Transport in Porous Media **22:** 181–193, 1996.

On Natural Convection from a Vertical Plate with a Prescribed Surface Heat Flux in Porous Media

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(Received: 5 December 1994; 30 June 1995)

Abstract. This paper presents a theoretical and numerical investigation of the natural convection boundary-layer along a vertical surface, which is embedded in a porous medium, when the surface heat flux varies as $(1+x^2)^\mu$, where μ is a constant and x is the distance along the surface. It is shown that for $\mu > -\frac{1}{2}$ the solution develops from a similarity solution which is valid for small values of x to one which is valid for large values of x. However, when $\mu \leqslant -\frac{1}{2}$ no similarity solutions exist for large values of x and it is found that there are two cases to consider, namely $\mu < -\frac{1}{2}$ and $\mu = -\frac{1}{2}$. The wall temperature and the velocity at large distances along the plate are determined for a range of values of μ.

Key words: natural convection, boundary-layers, surface heat flux.

0. Notation

g	Gravitational acceleration
k	Thermal conductivity of the saturated porous medium
K	Permeability of the porous medium
l	Typical streamwise length
q_w	Uniform heat flux on the wall
Ra	Rayleigh number, $= g\beta K(q_w/k)l/(\alpha v)$
T	Temperature
T_∞	Temperature far from the plate
u, v	Components of seepage velocity in the x and y directions
x, y	Cartesian coordinates

Greek Symbols

α	Thermal diffusivity of the fluid saturated porous medium
β	The coefficient of thermal expansion
γ	An undetermined constant
ϕ	Porosity of the porous medium
η	Similarity variable, $= y(1+x^2)^{\mu/3}/x^{1/3}$
μ	A preassigned constant
ν	Kinematic viscosity
θ	Nondimensional temperature, $= (T - T_\infty)\,\mathrm{Ra}^{1/3}k/q_w$
τ	Similarity variable, $= y(\log_e x)^{1/3}/x^{2/3}$
ξ	Similarity variable, $= y/x^{2/3}$
ψ	Stream function

1. Introduction

The subject of natural convection in porous media is of considerable practical and fundamental interest in geophysics, oil recovery, thermal insulation engineering, heat exchangers, ceramic processing and catalytic reactors to name a few. It has been a subject of active research over the past thirty years, and with an ever increasing scope of application it will continue to attract a good deal of attention. An excellent review of the extensive literature in this field, published prior to 1992, has been provided by Nield and Bejan [1] and these papers disclose the prominent characteristics of heat transfer in porous media.

The problem of natural convection flow from a heated vertical surface which is embedded in a porous medium provides probably one of the most fundamental studies in the area of convective flow in porous media. Cheng and Minkowycz [2] were the first to report the existence of similarity solutions for this problem when the surface temperature is proportional to x^λ (where x measures the distance along the plate from the leading edge and λ is a constant). They presented numerical results for values of λ between 0 and 1. The case $\lambda = 0$ corresponds to a uniform surface temperature and $\lambda = -\frac{1}{3}$ to a uniform surface heat flux. These similarity solutions are the leading-order terms in a series expansion for problems where other effects are present, for example in mixed convection [3–6] and higher-order boundary-layer theory [7, 8]. It was shown by Ingham and Brown [9] that the equations governing this flow configuration has a solution which satisfies all of the imposed boundary conditions only if $\lambda > -\frac{1}{2}$, with the solution becoming singular as $\lambda \to -\frac{1}{2}$. However, the question arises as to how does the boundary-layer solution develop if for small values of x the solution was given by a similarity form for which a solution is possible (i.e. $\lambda > -\frac{1}{2}$) but attempts to reach the asymptotic condition at large values of x for which a similarity solution were not possible (i.e. $\lambda \leqslant -\frac{1}{2}$). It is the answer to this problem that motivated the present investigation.

The purpose of this paper is to examine both theoretically and numerically the natural convection flow from a vertical surface which is embedded in a fluid-saturated porous medium, the surface being subject to a nondimensional heat flux of the form, see Merkin and Mahmood [10],

$$\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = -\left(1 + x^2\right)^{\mu}, \tag{1}$$

where θ and y are the nondimensional temperature and the coordinate normal to the plate, respectively, and μ is a preassigned constant. It is seen from expression (1) that, for $x \ll 1, (\partial\theta/\partial y)_{y=0} \simeq -1$, while for $x \gg 1$ we have $(\partial\theta/\partial y)_{y=0} \simeq -x^{2\mu}$. Thus, although it is possible to obtain similarity equations for both small and large values of x, in the latter case the equation possesses a solution only if we take $\mu > -\frac{1}{2}$. For $\mu \leqslant -\frac{1}{2}$ it is impossible to find similarity solutions for large values of x and by numerically solving the full governing partial differential equations, the behaviour of the solution for large values of x is discussed. It is found that there are two cases to consider, namely, $\mu < -\frac{1}{2}$ and $\mu = -\frac{1}{2}$. The details of the wall temperature and the velocity at large distances along the plate are presented for various values of the parameter μ. It is found that the computational results are in good agreement with the analytical predictions.

2. Basic Equations

Consider the steady, free convection flow from a vertical surface embedded in a fluid-saturated porous medium with a prescribed wall heat flux. We assume that the porous medium is isotropic and homogeneous and that the fluid is incompressible. Invoking the Boussinesq–Darcy approximation and assuming that the boundary-layer approximations hold, the free convection flow is described by the following nondimensional equation, see [5, 6],

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = \frac{\partial^3 \psi}{\partial y^3}, \tag{2}$$

since $\theta = \partial\psi/\partial y$. Here, ψ is the stream function, which is defined in the usual way, namely $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, with u and v being the velocity components along the x and y axes, respectively. The nondimensional variables in Equation (2) are defined as

$$\begin{aligned} &x = \bar{x}/l, y = \mathrm{Ra}^{1/3} y/l, \psi = \bar{\psi}/(\alpha\,\mathrm{Ra}^{1/3}), \\ &T = T_\infty + \mathrm{Ra}^{-1/3}(q_w/k)\theta, \end{aligned} \tag{3}$$

where l is a typical streamwise length, q_w is the uniform heat flux at the wall, T is the fluid temperature, α is the thermal diffusivity, k is the thermal conductivity, T_∞ is the ambient temperature and $\mathrm{Ra} = g\beta K(q_w/k)l/(\alpha\nu)$ is the Rayleigh number. Here g is the acceleration due to gravity, K is the permeability of the porous medium, β is the coefficient of thermal expansion and ν is the kinematic viscosity. Equation (2) must be solved subject to the following boundary conditions

$$\begin{aligned} &\psi = 0, \qquad \frac{\partial^2\psi}{\partial y^2} = -(1+x^2)^\mu \quad \text{on } y = 0, \\ &\frac{\partial\psi}{\partial y} \to 0 \quad \text{as } y \to \infty. \end{aligned} \tag{4}$$

The continuous transformation algorithm of Hunt and Wilks [11], or Kuiken [12], is applied to solve numerically Equation (2) along with the boundary conditions (4). This algorithm creates a single equation which effects a smooth transition between both the similarity equations which are valid for small and large values of x. In this case, the intermediate transformation is given by

$$\psi = x^{2/3}(1+x^2)^{\mu/3} f(x,\eta), \tag{5a}$$

where

$$\eta = y(1+x^2)^{\mu/3}/x^{1/3} \tag{5b}$$

and then Equation (2) becomes

$$\frac{\partial^3 f}{\partial \eta^3} + \frac{1}{1+x^2}[\tfrac{2}{3} + \tfrac{2}{3}(1+\mu)x^2] f \frac{\partial^2 f}{\partial \eta^2} - \\ - \frac{1}{1+x^2}[\tfrac{1}{3} + \tfrac{1}{3}(1+4\mu)x^2]\left(\frac{\partial f}{\partial \eta}\right)^2 \\ = x\left(\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial x\,\partial \eta} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial \eta^2}\right), \tag{6}$$

which has to be solved subject to the boundary conditions

$$f = 0, \quad \frac{\partial^2 f}{\partial \eta^2} = -1 \quad \text{on } \eta = 0, \\ \frac{\partial f}{\partial \eta} \to 0 \quad \text{as } \eta \to \infty. \tag{7}$$

For $f = f(\eta)$, we obtain from Equation (6), when $x = 0$,

$$f''' + \tfrac{2}{3} f f'' - \tfrac{1}{3}(f')^2 = 0, \tag{8}$$

where primes denote differentiation with respect to η. Equation (8) describes the free convection flow from a vertical surface which is immersed in a porous medium [2] and subject to a uniform heat flux. On the other hand, if we assume $f = f(\eta)$ and let $x \to \infty$, Equation (6) becomes

$$f''' + \tfrac{2}{3}(1+\mu) f f'' - \tfrac{1}{3}(1+4\mu)(f')^2 = 0, \tag{9}$$

which corresponds to a prescribed heat flux $(\partial\psi/\partial y)_{y=0} = -x^{2\mu}$.

It should be noted that when integrating Equation (9), subject to boundary conditions (7), it can be deduced that

$$\int_0^\infty \left(\frac{\mathrm{d}f}{\mathrm{d}\eta}\right)^2 \mathrm{d}\eta = \frac{1}{1+2\mu},$$

where the left-hand side is always positive but the right-hand side is positive or negative depending on whether $\mu \geqslant -\frac{1}{2}$ or $\mu < -\frac{1}{2}$ and, hence, a different mathematical approach is required for $\mu < -\frac{1}{2}$, $\mu = -\frac{1}{2}$ and $\mu > -\frac{1}{2}$. A complete numerical solution, accurate to $O(h^2)$ with h being the mesh spacing, is effected by using the method as described in [13] to integrate Equation (6). As a check of

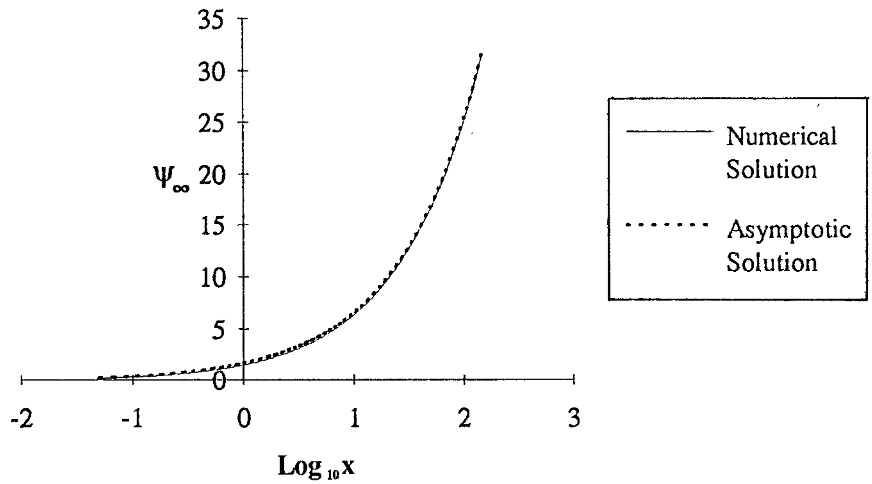

Fig. 1a. Nondimensional stream function as a function of the distance along the plate for $\mu = -0.125$.

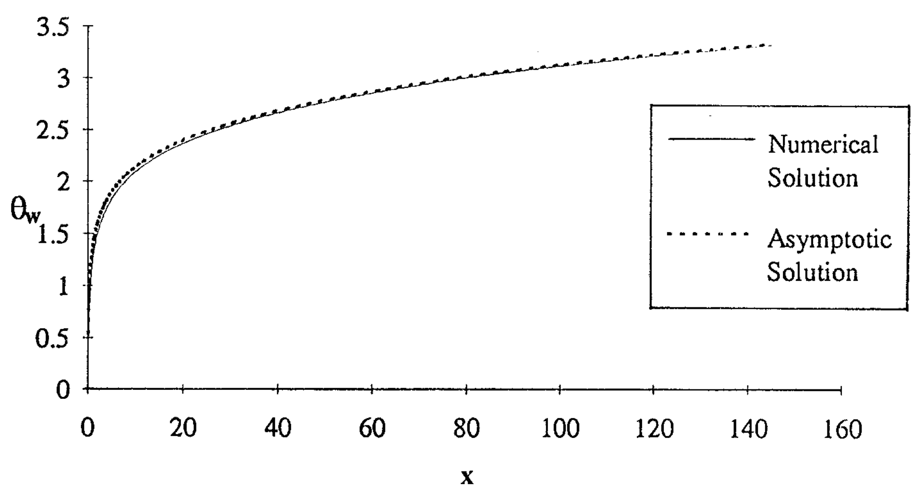

Fig. 1b. Nondimensional plate temperature as a function of the distance along the plate for $\mu = -0.125$.

the numerical procedure we start with the value of $\mu > -\frac{1}{2}$ and found, as expected, that the solution attained the asymptotic condition as given by Equation (9). This can be seen in Figure 1 where we present the variation of $(\partial f/\partial \eta)_{\eta=0}$, which is proportional to the temperature, and $f(x, \infty)$, which measures the amount of fluid entering the boundary-layer, for the case $\mu = -\frac{1}{8}$.

3. Asymptotic Solution for $\mu < -\frac{1}{2}$

In this case, we introduce the variables

$$\psi = x^{1/3} F(x, \zeta), \quad \zeta = y/x^{2/3}, \tag{10}$$

which gives Equation (9) for the critical case $\mu = \frac{1}{2}$. Substituting expressions (10) into Equation (2), we obtain

$$\frac{\partial^3 F}{\partial \zeta^3} + \tfrac{1}{3} F \frac{\partial^2 F}{\partial \zeta^2} + \frac{1}{3}\left(\frac{\partial F}{\partial \zeta}\right)^2 = x\left(\frac{\partial F}{\partial \zeta}\frac{\partial^2 F}{\partial x\,\partial \zeta} - \frac{\partial F}{\partial x}\frac{\partial^2 F}{\partial \zeta^2}\right) \tag{11}$$

and boundary conditions (4) become

$$\begin{aligned} &F = 0, \qquad \frac{\partial^2 F}{\partial \zeta^2} = -x^{1+2\mu}(1 + x^{1/2})^{\mu} \quad \text{on } \zeta = 0, \\ &\frac{\partial F}{\partial \zeta} \to 0 \quad \text{as } \zeta \to \infty. \end{aligned} \tag{12}$$

Boundary conditions (12) suggest looking for a solution of Equation (11) by expanding

$$F(x, \zeta) = F_0(\zeta) + x^{1+2\mu} F_1(\zeta) + \cdots \tag{13}$$

At leading order, we obtain

$$F_0''' + \tfrac{1}{3} F_0 F_0'' + \tfrac{1}{3}(F_0')^2 = 0 \tag{14a}$$

which has to be solved subject to the homogeneous boundary conditions

$$F_0(0) = 0, \qquad F_0''(0) = 0, \qquad F_0'(\infty) = 0, \tag{14b}$$

where primes are now used to denote differentiation with respect to ζ. Performing one integration of Equation (14a) and using boundary conditions (14b) results in the equation

$$F_0'' + \tfrac{1}{3} F_0 F_0' = 0. \tag{15}$$

A particular solution is given by

$$F_0(\zeta) = \sqrt{6}\tanh(\zeta/\sqrt{6}), \tag{16}$$

and the general solution (F_0, ζ) which will have $\mathrm{d}F_0/\mathrm{d}\zeta = k$ (say) on $\zeta = 0$ can then be found from Equation (15) by using the transformation

$$F_0 = k^{1/2} \bar{F}_0, \qquad \zeta = k^{1/2} \bar{\zeta}, \tag{17a}$$

where

$$\bar{F}_0 = \sqrt{6}\tanh(\bar{\zeta}/\sqrt{6}). \tag{17b}$$

To fix the leading order solution completely, we need to determine the value of the constant k. This is obtained by integrating Equation (2) and using the boundary conditions (4), which gives

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\int_0^\infty \left(\frac{\partial \psi}{\partial y}\right)^2 \mathrm{d}y\right] = (1+x^2)^\mu \tag{18}$$

or

$$\int_0^\infty \left(\frac{\partial \psi}{\partial y}\right)^2 \mathrm{d}y = \int_0^x (1+s^2)^\mu \,\mathrm{d}s \tag{19}$$

because, from the transformation (4), it follows that the integral (18) is zero at $x = 0$. Using expressions (10), (16) and (17), we obtain, for x large,

$$(\tfrac{8}{3})^{1/2} k^{3/2} = I_\infty, \tag{20}$$

where I_∞ can be expressed in terms of the gamma function as

$$I_\infty = \int_0^\infty (1+s^2)^\mu \,\mathrm{d}s = \frac{\sqrt{\pi}\,\Gamma(-\mu-\frac{1}{2})}{2\Gamma(-\mu)}. \tag{21}$$

Expressions (20) and (21) give

$$k = \left(\frac{3\pi}{32}\right)^{1/3} \left(\frac{\Gamma(-\mu-\frac{1}{2})}{\Gamma(-\mu)}\right)^{2/3} \tag{22}$$

and, thus, the leading order solution F_0 can be found using expression (17).

We are now able to consider the next term in expansion (13), which is given by the equation

$$F_1''' + \tfrac{1}{3}F_0F_1'' - (\tfrac{1}{3}+2\mu)F_0'F_1' + (\tfrac{4}{3}+2\mu)F_0''F_1 = 0, \tag{23a}$$

which has to be solved subject to the boundary conditions

$$F_1(0) = 0, \qquad F_1''(0) = -1, \qquad F_1'(\infty) = 0. \tag{23b}$$

Due to the arbitrary location of the leading edge of the expansion (13), there exists an eigensolution

$$F_e = \gamma(F_0 - 2\zeta F_0') \tag{24}$$

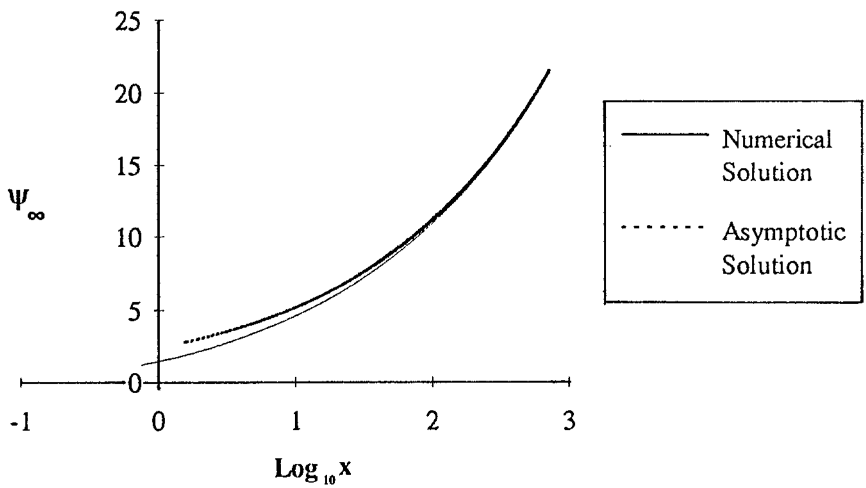

Fig. 2a. Nondimensional stream function as a function of the distance along the plate for $\mu = -1$.

at $\mathrm{O}(x^{-1})$. This is effectively a solution to Equation (23) with $\mu = -1$ and in general γ is an undetermined constant. However, for $\mu = -1, \gamma$ can be determined as the eigensolution and the perturbation to the leading-order due to the application of the wall heat flux boundary condition are both $\mathrm{O}(x^{-1})$ and, hence, this requires the expansion of $F(x, \zeta)$ to take the form suggested by Merkin and Mahmood [10], namely

$$F(x, \zeta) = F_0(\zeta) + \frac{\log_e x}{x} F_e(\zeta) + \frac{1}{x} F_1(\zeta) + \cdots \tag{25}$$

We then obtain for the equation at $\mathrm{O}(x^{-1})$, namely,

$$\begin{aligned} &F_1''' + \tfrac{1}{3}F_0F_1'' + \tfrac{5}{3}F_0'F_1' - \tfrac{2}{3}F_0''F_1 \\ &\quad = -\gamma(F_0F_0'' + F_0'^2). \end{aligned} \tag{26}$$

However, expression (25) is not unique as arbitrary multiples of the eigensolution, F_e can be added.

Results for the nondimensional plate temperature, $\theta_w = (\partial\psi/\partial y)_{y=0}$ and $\psi_\infty = \psi(x, \infty)$, as calculated from the numerical solution of Equation (6) are compared with those obtained from the asymptotic series for the case $\mu = -1$, see Figure 2. From expression (10), using the solution for F_0 and the value of k given by (21) (for $\mu = -1, I_\infty = \frac{1}{2}\pi$), we obtain

$$\theta_w \sim k\bar{F}'(0)x^{-1/3}, \qquad \psi_\infty(x, \infty) \sim k^{1/2}\bar{F}_0(\infty)x^{1/3}, \tag{27a}$$

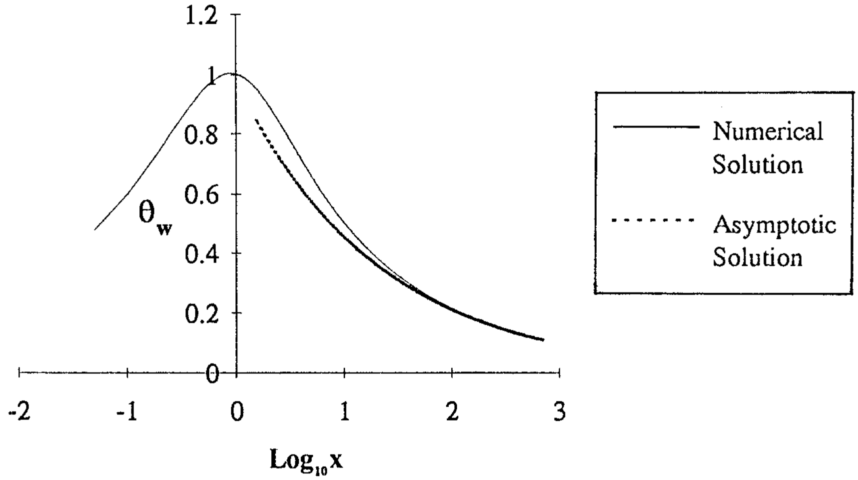

Fig. 2b. Nondimensional plate temperature as a function of the distance along the plate for $\mu = -1$.

i.e.

$$\theta_w \sim kx^{-1/3}, \qquad \sim \psi_\infty(x, \infty) \sim k^{1/2}\sqrt{6}x^{1/3}, \tag{27b}$$

for $x \gg 1$. The asymptotic expressions given by (27) are also shown in Figure 2 (by the broken lines) and they can be seen to be in good agreement with the results obtained from the full numerical solution. The $x^{-1/3}$ dependence of the plate temperature for large values of x also arises in wall plumes on adiabatic walls embedded in porous media [8] and this can be thought of as the limiting form of our solution since $(\partial T/\partial y)_{y=0} \to 0$ as $x \to \infty$ for all $\mu < 0$. However, we find that this power-law variation of the plate temperature for large values of x arises only for the case when $\mu < -\frac{1}{2}$. We shall further consider the case $\mu = -\frac{1}{2}$.

4. Asymptotic Expansion for $\mu = -\frac{1}{2}$

Since the expansion (13) in powers of $x^{1+2\mu}$ breaks down when $\mu = -\frac{1}{2}$ then an alternative approach is required, see Ingham and Brown [9]. Thus, following [10], we define the new variables

$$\psi = x^{1/3}(\log_e x)^{1/3}\Phi(x, \tau), \quad \tau = yx^{-2/3}(\log_e x)^{1/3}. \tag{28}$$

and then Equation (2) becomes

$$\frac{\partial^3 \Phi}{\partial \tau^3} + \left(\frac{1}{3} + \frac{1}{3\log_e x}\right)\Phi\frac{\partial^2 \Phi}{\partial \tau^2} + \left(\frac{1}{3} - \frac{2}{3\log_e x}\right)\left(\frac{\partial \Phi}{\partial \tau}\right)^2$$

$$= x\left(\frac{\partial\Phi}{\partial\tau}\frac{\partial^2\Phi}{\partial x\,\partial\tau} - \frac{\partial\Phi}{\partial x}\frac{\partial^2\Phi}{\partial\tau^2}\right), \tag{29a}$$

which has to be solved subject to the boundary conditions

$$\begin{aligned}&\Phi = 0, \qquad \tfrac{\partial^2\phi}{\partial\tau^2} = -\tfrac{1}{\log_e x}\left(1+\tfrac{1}{x^2}\right)^{-1/2} \quad \text{on } \tau = 0,\\ &\tfrac{\partial\Phi}{\partial\tau} \to 0 \quad \text{as } \tau \to \infty.\end{aligned} \tag{29b}$$

Equation (29a) suggests looking for a solution of the form

$$\Phi(x,\tau) = \Phi_0(\tau) + (\log x)^{-1}\Phi_1(\tau) + \cdots \tag{30}$$

To leading order, we have

$$\Phi_0''' + \tfrac{1}{3}\Phi_0\Phi_0'' + \tfrac{1}{3}(\Phi_0')^2 = 0, \tag{31a}$$

which has to be solved subject to the homogeneous boundary conditions

$$\Phi_0(0) = 0, \qquad \Phi_0''(0) = 0, \qquad \Phi_0'(\infty) = 0, \tag{31b}$$

where the primes now denote differentiation with respect to τ. Equation (31a) can now be integrated twice and a third integration gives rise to the particular solution

$$\Phi_0 = \sqrt{6}\tanh(\tau/\sqrt{6}). \tag{32}$$

This problem is very similar to that described in [10] and the general solution (Φ_0, τ) can be obtained from the particular solution $(\bar{F}_0, \bar{\zeta})$ by the transformation

$$\Phi_0 = C^{1/2}\bar{F}_0, \qquad \tau = C^{-1/2}\bar{\zeta}, \tag{33}$$

where C is a constant which has to be determined.

To find C, we use the transformation (28) in the integral condition (19), which for $\mu = -\frac{1}{2}$ becomes

$$\begin{aligned}\int_0^\infty \left(\frac{\partial\Phi}{\partial\tau}\right)^2 \partial\tau &= \frac{1}{\log_e x}\log_e(x + \sqrt{x^2+1})\\ &= 1 + \frac{\log_e 2}{\log_e x} + \mathbf{O}\left(\frac{x^{-2}}{\log_e x}\right).\end{aligned} \tag{34}$$

Thus, for the leading-order terms in the expansion (30), we require

$$\int_0^\infty \Phi_0'^2\,\mathrm{d}\tau = 1, \tag{35}$$

which gives

$$C = \left(\int_0^\infty \bar{F}_0'^2 \, d\bar{\zeta} \right)^{-2/3} = \tfrac{1}{2}(3)^{1/3} \tag{36}$$

on using the result (17b).

The equation for Φ_1, from the expansion (30), is given by

$$\Phi_1''' + \tfrac{1}{3}\Phi_0\Phi_1'' + \tfrac{2}{3}\Phi_0'\Phi_1' + \tfrac{1}{3}\Phi_0''\Phi_1 = \tfrac{2}{3}\Phi_0'^2 - \tfrac{1}{3}\Phi_0\Phi_0'' \tag{37}$$

which has to be solved subject to the boundary conditions

$$\Phi_1(0) = 0, \qquad \Phi_1''(0) = -1, \qquad \Phi_1'(\infty) = 0 \tag{38a}$$

$$\int_0^\infty \Phi_0'\Phi_1' \, d\tau = \tfrac{1}{2}\log_e 2. \tag{38b}$$

The nondimensional quantities θ_w and ψ_∞ defined in the previous section can be calculated by using transformation (28) and we have

$$\theta_w \sim C x^{1/3}(\log_e x)^{2/3}, \qquad \psi_\infty \sim C^{1/2} F(\infty) x^{1/3}(\log_e x)^{1/3}, \tag{39}$$

as $x \to \infty$, where $F(\infty) = \sqrt{6}$. Again, the solution is not unique beyond leading-order due to the existence of the eigensolution

$$\phi_e = \sigma(\phi_0 + \tau\phi_1).$$

As a check on the above theory, we solved Equation (16) numerically for the case $\mu = -\frac{1}{2}$, allowing the solution to proceed to very large values of x. From this solution we calculated that

$$\theta_w^* = x^{1/3}(\log_e x)^{-2/3}\phi_w, \qquad \psi_\infty^* = x^{1/3}(\log_e x)^{-1/3}\psi_\infty, \tag{40}$$

with the results presented in Table I. In this case, we found that the numerical solutions do appear to be approaching their respective asymptotic limits, albeit slowly. This is to be expected as the perturbation to the leading-order solution is of $\mathbf{O}\,((\log_e x)^{-1})$, which at the final value of x given in Table I the value of $(\log_e x)^{-1}$ is 0.0564 and this is comparable with the difference between the values of θ_w^* and ψ_∞^* given at this value of x and their corresponding asymptotic limit.

5. Conclusions

In this paper, it has been shown that near the leading edge of the plate the boundary-layer develops from a similarity solution, given by solving equation (8) subject to the boundary conditions (7), and it is valid for all values of μ. At large distances from the leading edge of the plate, i.e. at large values of x, three specific cases

TABLE I. Variation of θ_w^* and ψ_∞^* as a function of the distance along the plate for $\mu = -1$.

	Plate temperature		Stream function	
	121 mesh points	241 mesh points	121 mesh points	241 mesh points
x	θ_w^*	θ_w^*	ψ_∞^*	ψ_∞^*
1.0100E+01	1.20055	1.19964	1.87962	1.87983
1.0030E+02	0.98868	0.98868	1.93770	1.93778
1.9950E+02	0.95872	0.95745	1.95079	1.95083
5.0030E+02	0.92691	0.92533	1.96523	1.96521
9.9950E+02	0.90804	0.90656	1.97408	1.97401
2.0235E+03	0.89208	0.89052	1.98585	1.98173
4.9931E+03	0.87525	0.87357	1.99027	1.99007
1.0113E+04	0.86424	0.86246	1.99585	1.99560
2.0148E+04	0.85494	0.85307	2.00007	2.00040
4.0628E+04	0.84664	0.84462	2.00506	2.00467
1.0125E+05	0.83731	0.83521	2.01010	2.00962
2.0119E+05	0.83116	0.82895	2.01342	2.01286
5.0266E+05	0.82395	0.82160	2.01741	2.01674
9.8762E+05	0.81920	0.81675	2.02002	2.01927
2.8226E+06	0.81270	0.81001	2.02373	2.02284
1.0163E+07	0.80558	0.80305	2.02762	2.02564
1.8761E+06	0.80301	—	2.02935	—
2.0858E+07	0.80253	0.79955	2.02963	2.02842
4.8960E+07	—	0.79580	—	2.03171
4.9379E+07	0.799084	—	2.03171	—
∞	0.72112	0.72112	2.08008	2.08008

have to be considered, namely, $\mu < -0.5, \mu = -0.5$ and $\mu > -0.5$. In order to illustrate the nature of the solution in these ranges of values of μ three typical values of μ in these ranges have been investigated, namely, $\mu = -0.125, -0.5$ and -1. For $\mu = -0.125$ a similarity solution which is valid for large values of x was obtained by solving equation (9) subject to boundary conditions (7). For $\mu \leqslant -0.5$ the boundary-layer solution, at large distances from the leading edge of the plate, had an asymptotic structure with the perturbation to the leading-order being $0(x^{1+2\mu})$ when $\mu < -0.5$ and $0(\{\log_e x\}^{-1})$ when $\mu = -0.5$. The physical explanation why no similarity solutions exist for large values of x when $\mu \leqslant -\frac{1}{2}$ is probably related to the presence of a negative temperature gradient along the plate, i.e. $(\partial T/\partial x)_{y=0} < 0$, which implies a heating from below mechanism. This constrasts with the situation when $\mu > -\frac{1}{2}$ and there is a positive temperature gradient along the plate. The transition of the boundary-layer solution between the two regions was obtained by numerically solving Equation (6) subject to boundary conditions (7). The agreement between the numerical solutions and the asymptotic solutions, for large values of x, when $\mu = -0.125$ and -1 can be clearly seen in Figures 1 and 2, whereas Table I shows that the numerical solution is approaching the asymptotic limit, for $\mu = -0.5$, albeit slowly. This gives confidence in both the numerical solution obtained and the theory presented in this paper.

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