to various stress intensity histories. As a result, the crack propagated at various velocities in each case and provided a wealth of data on crack propagation. The stress intensity factor and the crack extension histories were determined via high speed cinematography of the caustics such as represented in Fig. 1.

For a series of tests in which the loading rate was changed systematically, the stress intensity factor and crack extension histories are plotted in Figs. 2, 3 and 4 for six different experiments. Two important aspects are revealed by these results: first, one notes that although the stress intensity factor varies considerably with time the velocity of crack propagation is constant, † and second, one observes that the maximal observed crack velocity, usually referred to as the terminal velocity is on the order of 0.45 C_r where C_r is the Rayleigh wave speed. The following two sections deal with these two aspects.

2.1. Crack propagation with constant velocity

In order to determine the instantaneous crack tip velocity from the data, it is necessary to deduce the histories of crack tip position, being mindful of the accuracy of the data and possible sources of velocity transitions. Several schemes for data evaluation have been explored. However, our most successful one rests on a power law fit of the form

$$a = \sum_{i=1}^{N} A_i t^{i-1} \tag{1}$$

where a is the crack tip position and t denotes the time. The coefficients A_i are determined by minimizing the likelihood function:

$$\chi^2 = \sum_{i=1}^{N} \delta_i^2 \left[a_i - \sum_{j=1}^{M} A_j t_i^{j-1} \right]^2 \tag{2}$$

† The results presented here have prompted B. Freund (personal communication) to compute the time required to establish the singular stress field at the crack tip after the crack has started to propagate. It turns out that the square root singularity field grows outward from the moving crack tip rather slowly so that a considerable time is required before that field is valid in a domain from which the method of caustics derives information. This measurement technique may thus not yield accurate results for short times following a highly transient crack tip event and further experiments are indicated and are being performed. However, even if such a correction were to prove necessary it would not affect the results in this paper. The correction might affect the absolute magnitude of the stress intensity factor under certain conditions but the relative or percent change would still be the same as reported here.

where (t_i, a_i) represent experimentally measured time and crack position data, δ_i is the standard error on a_i, N is the number of data points and $M-1$ is the degree of polynomial fit. The degree of the polynomial fit is yet to be determined but will be accomplished using the "goodness of fit" criterion as determined by the χ^2 test. For the crack extension history illustrated in Fig. 3 (solid line), the polynomial curve fitting program was tested with fourth and second order as well as linear polynomial fits. The linear fit was also applied in two segments taking into account that a stress wave

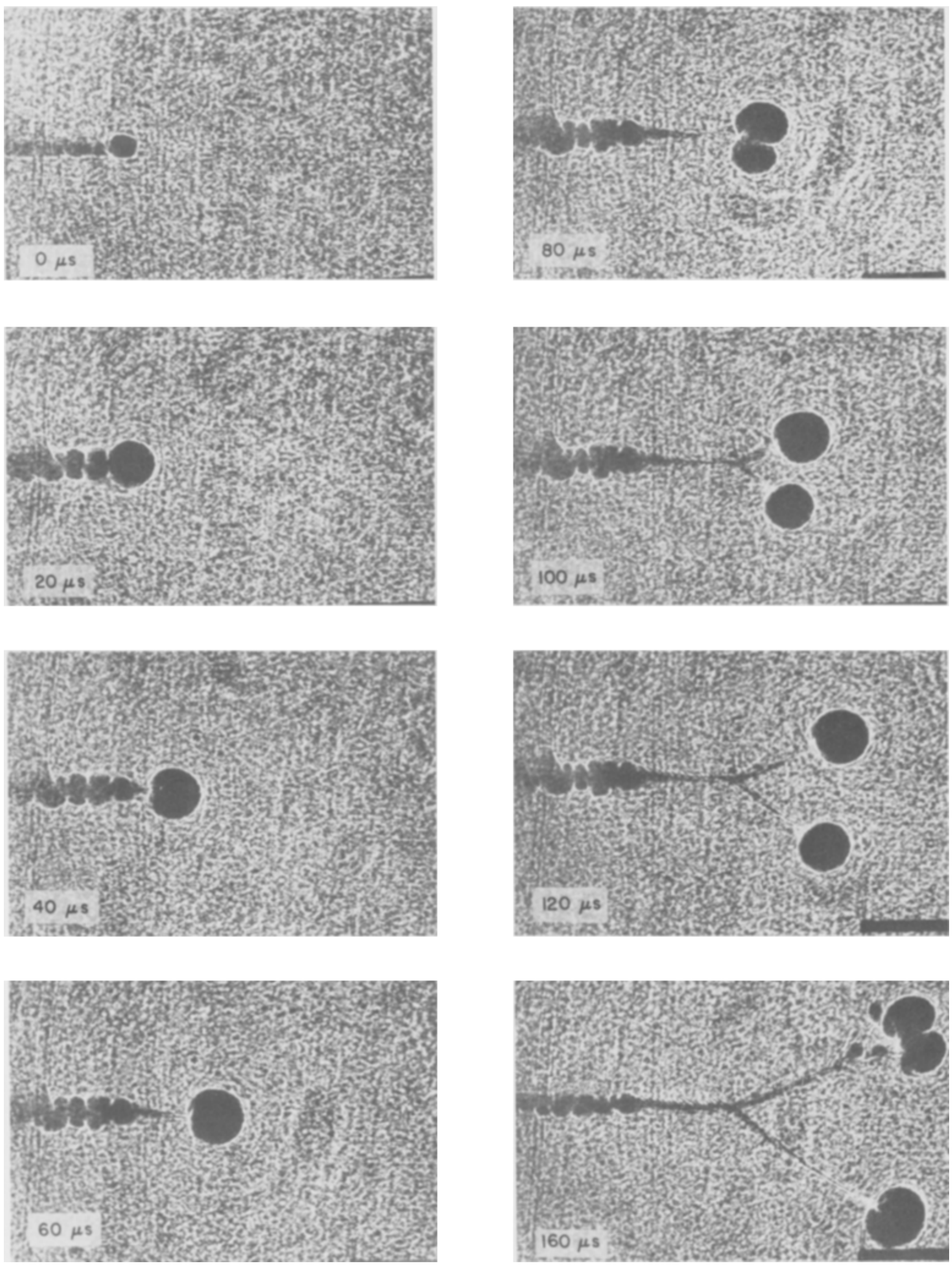

Figure 1. High speed cinematograph of crack propagation and branching.

interaction occurred at about 140 μsec. The two-segment, straight line provided the best estimate of the actual crack extension history. In all cases not involving arrested cracks or wave interactions, one straight line segment definitely provided the best data fit. When there was either crack arrest and reinitiation or stress wave interactions, these events had to be taken into consideration separately.

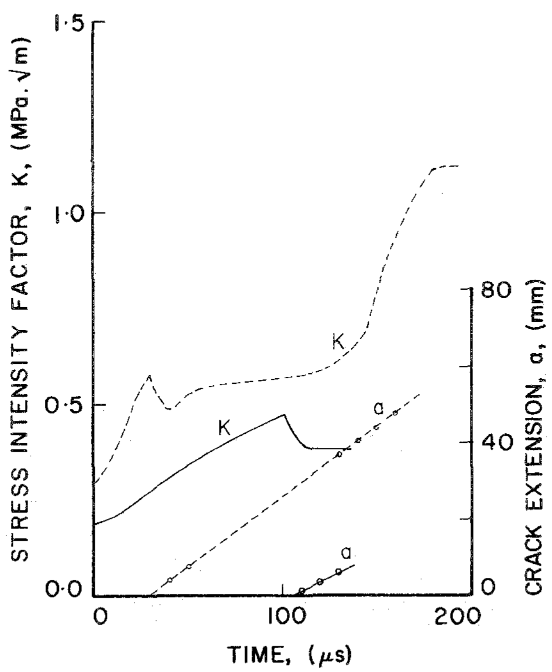

Figure 2. Stress intensity factor and crack extension history.

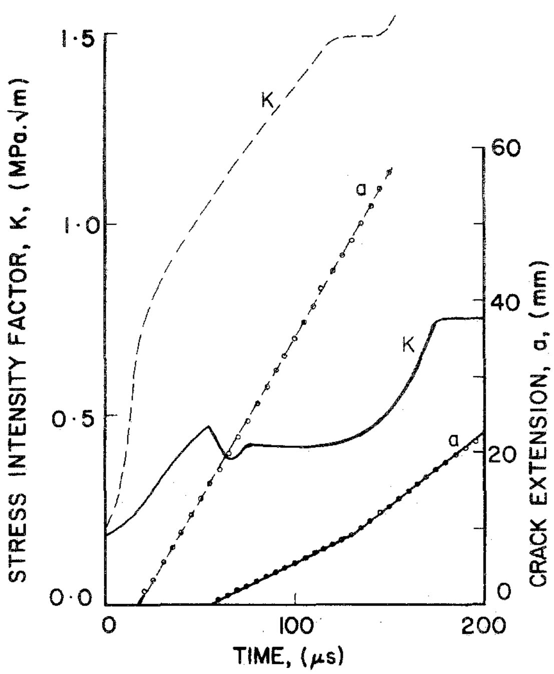

Figure 3. Stress intensity factor and crack extension history.

Thus these investigations lead to the following conclusions:

1. The crack velocity remains constant, and independently so of whether the stress intensity factor decreases, remains constant or increases – provided changes in the stress intensity factor did not occur due to rapid wave interaction.
2. The velocity with which the crack propagates is determined by the stress intensity factor at initiation.

Let us plot these data, as seems customary in dynamic fracture mechanics, on a graph of stress intensity factor versus (instantaneous) crack velocity. The record shown in Fig. 5 results. Also shown on this plot is the stress intensity factor-velocity plot as suggested by Dally and co-workers [5] who performed dynamic fracture experiments on Homalite-100 specimen using various geometrical configurations; the results of [5] are intended to suggest a unique relationship between the instantaneous stress intensity factor and the crack velocity. In contrast, the present experimental data reveal a definite lack of a one-to-one relation between the instantaneous stress intensity factor and the crack velocity because the crack travels at a constant velocity while the stress intensity factor varies.

The microscopic view of the crack propagation process outlined in [2] provides a possibly new way of interpreting the dynamic crack propagation problem and explains, temporarily at least, in a qualitative manner some of the salient features of crack growth. For purposes of continuity in presentation, we summarize here the important conclusions from the earlier papers:

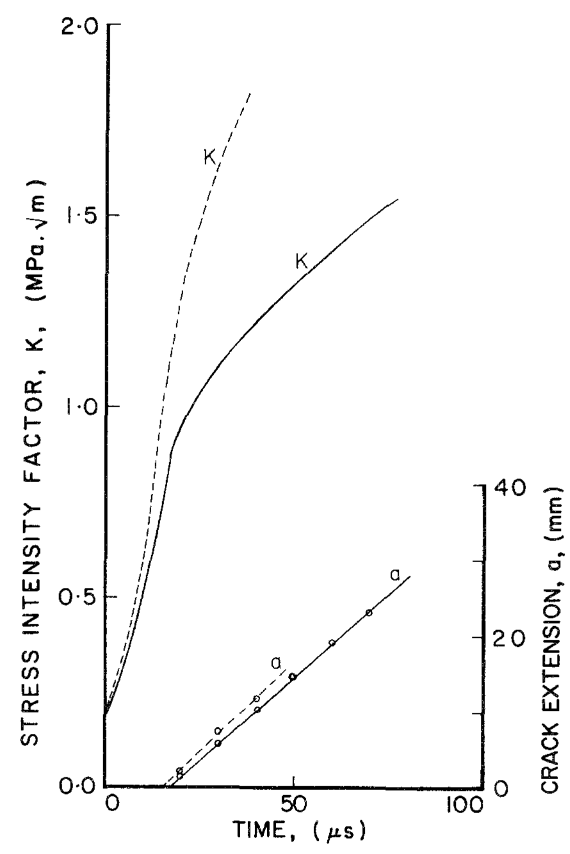

Figure 4. Stress intensity factor and crack extension history.

1. Crack propagation occurs by the linking up of many microcracks.
2. The number of growing microcracks that are activated is a function of the stress intensity factor and the distribution of voids in the material itself.
3. The size of the fracture process zone increases as the stress intensity factor increases (cf. Fig. 9 of [2]).
4. As the fracture process zone grows in size, the number of potential microcracks contributing to the fracture process zone increases statistically. Thus the probability of stimulating flaws into growth increases, thereby increasing the energy that is dissipated in the fracture process zone.
5. Also, as the number of initiated microcracks increases, the interaction of these microcracks becomes important in determining crack propagation behavior. An exact calculation of the nature of the interaction is very difficult and beyond the scope of this work.

We now apply these observations to suggest an explanation for the experimentally noted constancy of velocity of crack propagation. As the stress intensity factor increases prior to crack initiation, the high stresses ahead of the crack tip induce microfractures. The value of the stress intensity factor at initiation then establishes the size and general extent and geometry of the microcracks – that is the fracture process zone – at initiation. Therefore, the macrocrack propagates due to the interaction of and communication between these microcracks in the fracture zone. The nature of the subsequent interaction

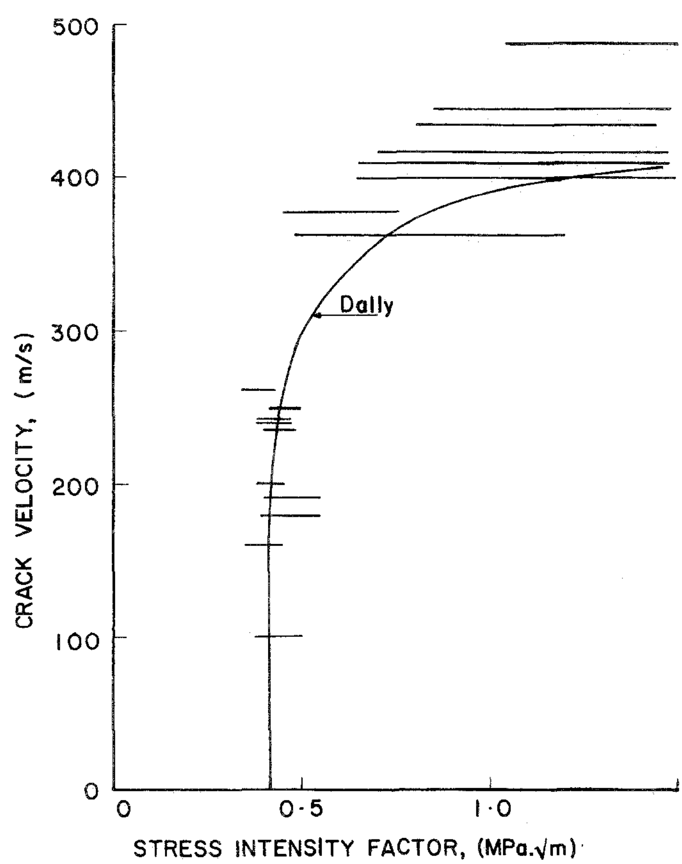

Figure 5. Crack velocity versus instantaneous stress intensity factor in Homalite-100.

is apparently determined by the size of the zone that existed when the zone started to propagate. With this viewpoint, it seems reasonable to expect the stress intensity factor at crack initiation to set the limit on the velocity with which the crack will travel subsequently, until new conditions arise through interaction with waves that produce a different microcrack geometry of the fracture process zone.

While gradual changes in the stress intensity factor do not seem to affect the velocity of crack propagation, the geometry and nature of the fracture process zone are apparently altered drastically by a large transient stress pulse so that a change in the propagation velocity is made possible. From Fig. 3 (solid line), it is seen that the crack travels at a constant velocity of 240 m/sec prior to the arrival of the reflected waves at about 150 μsec. The arrival of the waves reflected from the boundaries causes the stress intensity factor to increase as in Fig. 3 (solid line), and the crack velocity changes – within the resolution of the experiments – immediately to a new and constant velocity of 350 m/sec. On the other hand, from Fig. 2 (dotted line), it can be seen that the crack velocity remains constant at 363 m/sec even after the arrival of the waves reflected from the specimen boundaries, although the stress intensity factor increases. This behavior was observed consistently and leads to the conclusion that cracks travelling at low velocities, typically below 300 m/sec, can change their velocity of propagation upon encountering stress waves. If the interaction with the stress wave occurs when the crack velocity is above 300 m/sec, the crack velocity does not change, or branching occurs.

2.2. Terminal velocity of crack propagation

Using linear elastodynamic theory Freund calculated [6] the energy flux into the crack tip region. These computations show that when the energy required to create new surfaces, Γ, is taken to be a constant, the limiting crack velocity is the Rayleigh wave velocity (C_r). On purely analytical grounds, one would expect that the Rayleigh wave velocity would set the limit for crack propagation velocity since the energy input into the crack tip is through surface waves that travel with the Rayleigh wave velocity. The experimentally observed crack velocities in all kinds of materials are always significantly lower than the Rayleigh surface wave velocity. This observation is repeated in the present experiments where the maximum observed crack velocity was 487 m/sec, ($0.45C_r$). A survey of the velocity of propagation of cracks in various media is given in Table 1.

There have been a number of hypotheses put forward in order to explain the lower observed terminal velocities. It has been suggested that under increasing loads the phenomenon of crack branching intervenes and inhibits further increases in the crack propagation velocity. The implication is that if branching were suppressed, cracks might then travel at or close to the Rayleigh wave velocity. Once again, experimental data seem to contradict this viewpoint. When the applied stress is increased considerably, as indicated in Figs. 2, 3 and 4, the velocity does not increase above $0.45C_r$, even during the phase of propagation when the velocity is constant and when there is no evidence of crack branching at all. On this basis, one would have to assert the existence of a terminal velocity other than the Rayleigh wave velocity.

Along different lines of argument, it has been suggested that the experimentally observed low terminal velocity can be "explained" by allowing the energy required to create a new surface Γ to be rate dependent, namely, $\Gamma = \Gamma(V)$. If Γ would indeed vary with the velocity, it is conceivable that a limiting velocity other than the Rayleigh wave velocity may be obtained as suggested by Rose [7]. While this assumption might "explain" the existence of a lower terminal velocity, it meets with difficulties when one considers that the cracks in our experiments travel with a constant velocity in spite of variations in the stress intensity factor (increasing or decreasing). Computation of the energy flux into

Table 1. Survey of brittle crack velocities

Material	Author	ν	V/C_d	V/C_s	V/C_o	V/C_r
Glass	Bowden	0.22	**0.27**	0.42	0.29	**0.51**
	Edgerton	0.22	0.26	0.43	**0.28**	0.47
	Schardin	0.22	0.28	0.47	**0.30**	0.52
	Anthony	0.22	0.36	**0.60**	0.39	0.66
Plexiglas	Cotterell	0.35	**0.26**	0.54	0.33	0.58
	Paxson	0.35	0.28	0.58	**0.36**	0.62
	Dulaney	0.35	0.28	0.28	**0.36**	0.62
Homalite-100	Beebe	0.31	0.16	0.31	**0.19**	0.33
	Kobayashi	0.345	**0.17**	0.35	0.22	0.38
	Dally	0.31	**0.22**	0.35	0.24	0.38
	Smith	0.31	**0.22**	**0.38**	**0.25**	**0.41**

Note: Bold face numbers indicate values obtained from the references. Other values were calculated from Poisson's ratio shown in column 3.

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the region around an idealized crack tip in a brittle solid leads to the relation [6]:

$$\frac{E}{K^2}=\frac{g(V)}{\Gamma(V)} \tag{3}$$

where E is the modulus of elasticity, K is the stress intensity factor and $g(V)$ is a universal function of the velocity. Thus this energy equation predicts that the velocity of crack propagation should change under varying stress intensity factor situations. However, the stress intensity factor and crack extension histories presented in Figs. 2, 3 and 4 indicate that in spite of considerable changes (factors on the order of 2) in the stress intensity factor, the velocity remains constant. Thus if Γ is allowed to be a function of crack velocity, it creates a contradiction in explaining the constancy of crack velocity. The causes for the existence of a terminal velocity lower than the Rayleigh wave velocity has to be explained then in a different way. We suggest that a microcrack interpretation of the crack propagation process yields a satisfactory explanation.

If the crack propagation process is viewed as an ensemble propagation of many microcracks in a fracture process zone [2], then the speed of crack propagation must be governed by the nature of interaction between these microcracks. There is a finite time involved in this interaction process due to the communication of the stress fields in neighboring microcracks via stress waves. This crack interaction time is at least one source of the limitation on the maximum crack velocity because the time scale of interaction is determined by the number, size and separation distances of the individual microfracture sites and would be expected to depend on the particular material. Secondly, the micro-crack interaction is three dimensional in nature and the growth of these cracks need not always be in the direction of the main crack growth, but microcracks can also grow perpendicular to the direction of growth of the main crack front. These two factors together may contribute to, or explain why the observed maximum crack propagation velocity is substantially lower than the Rayleigh velocity.

Since microcrack growth is proposed here as the main cause of the (lower) limiting velocity, it would appear that in materials where the fracture process occurs through other mechanisms like cleavage along "weak" planes, in the absence of microcracking, the limiting velocities could approach the Rayleigh wave velocity. Experimental measurements on crack speeds in crystals [9], where cleavage dominates, indicate that cracks do indeed propagate at much higher fractions of the Rayleigh wave speed, typically around 0.8 C_r to 0.9 C_r, as compared to about 0.5 C_r in amorphous materials.

3. Crack branching

We now turn to the phenomenon of crack branching. By way of introduction we review first the literature on crack branching and then propose a mechanism for crack branching that is derived from the microscopic investigation outlined in [2] and is consistent with the crack propagation behavior just discussed.

3.1. Review of literature on crack branching

Branching of cracks in glass was recorded by Schardin [8] and other investigators have observed crack branching in crystalline as well as amorphous materials [8–11]. This phenomenon has not been explained by analytical means although notable attempts have been made by Achenbach [12], Burgers and Dempsey [13] and Burgers [14]. This lack of corroboration with experiments is due in part to the fact that in analyzing crack growth problems, the equations of linear elasticity are solved independently of the (non-linear) constitutive behavior of the material at the crack tip and by prescribing the crack path. If the equations of elasticity could be solved together with the details of microcracking as well as the localized non-linear constitutive behavior without a priori assumptions regarding the nature of crack motion, the solutions would reveal some additional or different features. Solutions of such general form are not in sight even for two dimensional problems.

Yoffe [15] attempted to explain the branching of cracks from an analysis of the problem of a crack of constant length that translates with a constant velocity in an unbounded medium. From this solution she found that the maximum $\sigma_{\theta\theta}$ stress acted normal to lines that make an angle of $60°$ with the direction of crack propagation when the crack velocity exceeded 60% of the shear wave speed. Therefore, she suggested that this fact might cause the crack to branch whenever the crack velocity exceeded that value. There are three major shortcomings with this argument. First, as pointed out by Baker [16], the $\sigma_{\theta\theta}$ stress is not a principal stress and in a (quasi)-brittle material it is reasonable to expect the crack to propagate perpendicular to the direction of the maximum principal stress. Second, the angle of $\pm 60°$ for crack branching by the Yoffe argument is large compared to the experimentally observed crack branching angles which range from $\pm 10°$ to $\pm 45°$. Finally, the velocity that is required for the realignment of the $\sigma_{\theta\theta}$ stress is considerably larger than the velocities at which crack branching is observed experimentally in many materials †. We conclude that the Yoffe argument, while very attractive due to its simplicity, is not likely to explain crack branching.

† In fact, for the velocities at which crack branching has been observed, there is very little realignment in the stress field that is so essential to the Yoffe argument.

Eshelby [17] approached the branching problem from the viewpoint of energy balance. It is known that the dynamic energy release rate, G, can be expressed as

$$G = G^* g(V), \tag{4}$$

where G^* is dependent on the geometry and loading and $g(V)$ is the velocity dependent function encountered in Eqn. (3), which may be approximated by [7]:

$$g(V) = 1 - (V/C_r) \tag{5}$$

Eshelby suggested that while the velocity factor $g(V)$ may change in a discontinuous manner, the factor G^* should be continuous at branching, at least for small angles of crack branching. Since after branching twice as much surface area is being created, the crack would not advance unless the velocity factor $g(V)$ could be doubled with a corresponding reduction in crack speed. From Eqn. (5), one deduces that in order for the factor $g(V)$ to double, the velocity before branching must be greater than $0.5C_r$ and it must drop upon branching. Although the main idea that the availability of sufficient energy at the crack tip is clearly a necessary condition, the conclusions about the discontinuous velocity behavior of the branched crack are not borne out by experiment in that cracks propagate hardly with speeds in excess of half the Rayleigh wave speed. Moreover, within the experimental resolution of our experiments on Homalite-100 the crack travels at a constant velocity prior to and after crack branching, with no apparent change in the velocity. The above energy argument is devoid of any mechanism by which the crack branching is achieved and merely poses a necessary condition on the energy release rate that must be satisfied if crack branching were to occur in the idealized manner.

Another attempt at making crack branching plausible rests on the stress wave hypothesis [18]. It is known that when stress waves interact with propagating cracks, the crack path may change abruptly. When the interaction is weak, the well known phenomenon of Wallner lines is observed. In this connection it has been suggested that when the interaction is strong, i.e. when the amplitude of the impinging stress wave is high, the resulting change in the crack path might lead to crack branching. However, this argument is also devoid of any specific mechanism by which crack branching is to be achieved. By contrast, the present experimental data show clearly that crack branching can occur without any interaction with stress waves, a situation which the stress wave hypothesis does not address. A detailed investigation of the effect of stress waves on propagating cracks will be presented in the fourth paper in this series [3].

To date, published analytical attempts at clarifying crack branching have all been directed to seek a necessary condition for branching through a comparison of the stress states prior to and after branching [12]. The stress state after branching is obtained via the following assumptions: a fast running crack comes to rest abruptly. In so doing it radiates an appropriate stress field in which two branches emanate instantaneously from the stopped crack tip at some arbitrary angles, but symmetically with respect to the original crack. More recently, Burgers and Dempsey [13] and Burgers [14] have analysed both mode III and mode I problems. However, no criterion based on the analytical solutions has yet been proposed for crack branching. Once again, this kind of analytical treatment does not provide us with a physical mechanism by which crack branching is achieved.

All the above "explanations" of branching address idealized two-dimensional situations. A common feature among all of these is that they assume a mathematically sharp plane crack propagating in a two dimensional solid which branches into two distinct, sharp, plane cracks. In a more physical vein, Congleton [19] tried to estimate the stress intensity factor at branching by considering a small Griffith crack placed ahead of the main crack. This idea clearly calls for microcracks ahead of the main crack and represents, to our knowledge, the only attempt at explaining dynamic branching † by considering what we perceive to be the real physical process of fracture. We turn next to consider the problem of crack branching in more detail.

3.2. Crack branching mechanism

As pointed out in [2], and summarized in Section 2.1, the mathematical model of a sharp crack propagating along a plane seems to be an unacceptable view for the real propagation process. The latter is essentially three dimensional in nature and governed by the microscopic phenomenon of microcrack interaction. From the microscopic model of the crack propagation process, together with direct observation, one deduces a mechanism for crack branching, which shows crack branching to be a process occasioned by the continuous interaction of the microcracks rather than an event that occurs when some critical state is reached.

Before proposing a mechanism for crack branching, let us summarize the main observations on branching from [2]. From both post-mortem and real-time examination of the branching phenomenon we observe that:

1. crack branching is the result of many interacting microcracks or microbranches;
2. only a few of the microbranches grow larger while the rest are arrested;
3. the branches evolve from the microcracks which are initially parallel to the main crack, but deviate smoothly from the original crack orientation;

† Congleton's work should not be confused with efforts to predict crack angling or kinking in quasistatic mixed mode problems.

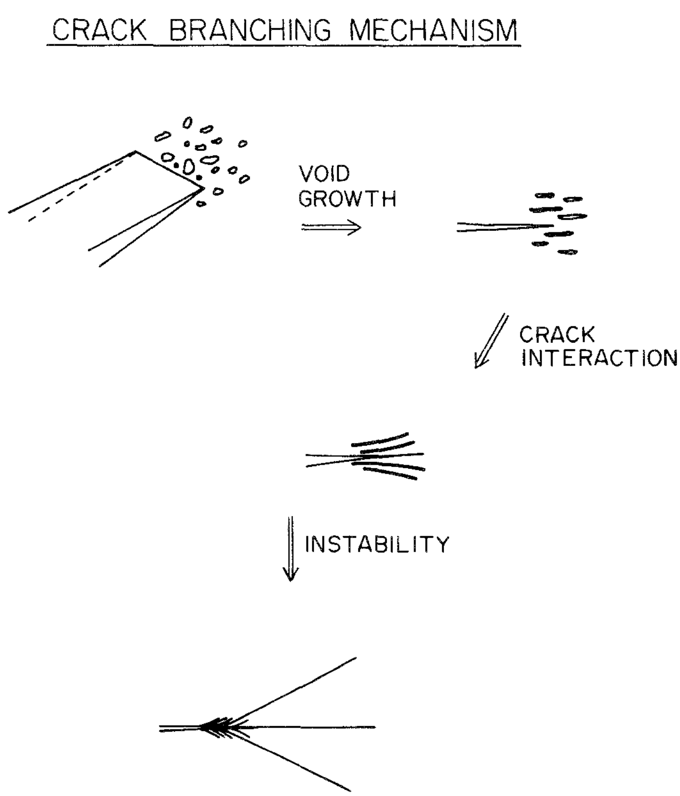

Figure 6. Crack branching mechanism.

4. the microbranches do not span the thickness of the plate, some occurring on the faces of the plate while others are entirely embedded in the interior of the plate.

These observations lead us to propose the following crack branching mechanism (illustrated in Fig. 6). Initially, a crack propagates at low stress intensity factor in a material that contains a number of voids. The crack cuts through these voids and some of the voids have the effect of diverting the crack to propagate along different planes [2]. When the stress intensity factor becomes sufficiently high the voids or other material flaws start to grow themselves into microcracks ahead of the major crack front, which now becomes really an ensemble crack front. The course of further crack propagation and its branching behavior is governed by the details of the interaction of these microcracks. The analytic solution to the problem of dynamic interaction of even the simplest multiple crack geometries is very difficult. However, there exist solutions to the static interaction problems and we shall draw upon these results to assess the nature of non-colinear crack propagation. For the present purpose we assume that the quasi-static solution provides a good insight into the dynamic interaction problem because the experimentally observed crack velocities are small enough to minimize the effect of inertia.

The solution to the problem of two static Griffith cracks interacting in a plane was provided by Pucik [20], who considered the crack centers to be located in a uniaxially tensioned plane at arbitrary points and the cracks themselves to be oriented at arbitrary angles with respect to one another, the uniform load at infinity being P. As part of the analysis, Pucik computed the maximum $\sigma_{\theta\theta}$ stress in the immediate crack tip vicinity and also the orientation along which this stress acted. For the case of two parallel cracks of equal length $2l$, the variation of the angle β at which the maximum $\sigma_{\theta\theta}$ acts is plotted in Fig. 7 as a function of the vertical separation Y_{12} and the horizontal separation X_{12} between the two cracks. From this plot one verifies that, depending on the proximity of the cracks, the two cracks may either "attract" or "repel" one another, that is, they either grow toward or away from one another. In particular, when Y_{12}/X_{12} approaches zero, we see from curve A in Fig. 7 that β increases, indicating repelling cracks. A similar solution to interacting cracks has been provided by Yokobori [21] which exhibits also this behavior

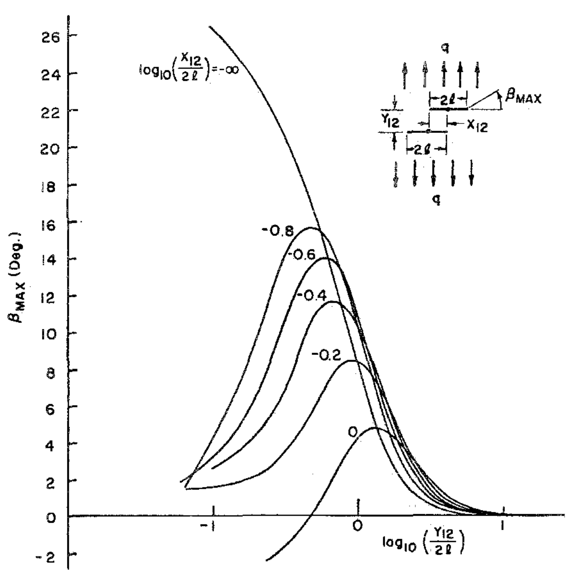

Figure 7. Variation of the angle at which the maximum $\sigma_{\theta\theta}$ acts. From Ref. [20].

of attracting or repelling cracks depending on the separation and the sizes of the cracks.

With this information in mind, let us now return to the crack branching problem. In this case there are several microcracks that interact with one another, and under certain conditions, the small cracks may deviate from the main crack plane. Here we note that while the quasi-static interaction solutions indicate a sharp deviation from the original propagation direction, one observes of course, in the real crack branching problem, a smooth deviation from the main propagation plane. We can account for this difference by considering the fact that the microcracks interact with one another nearly continuously through stress waves, unlike the quasi-static case in which the cracks "know" of each other instantaneously. Also, in our dynamic branching problem, we deal with a number of microcracks that interact with one another simultaneously, making the interaction more complex than in the quasi-static case treated in [20]. Thus, we have to consider the quasi-static solution as giving qualitative support to the argument of deviation of cracks due to interaction. This kind of continuous deviation of the microcracks from the major propagation plane is evident from the high speed photomicrographs of the branching process [2]. Once a number of microbranches (most of them only part-through) have been established, one would expect them to communicate through stress waves. If all of them were of equal size, geometrical statistical instability would permit only a few of the microbranches to propagate further.

In reality, the microbranches vary in size and thus the larger ones are most likely to develop into full fledged branches, while the others are arrested as a result of dynamic interaction with the growing ones. This observation points to statistical effects in branching which are indeed evident in our measurements. For example, we show in Fig. 8 the results of five tests on identical geometries subjected to identical load histories. We note that the location of the "point of branching" has a variation of only ± 1 mm. However, the details of the branch evolution vary considerably. In particular one observes that the number of successful branches varies between 2 and 4. If one considers also the unsuccessful branches the variation in the number of branches would be even greater. We see these features as evidence of the statistical nature of the distribution in size, number and location of microcracks.

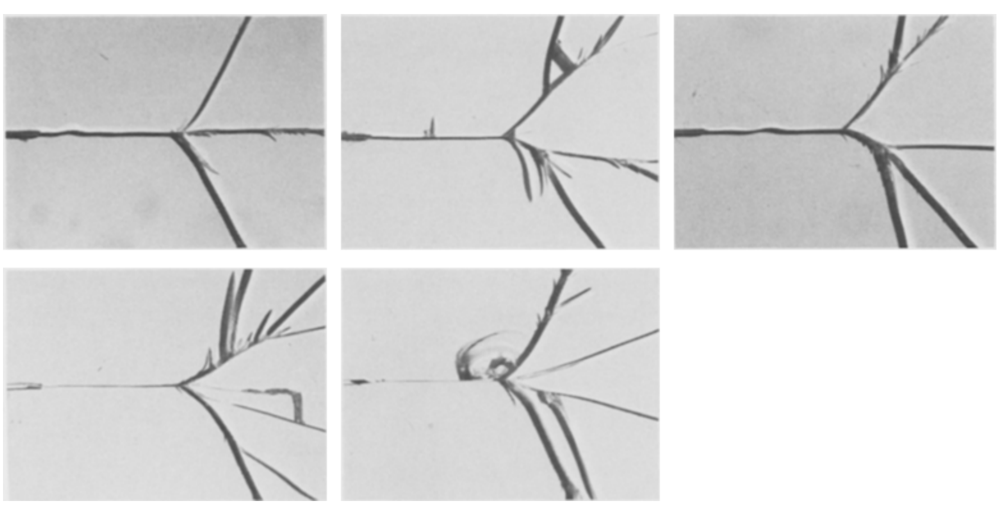

Figure 8. Appearance of branches in five identical tests ($K = 0.65$ MPa$\sqrt{m}$ at initiation, crack speed 400 m/sec).

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