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One atom in an optical cavity: Spatial resolution beyond the standard diffraction limit

G. Rempe

Fakultät für Physik der Universität Konstanz, D-78434 Konstanz, Germany
(Fax: + 49-7531/88-3072)

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Abstract. The position of a slow atom passing through a standing-wave light field in an ultrahigh-finesse optical resonator can be measured by observing either the intensity of the light transmitted through the cavity or its phase. Apart from the periodicity of the standing wave, both techniques allow to determine the position of the particle with a resolution much better than the standard classical diffraction limit $\Delta x \geq \lambda/2$. Position measurements with uncertainty $< \lambda/20$ seem to be possible with all-optical techniques.

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High-resolution position measurements of neutral atoms with optical techniques are of considerable interest both from a theoretical as well as from a practical point of view. This is due to recent experimental progress in using light forces to manipulate the motion of atoms [1]. For example, channeling of atoms at the nodes of a standing-wave light field was demonstrated [2] and sub-optical structures were produced by atomic deposition (atom lithography) [3]. However, while these techniques were used to create sub-micron spatial distributions of *many* atoms $N \gg 1$, optical methods for measuring the position of a *single* individual free atom $N = 1$ with a resolution better than the wavelength of light are being explored only recently.

For example, a novel scheme for the observation of the position of free atoms was demonstrated recently by Gardner et al. [4]. This method employs light-shift-induced Raman transitions in which an inhomogeneous light intensity causes a spatially varying atomic level shift which correlates an atomic resonance frequency with the atomic position. A different scheme has been suggested by Storey et al. [5] and, independently, by Marte and Zoller [6]. In that scheme, the atom is localized by measuring the phase shift of the optical field in a standing-wave cavity due to the spatially varying atom–field coupling. A similar scheme to determine atomic momentum using a running wave in a ring cavity has been proposed by Sleator and Wilkens [7]. However, these proposals did not take into account the decay of the cavity field which is necessary to actually get information on the atom's position. This disadvantage was pointed out by Kunze et al. [8] who proposed to use long-lived atomic states to store and read the position information in a Ramsey-type experiment. Only recently, a quantum Monte-Carlo technique including spontaneous emission and cavity losses was used to simulate a quantum trajectory of this open atom-cavity system [9]. Within this context, the following short notes are intended to give an intuitive discussion of the problem and outline some avenues which make possible to continuously observe the position of a *single* free atom. Moreover, some attempts to achieve an analytical order-of-magnitude estimation for the resolution Δx of the position measurement are made.

Before discussing the general case, it is instructive to review first the Heisenberg microscope [10], which is the prototype of an all-optical measuring device. Here, the particle is irradiated by light and its position is inferred from the scattered light. The resolving power of this particular measurement scheme is limited by the Heisenberg uncertainty relation $\Delta x \Delta p \geq \hbar/2$, which results if the random momentum exchange between the particle and the radiation field is taken into account. Because only one photon is scattered by the particle, the change of its momentum p is limited to $\Delta p \leq \hbar k$. Here, $k = 2\pi/\lambda$ with λ denoting the light wavelength. As a result, the precision of this position measurement is limited by the size of the optical wavelength, i.e., $\Delta x \geq \hbar/2\Delta p \geq 1/(2k) = (\lambda/2\pi)/2$, which corresponds to the classical diffraction limit. Note that this discussion suggests to decrease Δx by allowing for a larger momentum transfer Δp, which can be accomplished by exchanging many photons between the particle and the measurement apparatus (i.e., the light field). Spontaneous emission events (with $\Delta p \leq \hbar k$), which randomly

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These notes were prepared to celebrate H. Walther's 60th birthday and to honour his pioneering contributions to some of the most lively fields of quantum optics disturb the phase of the atomic dipole, can be eliminated by, e.g., detuning the light field from the atomic transition frequency. However, due to the small electric-dipole coupling, the presence of a single atom is difficult to detect when observing a high-intensity laser-light beam. But, note that the effect of a single atom on the light field increases in a small cavity made of high-reflectivity mirrors (and is largely proportional to the finesse of the resonator, as discussed now).

To be specific, consider a two-level atom in a high-finesse optical cavity which is driven by an external field. The effects to be discussed can most easily be understood by considering the Maxwell–Bloch equations [11] for the (dimensionless) mean intracavity field α, the atomic polarization p and inversion w (in a rotating frame)

$$\begin{aligned} \dot{\alpha} &= -\kappa(\alpha-\beta) - gp, \\ \dot{p} &= -g\alpha w - \gamma'(1+\mathrm{i}\delta)p, \\ \dot{w} &= 2g(\alpha p^* + \alpha^* p) - \gamma(w+1), \end{aligned} \tag{1}$$

where β (taken to be real) is the intracavity field strength in the absence of an atomic medium, κ the cavity-field decay rate, γ and γ' the longitudinal and transverse decay rates of the atomic inversion and polarization, respectively, δ is the normalized detuning $\delta = (\omega_a - \omega_l)/\gamma'$ between the atomic transition frequency ω_a and the light frequency ω_l (assumed to be resonant with the empty-cavity frequency), and g is the atom–field coupling constant, which, for a Gaussian standing-wave cavity mode $\psi(x,y,z) = \cos(kx)\exp[-(y^2+z^2)/\omega_0^2]$ with beam waist ω_0, length L and mode volume $V = \int\psi^2\,\mathrm{d}V = \pi\omega_0^2 L/4$, can be written as

$$g(\mathbf{r}) = \mu\sqrt{\frac{\omega_l}{2\hbar\varepsilon_0 V}}\,\psi(\mathbf{r}). \tag{2}$$

Here, μ is the dipole-matrix element of the atomic transition considered. For a purely radiatively broadened atom $\gamma = 2\gamma'$, and γ equals the free-space spontaneous-emission rate for an atom in a cavity with a small solid angle (as is considered here).

In the limit of sufficiently weak excitation β (see the discussion below) the probability of finding the atom in the upper level can be neglected. Setting the inversion $w = -1$ constant and solving for the steady-state solutions of (1), the intracavity field is

$$\alpha(\mathbf{r}) = \frac{\beta}{1 + \dfrac{g^2(\mathbf{r})}{\kappa\gamma'(1+\mathrm{i}\delta)}}, \tag{3}$$

which depends on the position of the atom. Note that α and β refer to intracavity fields with and without atom, respectively. Since both the incident field and the output field are related to the intracavity fields by the same factor which is the mirror transmission coefficient, the ratio $|\alpha|^2/\beta^2$ determines the cavity transmission. For a further discussion of (3) it is interesting to consider first exact resonance $\delta = 0$ and then large detuning $|\delta| \gg 1$.

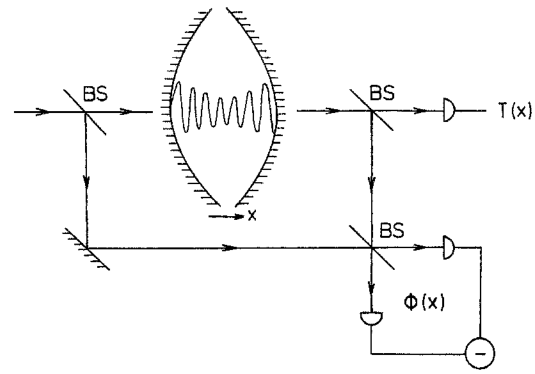

Fig. 1. Cavity geometry and relevant symbols; $T(x)$: transmission; $\phi(x)$: phase, BS: beam splitter

Case $\delta = 0$

In this case, α is purely real. Obviously, the position information is stored in the coupling constant $g(\mathbf{r})$ which should be made as large as possible. This can be achieved in a cavity with small volume ($g^2 \propto 1/V$) and high-reflectivity mirrors (small κ, i.e., high finesse $F = \pi c/2\kappa L$). Note that the condition $g > (\kappa, \gamma')$ corresponds to the limit of an atom strongly coupled to a single cavity mode, a requirement which was realized for the first time in experiments performed with the one-atom maser in the microwave domain [12]. Now, consider a symmetric Gaussian resonator mode between two identical concave optical mirrors (with a radius of curvature R larger than the mirror diameter D). The smallest cavity volume possible is given by "contacting" the mirrors (with only a small and therefore negligible spacing between them for the atoms to go through, see Fig. 1). Using $\omega_0^2 = (\lambda/\pi)(RL/2)^{1/2}$ for the beam waist of a Gaussian resonator mode with $R \gg L$ and $(RL)^{1/2} = D/2$ (which can be deduced from a straightforward geometrical consideration), (3) gives for the mean intensity-transmission coefficient through the cavity

$$\bar{T}(\mathbf{r}) = \frac{1}{\left(1 + \dfrac{g^2(\mathbf{r})}{\kappa\gamma'}\right)^2} = \frac{1}{\left[1 + \dfrac{6\sqrt{2}}{\pi^2}2\dfrac{F\lambda}{D}\psi^2(\mathbf{r})\right]^2}, \tag{4}$$

where $g^2(\mathbf{r}) = \gamma(3c\lambda^2)/(2\pi^2\omega_0^2 L)\,\psi^2(\mathbf{r})$ was used. Obviously, the position information is encoded in the intensity of the transmitted light field. Note that, apart from the resonance condition $\delta = 0$ and the specific value of λ, (4) depends on mirror parameters only. Therefore, different atoms can be studied with the same optical system, which is a necessary requirement for a useful device. Mirrors with losses of 1.6 ppm at the wavelength of the cesium D_2-resonance line at $\lambda = 852$ nm were demonstrated [13]. Two of these mirrors form an interferometer with a finesse $F \geq 10^6$ over a bandwidth larger than 40nm. Choosing $F = 10^6$, it follows that, with $D = 1$ cm diameter mirrors, the cavity transmission for an atom at an antinode $[\psi(\mathbf{r}) = 1]$ drops more than 10^4-fold below the empty-cavity transmission $[\psi(\mathbf{r}) = 0]$. This large effect can be explained by a shift of the normal-mode eigenfrequencies of the strongly coupled atom–cavity system with respect to the frequency of the incoming light. It can be understood intuitively by noting that the external driving field polarizes the atom, which, in turn, radiates into the cavity mode with a resulting phase shift of $-180°$. Therefore, β and the atom's polarization field interfere destructively giving rise to the ultralow transmission.

A drawback of the present device is that the transmitted intensity drops to a very small value and it is difficult to achieve an accurate position measurement with a reasonably large signal-to-noise ratio. This is due to the fact that, to avoid spontaneous emission, the atom was assumed to stay in the ground state. This limits the useful intensity, which scales proportional to the saturation photon number $n_s = \gamma\gamma'/4g^2$ [as can be calculated from (1) for $w = -1/2$]. In the regime of strong coupling, n_s is much smaller than one. In addition, note that quantum fluctuations of the transmitted light field can be quite significant for a high-finesse cavity [14]. This effect was not taken into account when starting from the semiclassical Maxwell-Bloch equations.

Case $|\delta| \gg 1$

In this case, and for $g^2/\kappa\gamma \gg 1$ (which is a reasonable assumption for a high-finesse cavity, as was discussed above), the presence of a single atom leads to a position-dependent change of the cavity transmission $T = |\alpha|^2/\beta^2$

$$T(\mathbf{r}) = \frac{1}{1 + \left(\dfrac{g^2(\mathbf{r})}{\kappa\gamma'\delta}\right)^2} = \frac{1}{1 + \left[\dfrac{6\sqrt{2}}{\pi^2}\dfrac{F\lambda}{D}\dfrac{\gamma}{\Delta}\psi^2(\mathbf{r})\right]^2}, \tag{5}$$

where $\Delta = \gamma'\delta = \omega_a - \omega_l$ and where the same assumptions were made as in the derivation of (4). With $F = 10^6$, $\lambda = 1$ µm and $D = 1$ cm, the transmission decreases by as much as 50% even for a detuning as large as $\Delta = 100\gamma$. But, note that, in contrast to the case of exact resonance, (5) is not independent of atomic parameters and that the effect of a single atom increases with the transition strength γ. However, an atom with large γ shows stronger absorption so that the intensity of the field has to be decreased to avoid spontaneous emission. A more careful analysis, therefore, requires a discussion of the signal-to-noise ratio which limits the resolution length of the position-measurement scheme. As the transmission depends on ψ^2, it is obvious that best resolution can be achieved for an atom which is localized halfway between a node and an antinode. The following order-of-magnitude estimation, therefore, concentrates on the one-dimensional case with $\psi(\mathbf{r}) = \psi(x) = \cos(kx) \propto 1/2^{1/2}$ only.

In part, the accuracy of the position measurement is determined by the fluctuations ΔN associated with the total number N of photons counted during the observation time τ (which will be specified below when discussing the motion of the atom). Neglecting technical noise sources, ΔN is given by the standard shot-noise limit, i.e., $\Delta N = N^{1/2}$. Obviously, the best signal-to-noise ratio is achieved with a high-power signal beam, i.e., large excitation β. However, the maximum possible number of photons is given by the requirement that no spontaneous-emission events occur during the observation time τ. The inversion can be calculated from the steady-state solution of (1) with the result $(w+1)/(-w) = 2(g/\Delta)^2|\alpha|^2$. It follows the condition that $(g/\Delta)^2|\alpha|^2\gamma \leq 1/\tau$ in the limit of small inversion $w \propto -1$. For given (τ, Δ), this establishes an upper limit for the intracavity photon number $|\alpha|^2$. Using $N \propto |\alpha|^2\tau/\tau_{cav}$, with $\tau_{cav} = 1/2\kappa$ as the photon storage time of the cavity, the maximum number of photons that can be detected is $N_{max} = (\Delta/g)^2(2\kappa/\gamma)$. Note that in this case $(w+1)/(-w) = 2/\gamma\tau$, so that for given γ, the limit $w \propto -1$ is approached only for an observation time $\tau \gg \gamma^{-1}$. Assuming $N = N_{max}$ (which can be realized experimentally by adjusting the driving-field strength), the resolution length $\Delta x = \Delta N/|dN/dx|$ is

$$\Delta x \propto \frac{\lambda/2\pi}{2T(\mathbf{r})\left(\dfrac{6\sqrt{2}}{\pi^2}\dfrac{F\lambda}{D}\right)^{3/2}\dfrac{\gamma}{\Delta}}. \tag{6}$$

It follows that $\Delta x = (\lambda/2\pi)/16 = \lambda/100 = 10$ nm (with $T = 0.8$ and $N_{max} = 400$) for $F = 10^6$, $\lambda = 1$ µm, $D = 1$ cm and $\Delta = 100\gamma$. Note that, for moderate atom–field detuning Δ (but with $|\delta| \gg 1$), the cavity transmission $T(\mathbf{r})$ is proportional to Δ^2 so that Δx decreases for increasing Δ. However, $T \propto 1$ for large Δ and Δx increases linearly with Δ.

To discuss a second but related measurement scheme, note that, for $|\delta| \gg 1$, the transmitted-field amplitude α is complex with respect to the empty-cavity field β, i.e., it accquires a phase shift ϕ with

$$\tan\phi(\mathbf{r}) = \frac{g^2(\mathbf{r})}{\kappa\gamma'\delta} = \frac{6\sqrt{2}}{\pi^2}\frac{F\lambda}{D}\frac{\gamma}{\Delta}\psi^2(\mathbf{r}), \tag{7}$$

which is valid for sufficiently large detuning $\delta^2 \gg g^2/\kappa\gamma'$. This phase shift $\phi(\mathbf{r})$ is due to the off-resonance refractive index of the atom which depends on the spatially varying atom-field coupling constant $g(\mathbf{r})$ and, therefore, on the position of the atom. It follows that a single atom behaves as a polarizable medium producing a phase shift as large as $\phi(\mathbf{r} = 0) \propto 1$ for $F = 10^6$, $\lambda = 1$ µm, $D = 1$ cm and a detuning $\Delta = 100\gamma$.

To determine the resolution length, let us suppose that the phase measurement is performed using a Mach–Zehnder interferometer with the atom–cavity system placed in one arm (homodyne phase detection). For equal intensities in both output ports, Δx can be taken as $\Delta x = \Delta\phi/|d\phi/dx|$, where $\Delta\phi$ is the uncertainty of an interferometric phase measurement, which can be approximated by the standard shot-noise limit $\Delta\phi = 1/N^{1/2}$. It follows that the resolution length is given by

$$\Delta x = \frac{\lambda/2\pi}{2T(\mathbf{r})\left(\dfrac{6\sqrt{2}}{\pi^2}\dfrac{F\lambda}{D}\right)^{1/2}}. \tag{8}$$

For an atom halfway between a node and an antinode, $\Delta x = (\lambda/2\pi)/16 = 10$ nm for $F = 10^6$, $\lambda = 1$ µm, $D = 1$ cm and $\Delta = 100\gamma$, which equals the result of the intensity measurement for this particular set of parameters. Note, however, that for $T(\mathbf{r}) \propto 1$ (i.e., large Δ) and in contrast to (6), the resolution achieved in the homodyne measurement is largely independent from atomic parameters and the atom-field detuning (if the factor $\lambda^{-1/2}$ is neglected and as long as $|\delta| \gg 1$). It follows that, for large Δ, the homodyne scheme has a smaller resolution length than the scheme based on measuring the intensity of the transmitted light (where Δx increases linearly with Δ).

In a realistic experiment both the intensity of the light transmitted and its phase will change simultaneously. Therefore, the decrease in cavity transmission can be used to enhance the position resolution of the homodyne measurement, if the phase shift from an atom moving towards an antinode also decreases the magnitude of the homodyne signal. This condition can be achieved by adjusting the phase of the reference beam in the Mach-Zehnder interferometer.

To discuss the observation time τ not yet specified, note that the steady-state solutions of (1) were used in the derivation of (4–8), which is strictly valid only for an atom at rest (with infinite mass). However, slow atoms can be used for which $g(\mathbf{r})$ does not change significantly on a time scale determined by κ^{-1} and γ'^{-1}. For example, cesium atoms with a wavelength $\lambda = 852$ nm for the resonance transition, mass $m = 2.2 \times 10^{-25}$ kg and speed 20 times the recoil velocity $\hbar k/m = 3.5$ mm/s move a distance of $\lambda/20$ in 600 ns, which is longer than the natural lifetime $\tau_{nat} = \gamma^{-1} = 32$ ns and the cavity lifetime $\tau_{cav} = 100$ ns for $F = 10^6$ and $L = 100$ μm.

To consider the effect of atomic motion in more detail, note that, even for an atom at rest initially, the wavepacket produced by the observation starts to move due to the mechanical force exerted on the particle by the off-resonant optical potential (i.e., light shift) $\hbar(g^2/\Delta)|\alpha|^2$. This force vanishes both at the nodes and antinodes of the standing wave and is largest halfway between them. In this region, the particle is accelerated and moves a distance $\Delta s \propto (\hbar k/m)(g^2/\Delta)|\alpha|^2\tau^2 \leq (\hbar k/m)(\Delta/\gamma)\tau$ during the observation time τ. For the measurement scheme to be useful, we require that the particle moves less than one resolution length within τ, i.e., $\Delta s \leq \Delta x$. A sufficient condition is $(\Delta/\gamma)\tau \leq \Delta x/(\hbar k/m)$ which establishes an upper limit for $(\Delta/\gamma)\tau$. Note that $\tau \geq \tau_{cav}$ is needed to use the results of the steady-state calculation, and $\tau \gg \tau_{nat} = 1/\gamma$ is necessary to achieve $w \propto -1$. These are lower limits for τ. Hence, it follows from $\Delta/\gamma \leq (\Delta x/\tau)/(\hbar k/m)$ that, for given $\tau \geq (\tau_{cav}, \tau_{nat})$, a moderate detuning Δ is required to achieve a small resolution length Δx. Choosing $\tau = 300$ ns (which corresponds to $\propto 3\tau_{cav}$, as in the example above, and $\propto 10\tau_{nat}$ for cesium), then $\Delta/\gamma \leq 10$ for $\Delta x = (\lambda/2\pi)/16 = \lambda/100 = 10$ nm. The mean velocity for an atom starting from rest is then $\Delta x/\tau \propto 10\, \hbar k/m = 3.5$ cm/s which is achievable with present state-of-the-art laser-cooling technology. The intracavity photon number is $|\alpha|^2 \propto 1$ (for $F = 10^6$, $\lambda = 1$ μm, $D = 1$ cm and $\psi^2 = 1/2$) and the atomic inversion $w = -0.83$, which is reasonably close to $w \propto -1$. Note that, in this example, all control parameters are optimized to achieve the smallest resolution length $\Delta x = (\lambda/2\pi)/16$. Otherwise, Δx increases because the intracavity photons impart excess momentum to the atom. For example, the back-action effect of the measurement-light field effectively limits the resolution to about $\Delta x \propto \lambda/20$ for $\Delta = 50\gamma$, $|\alpha|^2 \propto 30$ and identical parameters otherwise. This is larger than the limits stated in (6) and (8) but is still one order of magnitude smaller than the standard diffraction limit.

The discussion also shows that the natural spreading of the atomic wave packet does not limit the resolution length. For example, a cesium atom localized to within $\Delta x = \lambda/20$ propagates for a time $t \propto 2m(\Delta x)^2/\hbar = 8$ μs, before the wave packet doubles in size. This time is much longer than the typical time scale associated with the acceleration of the atom, as was discussed above.

Finally, note that the relation $N_{max} = (\Delta/g)^2(2\kappa/\gamma)$ depends on the position of the atom [due to $g = g(\mathbf{r})$]. Therefore, the condition $N = N_{max}$ for the number of photons detected (which was assumed throughout the discussion) implies that the largest allowed signal can only be achieved if the position of the atom is already known. Although $N = N_{max}$ can be realized experimentally by adjusting the incident field strength during the measurement, it is more realistic to assume that the system's excitation is constant and that the position of the atom is not known a-priori. Therefore (and to suppress spontaneous emission), the driving-field strength must comply with the requirement that $N \leq N_{max}$ for *any* position. Using $N = N_0 T$, where N_0 is the number of transmitted photons detected in the case the cavity is empty, together with (5) for the transmission coefficient T, it is straightforward to calculate that $N_0 \leq 4\Delta/\gamma$ gives a sufficient condition to achieve $N \leq N_{max}$. The equal-sign $N = N_{max}$ is valid for $\Delta/\gamma = g^2/\kappa\gamma$. This condition determines the optimum detuning for a given value of the coupling constant g, i.e., a given position. For example, $\Delta/\gamma = 50$ for $F = 10^6$, $\lambda = 1$ μm, $D = 1$ cm and an atom halfway between a node and an antinode with $\psi^2 = 1/2$. With $N_0 = 200$ and $T = 0.5$, one finds $N = N_0 T = 100 = N_{max}$. Note that, for an atom at a different position (close to an antinode, for example, with $T = 0.2$), the same incident intensity (i.e., $N_0 = 200$) gives $N = 40$, which is not too different from $N_{max} = 50$. This justifies the assumption $N \propto N_{max}$ for any position. The resolution length amounts to $\Delta x \propto \lambda/20$, as calculated above.

To conclude, we found that the continuous observation performed on the field enables a continuous position measurement, which makes possible to trace the atomic motion while the particle moves through the intracavity light field (and possibly oscillates in the potential wells produced by the standing wave for $\Delta = \omega_a - \omega_l > 0$) with a resolution about one order of magnitude more accurate than the standard diffraction limit. Note, however, that unless some other position-measurement scheme is used in addition to the cavity field (e.g., a mechanical slit), only a relative localization is possible due to the periodicity of the standing wave.

To mention an interesting application, the cavity system can be used to perform joint measurements on the position *and* momentum of a *single* quantum particle. When this traverses the resonator with a velocity which is large perpendicular to the cavity axis, then the phase shift observed on the optical field serves to determine the position while the far-field diffraction pattern can be used to measure the momentum uncertainty introduced by a measurement process. As has recently been pointed out by several authors [15], an initial minimum uncertainty wave packet with $\Delta x \Delta p = \hbar/2$ broadens (e.g., diffracts) more rapidly due to the joint measurement, an effect which can be described by a modified uncertainty relation

$\Delta x \Delta p \geq \hbar$ (with the product of standard deviations for the measurement outcomes at least twice as large as the lower bound implied by the usual uncertainty principle).

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