# POWER CONSUMPTION, INTENSITY OF AGITATION, AND EFFICIENCY OF RADIAL-BLADE STIRRERS

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The problem of assessing the power consumption of an agitator has not as yet been completely solved.

Equations are still being published which are based on the operation of stirrers in an infinite liquid medium with coefficients of resistance which are theoretically unfounded. In the present paper theoretical equations and a calculation equation are given which have been derived by the author from numerous experiments with laboratory and industrial radial-blade stirrers working in a dimensionally limited liquid.

Figure 1 shows two cylindrical flat-bottomed vessels which were filled with water and transformer oil at 20°C to a height of $H_1 = D = 240$ mm.

The vessels were fitted with a standard plate stirrer with $h_s = L = 2R_b$ and a two-blade stirring element $L = 2R_b$ (here $R_b$ is the radius of the stirrer blade) and $h = 0.079$; in both cases

$$\frac{L}{D} = 0{,}66.$$

By varying the stirrer speed vortex cones, funnels of equal size were produced in the liquids (measured with a tracer device in a meridianal section) which was an indication of a steady motion of liquids in the vessels. Experiments showed that this result is achieved at a lower speed with a plate-type stirrer, since it has a smaller blade area than a two-blade stirrer.

It should be pointed out that, in the case of identical profiles of the vortex cone in both vessels, the reaction moments caused by the interaction between the liquid being agitated and the walls and bottoms of vessels measured

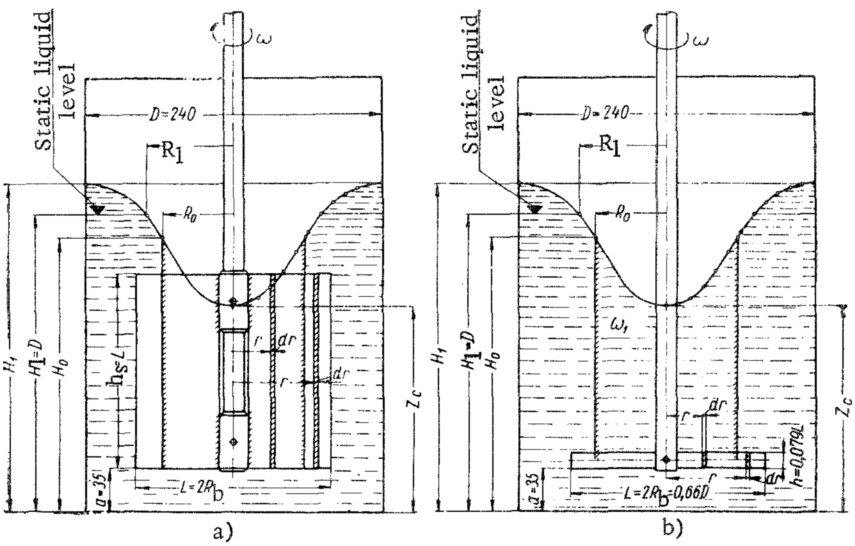


Fig. 1. Schematic diagrams of stirrers. a) Standard plate stirrer; b) two-blade stirrer.

TABLE 1. Results Obtained in Testing Blade and Plate Stirrers with L/D = 0.5

| Test No. | Radius r, mm | Calculation equations | $z_c$, $z$, $H_0$, $H_y$, $H_1$ in mm, calc. | $z_c$, $z$, $H_0$, $H_y$, $H_1$ in mm, exper. | Deviation of computed data from experimental values, % |
|---|---|---|---|---|---|
| 1 | 0 | $z_c = z_y - \frac{\beta c^2}{g}\left(\frac{1}{R_0^2} - \frac{1}{R_y^2}\right) - \frac{\omega_1^2 R_0^2}{2g}$ | 167 | 167 | 0 |
| 2 | 10 | $z_{2-6} = z_c + \frac{\omega_1^2 r^2}{2g}$ | 169,1 | 169 | 0 |
| 3 | 20 | $c = \omega_1 R_0^2$ | 175,5 | 176 | 0,28 |
| 4 | 30 | Range of vortex flow | 186 | 190 | 2,1 |
| 5 | 40 | | 201 | 211 | 4,7 |
| 6 | $R_0 = 48$ | $H_0 = z_c + \frac{\omega_1^2 R_0^2}{2g}$ | 216 | 216 | 0 |
| 7 | 50 | $\omega_1 = \omega\left(\frac{h}{2R_л}\right)^{0,115}$ | 220 | 224 | 1,78 |
| 8 | 60 | $H_y = \frac{\beta c^2}{2g}\left(\frac{1}{R_0^2} - \frac{1}{R_y^2}\right) + H_0$ | 234,4 | 234,5 | 0 |
| 9 | $R_y = 66$ | | 240 | 240 | 0 |
| 10 | 70 | $z_{6-14} = \frac{\beta c^2}{2g}\left(\frac{1}{R_0^2} - \frac{1}{r^2}\right) + H_0$ | 243 | 242 | 0,41 |
| 11 | 80 | | 248,7 | 247 | 0,68 |
| 12 | 90 | Range of nonvortex flow | 252,6 | 250 | 1,03 |
| 13 | 100 | | 255,3 | 253 | 0,91 |
| 14 | 110 | | 257,3 | 255 | 0,9 |
| 15 | 120 | $H_1 = \frac{\beta c^2}{2g}\left(\frac{1}{R_0^2} - \frac{1}{R^2}\right) + H_0$ | 258,5 | 257 | 0,58 |

Note. Stirrer blades had the following dimensions: L = 120 mm; h = 12.5, 15, 30, 40, 50, 80, 90, 105, and 120 mm; a = 35 mm. Vessel D = $H_1$ = 240 mm; t = 20°C.

by means of a torque dynamometer were equal. This suggests that the energy consumption for agitating the liquid was the same for both vessels. Tables 1 and 2 compare the experimental and the theoretical heights of parametric points on the free surface of the liquid during the operation of the two-blade and plate stirrers in water and transformer oil with the same ratio L/D, but with different areas of working surfaces.

Investigations established that both stirrers produce, at different speeds but with equal power consumption, identical hydrodynamic conditions in the liquids being stirred. The tables show that the experimental data are in a good agreement with those calculated from equations with which all parameters of the free surface of the liquid produced by various types of radial-blade stirrers can be calculated. Similar results were obtained during the investigation of two-blade stirrers in industrial vessels during the agitation of liquids with a viscosity of $\mu$ = 1-75 cP.

Numerous experimental investigations show that the complex flow of the liquid in a vessel of limited size rotates in the same direction as the blades of the stirring element but with different velocities at different points of the blade's length. The force acting on the blades of the stirrer is produced mainly by the head resistance of the liquid moving in the vessel at relative angular speeds directed normal to the working surfaces of the blades. The fact that part of the liquid flows at radial and axial velocities produces an insignificant sliding friction on the surfaces of the blades, which has no appreciable effect on the change of the main portion of the head resistance.

Figure 2 gives as an example the angular velocity curves for transformer oil stirred in the vessel and also of the blades of the plate and the two-blade stirrers. The angular velocity of the two-blade stirrer is much higher than that of the plate-type stirrer. This can be attributed to the fact that the volume of liquid moved by the two-blade agitator per second is smaller than that of the plate-type stirrer, as a result of which two stirrers which produce the same hydrodynamic conditions in two vessels must have different angular velocities.

Figure 3 gives the diagrams of the relative angular velocities of transformer oil particles in the vortex and nonvortex zones of the flow.

According to these curves the angular velocities of moving particles of the liquid being agitated in the vortex and the nonvortex zones may be expressed as follows:

$$v_{rel_1} = (\omega - \omega_1)\, r;$$

$$v_{rel_2} = \omega r - \frac{c}{r},$$

where c is the tension or the intensity of circulation, $\omega$ the angular velocity of the blade, and r the variable current radius.

From the kinematic investigation [2], we know that

$$c = \omega_1 R_0^2.$$

TABLE 2. Results Obtained in Testing Blade and Plate Stirrers with L/D = 0.66

| Test No. | Radius r, mm | Calculation equations | $z_c$, $z$, $H_0$, $H_1$ in mm calc. | $z_c$, $z$, $H_0$, $H_1$ in mm exper. | Deviation of computed data from experimental values, % |
|---|---|---|---|---|---|
| 1 | 0 | $z_c = H_y - \frac{\beta c^2}{2g}\left(\frac{1}{R_0^2} - \frac{1}{R_y^2}\right) - \frac{v_0^2}{2g}$ | 164,7 | 165,0 | 0,12 |
| 2 | 10 | $z_{2-6} = z_c + \frac{\omega_1^2 r^2}{2g}$ | 166,4 | 167,0 | 0,36 |
| 3 | 20 | $c = \omega_1 R_0^2$ | 171,5 | 171,0 | 0,30 |
| 4 | 30 | Range of vortex flow | 179,9 | 178,0 | 1,06 |
| 5 | 40 | | 191,9 | 194,0 | 1,10 |
| 6 | 50 | $\omega_1 = \omega\left(\frac{h}{2R_A}\right)^{0,143}$ | 207,2 | 210,0 | 1,30 |
| 7 | $R_0$=57,8 | $H_0 = z_c + \frac{\omega^2 R_0^2}{2g}$ | 221,5 | 222,0 | 0,22 |
| 8 | 60 | | 226,0 | 225,7 | 0,13 |
| 9 | $R_y$=70 | $H_y = \frac{\beta c^2}{2g}\left(\frac{1}{R_0^2} - \frac{1}{R_y^2}\right) + \frac{\omega_1^2 R_0^2}{2g}$ | 240,2 | 239,5 | 0,30 |
| 10 | 80 | $z_{8-13} = \frac{\beta c^2}{2g}\left(\frac{1}{R_0^2} - \frac{1}{r^2}\right) + H_0$ | 249,5 | 248,5 | 0,40 |
| 11 | 90 | | 256,0 | 256,5 | 0,20 |
| 12 | 100 | Range of nonvortex flow | 260,5 | 260,7 | 0,10 |
| 13 | 110 | | 263,8 | 263,0 | 0,30 |
| 14 | 120 | $H_1 = \frac{\beta c^2}{2g}\left(\frac{1}{R_0^2} - \frac{1}{R^2}\right) + H_0$ | 264,8 | 265,0 | 0,10 |

Note. Stirrer blades had the following dimensions: L = 158 mm; h = 12.5, 25, 37.5, 50, 62.5, 79, 100, 120, and 158 mm; a = 35 mm. Vessel D = $H_1$ = 240 mm; t = 20°C.

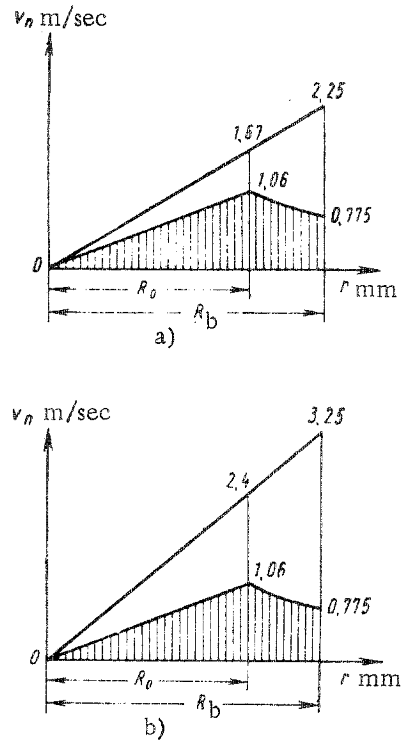


Fig. 2. Angular velocity curves for agitated transformer oil (shaded area) and for various stirring devices. a) Plate-type; b) two-blade type.

A point of great importance for these equations is to determine for real conditions the angular velocity of the liquid in the zone of the vortex flow $\omega_1$ and the radius of the core of the vortex cross section $R_0$.

Experimental investigations carried out with the radial-blade stirrers in cylindrical flat-bottom vessels with a diameter of 600-1250 mm filled to a height $H_1$ = D with liquids of different viscosities produced the following relationship:

$$\omega_1 = A\left(\frac{s_x}{s_s}\right)^n \cdot \left(\frac{\mu_w}{\mu_{li}}\right)^m \left(\frac{R_s}{R}\right)^p \omega, \tag{1}$$

where the coefficient A = 3.02, $s_x$ the working area of the blades being investigated submerged in the liquid, $s_s = L^2 = 4R_b^2$ the standard working area of the plate-type stirrer, $\mu_w$ and $\mu_{li}$ the absolute viscosities of water and of the liquid being investigated at 20°C in cP, $R_s$ = 120 mm the radius of the standard vessel, and R the radius of various vessels in mm.

For the blades of stirring elements with a ratio L/D = 0.5, the following data were obtained: n = 0.115, m = 0.105, and b = 1.35. For L/D = 0.66, the values of m and p remained the same while n = 0.143.

Thus, for plate-type stirrers with L/D = 0.5 and 0.66 (D = 600 mm, $H_1$ = D) we obtained for low-viscosity liquids the following calculation equation:

$$\omega_1 = 3.02\,\omega\left(\frac{\mu_w}{\mu_{li}}\right)^{0.115}\left(\frac{R_s}{R}\right)^{1.35}.$$

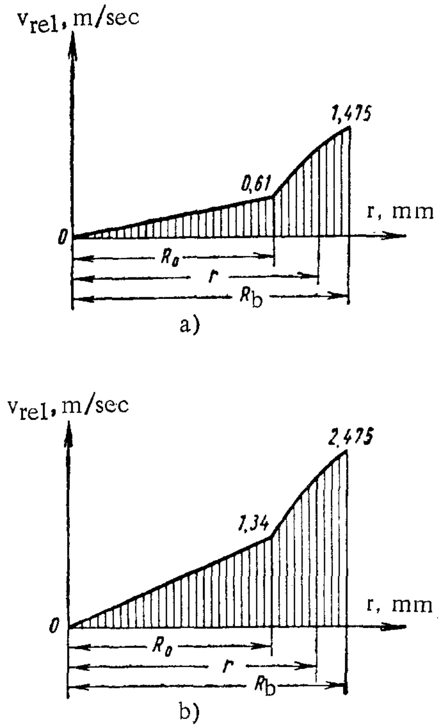


Fig. 3. Diagrams of the relative angular velocities for transformer oil in a direction normal to the working surfaces of the stirrer. a) Plate-type stirrer; b) two-blade propeller stirrer.

For two-blade stirrers working in the same conditions with L/D = 0.5,

$$\omega_1 = 3.02\,\omega \left(\frac{h}{2R_b}\right)^{0.115} \left(\frac{\mu_w}{\mu_{li}}\right)^{0.115} \left(\frac{R_s}{R}\right)^{1.35},$$

and with L/D = 0.66,

$$\omega_1 = 3.02\,\omega \left(\frac{h}{2R_b}\right)^{0.143} \left(\frac{\mu_w}{\mu_{li}}\right)^{0.115} \left(\frac{R_s}{R}\right)^{1.35}.$$

For determining the core radius of the vortex section we use the equation [3] obtained earlier:

$$R_0 = R_{\exp}\left[0.5 + \frac{0.25}{1 + \frac{\mathrm{tg}^2\gamma}{n_1^2}} - \frac{R^2}{2R_1^2}\right], \tag{2}$$

where $n_1$ is the total number of sinks and sources, R the radius of the vessel, and $R_1$ the radius of the cross section of the vortex cone at the line of the static level of the liquid.

In this case,

$$R_1 = R\left[0.508 + 0.215\left(\frac{R_b}{R} - 0.3\right)\right], \tag{3}$$

and, finally,

$$\tan\gamma = 0.527\left(-\frac{R_b}{R}\right)^{-0.73}. \tag{4}$$

Equations (2)-(4) can be used for calculating radial-blade stirrers with a ratio $R_b/R$ of 0.3-1.

The masses of the liquid being agitated in the zones of the vortex and nonvortex flows with which the blades of the two-blade stirrer make continuous contact can be determined from the following equation:

$$dm_1 = \frac{\gamma_{li} h}{g}(\omega - \omega_1)\,r\,dr; \quad dm_2 = \frac{\gamma_{li} h}{g}\left(\omega r - \frac{c}{r}\right)dr,$$

where c is the intensity of circulation.

Using the theorem on the kinetic moment of the system we determine the maximum bending moment due to external forces acting on one blade of the two-blade stirrer:

$$M_{be}^{\max} = \frac{\gamma_{li} h}{g}\left[(\omega - \omega_1)^2 \int_0^{R_0} r^3 dr + \int_{R_0}^{R_b}\left(\omega r - \frac{c}{r}\right)^2 r\,dr\right] = \frac{\gamma_{li} h}{g}\left[\frac{\omega^2 R_b^4}{4} + \omega c\left(\frac{R_0^2}{2} - R_b^2\right) + c^2\left(0.25 + \ln\frac{R_b}{R_0}\right)\right]. \tag{5}$$

The torque on the stirrer shaft is determined from the equation

$$M_{to} = 2M_{be}^{\max}$$

The power consumption of the two-blade stirrer during steady motion is determined from the equation

$$N = \omega M_{to} \tag{6}$$

The volumes of the liquid thrust by the elementary areas of blades (Fig. 2) per second during steady motion in the vortex and nonvortex zones of the flow are

$$dQ_1 = h(\omega - \omega_1)\,r\,dr; \quad dQ_2 = h\left(\omega r - \frac{c}{r}\right)dr.$$

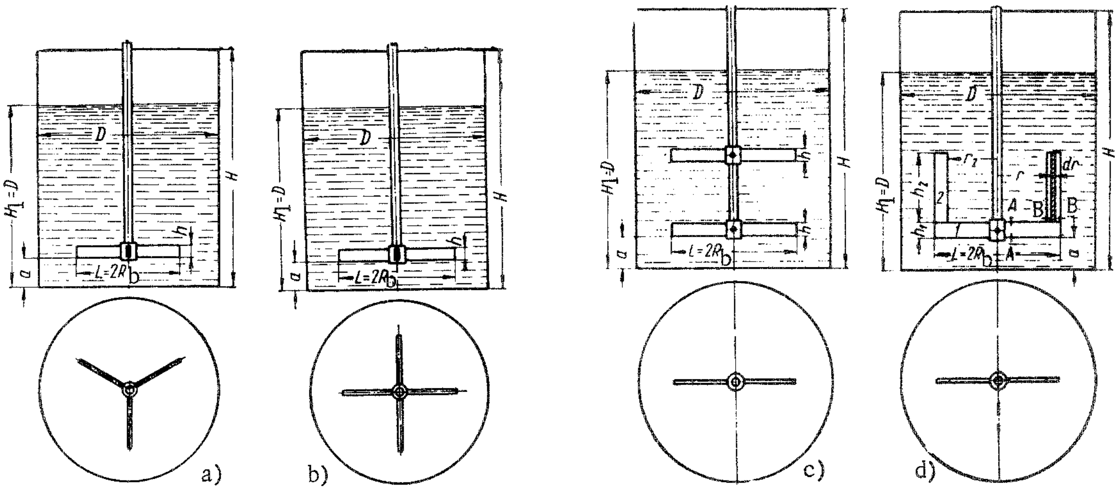


Fig. 4. Schematic diagrams of the stirrers being investigated. a) Three-blade propeller; b) four-blade propeller; c) two-blade two-stage type; and d) rectangular armature type.

The total volume of the liquid thrust in the vessel by the two-blade stirrer per second during steady motion is

$$Q = 2h\left[(\omega - \omega_1)\int_0^{R_0} r\,dr + \int_{R_0}^{R_b}\left(\omega r - \frac{c}{r}\right)dr\right] = h\left[\omega R_b^2 - c\left(1 + 2\ln\frac{R_b}{R_0}\right)\right].$$

The masses of the agitated liquid $dm_1$ and $dm_2$ in the vortex and nonvortex zones of the flow with which the elementary areas of the plate-type stirrer make contact during rotation for one second can be determined from the following equations

$$dm_1 = \frac{\gamma_{li} L}{g}(\omega - \omega_1)\,r\,dr;$$

$$dm_2 = \frac{\gamma_{li} L}{g}\left(\omega r - \frac{c}{r}\right)dr.$$

The maximum bending moment due to external forces acting on one blade of the plate-type stirrer is determined from the theorem on the kinetic moment of the system:

$$M_{be}^{max} = \frac{\gamma_{li} L}{g}\left[(\omega - \omega_1)^2\int_0^{R_0} r^3 dr + \int_{R_0}^{R_b}\left(\omega r - \frac{c}{r}\right) r\,dr\right]$$

$$= \frac{\gamma_{li} L}{g}\left[\frac{\omega^2 R_b^4}{4} + \omega c\left(\frac{R_0^2}{2} - R_b^2\right) + c^2\left(0.25 + \ln\frac{R_b}{R_0}\right)\right].$$

The torque acting on the stirrer shaft is found from the equation

$$M_{to} = 2M_{be}^{max}.$$

The power consumption of the plate-type stirrer for low-viscosity liquids during established motion is determined from Eq. (6).

The above equations show that the formula of the plate-type stirrer is a general equation and can produce as special cases the equations for calculating the two-blade and other stirrers.

For the plate-type stirrer the author obtained

$$Q = 2R_b\left[\omega R_b^2 - c\left(1 + 2\ln\frac{R_b}{R_0}\right)\right].$$

In a similar way the values of $\omega_1$, $M_{be}$, $M_{to}$, N, and Q were determined for stirrers of other designs.

We consider the operation of four stirrer types, the sketches and dimensions of which are given in Fig. 4. The following data were obtained for the three-blade stirrer:

for L/D = 0.5:

$$\omega_1 = 3.02\,\omega\left(\frac{3h}{4R_b}\right)^{0.115}\left(\frac{\mu_w}{\mu_{li}}\right)^{0.115}\left(\frac{R_s}{R}\right)^{1.35},$$

and for L/D = 0.66:

$$\omega_1 = 3.02\,\omega\left(\frac{3h}{4R_b}\right)^{0.143}\left(\frac{\mu_w}{\mu_{li}}\right)^{0.115}\left(\frac{R_s}{R}\right)^{1.35}.$$

$M_{be}^{max}$ is determined from Eq. (5) while $M_{to} = 3M_{be}^{max}$. N is found from Eq. (6):

$$Q = 1.5h\left[\omega\,R_b^2 - c\left(1 + 2\ln\frac{R_b}{R_0}\right)\right].$$

For the four-blade stirrer with L/D = 0.5,

$$\omega_1 = 3.02\,\omega\left(\frac{h}{R_b}\right)^{0.115}\left(\frac{\mu_w}{\mu_{li}}\right)^{0.115}\left(\frac{R_s}{R}\right)^{1.35}, \quad (7)$$

and for L/D = 0.66,

$$\omega_1 = 3.02\,\omega\left(\frac{h}{R_b}\right)^{0.143}\left(\frac{\mu_w}{\mu_{li}}\right)^{0.115}\left(\frac{R_s}{R}\right)^{1.35}. \quad (8)$$

$M_{be}^{max}$ is determined from Eq. (5), and

$$M_{to} = 4M_{be}^{max} \quad (9)$$

N is found from Eq. (6):

$$Q = 2h\left[\omega\,R_b^2 - c\left(1 + 2\ln\frac{R_b}{R_0}\right)\right]. \quad (10)$$

For the two-blade two-stage stirrer $\omega_1$ (for L/D = 0.5) is determined from Eq. (7) and for L/D = 0.66 from Eq. (8).

$M_{be}^{max}$ is found from Eq. (5) and $M_{to}$, N, and Q from Eqs. (9), (6), and (10), respectively.

For the rectangular armature-type stirrer the bending moment of blade 1 in the cross section AA is determined from Eq. (5) adapted for the calculation of the two-blade stirrer:

for L/D = 0.5

$$\omega_1 = 3.02\,\omega\left[\frac{h_1R_b + h_2(R_b - r_2)}{2R_b^2}\right]^{0.115}\left(\frac{\mu_w}{\mu_{li}}\right)^{0.115}\left(\frac{R_s}{R}\right)^{1.35};$$

for L/D = 0.66

$$\omega_1 = 3.02\omega\left[\frac{h_1R_b + h_2(R_b - r_2)}{2R_b^2}\right]^{0.143}\left(\frac{\mu_w}{\mu_{li}}\right)^{0.115}\left(\frac{R_s}{R}\right)^{1.35};$$

$$M_{be-2} = \frac{\gamma_{li}h_2}{g}\left[\omega^2\frac{R_b^4 - r_2^4}{4} - \omega c\left(R_b^2 - r_2^2\right) + c^2\ln\frac{R_b}{r_2}\right].$$

The maximum bending moment for blade 1 in the section AA about the axis of rotation is determined from the equations

$$M_{be}^{max} = M_{be-1} + M_{be-2},$$

and

$$M_{\text{to}}=2M_{\text{be}}^{\text{max}}$$

The power consumption N by a rectangular armature-type stirrer during a working period is found from Eq. (6):

$$Q=h_1\left[\omega R_{\text{b}}^2-c\left(1+2\ln\frac{R_{\text{b}}}{R_0}\right)\right]+h_2\left[0.5\,\omega\left(R_{\text{b}}^2-r_2^2\right)-c\ln\frac{R_{\text{b}}}{r_2}\right].$$

For calculating the strength of blades 1 and 2, it is necessary to find the bending moment of blade 2 in cross section BB:

$$M'_{\text{be}}=\frac{\gamma_{\text{li}}h_2^2}{2g}\left[\omega^2\,\frac{R_{\text{b}}^3-r_2^3}{3}-2\omega\,c\,(R_{\text{b}}-r_2)-\frac{c^2}{r}\right].$$

It may be seen from the design of the rectangular armature-type stirrer that blade 2 is subjected to bending in section BB while section AA of blade 1 is subjected to bending by $M_{\text{be}}^{\text{max}}$ and to torsion by $M'_{\text{be-2}}$. Consequently, blade 1 must be proportioned for the combined stress while blade 2 must be proportioned for bending stress only.

Using the theoretical and experimental investigation of the power consumed by the radial-blade stirrers and of the volume of liquid thrust per second,we now proceed to the consideration of the problems of intensity and efficiency of agitation.

The author believes that the intensity of agitation must be determined from the amount of energy rationally introduced per unit volume of liquid moved by the stirrer per unit time:

$$J=\frac{N}{Q}.$$

Thus, the intensity of agitation is characterized by the pressure of the liquid produced by the stirrer in the rotating flow enclosed in the vessel. A change of stirrer speed instantaneously changes the pressure in the flow of the liquid being stirred as well as the profile of the vortex cone which characterizes the magnitude of the velocity head and of the power consumption.

It should be pointed out that the increasing intensity of agitation correspondingly increases the power consumption, but the technological effect of the increasing intensity of agitation has its strict limitations. For this reason the intensity of agitation in vessels with stirrers must be selected on the basis of the experimental data and the conditions of agitation at minimum power consumption.

The author believes that the efficiency of agitation is best characterized by the attainable degree of uniformity of a unit volume and the intensity requiring minimum power consumption and time for performing the process. In this case the efficiency of agitation can be expressed by the equation

$$\text{E}=\frac{N}{Q}\cdot\frac{V}{Q},$$

where V is the unit volume of the liquid being agitated in $m^3$.

Consequently, the efficiency is characterized by the intensity of agitation and the time spent for the circulation of a unit of the volume being agitated. In this case, the aspects of efficiency and intensity of agitation are closely interrelated.

The results of investigation [1] are of particular practical interest since it was carried out for determining the most rational designs of radial-blade (propeller) mixers by studying their performance in low-viscosity liquids at equal levels of power consumption. Experiments showed that various types of stirrer require, at the same values of the ratio L/D, viscosity, and power consumption, different speeds to bring the liquid contained in the vessel to the same hydrodynamic condition characterized by a definite deformation of the free surface (profile of the vortex cone). Consequently, the intensity and efficiency of simple and more sophisticated designs of radial-blade stirrers are equal.

These considerations are in good agreement with the experimental results given in Fig. 5.

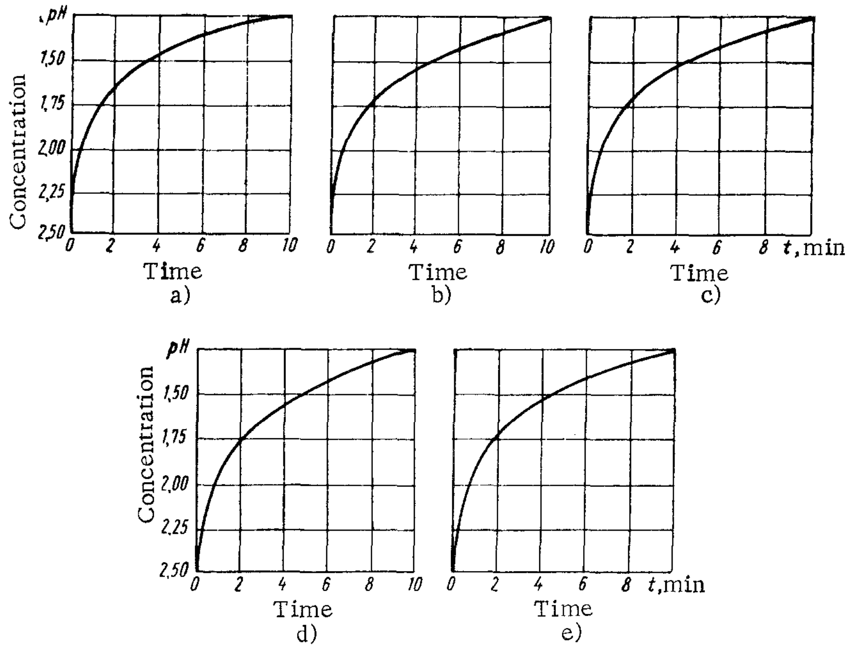


Fig. 5. Curves representing the dissolution of solid oxalic acid in water during the stirring of the solution with various propeller stirrers. a) Two-blade stirrer at h = 12.5 mm, n = 230 rpm; b) the same at h = 12.5 mm, n = 200 rpm; c) four-blade stirrer at n = 235 rpm; d) six-blade stirrer at n = 218 rpm; e) armature-type rectangular stirrer at n = 195 rpm.

Thus, this investigation shows that, during the stirring of low-viscous liquids of $\mu$ = 1-75 cP, the simplest stirrers should be used with their number reduced to a rational minimum.

From the point of view of a correct distribution of power, the three-blade stirrer with a blade spacing of 120° is the most satisfactory since it produces the most favorable conditions for the operation of the shaft and the drive.

For stirrers working in low-viscosity liquids, it is recommended to use the ratio L/D = 0.50, since in this case the power consumption is most efficient. In stirring more viscous liquids ($\mu$ > 75 cP), this ratio should be made 0.66 because of the reduced size of the core of the vortex section.

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