THE DETERMINATION OF THE CRITICAL SPEEDS OF MIXER SHAFTS IN VISCOUS LIQUIDS

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In order to calculate how far a stiff high-speed mixer shaft is from resonance, at its operational angular velocity ω, it is necessary to calculate the first critical velocity ω_0.

For this purpose the authors of the present paper derived equations which take into account the design features of shafts and showed the effect of some properties of the substance being mixed on the calculated values of ω_0. These equations, which contain the mass of the shaft itself, are based on paper [1].*

Owing to the complexity of the final equations for determining ω_0, paper [1] suggests that this problem should be solved for each case separately. This is a lengthy process. However, in our case, the stirrer mass can be neglected (with mixer sizes normally used the moment of gyration increases ω_0 by about 3% which improves the stability of a rigid shaft). With this assumption, we obtained final equations for determining ω_0 for the main types of shaft supports (see table). These methods of supporting the shaft are used in the special standard issued by NIIkhimmash (Scientific Research Institute of Chemical Engineering) [3] for the drives of mechanical mixing devices, and are scheduled for use in the general-engineering standard which is now being prepared for vertical mixer drives.

Let us now consider the derivation of equations for determining ω_0; we use, as an example, a single-span constant-cross-section shaft which carries a concentrated stirrer mass and rests on hinged (top) and rigidly clamped (bottom) supports (Fig. 1). Since the concentrated mass M of the stirrer divides the shaft into two parts $0 \leq x \leq a$ and $a \leq x \leq 1$, where $x = x_1/L$; $a = l_1/L$, we can, following [1], write the equation of equilibrium for the shaft in the following form

$$EJ\frac{d^4y}{dx_1^4} - m\,\omega^2 y = F(x_1), \tag{1}$$

where $F(x_1)$ is the additional load due to the concentrated mass of the stirrer. This is given by the mass of the stirrer and the deflection of the shaft at the point where it is clamped; m is the mass of a unit length of shaft in kg/m; ω is the angular velocity of the shaft in rad/sec; E is the modulus of elasticity of the shaft material in N/m^2; and J is the moment of inertia of the shaft cross section in m^4.

After introducing the notation

$$\alpha^4 = \frac{L^4 m\,\omega^2}{EJ}, \tag{2}$$

we rewrite Eq. (1) and obtain:

$$y^{IV} - \alpha^4 y = f(x), \tag{3}$$

*The methods for calculating ω_0, which are used in general engineering practice and do not contain the mass of the shaft [2], produce in our case a considerable error, since the mass of the shaft is usually several times greater than that of the stirrer.

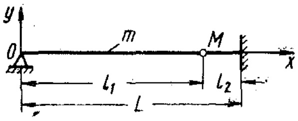


Fig. 1. Shaft loading diagram.

where

$$f(x) = \frac{L^4}{EJ} F\left(\frac{x_1}{L}\right).$$

According to [1] the general solution of Eq. (3) for the first section can be written as follows:

$$y_1 = BT(\alpha x) + DV(\alpha x),$$

and the equation for the second section is

$$y_2 = BT(\alpha x) + DV(\alpha x) + \varphi(\alpha x), \tag{4}$$

where B and D are arbitrary constants: $T(\alpha x)$, $V(\alpha x)$ are Krylov's functions and $\phi(\alpha x)$ is the partial Krylov integral [1, 4].

According to reference [1], we have for the concentrated mass M of the stirrer

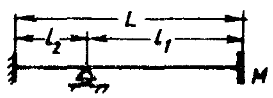
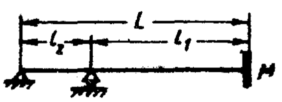
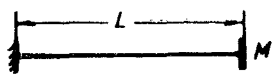
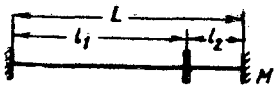
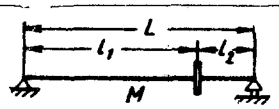
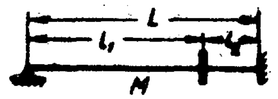
$$\varphi(\alpha x) = \frac{L^3}{\alpha^3 EJ} M\omega^2 y_1(a) V[\alpha(x-a)]$$

and, using the notation

$$k = \frac{M}{mL}$$

we obtain, with Eq. (2), from Eq. (4) the following expression

$$y_2 = BT(\alpha x) + DV(\alpha x) + \alpha k[BT(\alpha a) + DV(\alpha a)] \times V[\alpha(x-a)]$$

No.	Diagram	Equation of the critical speed	No. of the shaft
1		$k = \frac{1}{\alpha} \cdot \frac{D(\alpha b) B(\alpha a) + B(\alpha b) E(\alpha a)}{B(\alpha b) B(\alpha a) - D(\alpha b) S_1(\alpha a)}$	
2		$k = \frac{1}{\alpha} \cdot \frac{S_1(\alpha b) E(\alpha a) - B(\alpha b) B(\alpha a)}{S_1(\alpha b) B(\alpha a) + B(\alpha b) S_1(\alpha a)}$	
3		$k = \frac{1}{\alpha} \cdot \frac{E(\alpha)}{B(\alpha)}$	
4		$k = \frac{1}{\alpha} \cdot \frac{U^2(\alpha) - V(\alpha) T(\alpha)}{U(\alpha b)[V(\alpha) U(\alpha a) - U(\alpha) V(\alpha a)] + V(\alpha b)[T(\alpha) V(\alpha a) - U(\alpha) U(\alpha a)]}$	
5		$k = \frac{1}{\alpha} \cdot \frac{T^2(\alpha) - V^2(\alpha)}{T(\alpha b)[V(\alpha) T(\alpha a) - T(\alpha) V(\alpha a)] + V(\alpha b)[V(\alpha) V(\alpha a) - T(\alpha) T(\alpha a)]}$	
6		$k = \frac{1}{\alpha} \cdot \frac{T(\alpha) U(\alpha) - V(\alpha) S(\alpha)}{U(\alpha b)[V(\alpha) T(\alpha a) - T(\alpha) V(\alpha a)] + V(\alpha b)[S(\alpha) V(\alpha a) - U(\alpha) T(\alpha a)]}$	

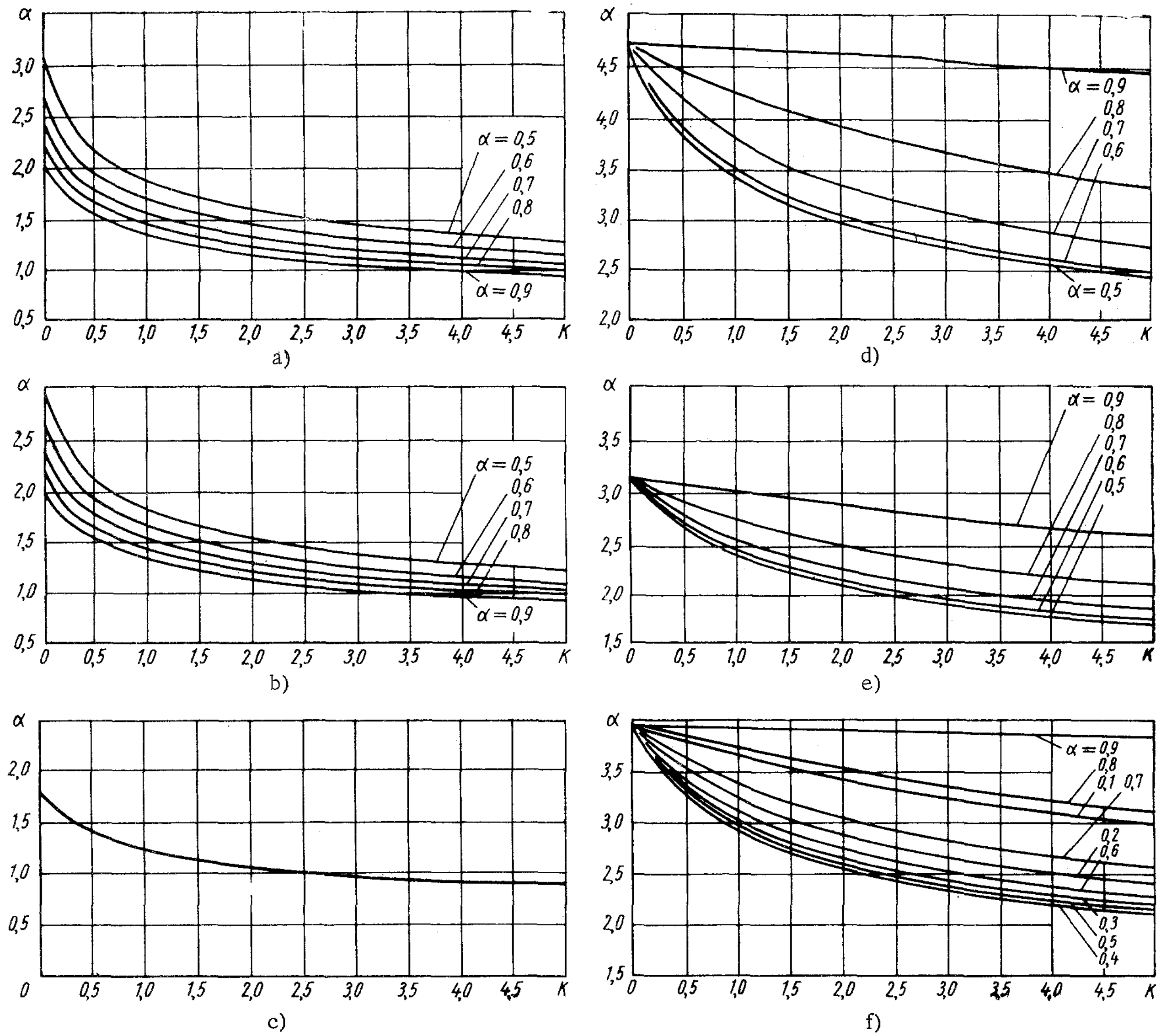


Fig. 2. Dependence of roots α of the equation of critical mixer shaft speeds on k = M/mL for various l_1/L values; shaft arrangements of the table: a-f) for shafts No. 1-6 respectively.

or
$$y_2 = B\{T(\alpha x) + \alpha k T(\alpha a) V[\alpha(x-a)]\} + D\{V(\alpha x) + \alpha k V(\alpha a) V[\alpha(x-a)]\}.$$

The boundary conditions for the bottom support with x = 1 are as follows: $y_2(1) = 0$ and $y_2(1) = 0$ or, in an expanded form*

$$y_2(1) = 0 = B[T(\alpha) + \alpha k T(\alpha a) V(\alpha b)] + D[V(\alpha) + \alpha k V(\alpha a) V(\alpha b)] \tag{5}$$

and

$$y_2'(1) = 0 = B[S(\alpha) + \alpha k T(\alpha a) U(\alpha b)] + D[U(\alpha) + \alpha k V(\alpha a) U(\alpha b)], \tag{6}$$

where

$$b = (1-a) = \frac{l_2}{L}.$$

*It should be remembered that in paper [1], Eq. (26) must be replaced by the second equation of (22) with x = 1 (this was mentioned in paper [5]).

We now obtain from the coefficients of Eqs. (5) and (6) with arbitrary constants the determinant and, after reducing it to zero, also the transcendent equation (shaft No. 6 in the table) which provides the roots α determining the critical speeds of the shaft illustrated in Fig. 1 and in the table (No. 6).

The same method can be used to obtain equations for the other methods of supporting the shaft in bearings given in the table. The following notation has been used in these equations: $S(\alpha)$, $T(\alpha)$, $U(\alpha)$, $V(\alpha)$, $T(\alpha a)$, $V(\alpha a)$, $T(\alpha b)$, $U(\alpha b)$, $V(\alpha b)$ are Krylov's functions of corresponding arguments α, αa, αb tabulated in paper [6] for arguments from 0 to 5 rad: $B(\alpha)$, $E(\alpha)$, $B(\alpha a)$, $E(\alpha a)$, $S_1(\alpha a)$, $B(\alpha b)$, $D(\alpha b)$, $S_1(\alpha b)$ are the functions of arguments α, αa, αb (in Praeger–Hohenemser notation) tabulated in paper [6] for arguments from 0 to 10 rad: they are combinations of circular and hyperbolic functions.

In order to facilitate their use, our equations have been solved for a wide range of k and a values actually used in the design of mixers. These solutions are given in form of graphs in Fig. 2a-f.

It should be pointed out that the transcendent equation for the No. 3 shaft is given in paper [6], for the No. 5 shaft in paper [5], and for the No. 2 shaft in paper [7]. However, paper [7] gives the values of roots α only for a values for which the span l_2 is longer than cantilever l_1, which case only seldom occurs with cantilever mixer shafts. In paper [8], the equation for No. 2 shaft is given in a slightly different form, with the table of roots for $k=0$ (zero concentrated mass), which is also unacceptable for our case.

The substitution of the value of root α from the corresponding curve (Figs. 2a-f) into Eq. (2) produces, for given k and a values, the first critical speed* for any of the shafts shown in the table:

$$\omega_0 = \frac{\alpha^2}{L^2} \sqrt{\frac{EJ}{m}}. \tag{7}$$

After calculating ω_0 from Eq. (7) and with the condition

$$\omega \leqslant 0.7\, \omega_0 \tag{8}$$

satisfied we obtain for a given shaft the diameter of the rigid shaft which is stable with respect to the dangerous bending vibrations.

Thus, the calculation equations for ω_0 obtained in [1] and the corresponding graphs (Figs. 2a-f) relieve the designer from complex mathematical calculations.

Let us now consider the variation of the ω_0 value obtained above during the mixing of a material with certain physical properties (viscosity, density).

The medium in which the shaft is vibrating has a considerable effect on the vibration parameters of the shaft as calculated for a shaft vibrating in air. In particular, the critical speed of the shaft during its rotation in liquid ω_{li}, decreases compared with the critical speed of a shaft rotating in air ω_0. The reason is that the vibration of a solid body in a liquid produces an effect called the effect of the attached mass (as if the mass of the body were increased without affecting its elastic properties [9]).

A theoretical determination of the attached mass of a solid body performing a nonuniform motion in a limited volume of liquid is very complicated [10]. In the case of a shaft with a stirrer of a complex shape (a three-blade propeller, closed turbine) the size of the attached mass and, consequently, the critical speed of the shaft rotating in the liquid, can be obtained only experimentally.

The authors of this paper used a special apparatus to determine the critical speeds of shafts with turbine and propeller stirrers rotating in air and in liquids with different viscosities. Glycerine was used as the working liquid, its viscosity being varied by heating. The temperature had practically no effect on the density of glycerine.

* The transcendent equations of critical speeds have a multitude of roots α_1, α_2, α_3, ..., α_i. There is a definite critical speed corresponding to each value of α_i. However, in the case of stiff shafts considered in [11], only root α_1 is of interest.

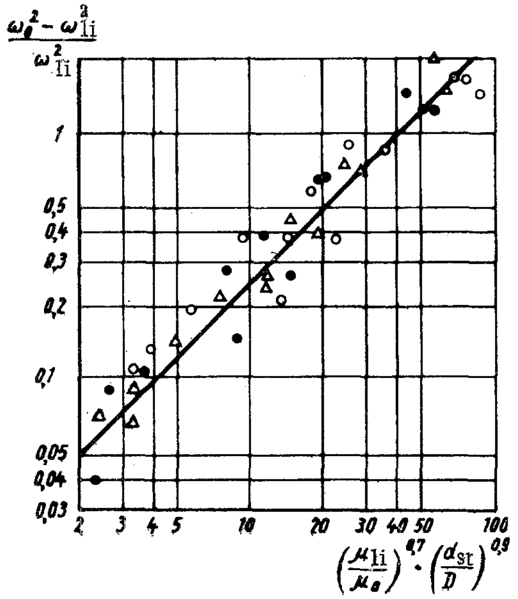


Fig. 3. Relation $\frac{\omega_0^2-\omega_{1i}^2}{\omega_{1i}^2}=f\left(\frac{\mu_{1i}}{\mu_0},\frac{d_м}{D}\right)$ for cantilever shafts of closed turbine mixers: for d_{st} = 250 mm; △) for d_{st} = 200 mm; ●) for d_{st} = 150 mm.

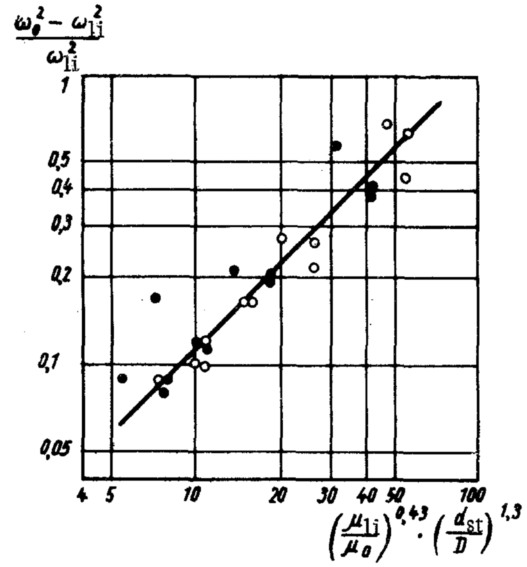


Fig. 4. Relation $\frac{\omega_0^2-\omega_{1i}^2}{\omega_{1i}^2}=f\left(\frac{\mu_{1i}}{\mu_0},\frac{d_м}{D}\right)$ for cantilever shafts of propeller stirrers: ●) for d_{st} = 200 mm; ○) for d_{st} = 150 mm.

The natural frequency of a shaft rotating in air was adopted as the critical speed ω_0 of the shaft rotating in air, while the working angular velocity of the shaft running with maximum dynamic vibration amplitudes was adopted as the critical speed of a shaft rotating in the liquid ω_{1i}. The investigations were carried out on a special experimental apparatus described in paper [11].

The results of investigations of cantilever shafts with turbine and propeller mixers are given in Figs. 3 and 4, where d_{st} is the diameter of the stirrer in m; D is the diameter of the vessel in m; μ_{1i} is the viscosity of the liquid being stirred in N sec/m^2; and μ_0 is the viscosity of water in N sec/m^2.

We use the simplex $\omega_0^2-\omega_{1i}^2/\omega_{1i}^2$ suggested in papers [9, 12, and 13] since it provides information on the attached mass of the mixer and on the reduction of the critical speed of the shaft in the liquid (compared with ω_0 in air); the simplex μ_{1i}/μ_0 is selected on the basis of experiments. These experiments showed that, when working in water, the critical speed ω_{1i} of shafts with propeller or turbine stirrers is only 3-5% lower than ω_0 in air, i.e., for this case it may be assumed that $\omega_{1i}\approx\omega_0$. This relationship has been noted by several investigators, e.g., in [14]; it follows from Eqs. (10) and (12) given below, for $\mu_{1i}=\mu_0$.

During the tests, the initial values varied as follows: μ_{1i} = 0.016-1.4 N sec/m^2; $\rho\approx$ const $\approx$ 1230 kg/m^3; d_{st} = 150-250 mm; D = 500 mm, 650 mm. All results were obtained with a cantilever stainless steel shaft supported by the bearings of a column (No. 2 in the table); its dimensions were as follows: L = 2000 mm, L_1 = 1400-1800 mm, d_{in} = 11.10 mm, and d_{ex} = 17.5 mm.

After mathematical processing of the experimental data, we obtain the following equations:

a) for cantilever shafts of closed turbine mixers

$$\frac{\omega_0^2-\omega_{1i}^2}{\omega_{1i}^2}=0.025\left(\frac{\mu_{1i}}{\mu_0}\right)^{0.7}\left(\frac{d_{st}}{D}\right)^{0.9} \qquad (9)$$

or

$$\omega_{1i} = \frac{\omega_0}{\sqrt{1 + 0.025\left(\frac{\mu_{1i}}{\mu_0}\right)^{0.7}\left(\frac{d_{st}}{D}\right)^{0.9}}}; \tag{10}$$

b) for cantilever shafts of three-blade propeller mixers with a pitch ratio equal to unity,

$$\frac{\omega_0^2 - \omega_{1i}^2}{\omega_{1i}^2} = 0.115\left(\frac{\mu_{1i}}{\mu_0}\right)^{0.43}\left(\frac{d_{st}}{D}\right)^{1.3} \tag{11}$$

or

$$\omega_{1i} = \frac{\omega_0}{\sqrt{1 + 0.115\left(\frac{\mu_{1i}}{\mu_0}\right)^{0.43}\left(\frac{d_{st}}{D}\right)^{1.3}}}. \tag{12}$$

The greatest reduction (about 30%) of the critical speed of a shaft working in a liquid compared with ω_0 was observed at maximum values of viscosity and stirrer diameter. Consequently, for a stable operation of a high-speed mixer shaft in a viscous liquid, expression (8) must be changed to

$$\omega \leqslant 0.7\,\omega_{1i}. \tag{13}$$

These investigations showed that the deviation of experimental values of ω_{1i} from the data calculated from Eqs. (10) and (12) does not exceed 10%. The effect of the density of the liquid ρ_{1i} and of the density of the solid ρ_{so} vibrating in this liquid, has been noted by the introduction of the additional simplex ρ_{1i}/ρ_{so} into the right-hand side of Eqs. (9) and (11). In our investigations for ρ_{1i}/ρ_{so} = const, the effect of this simplex is shown by the constant coefficient of Eqs. (9) and (11).

Thus, after calculating ω_0 from Eq. (7) and the curves of Fig. 2, and determining ω_{1i} from Eqs. (10) and (12), we obtain [with Eq.(13)] that diameter of the cantilever shaft which is stable with respect to bending vibrations in a viscous liquid.

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