INVESTIGATION OF THE SPECTRUM OF NATURAL OSCILLATION FREQUENCIES OF GASES IN COMPRESSOR PIPING

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In recent years, extensive investigations devoted to the problem of pulsations in piping were performed in our country and abroad. The theoretical [1] and experimental [2, 3] investigations of the causes and consequences of pulsations made it possible to develop new methods for controlling this phenomenon.

The problems in determining the spectrum of natural frequencies of gas oscillations are especially important. The knowledge of this spectrum would provide a better basis for devising measures for pulsation prevention.

The spectrum of natural frequencies is calculated by means of the equations derived for relatively simple piping systems. An equation of the spectrum of natural frequencies without an allowance for the hydraulic losses for systems containing different loads z_1 and z_3 was derived in [1]:

$$\frac{z_0(z_3+z_1)}{z_1 z_3}\cos\frac{\omega l}{c}+\frac{zl}{c}+\frac{z_0}{z_2}\cos\frac{\omega l_1}{c}\cos\frac{\omega l_2}{c}-\frac{z_0^3}{z_1 z_2 z_3}\sin\frac{\omega l_1}{c}\sin\frac{\omega l_2}{c}+\left[\frac{z_0^2+z_1 z_3}{z_1 z_3}\sin\frac{\omega l}{c}\right.$$
$$\left.+\frac{z_0^2}{z_1 z_2}\cos\frac{\omega l_2}{c}\sin\frac{\omega l_1}{c}+\frac{z_0^2}{z_2 z_3}\sin\frac{\omega l_2}{c}\cos\frac{\omega l_1}{c}\right]=0,$$

where l, l_1, and l_2 are the spacings between the loads in the piping system, c the velocity of sound in gas, and ω the angular frequency.

The general form of the solution of Eq. (1) is rather complex. In certain particular cases, the solution can readily be obtained by analytical means.

a) Assume that a pipe with a mounted chamber, whose elastic resistance is

$$z_2=\frac{\rho_0 c^2}{i\omega V_2},$$

is first loaded with a purely reactive resistance

$$z_1=\frac{\rho_0 c^2}{i\omega V_1},$$

while its end is open (here, ρ_0 is the gas density and $i=\sqrt{-1}$). The load for the open end tends to zero at low frequencies ($z_3 \rightarrow 0$). Then, after the substitution of the z_1, z_2, and z_3 values in Eq. (1) and suitable simplifications, we obtain the resonance condition for a system consisting of two chambers with volumes V_1 and V_2, which are spaced at distances l_1 and l_2:

$$\frac{\omega l_1}{fc}\left(\sin\frac{\omega l}{c}-\frac{\omega V_2}{fc}\sin\frac{\omega l_1}{c}\sin\frac{\omega l_2}{c}\right)=\cos\frac{\omega l}{c}-\frac{\omega l_2}{fc}\sin\frac{\omega l_2}{c}\cos\frac{\omega l_1}{c},$$

where f is the cross-sectional area of the pipe.

b) If one end of the pipe is open ($z_3=0$) and a volume V_1 is connected to its other end, the dimensions of this volume are small in comparison with the pipe length l and the wavelength λ. Then, it can be considered that

$$z_1 = \frac{\rho_0 c^2}{i \omega V_1}.$$

The absence of the V_2 chamber means that $z_2 = \infty$. In this case, on the basis of the general Eq. (1), the resonance condition for the above system will be

$$\frac{z_3 z_0}{z_1} \cos\frac{\omega l}{c} + z_0 \cos\frac{\omega l}{c} - \frac{z_0^2}{z_1 z_2} \sin\frac{\omega l_1}{c} \sin\frac{\omega l_2}{c} + i\left(\frac{z_0^2}{z_1} \sin\frac{\omega l}{c} + \frac{z_0^2}{z_2} \sin\frac{\omega l_2}{c} \cos\frac{\omega l_1}{c}\right) = 0.$$

After substituting z_1, z_2, and z_3, we obtain

$$\frac{\rho c}{f} \cos\frac{\omega l}{c} = \frac{\rho \omega V_1}{f} \sin\frac{\omega l}{c},$$

whence,

$$\tan\frac{\omega l}{c} = \frac{fc}{\omega V_1}.$$

In order to find the roots of this equation, it is necessary to plot a family of tangent curves as function of $\omega l/c$ and the hyperbola $f l[(\omega l/c) V_1]^{-1}$. Then, the points of intersection between the hyperbola and the tangent curves will correspond to the roots of the equation for the natural frequencies. If the pipe is short, the first root will be much smaller than the others, and we can write approximately

$$\tan\frac{\omega l}{c} \approx \frac{\omega l}{c} = \frac{fl}{V_1};$$

whence,

$$\omega = c\sqrt{\frac{f}{V_1 l}}. \tag{2}$$

Equation (2) constitutes the well-known relationship for the Helmholtz resonator.

c) For a pipe without a chamber which is open at the far end and closed at the other, $z_1 = \infty$, $z_2 = \infty$, and $z_3 = 0$; then, from Eq. (1), it follows that

$$\cos\frac{\omega l}{c} = 0. \tag{3}$$

From Eq. (3), we obtain

$$\frac{\omega l}{c} = \frac{2n-1}{2}\pi, \quad n = 1, 2, \ldots$$

or

$$l = \frac{2n-1}{2} \cdot \frac{\pi c}{\omega}. \tag{4}$$

After the wavelength is taken into account, Eq. (4) will assume the following form:

$$l = (2n-1)\frac{2\pi c}{4\omega} = (2n-1)\frac{\lambda}{4}.$$

Thus, resonance will set in when the pipe length is equal to an odd number, multiplied by a quarter of the wavelength. This fact is also well known in acoustics.

d) For a system consisting of a pipe, a chamber, and a secondary pipe, the loads will have the following values: $z_1 = \infty$; $z_2 = \rho_0 c^2/i\omega V_2$ (the elastic resistance of the chamber) and $z_3 = \rho c/f = z_0$ (the piping load beyond which there is an infinite-length pipe).

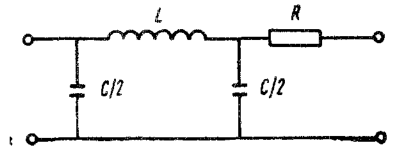

Fig. 1. Circuit of a cell of the electric analog simulator.

After substituting the z_1, z_2, and z_3 values in Eq. (1), we obtain the following expression for calculating the natural frequencies of gas oscillation:

$$\cot\frac{\omega l}{c}=\frac{\dfrac{\omega V_2}{fc}}{\tan\dfrac{\omega l_1}{c}\cot\dfrac{\omega l_2}{c}+1}.$$

Thus, Eq. (1) constitutes a general equation; all the particular solutions can be obtained from it. This equation makes it possible to calculate the spectrum of natural frequencies for a system containing different loads if the values of the latter are known.

In the case of more complex piping systems, the calculation of the frequency spectrum becomes more complicated and requires artificial division of the system into a number of simple sections. Calculations provide the possibility of determining the approximate range of gas oscillation frequencies in the entire system, which is an approximate result that does not make it possible to estimate the mutual effect of individual sections. Moreover, this method is exceedingly cumbersome, while, in certain cases, it may present an insoluble problem.

The electric analogy method, based on the mathematical similarity between gas pressure fluctuations in piping and voltage oscillations in electric circuits, is a more promising method for investigating complex branched piping systems. This makes it possible to apply the basic simulation principles to the investigation of the dynamic processes occurring in complex piping systems and to develop new analytical design methods on this basis.

Material on investigations of the spectrum of natural frequencies of piping systems, based on the analogy method and the use of the electroacoustic analog [4-6], is available in the literature.

The authors have developed an electric analog simulator for investigating natural frequency spectra in piping systems of compressor stations. Among the advantages of this simulator are the simplicity with which any complex system can be assembled, the possibility of varying the gas-dynamics parameters, the convenience and accuracy of electric measurements, oscilloscope recording of the processes in time, etc.

The piping unit simulator (Fig. 1) consisted of a number of series-connected electric filter cells with an inductance L, a capacitance C, and a resistance R, calculated per unit length of conductor.

As is known, the inductance, the capacitance, and the resistance of a conductor are related by the following equations:

$$u=L_{\mathrm{el}}\frac{di}{d\tau};\quad u=\frac{1}{C_{\mathrm{el}}}q=\frac{1}{C_{\mathrm{el}}}\int i\,d\tau;\quad u=Ri, \tag{5}$$

where u is the voltage, q the charge, i the current intensity, and τ the time.

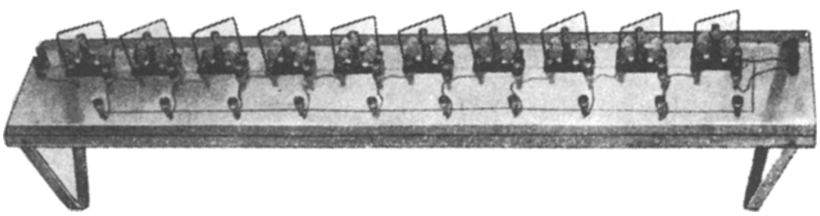

Fig. 2. Assembly panel.

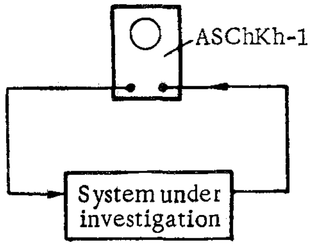

Fig. 3. Connection of the ASChKh-1 device to the line.

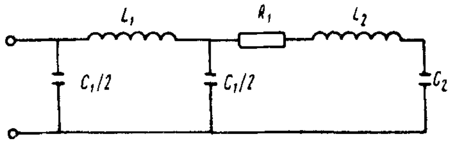

Fig. 4. Circuit diagram of the simulator for a pipe with a diaphragm at its end.

The following analogous equations hold in acoustics:

$$p = M_A \frac{dvf}{dt}; \quad p = \frac{x}{C_A}; \quad p = r_A f v, \tag{6}$$

where p is the pressure, M_A the acoustic mass, v the velocity, x the displaced volume, C_A the acoustic capacitance, r_A the acoustic resistance, and f the cross-sectional area of the pipe.

For cases of practical interest, the M_A, C_A, and r_A values have been calculated and can be considered as known in most cases.

Relationships (5) and (6) are referred to as "electroacoustic analogies;" they justify the application of Kirchhoff laws, which constitute the boundary and the nodal conditions, and of the d'Alembert principle in acoustics as a consequence of the equality of action and reaction.

In order to apply the method of electroacoustic analogies to piston devices, it is necessary to define the elements of which an ordinary acoustic system is composed (a piping system with the equipment is contemplated) and to determine their combinations. Since the basic parameters of acoustic systems correspond to the basic elements L, R, and C of a quadripole, they can be termed acoustic quadripoles.

Any acoustic system constitutes a combination of expansion chambers which are connected to each other by communication pipes. In expansion, a gas is characterized by elastic reaction, and its electric analog is the capacitance. Thus, the expansion chamber's volume plays the role of the acoustic elasticity of gas, which is expressed by

$$C_A = \frac{V}{\rho_0 c^2}.$$

In constriction elements, a gas produces an inertial reaction, characterized by the mass, and its electric analog is the inductance. Thus, gas in a connecting pipe plays the role of the acoustic mass, which is expressed by the equation

$$M = \frac{\rho_0 l}{f}.$$

By means of conversion coefficients, which relate the inductance, the capacitance, and the resistance to the corresponding parameters of the piping (the acoustic mass, the acoustic capacitance, and the hydraulic resistance), we can simulate piping sections with a certain given length and diameter. We investigated piping sections of relatively small extent and with low hydraulic resistance (communicating piping systems of piston compressors). Therefore, the problem of determining the spectrum of natural frequencies was solved without taking into account friction. The individual elements included in the piping system (chambers, coolers, oil and moisture separators, diaphragms, etc.) were simulated by means of various electric elements and devices.

Figure 2 shows the assembly panel, where each of the ten cells simulates a pipe section with a length 3 m. A voltage whose frequency can be varied from 0 to 20 kc is supplied to the input of the assembled simulator. Since the simulator operates on an artificial time scale, a change in frequency from 0 to 20 kc corresponds to the actual operating frequencies of piston compressors.

If the frequency supplied to the input of the system under investigation coincides with the natural oscillation frequency, voltage resonance will arise in the system. Consequently, each value of the input signal frequency corresponds to a strictly defined amplitude of the output voltage, i.e., the amplitudes at the system's output will periodically change in correspondence with its frequency characteristic. Thus, the spectrum of natural frequencies of the system under investigation can be obtained.

The variable frequency can be most conveniently supplied to the simulator by means of a device designed for investigating the frequency characteristics of electric quadripoles, for instance, an ASChKh-1 device (Fig. 3). In investigating the frequency characteristic of the simulated piping system, a luminous figure, whose upper envelope periodically repeats the system's frequency characteristic, appears on the screen of the electron-beam tube of the ASChKh-1 device.

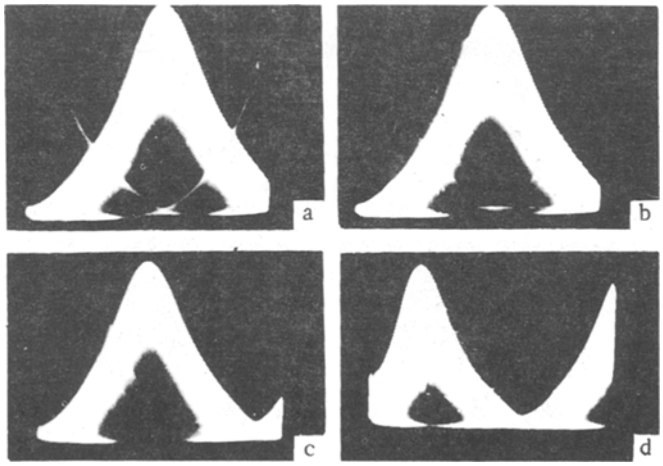

Fig. 5. Frequency characteristics of a system consisting of a pipe with a length of 6 m and a diameter of 62.5 mm. a) Without diaphragm; b to d) with diaphragms having diameters of 20, 15, and 3 mm, respectively.

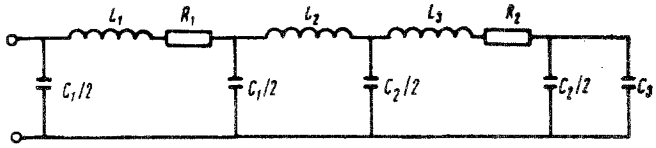

Fig. 6. Electric circuit of the simulator for a pipe which has a diaphragm installed at the middle and is connected to a large-volume chamber.

Investigations of systems whose spectra are well known, for instance, a pipe whose one end is closed while the other is open and a pipe with two closed ends, have shown that the accuracy of investigations based on the electric analog simulator attains 2.5%, which is entirely acceptable in solving technological problems.

By using the electric analog simulator, we succeeded in determining the natural frequency spectra of a number of systems whose analytical determination is difficult and sometimes impossible because of the lack of analytical expressions.

We shall now give the results of the simulation of some piping systems.

Pipe with a diaphragm installed at the end. The cells simulating a pipe of a certain given length were mounted on the assembly panel. The diaphragm was simulated by an inductance L_2, which was connected in series to the circuit. The open end beyond the diaphragm was simulated by means of a sufficiently large electric capacitance C_2 (Fig. 4).

Figure 5 shows the frequency characteristics of this system, obtained on the screen of the ASChKh-1 device. It is obvious that the connection of a diaphragm at the open end of the pipe affects the natural frequency spectrum, causing a shift of the resonance frequencies. The shift is limited on the high-frequency side by the resonance frequency of a pipe whose one end is open and the other is closed, while, on the low-frequency side, the shift is limited by the resonance frequency of a pipe with both ends closed.

Pipe with a diaphragm installed at the middle and connected to a large-volume chamber. The simulator circuit is shown in Fig. 6. Here L_1, L_3, C_1, C_2, R_1, and R_2 are the respective inductances, capacitances, and resistances simulating the pipe, L_2 the inductance simulating the diaphragm, and C_3 the capacitor simulating the chamber's volume.

Figure 7 shows the frequency characteristics obtained on the screen of the ASChKh-1 device for the given system. It is obvious from the characteristics shown that the installation of diaphragms affects the natural frequency spectrum. The investigation results indicate the fallacy of the opinion [3] that local lumped resistances do not affect the natural oscillation frequency and, consequently, the position of resonance frequencies.

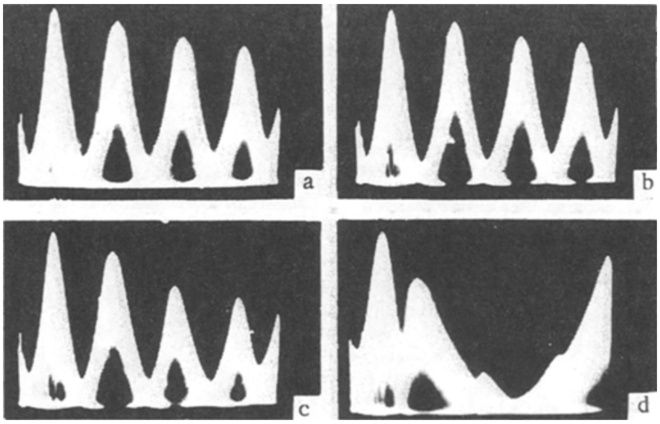

Fig. 7. Frequency characteristics of a system consisting of a pipe with a length of 27 m which is connected to a 5-m^3 chamber. a) Without diaphragm; b to d) with diaphragms having diameters of 20, 15, and 3 mm, respectively, installed at a distance of 6 m from the front end.

An important advantage of the proposed method is its clarity. By using the proposed simulator, one can quickly estimate any change introduced in the design of a piping system and choose the optimum piping system which would guarantee operation outside the zone of resonance conditions.

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