APPROXIMATE METHOD FOR CALCULATING THE AMPLITUDE OF LIQUID PULSATIONS IN EXTRACTION COLUMNS

(UDC 66.048.6)

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Translated from Khimicheskoe i Neftyanoe Mashinostroenie, No. 3,
pp. 23-24, March, 1965

The pulsation column constitutes one of the most efficient types of extraction equipment. However, certain difficulties, like the problem of imparting pulsating motion to the liquid filling the column, are encountered in designing industrial models of such columns.

The present article describes an approximate method for calculating the amplitude of pulsations of the liquid in the column as a function of the design characteristics of the column, the air cushion volume, and the pulsation conditions.

The figure shows the diagram of an extraction column, in the pulsation main of which the operating and the auxiliary liquids are separated by an air cushion. This layout can be considered as a peculiar oscillatory circuit, where the air cushion plays the role of a rigid body, the liquid in the column and in the communicating tube plays the role of the oscillating body, while the hydrodynamic resistance to the liquid's motion plays the role of friction. The compelling force acts on the system through the upper surface of the auxiliary liquid. The motion of this surface accurately reproduces the pulsator's motion.

The column and the pipe usually consist of series-connected sections with different diameters, lengths, and hydrodynamic resistance values.

The pressure drop in the i-th section can be expressed by means of the equation

$$\Delta P_i = \frac{m_i}{S_i}\ddot{x}_i + \beta_i |\dot{x}_i| \dot{x}_i,$$

where the first term on the right-hand side accounts for the inertial quality of the liquid, while the second term expresses the hydrodynamic resistance to its motion.

After performing summation with respect to all n sections, we obtain

$$\Delta P = \sum_{i=1}^{n} \Delta P_i = \sum_{i=1}^{n} \frac{m_i}{S_i}\ddot{x}_i + \sum_{i=1}^{n} \beta_i |\dot{x}_i| \dot{x}_i, \tag{1}$$

where $\Delta P = P - P_a$; P is the pressure in the air cushion and P_a the atmospheric pressure.

In the general case of a real gas in the air cushion, the Poisson law $PV^{\varkappa} = \text{const}$ holds.

In this case,

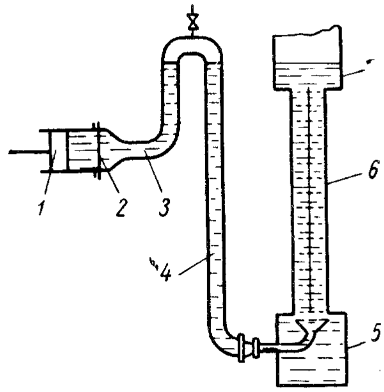
$$\frac{P}{P_0}\left(\frac{V_0 + S_1 x_1 - S_n x_n}{V_0}\right)^{\varkappa} = 1,$$

where V_0 is the initial volume of the air cushion, S_1 the cross-sectional area of the first section, S_n the cross-sectional area of the auxiliary liquid, x_1 the liquid's displacement in the first section, and x_n the displacement of the auxiliary liquid.

The exponent $\varkappa$ constitutes a complex function of the conditions of heat exchange between the gas filling the air cushion and the surrounding medium. However, under actual conditions, for relatively low pulsation

Amplitude of Liquid Pulsations in the Tube Communicating with the Column (cm)

V_0, cm^3	Calculation method	Pulsation frequency, cpm					
		98		136		191	
		number of plates					
		60	30	60	30	60	30
1600	Calculation based on data by Roshchin, Baranov, and Chemezov.	3.7	3.7	2.0	2.0	1.0	1.0
	Experimental data.	4.8	6.1	2.4	2.4	–	1.0
	Calculation based on the author's method	4.97	5.9	2.4	2.5	1.1	1.11
2000	Calculation based on data by Roshchin, Baranov, and Chemezov.	3.1	3.1	1.6	1.6	0.8	0.8
	Experimental data.	4.3	4.7	2.0	2.1	0.6	0.8
	Calculation based on the author's method	4.19	4.78	1.93	1.99	0.87	0.87
2500	Calculation based on data by Roshchin, Baranov, and Chemezov.	2.6	2.6	1.3	1.3	0.7	0.7
	Experimental data.	3.6	4.1	1.7	1.7	0.4	0.5
	Calculation based on the author's method	3.42	3.74	1.62	1.65	0.72	0.72
2900	Calculation based on data by Roshchin, Baranov, and Chemezov.	2.3	2.3	1.2	1.2	0.6	0.6
	Experimental data.	3.2	–	1.4	1.5	0.4	0.45
	Calculation based on the author's method	2.95	3.15	1.33	1.34	0.62	0.62



System for imparting pulsating motion to the liquid in the column. 1) Pulsator; 2) membrane; 3) auxiliary liquid; 4) pipe communicating with the column; 5) lower settling tank; 6) operating part of the column; 7) upper settling tank.

frequencies and amplitudes, it can be considered that the processes of gas compression and expansion occur isothermally, i.e., $\varkappa = 1$.

In this case, assuming that $P_0=P_a$, we obtain

$$P - P_0 = \frac{P_0(S_n x_n - S_1 x_1)}{V_0 - S_n x_n + S_1 x_1}$$

and we can write Eq. (1) in the following form:

$$\sum_{i=1}^{n} \frac{m_i}{S_i}\ddot{x}_i + \sum_{i=1}^{n}\beta_i|\dot{x}_i|\dot{x}_i + \frac{P_0 S_1}{V_0 + S_1 x_1 - S_n x_n}x_1 = \frac{P_0 S_n}{V_0 + S_1 x_1 - S_n x_n}x_n. \tag{2}$$

According to the law of the continuity of mass, the liquid's motion in any section is rigidly bound to its motion in the first section

$$x_i = \frac{S_1}{S_i}x_1;\ \dot{x}_i = \frac{S_1}{S_i}\dot{x}_i;\ \ddot{x}_i = \frac{S_1}{S_i}\ddot{x}_1,$$

whence,

$$\sum_{i=1}^{n}\frac{m_i}{S_1}\ddot{x}_i = \rho S_1 x_1 \sum_{i=1}^{n}\frac{L_i}{S_i} = m\ddot{x}_1;\quad \sum_{i=1}^{n}\beta_i|\dot{x}_1|\dot{x}_i = |\dot{x}_1|\dot{x}_1\sum_{i=1}^{n}\beta_i\left(\frac{S_1}{S_i}\right)^2 = \beta|\dot{x}_1|\dot{x}_1;$$

then, Eq. (2) assumes the following form:

$$\frac{m}{P_0 S_1}(V_0 + S_1 x_1 - S_n x_n)\ddot{x}_1 + \frac{\beta}{P_0 S_1}\times(V_0 + S_1 x_1 - S_n x_n)|\dot{x}_1|\dot{x}_1 + x_1 = \frac{S_n}{S_1}x_n. \tag{3}$$

In order to simplify this solution, we shall assume that $S_n = S_1 = S$, $x_1 = x$ and denote $V_0/S = L_0$. After this, Eq. (3) can be written thus:

$$\frac{m}{P_0}(L_0 + x - x_n)\ddot{x} + \frac{\beta}{P_0}(L_0 + x_1 - x_n)|\dot{x}|\dot{x} + x = x_n. \tag{4}$$

If x_n constitutes a harmonic function of time, this equation has the form of the ordinary equation of forced oscillations, and it differs from the latter only by the fact that it contains nonlinear terms.

By using the harmonic balance method, we shall find the solution of Eq. (4) in the form of $x = a_0 + a \sin \omega t$ without considering the higher harmonics. Then, the constants a_0, a, and ψ will be related by the equations

$$\begin{gathered} a\left[1 - \frac{m}{P_0}(L_0 + a_0)\omega^2\right] = A\cos\psi; \\ a^2 \cdot \frac{8}{3\pi}\,\frac{\beta(L_0 + a_0)\omega^2}{P_0} = A\sin\psi; \\ \frac{L_0}{L_0 + a_0} = 1 - \frac{a^2\omega^2\beta}{P_0}\left(\frac{8}{3\pi}\cdot\frac{a^2\omega^2\beta}{P_0} - 1 + \frac{8}{3\pi}\right). \end{gathered} \tag{5}$$

The consideration of a_0 introduces certain difficulties in the solution. However, an approximate estimate of the a_0 value on the basis of Eq. (5) indicates that this value does not exceed a few percent of the L_0 value under actual conditions. If we neglect a_0, we can write Eq. (5) in the following form:

$$\left.\begin{aligned} a\left(1 - \frac{m\omega^2}{P_0}L_0\right) &= A\cos\psi; \\ a^2\frac{8}{3\pi}\cdot\frac{\beta\omega^2 L_0}{P_0} &= A\sin\psi. \end{aligned}\right\} \tag{6}$$

After introducing the notation

$$\frac{P_0}{mL_0} = \omega_0^2 \text{ and } \frac{8}{3\pi}\cdot\frac{\beta}{m} = \frac{1}{B},$$

Eq. (6) can be simplified:

$$\left.\begin{aligned} a\left(1 - \frac{\omega^2}{\omega_0^2}\right) &= A\cos\psi; \\ \frac{a^2}{B}\cdot\frac{\omega^2}{\omega_0^2} &= A\sin\psi. \end{aligned}\right\} \tag{7}$$

Then, after eliminating $\cos\psi$ and $\sin\psi$, we obtain the equation

$$\frac{a^4}{B^2}\cdot\frac{\omega^4}{\omega_0^4} + a^2\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 - A^2 = 0,$$

the solution of which can be written as the following equation:

$$a^2 = -\frac{B^2}{2}\left(\frac{\omega_0^2}{\omega^2} - 1\right)^2 + \sqrt{\frac{B^4}{4}\left(\frac{\omega_0^2}{\omega^2} - 1\right)^4 + \frac{\omega_0^4}{\omega^4}B^2A^2}. \tag{8}$$

The phase shift of liquid oscillations is determined by Eq. (7):

$$\tan\psi = -\frac{a}{B\left(1 - \frac{\omega_0^2}{\omega^2}\right)}. \tag{9}$$

In Eqs. (8) and (9), we have

$$\omega_0^2 = \frac{P_0}{\rho V_0 \sum_{i=1}^{n} \frac{L_i}{S_i}}; \quad B = \frac{3\pi}{8} \cdot \frac{\rho S_1 \sum_{i=1}^{n} \frac{L_i}{S_i}}{\sum_{i=1}^{n} \beta_i \left(\frac{S_1}{S_i}\right)^2}.$$

A similar problem was solved in a paper by Roshchin, Baranov, and Chemezov [1]. The theoretical and experimental data obtained by these authors differ from each other to a considerable extent (see table). These discrepancies can obviously be explained by the erroneous assumption in calculations concerning the coincidence of the oscillation phases of the liquid in front of and beyond the air cushion.

As is shown by calculations based on Eq. (9), the minimum phase lag of the liquid oscillations in the column with respect to the pulsator oscillations is equal to ~120°.

The results obtained by the author in calculating the pulsation amplitude by means of Eq. (8) and using the data provided by Roshchin, Baranov, and Chemezov are given in the table.

It is seen that the calculation results are in good agreement with the experimental data.

LITERATURE CITED

1. A. N. Roshchin, G. P. Baranov, and V. A. Chemezov, Investigation and Design of Chemical Technology Equipment, NIIKhimmash transactions, 35 and 61 (1960).