Configuration Factor for Radiation in a Tunnel or Partial Cylinder

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Abstract

The configuration factor for thermal radiation from a section of the ceiling of a tunnel or the inner surface of a partial cylinder to a point on the axis has been derived. Calculations indicate the possibility of high incident radiative heat fluxes from the ceiling to the center. Configuration factors have been calculated for a column furnace used for fire resistance testing.

Introduction

Concern about the risks in tunnels has become greater recently, in particular because of the construction of the tunnel under the English Channel.¹ This concern has prompted considerations of the fire risk in such cases. As a part of this, an expression for the configuration factor for radiative heat transfer from a section of tunnel ceiling to a point on the axis of the associated cylinder has been derived and is given below. The configuration factor derived here should be clearly distinguished from an integrated configuration factor which might be calculated for radiative heat transfer between two finite surfaces.

Configuration Factor

An expression may be derived for the configuration factor from a section of the ceiling of a tunnel to an infinitesimal area on the horizontal plane through the axis of the cylinder associated with the tunnel section, at one end (see Figure 1).

Here, "ceiling" is taken to mean the upper surface of the tunnel, above the horizontal plane through the axis. The tunnel is of radius R, and the distance from the end of the tunnel section to an elemental area of ceiling is x. The distance from the elemental area, $R \,\delta\theta \,\delta x$, to the receiving infinitesimal area under consideration is ρ . The configuration factor in general terms is given in several texts.² Here, the configuration factor from the ceiling of a section of tunnel of length, L, subtending an angle Θ (that is, the maximum of the integration variable, θ , as indicated in Figure 1) is given by:

$$\Phi_{\Theta} = \frac{1}{\pi} \int_{x=o}^{x=L} \int_{\theta=o}^{\theta=\Theta} \frac{Cos\beta Cos\alpha Rd\theta dx}{\rho^2}$$
(1)



Figure 1. The tunnel arrangement assumed.

The integrations are to be carried out from $x = \theta = 0$ to the maximum values of each—that is, L and Θ , respectively.

The cosine terms may be replaced by:

$$Cos\alpha = (RCos\theta) / \rho_{and} \quad Cos\beta = R / \rho$$
 (2)

Integrating over θ , this gives:

$$\Phi_{\Theta} = \frac{Sin\theta}{\pi} \int_{o}^{L} \frac{R^{3}dx}{\rho^{4}}$$
(3)

Replacing ρ^2 by $x^2 + R^2$ and integrating over x from 0 to L results in the expression:¹³

$$\Phi_{\Theta} = \frac{Sin\theta}{2\pi} \left\{ \frac{G}{1+G^2} + \tan^{-1}(G) \right\}$$
(4)

where G = L/R.

For the entire quadrant (that is, where $\Theta = \pi/2$) Equation (4) becomes:

$$\Phi = \frac{1}{2\pi} \left\{ \frac{G}{1+G^2} + \tan^{-1}(G) \right\}$$
(5)



Figure 2a. Variation of the configuration factor, Φ , with the geometrical ratio, G.

The configuration factor depends not on the absolute values of the tunnel radius, R, and the tunnel section length, L, but on the geometrical ratio, G=L/R.

Variation of Φ with G is given in Figure 2a. This figure shows that, for a tunnel of usual length (that is, L > 1.5R) the configuration factor is high and typically more than 90% of its maximum possible value of 0.25. Adding the quadrant that corresponds to $\theta = 0$ to $-\pi/2$, the configuration factor would be double this. If a mirror image section of tunnel were also considered, the maximum possible configuration factor to the center of the entire section would be 1.

Figure 2a shows the effect of varying the length of the tunnel section at a given radius. In order to see the effect of varying the tunnel's radius at a fixed section length, the configuration factor Φ has been plotted against 1/G (that is, R/L) in Figure 2b. This shows that Φ declines quite rapidly with increasing radius for a given tunnel section length.



Figure 2b. Variation of the configuration factor, $\Phi,$ with the Inverse of the geometrical ratio, 1/G.

The variation of Φ_{Θ} with Θ , for given values of *G*, is given in Table 1. It should be stressed that Equations (4) and (5) give the configuration factor for radiation from a finite area to an infinitesimal element and may be used to calculate incident radiant flux at a point. This should not be confused with an integrated configuration factor, which may be used to calculate radiation from a finite to a finite area.⁴

TABLE 1

Values of Configuration Factor $\Phi_{_{\Theta}}$ for Given Values of G and Θ from Equation (4)

	G Ø degs.	0.2	0.4	0.6	0.8	1.0	2.0	3.0	4.0	5.0
	90	0.062	0.115	0.156	0.185	0.205	0.240	0.247	0.248	0.249
	75	0.060	0.112	0.151	0.179	0.198	0.232	0.238	0.240	0.241
	60	0.054	0.100	0.135	0.160	0.177	0.208	0.214	0.215	0.216
	45	0.044	0.082	0.110	0.131	0.145	0.170	0.174	0.176	0.176
	30	0.031	0.058	0.078	0.093	0.102	0.120	0.123	0.124	0.125
and a second sec	15	0.016	0.030	0.040	0.048	0.053	0.062	0.064	0.064	0.064
					l					

British Standard Fire Resistance Test

The expressions derived for the configuration factor, as given by Equations (4) and (5), are valid for the case of a partial cylinder in general, not just for a tunnel. They may therefore be used, for example, to calculate the configuration factor for radiation from the cylindrical lining of a vertical furnace to the axis of the associated cylinder. Such a furnace is used as part of the British Standard fire resistance test for columns, as specified in BS 476:Part 20;1987.⁵ The Standard specifies that the length of the exposed face of the specimen and the face of the furnace lining immediately opposite the specimen be not less than 0.6 m and not more than 1.3 m. That is, the radius of the furnace is not specified uniquely by the standard.

In the column furnace used at the testing center of the Loss Prevention Council at Borehamwood in the U.K., the radius of the furnace has been stated⁶ to be approximately 0.69 m and the furnace length to be approximately 3.44 m. It has also been said that the furnace is approximately a semi-cylinder. Using R=0.69 and L=3.44/2=1.72, Equation (5) produces a value of 0.244 as the configuration factor for the quadrant to the axis at one end. Considering the semi-circle and the entire length of the furnace, the configuration factor to the center of the furnace on the axis is 0.976.

If a point on the axis slightly away from one end of the furnace is considered, one may assume, say, R=0.69 and L=3.4. This produces a value of the configuration factor of 0.498; that is, there is a dramatic fall off in the calculated value. However, this assumes the end of the furnace to be open. In reality, there would be a contribution at such a receiving point from those parts of the furnace beyond the end of the semi-cylindrical section assumed here. This extra contribution would increase the effective configuration factor to a point slightly away from one end.

Overall, one may infer that, if calculations of radiative heat transfer are to be carried out in relation to such a column furnace, it would be important to bear these considerations in mind.

Radiative Heat Flux

The incident radiative heat flux from the ceiling to a point on the axis of a tunnel section at one end is given by:

$$\dot{q}'' = \Phi_{\Theta} \varepsilon_{us} \sigma T^{4}$$
(6)
where
$$\begin{aligned} \Phi_{\Theta} &= \text{ configuration factor of Equation (4)} \\ \varepsilon_{us} &= \text{ emissivity of the upper surface} \\ \sigma &= \text{ Stefan-Boltzman constant} \\ T &= \text{ temperature of the upper surface in degrees Kelvin} \\ \dot{q}'' &= \text{ radiative heat flux per unit area} \end{aligned}$$

Two calculations are given below as examples. These afford some idea of the contribution to the incident heat flux which may be expected.

Hot Upper Surface

Consider the case with G=0.75, T=600 deg K and $\mathcal{E}_{us} = 1$.

For a tunnel of 4 m radius, which is the approximate radius of the tunnel under the English Channel, this is equivalent to a heated section of tunnel 3 m long. Consider a section of ceiling with $\Theta = 60$ degrees. The configuration factor to a point on the axis at the end of the heated section is given by Equation (4), which produces:

$$\Phi_{\Theta} = 0.155$$

This gives a value of $\dot{q}^{"}$ from Equation (6), of:

 $\dot{q}''_{=1.14 \text{kW/m}^2}$

If we consider a point on the axis at the center of a section of tunnel of length 6 m, the incident radiative heat flux would be four times this value, or 4.56 kW/m^2 .

Radiation in a Tunnel or Cylinder

Given that a value of 20 kW/m² has been generally associated with the onset of flashover,⁴ it can be seen that the radiative heat flux from a tunnel ceiling would, overall, be expected to make a significant contribution. Together with heat transfer from other sources, especially from a hot smoke layer, the total incident heat flux could be very high. It should also be borne in mind that the above example considers only a 6 m section of tunnel.

Flaming Upper Surface

The second case is that of a flaming ceiling. This might be relevant should the surface material ignite. The pre-conditions for ignition will not be considered here. Instead, it will be assumed that the ceiling is flaming and that the surface of the "flame sheet" has the same configuration factor to a point on the tunnel axis at one end of the flaming section as is given by Equation (5). In this case, the temperature, *T*, may be replaced by a typical flame temperature of, say, 1300 deg K. The radiative hot flux to a point on the axis at one end for a tunnel section of length 3 m and radius 4 m, with Θ =60 degrees, as given by Equation (6), is 25.1 kW/m². For a section of tunnel 6 m long with radius 4 m, the flux to a point on the axis at the center would be four times this value—that is, approximately 100 kW/m².

This calculation suggests the possibility of very high incident fluxes at the center and the likelihood of ignition of a fuel at the center within seconds. For example, fiberboard would be expected to ignite spontaneously within 5 seconds of exposure to a flux of $52 \text{ kW/m}^{2.4}$ In addition, Rasbash has stated that a heat flux of 100 kW/m^2 would typically be expected to ignite a material about 25 times faster than a flux of 20 kW/m^{2.7} These considerations are also interesting given Rabash's suggestion that direct radiative heat transfer from a large body of flame to a non-burning fuel item may be an important element in bringing about flashover and, in some cases, may be dominant.⁷

Radiation and Flashover

We do not address the question of a general definition of flashover in a tunnel. For the purposes of this discussion, we consider flashover to be a transition to generalized burning within the section of tunnel associated with the heated ceiling.

In addition to the radiation from the ceiling of a tunnel to a fuel at its center, there would be radiation from other sources, particularly the hot smoke layer which would be expected to form above a fire. While being a source of radiation to the center, the smoke layer would also absorb some of the radiation from the ceiling. Unless the absorptivity of the layer were very high, though, one would still expect a significant contribution from the ceiling. Further, radiation from the ceiling would tend to elevate the temperature of a smoke layer, even if it did not pass through the layer to fuel below. There is also the possibility that a smoke layer would only extend a small distance below the top of the ceiling while, lower down, the "sides" of the ceiling would be hot and directly transmit radiation to the center. There also exists the possibility of a

section of the tunnel's side, below the horizontal through the tunnel's center, transmitting radiation to the center of the tunnel. Overall, it would seem that it would be wise not to ignore the radiation from the ceiling of a tunnel to its center.

Conclusion

An expression for the configuration factor for radiative heat transfer from the ceiling of a tunnel to a point on its axis at one end of the tunnel section has been derived. This shows that the geometrical ratio, G, of tunnel section length (L) to the tunnel's radius (R) is the determining factor, rather than the absolute values of L and R as such.

Two simple, illustrative calculations indicate that the incident radiative heat flux from the ceiling of a tunnel could be a significant part of the total heat flux to the center. Further, if a section of the tunnel's ceiling were flaming, very high fluxes might be produced which, in some conditions, might be sufficient to produce flashover within seconds.

The expression for the configuration factor derived is not limited to tunnels. It applies in general for the internal surface of a partial cylinder to the axis of the associated cylinder.

Calculating configuration factors for a furnace used in the fire resistance testing of columns shows the configuration factor to the center of the axis of the associated cylinder to be close to 1 but to fall off rapidly towards the ends, assuming those ends to be open.

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