CRITERION OF FORMATION OF A THERMONUCLEAR BURN WAVE IN DEUTERIUM–TRITIUM TARGETS

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1. INTRODUCTION. PROBLEM OF SPARK-IGNITION CRITERION

One of the conditions for implementing high-efficiency thermonuclear burning in laser thermonuclear fusion (LTF) is the formation of a thermonuclear (TN) burn wave in an inhomogeneous plasma [1, 2]. An energy gain $K \sim 10^3$ can be reached by producing at the center of the target a high-temperature region having a mass much lower than the total mass of the relatively cold fuel. When a self-sustaining burn wave is produced, the efficiency K can be substantially increased compared with uniform burning.

A preliminary theoretical analysis of the conditions for the onset of a burn wave was made in [1-6]. Analytic self-similar solutions for a burn wave with various energy-transport mechanisms were obtained in [4, 5, 7-9]. Many numerical calculations of a burn wave were made (e.g., [1-13]).

Complicating somewhat the optimization of the burn propagation is the choice of a criterion of the very occurrence of a thermonuclear flash.

Thus, a thermonuclear flash is taken in [6, 8, 9] to mean a burn wave with hyperbolic time dependences of the temperature and of the front radius (regime with a peak): $T_f(t) \sim 1/(t_0 - t)^a$, $R_f(t) \sim 1/(t_0 - t)^b$. Such a regime is theoretically characterized by a fast temperature rise and a high front velocity, which tend to infinity within a finite time interval: $t \to t_0$. At $t << t_0$, however, such a regime hardly differs from one linear in time, $T \sim 1 + at/t_0$, and at $t \sim t_0$ the temperature reaches rapidly 20-30 keV, when the dependence of the rate of energy release on the temperature weakens and the wave is defined by a simple power law $T \sim t^a$ [9]

It was shown in [10] by numerical methods that for chosen initial conditions a burn wave goes through two stages: initial "subsonic" characterized by a dropping or slightly rising temperature behind the front and determining the wave-generation conditions, and a "supersonic" stage of intense burning, which contributes up to 80% to the TN target output. The transition to the intense energy release state is in this case quite rapid.

A more general criterion of a spark-ignited TN flash is therefore the condition that the burning be self-sustained, i.e., a burn wave with a temperature that grows behind the front ($\dot{T}_f > 0$ [3-5, 7, 10-14], when the energy contribution from the reaction products in the burn zone exceeds the energy lost to heat conduction and radiation.

Thus, qualitative estimates [3] yielded for an isochoric wave profile, i.e., for equal cold- and hot-region densities, $p_a = \rho_f$, yielded the conditions $T_f > 15$ keV and $\rho R_f > 0.6$ g/cm^2. Similar results were obtained numerically in [11, 12].

More accurate ignition conditions, close to those of [3] were obtained for a plasma of inhomogeneous density and for shock-wave driven TN burning [4]:

$$T_f > 20 \text{ keV}, \quad R_f(\rho_f \rho_a)^{1/2} > 0.5 \text{ g/cm}^2.$$

To optimize the target burning it is necessary to study the relative contributions of various energy-transport mechanisms — hydrodynamic compression and expansion, thermal conductivity, energy and momentum transport by fast reaction products as well as ion–electron energy exchange — in the region of the burn-wave front during each of the stages of its development. A joint allowance for these mechanisms requires the use of numerical methods [10-13, 15-18].

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The most adequate for the description of energy transport by reaction products and by radiation is the Monte Carlo method, implementation of which in the TERA program complex [10, 13, 15-18] was used in our present study. A numerical calculation of the conditions for target self-heating, in the form of a curve that separates on the (ρR_f, T_f) plane the ignition region from the subcritical region, was carried out in [13] and is in good agreement with the qualitative estimates of [14].

Another aspect of spark ignition of a target is the very possibility of producing a hot ignitor in the target center. To this end it is necessary that the time t_c of the spark decay via radiation and heat conduction be longer than the time needed to produce the spark. As noted in [14], when a target is hydrodynamically compressed by converging shock waves the time of spark production cannot be shorter than the time t_s for the passage of sound through the ignitor. We can thus assume as a rather rough criterion of the formation of the initiation zone the condition that the spark decay time be equal to the sound decay time [14], $t_c = t_s$.

A calculation of this criterion without allowance for the energy transport by the fast particles is given in [14]. Since no allowance was made for the reaction-products' energy loss, the region of possible parameters of the spark turned out to be too narrow. For example, it follows from [14] that certain regions above the plot of the ignition condition cannot be produced by hydrodynamic compression, even though the temperature of the burn wave rises in this region, i.e., there is no spark decay.

A more adequate estimate of the possible ignitor parameters for spark ignition, with allowance for the energy transport by fast particles in the calculation of the characteristic spark-decay time, is obtained in the present paper.

The limits of ρR and T of the combustion zone, calculated in [13], ensure self-ignition of the target if no account is taken of the prior history of the process. As shown in [10-13, 17, 18], although the wave-formation stage can be characterized by a dropping temperature, since the ignition conditions have not been met, later on, after redistribution of the temperature, a situation can arise in which the self-heating conditions begin to be satisfied and a TN flash develops. That is to say, in the course of the spark decay, during the time of hydrodynamic expansion of the target from the region under the ignition curve, the "trajectory" of the wave on the (ρR, T) plane lands in the self-sustaining-wave region.

To calculate the wave trajectory from the subcritical region we must know the energy-transport mechanism that determines the front propagation, and the type of wave, isobaric or isochoric, close to the considered situation. As noted in [1, 14], when the wave is formed an isobaric profile is closer, and during the stage of intense burning the wave propagates in the plasma at a constant density.

It was proposed in [14] that an isobaric spark expands through heat conduction, at a constant pressure, and with the total energy conserved: $\rho_f T_f = \text{const}$, $\rho_f T_f R_f^3 = \text{const}$. In this case $\rho R_f T_f = \text{const}$, and to land in the ignition region the condition $\rho R_f T_f > 2$ g·keV/cm^2 must be satisfied [14].

In a real situation, however, with considerable energy transported by fast particles, the energy conservation condition is not satisfied, and the wave trajectory is not described by the relation $\rho R_f \sim 1/T_f$.

We obtain in the present paper, as a result of qualitative estimates and a direct numerical simulation of the dynamics of burn-wave formation, the conditions of the density distribution and the active-zone temperature at the instant of the maximum compression, whereby a self-sustaining TN burn wave is inevitably obtained in the target during the time of its expansion.

2. CONDITIONS FOR SELF-SUSTAINING BURNING OF DT TARGETS

In this section we investigate the conditions for igniting a thermonuclear target in the presence of an isobaric spark, using a qualitative analysis of the burn-wave formation process and a direct numerical simulation of the burn dynamics.

The condition, calculated using the TERA program [13], for a self-sustaining burn wave, is shown in Fig. 1. Let us examine in greater detail the burn-wave formation condition.

The characteristic time of spark decay can be estimated by using the qualitative relations

$$t_c \sim T_f/(q_e+q_g-q_a-q_{dt}) \tag{1}$$

where $q_e \cong k_e \varkappa_e(T_f) T_f/\rho R_f^2$ is the intensity of the energy lost to heat conduction per plasma particle,

$q_g \cong k_g W_g(T_f)\rho(1 - \eta_g)$ the loss to radiation,

$q_\alpha \cong k_\alpha W_{dt}(T_f)\rho E_\alpha \eta_\alpha$ the energy loss of primary α particles in the burn zone,

$q_{dt} \cong k_{dt} W_{dt}(T_f)\rho \Delta E_n \eta_{dt}$ the energy loss of recoil nuclei in the burn zone,

$W_{dt} \sim \langle \delta_{dt} v\rangle$ the rate of thermonuclear DT reactions,

$W_g = C_g \sqrt{T_f}$ the bremsstrahlung intensity, with $C_g \cong 1.21 \cdot 10^9$ keV$^{1/2}$cm^3/g·sec for a DT plasma,

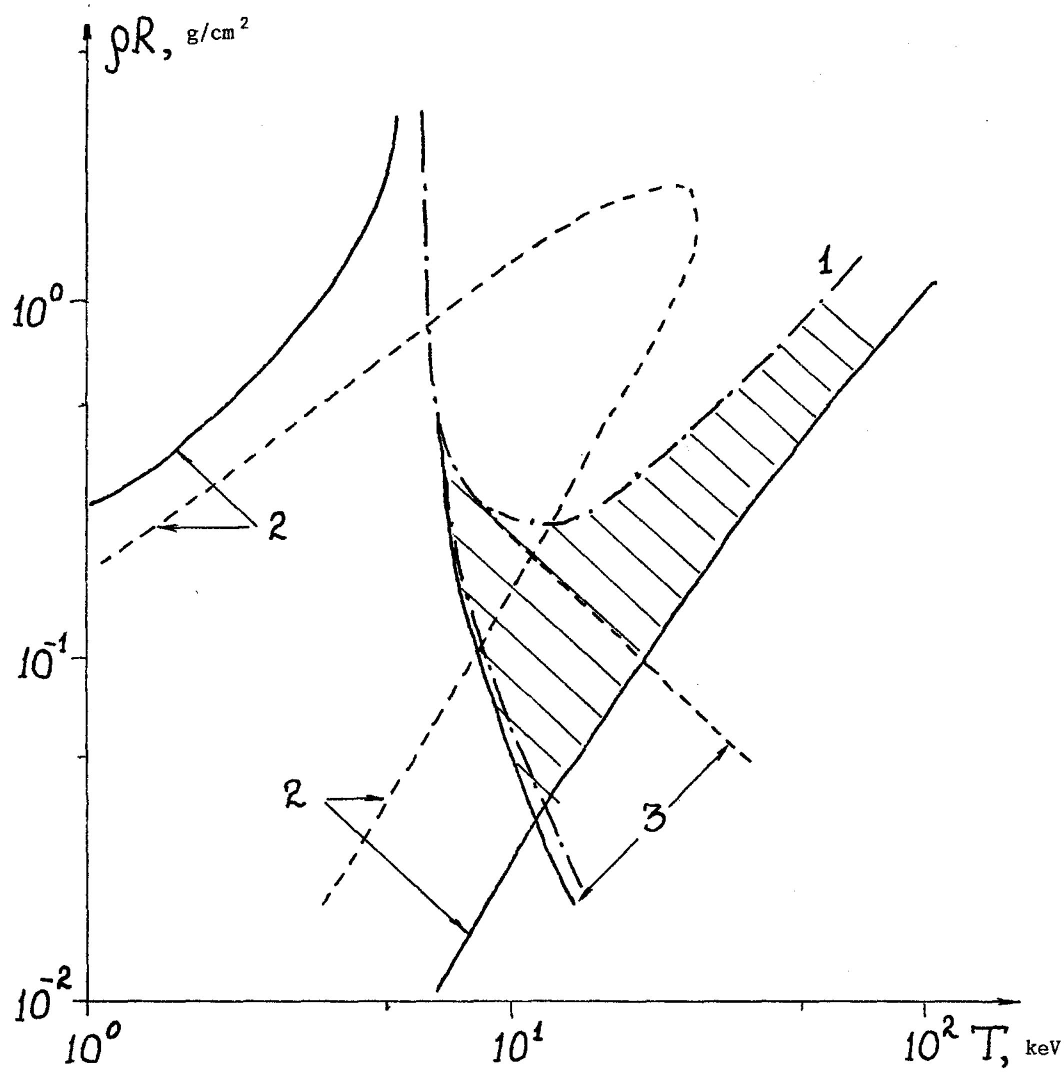

Fig. 1. Regions of possible parameters of initial initiation zone for an isobar spark ignition of a laser target: qualitative calculation with (solid line) and without (dashed) allowance for particle transfer; dash-dot line) complete numerical calculation using the TERA program.

Above curve 1 — region of self-sustaining burn wave.

Between curves 2 — region of ignitor parameters that can be obtained for direct hydrodynamic compression.

Curve 3 — left boundary of region from which the burn wave can land in the course of its development in the ignition region above curve 1. Dashed curve) region of possible subcritical ignition.

η_j, $\eta_g(\rho R_f, T_f)$ the energy fraction left by the charged particles and photons in burn zone,

k_e, k_α, k_g, k_{dt} coefficients less than 1 and dependent on the form of the temperature profile of the wave. It follows from numerical and analytic calculations [7, 13, 17] that $k_e \cong k_g \cong 1$, $k_\alpha \cong k_{dt} \sim 0.3\text{-}0.4$,

E_α is the α-particle energy, ΔE_n is the average variance of the neutrons in the burn zone.

Obviously, $t_c < 0$ in the region above the ignition curve (see Fig. 1). Therefore the spark formation conditions can be written, following [14], in the form

$$t_c > t_s, \quad \text{or} \quad t_c < 0 \tag{2}$$

where $t_s = R_f/v_s$ is the time of sound propagation through the ignitor,

$v_s = \sqrt{T_f}/C_s$ is the speed of sound, $C_s = 1.76 \cdot 10^{-8} (\text{A keV})^{1/2}$sec/cm, A is the average atomic weight of the plasma nuclei.

As a result, using the definition (1) we can rewrite the relations in (2) in the form

$$2F\rho R_f/(1 - (\rho R_f)^2 W) > 1, \tag{3a}$$

or

$$(\rho R_f)^2 W > 1 \tag{3b}$$

here: $F(T_f) = \sqrt{T_f}/2k_e æ_e C_s$,

$$W(\rho R_f, T_f) = [W_{dt}(k_\alpha E_\alpha \eta_\alpha + k_{dt}\Delta E_n \eta_{dt}) - k_g W_g(1-\eta_g)]/T_f k_e æ_e$$

Relations (3) specify on the (ρR_f,T) plane the region of spark parameters that can be obtained by hydrodynamic compression of the active zone of the target by converging shock waves.

In the region $T_f < 10$ keV, where the loss to radiation exceeds the energy input from the fast charged particles ($W < 0$), the region of the possible spark has the following ρR limits:

$$1/(F+(W+F^2)^{1/2}) < \rho R_f < 1/(F-(W+F^2)^{1/2}) \tag{4a}$$

In the $T_f > 10$ keV region ($W > 0$) the spark region has the lower limit

$$\rho R_f > 1/(F+(W+F^2)^{1/2}) \tag{4b}$$

The calculation of the regions (4) by iteration is illustrated in Fig. 1. The same figure shows the calculation of the spark region without allowance for the energy lost by the reaction products in accordance with the data of [14]. Evidently, if the particle energy transport is taken into account, the total ignition zone lies in the possible spark region.

Let us consider the possibility of igniting a target with spark parameters in the post-critical region. For qualitative estimates of the wave trajectory on the (ρR_f,T_f) plane we must make a number of assumptions concerning the character of the energy transport in an isobar spark during the initial stage.

As noted in [13, 14], expansion of the isobar spark is due to energy transport from the zone of combustion by electrons, fast particles, and radiation. No shock waves are formed. Assuming that the density and temperature discontinuity on the wave front is large enough, we can assume that the characteristic dimensions ΔR_f on the front are small: $\Delta R_f << R_f$, for all the considered energy-transport mechanisms.

The temperature rise ΔT, due to energy transport from the burn zone, of the cold region ahead of the wave front within a characteristic time Δt, can then be estimated by using the relation:

$$\Delta T = \Delta t q_s/\rho_a \Delta R_f \tag{5}$$

here: q_s is the total density of the energy flux through the wave front, ρ_a is the density of the cold region of the active zone of the plasma. Putting $\Delta T \sim T_f$ we obtain from (5) an estimate of the front velocity:

$$\dot{R}_f \simeq q_s/\rho_a T_f \tag{6}$$

As a result, using relations (1), we can write for the rate of change of the burn-zone mass M_f:

$$\dot{M}_f = \rho_f M_f Q_r(\rho_f R_f, T_f)/T_f \tag{7a}$$

where

$$Q_r = 3k_e æ_e T_f/(r_f R_f)^2 + k_g W_g(1-\eta_g) + W_{dt}[k_\alpha E_\alpha(1-\eta_\alpha)+k_{dt}\Delta E_n(1-\eta_{dt})]$$

By analogy with (7a), Eq. (1) leads to an equation describing the evolution of the average temperature behind the front:

$$\dot{T}_f = \rho_f Q_T(\rho_f R_f, T_f) \tag{7b}$$

where

$$Q_T = -k_e æ_e T_f/(\rho_f R_f)^2 - k_g W_g(1-\eta_g) + W_{dt}(k_\alpha E_\alpha \eta_\alpha + k_{dt}\Delta E_n \eta_{dt}).$$

Since the initial parameters of the ignitor are located in a region defined by the condition (4), in which the speed of sound exceeds the rate of spark expansion, $v_s > \dot{R}f$, it follows, as noted in [14], that the front propagation should proceed with conservation of the equal-pressure condition:

$$\rho_a T_a = \rho_f T_f \equiv P = \text{const} \tag{8}$$

where: ρ_a and T_a are the density and temperature of the cold region.

The condition (8) specifies a connection between the ignitor density and temperature, and hence between $\rho_f R_f$, M_f, and T_f:

$$\rho_f R_f = (3M_f/4\pi)^{1/3}(P/T_f)^{2/3} \tag{9}$$

Allowance for (9) makes it possible write down Eqs. (7) with parametrization of one variable, i.e., in a form that describes uniquely the shape of the wave trajectory on the (ρR_f, T_f) plane:

$$\frac{d}{d\xi}(\rho_f R_f) = (Q_r - 2Q_T)\frac{\rho_f R_f}{3T_f^2} \tag{10a}$$

$$\frac{dT_f}{d\xi} = \frac{Q_T}{T_f} \tag{10b}$$

Figure 1 shows a calculation of the left-hand boundary of the zone of subcritical ignition, from which the trajectory of the burn wave can, in the course of expansion and cooling, land in accordance with Eqs. (10) in the region of self-sustaining burning above the ignition curve. The same figure shows this boundary obtained by direct numerical simulation of the wave evolution in accordance with the TERA program. The qualitative agreement between these calculations attests to the adequacy of the assumptions concerning the character of the development of the equal-pressure spark, assumptions made in the derivation of Eqs. (10). It can be seen that allowance for the energy lost by the fast particle broadens substantially the zone of the possible subcritical ignition compared with the data of [14].

Another aspect of igniting a thermonuclear plasma from the region below the firing curve is the ability of the wave trajectory to reach the self-flareup region within a time shorter than the characteristic time t_0 for inertial retention of the target. For targets without shells this time coincides with the time $t_0 \equiv t_s(\rho_a, T_a)$ of sound propagation in the cold region ahead of the front, while t_0 for a target with a heavy ablator takes the form [19]:

$$t_0 \simeq t_s\sqrt{1+M_{sh}/m_{dt}} \tag{11}$$

where m_{dt} and m_{sh} are the masses of the active fuel and of non-evaporated part of the shell.

To estimate the subcritical-zone regions from which a target can be ignited during the time of its expansion, we use the system of equations (10) for the functions $\rho_f R_f(\zeta)$, $T_f(\zeta)$ with the timing reversed ($\zeta \to -\zeta$). Solution of such a system with the initial conditions for the ignition curve makes it possible, when the variable ζ reaches values $\zeta = \zeta_{cr} = Pt_0$, to obtain the $F_{cr}(\rho_f R_f, T_f)$ curve that specifies the set of ignitor-parameter values for the instant of maximum compression, at which the burn wave reaches in the course of its development the conditions for ignition during the lifetime of the target.

Using (2.11), we represent the expression for ζ_{cr} in the form

$$\xi_{cr} = t_s R_0 T_a\sqrt{1+M_{sh}/m_{dt}} \equiv C_s\beta \tag{12}$$

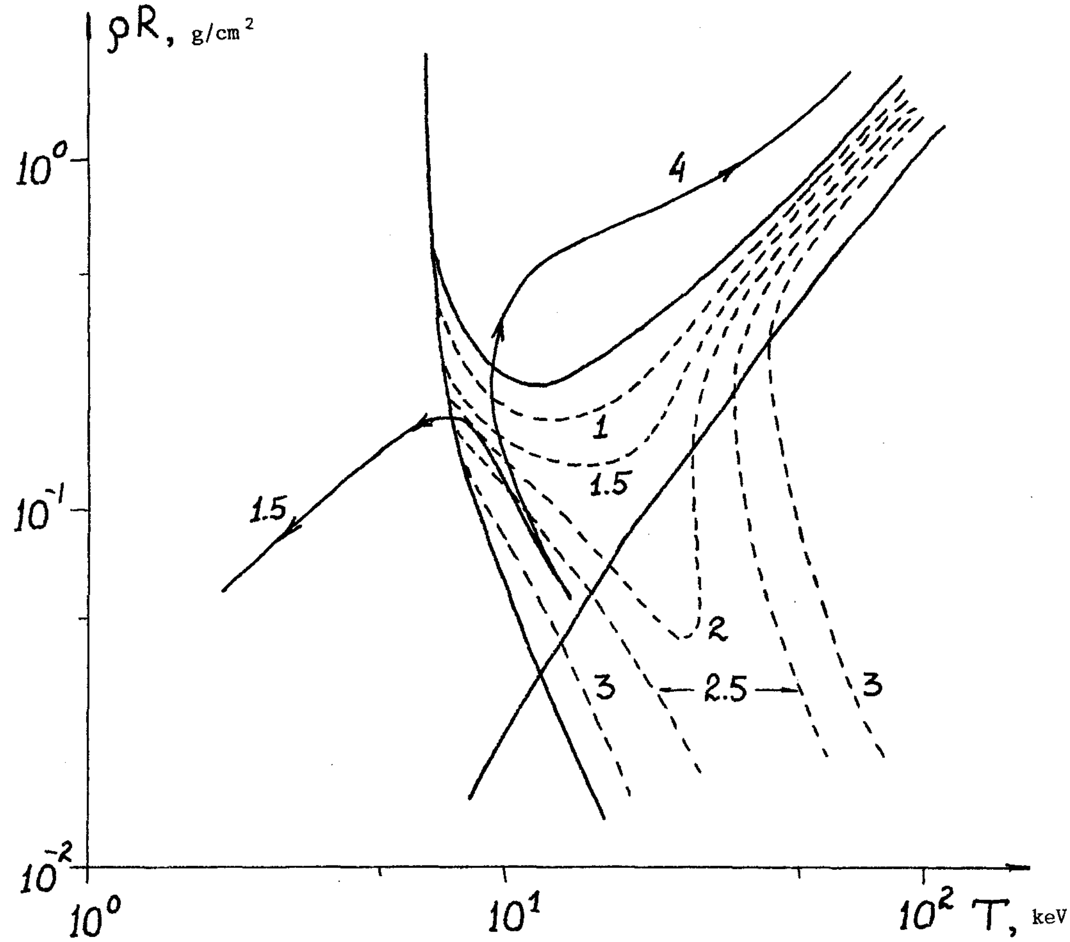

Fig. 2. Regions of subcritical spark ignition of a laser target:

Between the solid curves) region of possible parameters of subcritical ignitor in hydrodynamic compression of the target.

Dashed curves) boundaries of ignitor parameters for targets with different parameters $\beta = \rho_a R_0 \sqrt{T_a (1 + M_{sh}/m_{dt})}$ (T is measured in keV and ρR in g/cm^2), above which a TN burst is possible in the corresponding targets during the time of its hydrodynamic spreading.

—>—) TERA-program calculation of the trajectories of the burn-zone parameters for spark ignition of targets with different values of β.

here: $\beta = \rho_a R_0 \sqrt{T_a (1 + M_{sh}/m_{dt})}$ and R_0 is the radius of the active zone.

For specified values of the ignition criterion ($f_0 = 1$), the parameter β is the only one pertinent to the problem. Figure 2 illustrates the calculation of the F_{cr} curves for different values of β. Spark ignition of a target with a given β from the region below the corresponding $F_{cr}(\beta)$ is impossible.

The qualitative estimates of spark evolution in the subcritical region, obtained above with the aid of Eq. (10), agree well with a direct numerical simulation of the plasma burning dynamics [10, 13, 17]. Examples of the calculation of the trajectories of the burn-zone development on the ($\rho_f R_f, T_f$) plane in targets with different values of the parameter β are shown also in Fig. 2. It can be seen that an attempt of spark ignition from the region below the $F_{cr}(\beta)$ curve disintegrates the target prior to formation of a TN flash.

From an analysis of the calculations shown in Figs. 1 and 2 we can draw the following conclusions concerning the feasibility of a self-sustaining burn wave in a DT target:

• Target self-heating condition — production in the target of a central hot region with parameters above the ignition curve.

• The region of possible subcritical ignition (with the ignitor from the hatched region of Fig. 1) is restricted by the condition for spark production by hydrodynamic compression, and by the condition that the wave trajectory land in the ignition region.

• Spark ignition of a target by an ignitor from the subcritical region (below the ignition curve), i.e., the wave reaching the self-heating conditions within the time of inertial containment is possible only for targets with the parameter $\beta = \rho_a R_0 \sqrt{T_a}\,(1 + M_{sh}/m_{dt}) > 1$ (with T measured in keV and ρR in g/cm^2).

• Spark ignition of targets with parameter $\beta > 3$ is possible for all admissible values of $\rho_f R_f$ and T_f of the ignitor in the subcritical region.

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