3. V. A. Gurashvili, A. V. Dem'yanov, et al., Inzh. Fiz. Zh., **55**, 37 (1988).
4. M. Iyoda, S. Sato, H. Saito, et al., Appl. Phys., **47**, 251 (1988).
5. P. W. Milonni and A. N. Paxton, J. Appl. Phys., **49**, 1012 (1978).
6. Z. M. Benenson, I. V. Kochetov, A. K. Kurnosov, et al., Kvantovaya Elektron. (Moscow), **14**, 2457 (1987).
7. R. Sh. Islamov, Yu. B. Konev, et al., ibid., **11**, 142 (1984).
8. M. Lax, G. P. Agrawal, et al., J. Opt. Soc. Am., **2**, 731 (1985).
9. L. R. Rabiner and C. M. Rader, Digital Signal Processing, IEEE, N. Y. (1972).
10. N. N. Elkin, I. V. Kochetov, A. K. Kurnosov, and A. P. Napartovich, Kvantovaya Elektron. (Moscow), **17**, 313 (1990).

NUMERICAL INVESTIGATION OF MODES OF A PLANE-PARALLEL CAVITY WITH PERIODIC MIRRORS

S. N. Kozlov and A. P. Napartovich

Selection of the transverse modes of wide-aperture cavities is of great importance when it comes to obtaining high-power laser emission with low divergence. It was proposed in [1] to improve the directivity pattern of a laser by replacing one of the mirrors of a plane-parallel cavity by a periodic reflecting grating. The Talbot effect of reproducing periodic field at a certain distance [2] was used to maximize the selectivity of the method. If the cavity length is half the Talbot length, a minimum-loss periodic mode which is self-reproducing on a grating mirror is established in the cavity.

The experiments of [1] have shown that the proposed method makes it possible to fill completely the active medium and to make the divergence of the individual lobes of the directivity pattern as close as possible to diffractive pattern on the full aperture of such a cavity.

Further investigations of these, for short, Talbot cavities were based on the theory developed for infinite gratings [3, 4], although the experiments were performed with gratings not larger than 30 periods (see, e.g., [4-7]).

Under the influence of the boundaries, the image of each reflecting groove of the grating is "smeared out" by an amount $\delta \approx \phi L_T$, where $\phi \approx \lambda/Na$ is the diffraction-divergence angle, $L_T = 2a^2/\lambda$ the Talbot length for a square grating (N is the number of rules, a the grating period, and λ is the wavelength). It follows hence that for $N \approx 10$ the diffraction causes the grating image to become "smeared" by approximately 1/5 of its period and fill the gaps between the grooves. After many passes there is established in a Talbot cavity a certain periodic mode subject to definite diffraction losses both through the gap and through the edges of the cavity.

We report here a detailed study of the influence of the grating boundaries and of a continuous mirror and the mode structure of a Talbot cavity.

Consider for simplicity a cavity in which one mirror is a square bounded grating and the other is not ruled and is of sufficiently large size. The equation that determines the modes of this cavity is then separable with respect to the variables in the mirror planes, and the problem reduces to determination of the modes of a two-dimensional Talbot cavity.

The corresponding equation is

$$\gamma u(x) = \frac{exp(-i\pi/4)}{\sqrt{2\lambda L}} \sum_{n=0}^{N-1} \int_{na}^{na+\Delta} exp\left(i\pi\frac{(x-x')^2}{2\lambda L}\right) u(x')dx', \quad (1)$$

where L is the cavity length and Δ is the dimension of the reflecting part of the rule. Since the replica is observed at distances that are multiples of $L_T/2 = a^2/\lambda$, the cavity length is [1]

Translation of Preprint IAE-4748/16, Kurchatov Atomic Energy Institute, Moscow (1988).

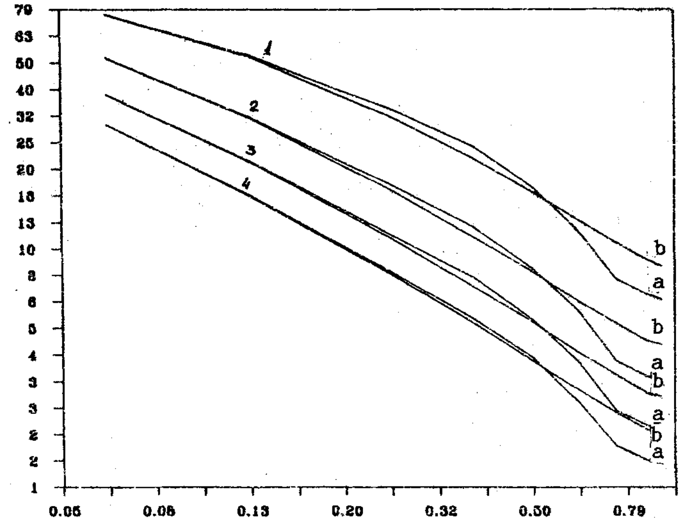


Fig. 1

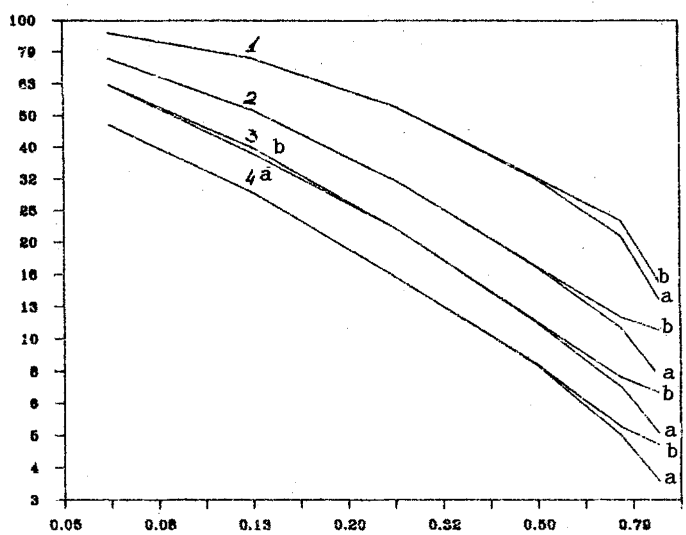


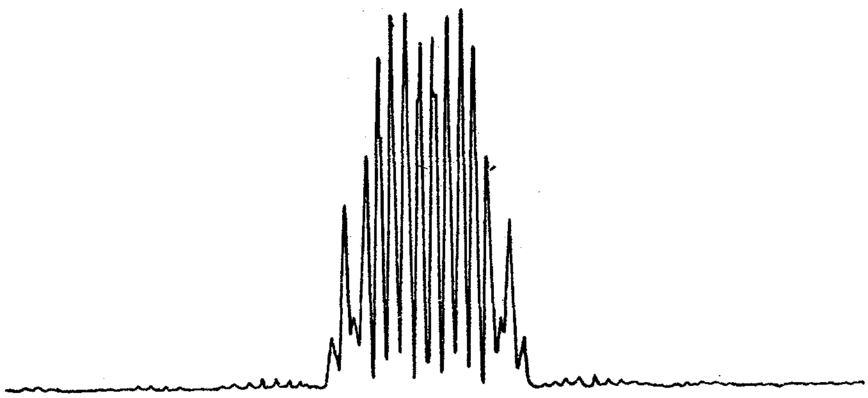
Fig. 2

Fig. 1. Dependence of the relative loss W (in %) on s for m = 1 and N = 10 (1), N = 20 (2), N = 30 (3), N = 40 (4), in logarithmic coordinates: a — bounded continuous mirror, b — unbounded continuous mirror.

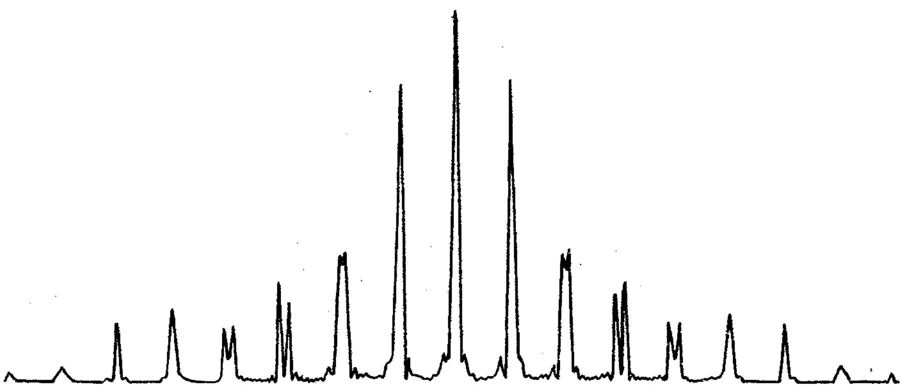
Fig. 2. Dependence of relative loss W (in %) on s for m = 2 and N = 10 (1), N = 20 (2), N = 30 (3), N = 40 (4), in logarithmic coordinates: a — in phase mode, b — antiphase mode.

$$L = \frac{ma^2}{2\lambda}, \quad m=1,2,\ldots \quad . \tag{2}$$

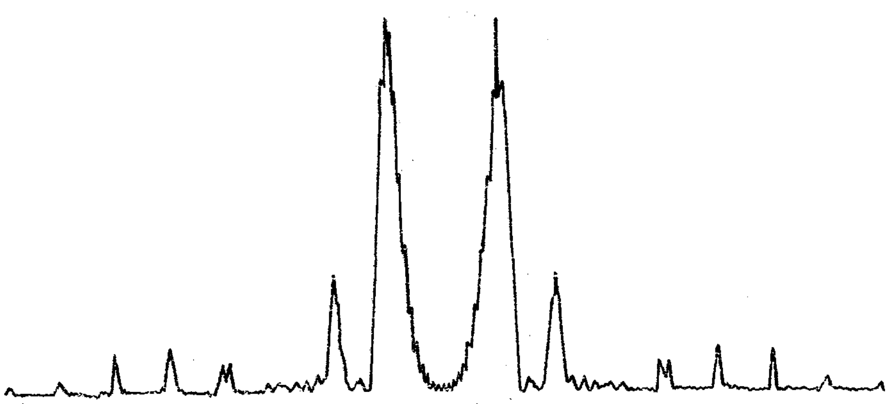
One, antiphase mode exists at m = 1, and two, in- and antiphase, are produced at m = 2, having minimal losses and competing with each other. As m increases, the number of competing periodic modes increases, and the proposed transverse-mode selection method efficacy decreases. It is advantageous therefore to consider a cavity with m = 1, in which mode selection is most effective, and a cavity with m = 2, in which the selection is worse but on the other hand the far-field of the in-phase mode has one central lobe containing a significant fraction of the energy.



a

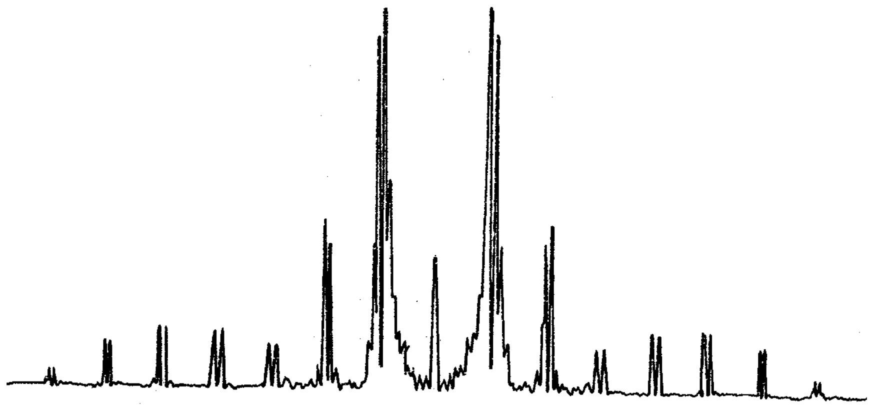


b



c

Fig. 3. Distribution of in-phase mode emission intensity for m = 2, N = 10, s = 0.5 behind a periodic mirror in the near field (a) and far field: passing through the gaps (b), over the edges of the mirror (c), and the total (d).



d

Fig. 3 (continued)

Substituting (2) in (1) and making all distances dimensionless with respect to a, we obtain the following equation for the natural modes of a Talbot cavity:

$$\gamma u(\xi) = \frac{exp(-i\pi/4)}{\sqrt{m}} \sum_{n=0}^{N-1} \int_{na}^{n+s} exp\left(i\pi\frac{(\xi-\xi')^2}{m}\right) u(\xi')d\xi', \tag{3}$$

where $s = \Delta/a$ is the diffraction ruling ratio, $\xi = x/a$, $u(\xi)$ is an eigenfunction, and γ is the eigenvalue connected with the relative losses w per pass by the relation $w = 1-|\gamma|^2$.

As seen from (3), the loss in a Talbot cavity depends only on the three parameters m, s, and B. This dependence can be described approximately by starting from the considerations advanced above: The image of the groove is "smeared" by an amount $\delta \approx 2\phi L$, where $\phi \approx \lambda/Na$ and L is determined from (2), i.e., $\delta \approx ma/N$. The relative loss is therefore

$$w \sim \delta/\Delta \sim \frac{m}{Ns} . \tag{4}$$

An analytic solution of (3) meets with certain difficulties, therefore the principal modes of the cavity were determined here by numerical iteration. To separate an in-phase or antiphase mode, an in- and anti-phase initial field was specified on the periodic mirror. Measures were taken to preserve parity of the computation scheme.

The problem connected with radiation propagation from one mirror to the other was solved in the parabolic approximation. The corresponding differential equation was solved by the Fourier method using the fast-Fourier-transformation (FFT) algorithm [8]. To use this method, the computation range must be chosen large enough to prevent the radiation inside the cavity from touching the boundaries of the region.

In the present study, the subdivision and size of the computation region were determined, by trial and error, to keep the loss-determination error from exceeding several per cent. Specifically, the period of the grating mirror was subdivided into 64 parts, and the computation region was 5-6 times the dimensions of the periodic mirror.

Computations have shown that a repeated antiphase mode, $\arg \gamma \approx -\pi/4$ exists for m = 1, just as in case of an infinite periodic mirror, but $|\gamma| < 1$, i.e., this mode is subject to definite losses. The computations were made for two cases, infinite and bounded continuous mirrors (the dimensions of the latter are equal to those of the periodic mirror). The corresponding dependences of the losses on N and s are shown in Fig. 1. It is seen from the curves that the loss of a cavity with a bounded continuous mirror is larger than that of a cavity with an unbounded continuous mirror only if $s < 0.5$.

Finally, numerical calculations yielded a more accurate Eq. (4) for the relative losses. For an unbounded continuous mirror this dependence is given, in a wise range of parameters (for $1 - s \gg 1/N$ and $Ns \gg 1$) by

(5)

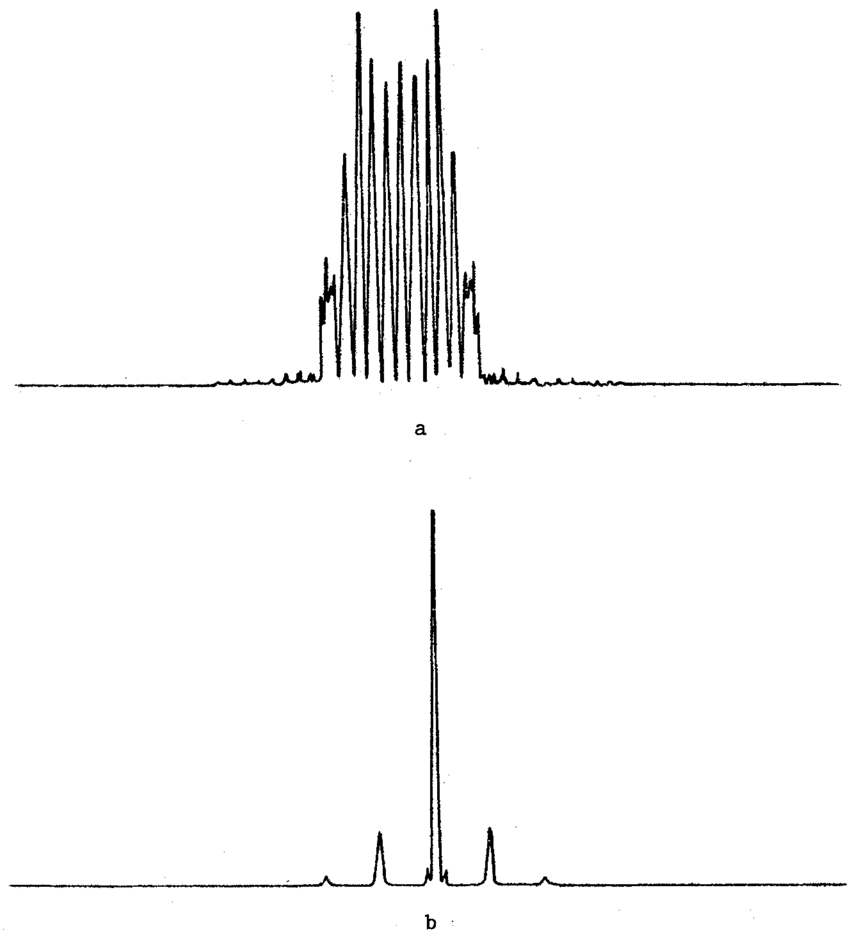


Fig. 4. Distribution of in-phase mode intensity at m = 2, N = 10, s = 0.5 behind a continuous mirror in the near (a) and far (b) fields.

$$w = \frac{0.78}{Ns} .$$

If the continuous mirror is bounded, the loss, as seen from Fig. 1, is given by Eq. (5) in a narrow range of parameters (actually for $s < 1/2$).

A cavity with an unbounded continuous mirror was investigated for m = 2. The computations have shown that the corresponding periodic modes have the highest Q and that $\arg \gamma \approx 0$ and $\arg \gamma \approx -\pi/2$ for the in-phase and antiphase modes, just as for an infinite periodic mirror. The dependences of the losses of both modes on N and s are shown in Fig. 2, just as for m = 1, they are well approximated by the equation

$$w = \frac{1.56}{Ns} . \tag{6}$$

and only for $s > 0.5$ does the loss of the even mode tend to decrease. Combining (5) and (6), we can conclude that the loss of an antiphase mode is well enough described in a sufficiently large range of parameters (at $1 - s \gg m/N$ ans $Ns \gg m$) by the equation

$$w = 0.78 \frac{m}{Ns} . \tag{7}$$

The loss of an in-phase mode is also described by (7), but in a narrower range of parameters (at $s < 0.5$).

It is seen from Fig. 2 that the in-phase mode has a higher Q at $s > 0.5$. It must be noted that, as follows from the qualitative derivation of (7), the losses of a higher-order modes not investigated here should be proportional to the number of the mode.

Let us examine the best way of extracting the radiation from the cavity. From the standpoint of energy, it is expedient to use an ideally reflecting grating and a continuous mirror. Figure 3a shows the intensity distribution of an in-phase mode in the plane of a periodic mirror (prior to reflection) at cavity parameters m = 2, N = 10, and s = 0.75 (unbounded solid mirror). It is seen that the gaps between the grooves are almost completely filled with radiation; a significant part of the energy emerges also from the edges of the mirror. Figure 3 shows the field intensity distribution, in the far zone, of radiation emerging only through the gaps between the grooves; Fig. 3a shows the result in the far field when the radiation is emitted only from the edges of the periodic mirror; for radiation emitted both from the edges and through the gaps, the corresponding intensity distribution in the far field is shown in Fig. 3d.

Evidently, radiation emerging from the edges of a periodic mirror complicates the far-field picture, whereas radiation emerging through the slits has an appreciable number of peaks among which the entire emitted energy is distributed. In the latter case the divergence of an individual lobe of the directivity pattern reaches the diffraction value for the total aperture of the cavity.

Figure 4a shows the intensity distribution of an in-phase mode on a continuous mirror for the same cavity parameters. If this mirror is semitransparent, the field intensity distribution, in the far field, of the radiation emerging through this mirror takes the form shown in Fig. 4b. The divergence of the central lobe is determined by the total aperture of the periodic mirror, and contains an appreciable fraction ($\approx$ 77%) of the energy emerging through the continuous mirror.

It is thus advantageous to use the openings of a periodic mirror to extract radiation. To improve the selectivity and to increase the energy fraction contained in the central lobe it is desirable to have a groove to peak ratio of the periodic mirror larger than one-half. Determination of the optimum value of this ratio calls for a more detail investigation of the modes of a Talbot cavity.

In conclusion, the authors thank N. N. Ëlkin for help in organizing the computation program.

LITERATURE CITED

1. V. P. Ablekov, V. S. Belyaev, V. M. Marchenko, and A. M. Prokhorov, DAN SSSR, **230**, No. 5, 1066 (1976).
2. H. F. Talbot, Phil. Mag., **9**, 401 (1836).
3. A. P. Smirnov, Opt. Spektrosk., **42**, No. 4, 755 (1977).
4. V. M. Marchenko, T. M. Makhviladze, A. M. Prokhorov, and M. E. Sarychev, Zh. Éksp. Teor. Fiz., **74**, No. 3, 872 (1978).
5. A. S. Koryakovskii and V. M. Marchenko, FIAN Preprint No. 89 (1979).
6. V. K. Ablekov, V. S. Belyaev, V. I. Vinogradov, V. G. Marchenko, V. M. Marchenko, and A. M. Prokhorov, Opt. Spetkrosk., **44**, No.6, 1208 (1978).
7. V. G. Marchenko, Kvantovaya Elektron. (Moscow), **8**, No. 5, 1037 (1981).
8. E. A. Sziglas and A. E. Siegman, Appl. Opt., **14**, 1874 (1975).