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Mixed mode plane stress ductile fracture

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ABSTRACT
Mixed mode ductile fractures in thin sheets are shown to be possible. The staggered deep edge notch tension specimen enables J_p, the plane stress propagation value of the J integral, and dJ/da, the rate of increase in J with crack growth to be measured from the specific work of fracture. The J integral can also be separated into its two component modes J_I and J_{II}.

For the particular low alloy steel tested J_p is virtually independent of the mode of fracture, but for other materials J_p may be dependent on the fracture mode.

1. Introduction

The initiation of fracture from a notch in a brittle material under mixed mode loading has received considerable attention and a variety of initiation criteria have been advanced [1–11]. However, once initiated the path taken by a fracture, in a homogeneous isotropic brittle material, is that for which the local stress field at its tip is mode I [12]. In contrast fractures in thin ductile sheets are not pure mode I, because of out of plane mode III shear in the fracture process zone. Ductile fracture in thin sheets where mode II displacements are absent at the tip of the crack are usually treated as generalized mode I fractures, where the fracture criterion is dependent on the sheet thickness. The propagation of mixed mode I and II fractures is possible in ductile sheets because localized necks can form that in general are at an angle to the principal stresses. The attempts that have been made to investigate mixed mode ductile fracture have not been very successful. Pook [13] performed a series of tests on ductile tension specimens with slits at an angle to the applied stress, but the fracture propagated essentially in a mode I direction. Jones and Chisholm [14, 15] did achieve a mode II fracture in a compact shear specimen; but their linear elastic analysis was inappropriate because in one case [14] the net section shear stress at fracture was greater than the yield shear strength and in the other [15] the plastic particle was of the order of half the ligament or more.

True mixed mode ductile fractures can only occur in specimens that are completely yielded and where post yield fracture mechanics is necessary and the J integral [16] is appropriate. Mixed mode ductile fractures can be obtained in thin sheets by the use of staggered deep edge norched tensile specimens (see Fig. 1) similar to those originally suggested by Hill [17] for determining the yield criterion of ductile metals. In these specimens localized necks form along the line joining the notches which develop into fractures and enable the mixed mode J integral to be calculated.

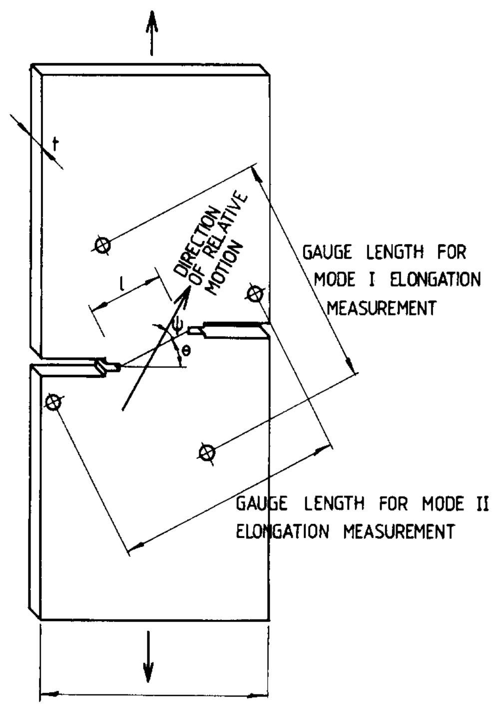

Figure 1. The staggered deep edge notch tension specimen.

1.1. *The fracture of deep edge notch specimens*

The initial notch provides a greater constraint against plastic flow than does the tip of the propagating crack. As a consequence the crack tip opening displacement ahead of a propagating crack is in general slightly greater than the initiation value becoming constant after a crack growth of about twice the sheet thickness [18]. This change in plastic constraint also generally causes a slight increase in J which is distinct from the much larger increase observed in J_R curves. We have previously argued [18–20] that the total work performed in fracturing a deep edge notch tension specimen (W_f), that yields completely before tearing can be separated into two parts:

(i) The work performed within the process zone of about the sheet thickness (t), which eventually forms the necked zone, called by us the essential work of fracture (W_e) and assumed to be proportional to the ligament length (l).

(ii) The work performed in the surrounding plastic region (W_p), which similarity arguments show is proportional to the square of the ligament length (l).

Thus, if the specific work of fracture $w_f = W_f/lt$ is plotted against the ligament length we obtain a straight line whose intercept is the specific essential work $w_e = W_e/lt$. Similar arguments [19, 20] show that the elongation at fracture is a linear function of the ligament length and that the intercept is the crack tip opening displacement of the propagating crack. We now wish to argue that the specific essential work can be identified with J_p the value of the J integral for a crack starting to grow under the plastic constraint appropriate to a propagating crack. The J_R curve is approximately a linear function of crack growth [21] and is given by

$$J_R = J_p + \frac{dJ}{da} a \qquad (1)$$

where J_p is normally the initiation value for plane strain fracture, but for plane stress fracture must be identified with the slightly larger value appropriate to propagation, $\mathrm{d}J/\mathrm{d}a$ is a constant and a is the crack growth. J_R can be obtained from the energetic formulation [16]

$$J_\mathrm{R} = -\frac{\partial V}{\partial a} \tag{2}$$

where $\mathrm{d}V$ is the energy released per unit thickness during crack growth $\mathrm{d}a$. Now if we integrate J_R with respect to a until the two fractures meet we have

$$\frac{W_f}{2t} = J_p\frac{l}{2} + \frac{\mathrm{d}J}{\mathrm{d}a}\frac{l^2}{8} \tag{3}$$

and

$$w_f = J_p + \frac{\mathrm{d}J}{\mathrm{d}a}\frac{l}{4} \tag{4}$$

We can, therefore, identify J_p with the specific essential work, w_e, and observe that $\mathrm{d}J/\mathrm{d}a$ is four times the slope of the curve of w_f against ligament length l. There are inaccuracies in this approach, because for strict J controlled crack growth [22]

$$\frac{l\sigma_0}{J_\mathrm{R}} \gg 1 \tag{5}$$

and

$$\frac{l}{J_\mathrm{R}}\frac{\mathrm{d}J}{\mathrm{d}a} \gg 1 \tag{6}$$

where σ_0 is the flow stress, but J_p should be well defined and $\mathrm{d}J/\mathrm{d}a$ defined to perhaps somewhat less accuracy.

If the deep edge notches are staggered (see Fig. 1) the fracture occurs under both normal and shear stresses. A neck precedes fracture and the velocity discontinuity across the neck is at angle ψ to it which is given by [23]

$$\tan\psi = \frac{1}{4}\cot\theta \tag{7}$$

Consequently the specific work of fracture has both mode I and mode II components which can be separated [24] so that

$$w_f = w_{\mathrm{I}f} + w_{\mathrm{II}f} \tag{8}$$

and likewise

$$J_p = J_{\mathrm{I}p} + J_{\mathrm{II}p} \tag{9}$$

$$\frac{\mathrm{d}J}{\mathrm{d}a} = \left(\frac{\mathrm{d}J}{\mathrm{d}a}\right)_\mathrm{I} + \left(\frac{\mathrm{d}J}{\mathrm{d}a}\right)_\mathrm{II} \tag{10}$$

1.2. *Experiments on staggered deep edge notch specimens*

The material chosen for these experiments was Lyten, a low alloy steel sheet of nominal thickness 1.6 mm which has been used for similar mode I fracture [18, 20, 21] experiments.

TABLE 1
Chemical composition of Lyten

Carbon	0.10%
Phosphorus	0.09%
Manganese	0.85%
Silicon	0.40%
Sulphur	0.03%
Nickel	0.25%
Chromium	0.8 %
Copper	0.25%

TABLE 2
Mechanical properties of Lyten

	Direction relative to rolling		
	Transverse	45°	Longitudinal
Yield strength (MPa)	365	359	324
Ultimate strength (MPa)	524	514	511
Elongation on 50° (%)	32.3	-	31.0
Anisotropy coefficient r	1.28	1.09	0.96

The composition and mechanical properties of the material are shown in Tables 1 and 2. From Table 2 it is seen that the degree of plastic anisotropy is not high. To ensure that there are no anisotropic fracture effects, all specimens were cut so that the rolling direction was aligned to the line joining the staggered notches.

The gross width of the specimens was made large enough to ensure that yielding was confined to the ligament and brittle lacquer coatings were used to confirm that yielding did not spread to the outer edges of the specimens. It was previously observed [18] that in mode I loading there was a partial transition to plane strain fracture if the ligament was less than 10 mm, caused by the constraint against free neck formation imposed by the notch. Thus, for stagger angles 0°–36°, the ligament length used ranged from 10–60 mm. However for larger stagger angles, the cracks growing from the two notches did not join up for large ligaments and their length had to be limited to 40 mm for $\theta = 54°$, 20 mm for $\theta = 72°$ and 10 mm for $\theta = 90°$. The smallest ligament tested had, therefore, to be reduced for $\theta = 72°$, and 90° to 5 mm respectively. Since the necking for fractures with a high mode II component was small, there was no apparent transition to plane strain fracture with these small ligament lengths. In materials with a smaller strain hardening exponent more intense necks form and the zone is much longer, and fractures follow the line joining the notches more readily. The last 2–3 mm of the notches were finished with a 0.15 mm saw to give a sharp initiating notch.

The elongations normal to and along the ligaments were measured with strain gauged clip gauges from points outside the plastic region. These clip gauges, which were mounted in pairs on each side of the specimen to eliminate any effects due to bending, were attached to the specimen by lugs that were free to rotate. The mode I and II works of fracture were obtained from the area of the load-elongation curves recorded on an X–Y plotter.

1.3. *Results of experiments on staggered deep edge notch specimens*

All specimens with notches staggered at 18°–54° showed a definite yield point. Whereas only a few specimens with stagger angles greater than 54° showed a definite yield and none was observed for zero stagger. A well defined yield point is to be expected for stagger angles close to 36° because in an unnotched specimen a neck forms at an angle of 35° 16′ in an isotropic material [19] and the staggered notches help promote this neck which forms almost simultaneously over the whole ligament. At stagger angle far from 35° 16′ yielding develops gradually from the notch tips and a definite yield point is not observed. The brittle lacquers very clearly illustrated this difference in the yield behaviour.

The direction of the relative motion across the notched section illustrated in Fig. 2 conform well with that predicted by the theory of Hill [17] for both measurements at yield and at complete fracture. The yield locus shown in Fig. 3 also agrees well with Hill's theory [17]. The maximum stress cannot be compared with Hill's theory because necking is limited to a very small zone at the tip of the notch less than 3 mm long, also tearing starts before the maximum load is reached.

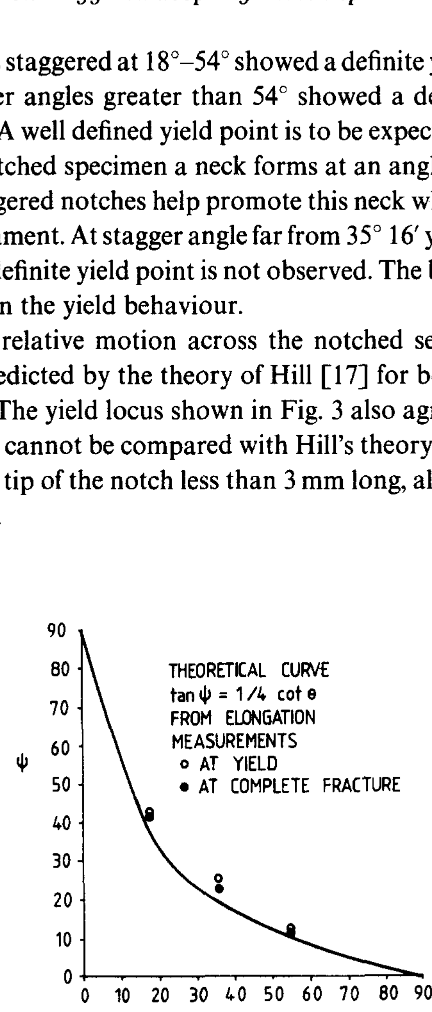

Figure 2. Direction of relative motion across the ligament.

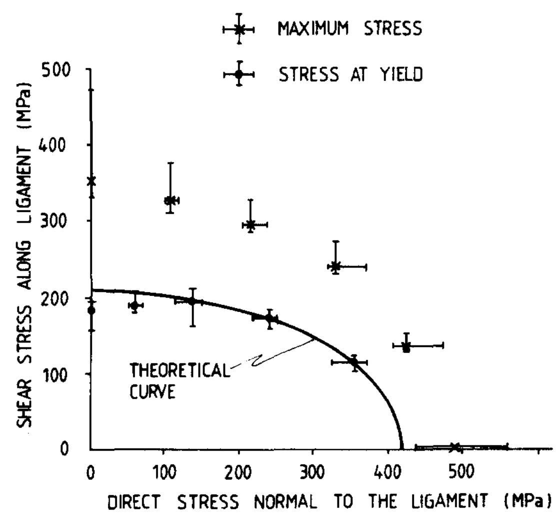

Figure 3. Yield and maximum stress loci.

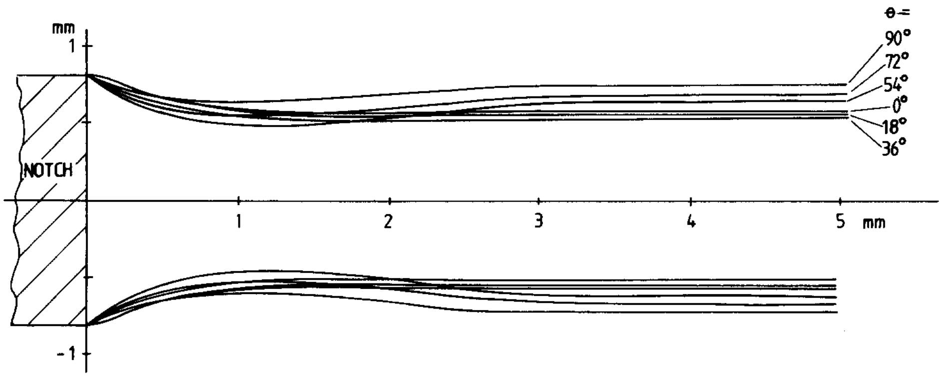

Figure 4. Reduction in thickness across the ligament.

The reduction in thickness in the fracture surface is shown in Fig. 4. For stagger angles 0°–36° the greatest reduction in thickness occurred over the central portion of the ligament, but for larger stagger angles the maximum occurred within 1 mm of the notch. Not surprisingly the greatest reduction over the central portion of the ligament occurs for a stagger angle of 36°, because, as already noted, this angle greatly favours neck formation. It is not clear why the maximum reduction occurs close to the notch for the larger stagger angles.

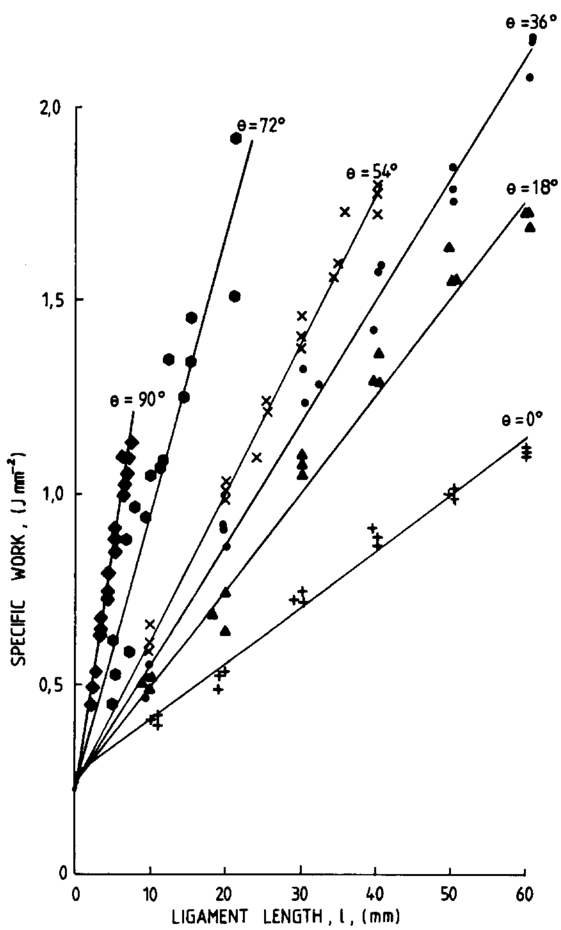

Figure 5. Specific work of fracture.

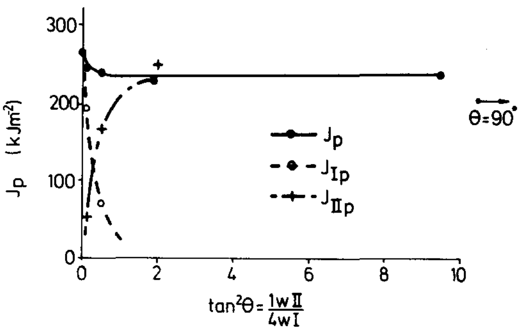

Figure 6. J_p as a function of $\tan^2 \theta$.

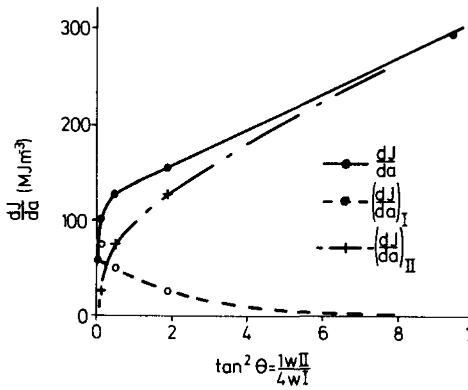

Figure 7. dJ/da as a function of $\tan^2 \theta$.

The specific total work of fracture is shown in Fig. 5 as a function of ligament length for a range of stagger angles. It is seen that the specific total work is a linear function of the ligament length with a zero intercept, that we interpret as J_p, almost independent of stagger angle. Since we measured both the mode I and the mode II displacements, we are able to separate both J_p and dJ/da (4 × slope of the lines in Fig. 5) into their two components which we display in Figs. 6 and 7 as a function of $\tan^2 \theta$ which is a quarter of the theoretical ratio of mode II work to that of mode I. At a stagger angle of 54° we found a negative intercept for J_p which may possibly be an artifact caused by the reduction in thickness being larger near the notch tip than in the steady state propagation condition away from the tip or by inaccuracies in our measurements.

2. Conclusions

The measurement of the work of fracture may provide an easy approximate method of measuring J_p and dJ/da. The staggered deep edge notch specimen enables a mixed mode ductile fracture to occur. It appears for the particular material chosen for these tests that J_p is insensitive to the mode of fracture, but, since we cannot suggest any theoretical grounds for this insensitivity, it may not be a general result, and we suspect that J_p would be more sensitive to the mode of fracture in more ductile materials. There is quite strong dependence of dJ/da on the mode of fracture.

Acknowledgements

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REFERENCES

[1] F. Erdogan and G.C. Sih, *Journal of Basic Engineering* 85 (1963) 519–527.
[2] J.G. Williams and P.D. Ewing, *International Journal of Fracture Mechanics* 8 (1972) 441-446.
[3] I. Finnie and A. Saith, *International Journal of Fracture* 9 (1973) RCR 484–486.
[4] P.D. Ewing and J.G. Williams, *International Journal of Fracture* 10 (1974) RCR 135.
[5] P.D. Ewing, J.L. Swedlow and J.G. Williams, *International Journal of Fracture* 12 (1976) 85–93.
[6] B. Cotterell, *International Journal of Fracture Mechanics* 1 (1965) 96–103.

[7] M.C. Hussain, S.L. Pu and J. Underwood, in *Fracture Analysis*, Proceedings of the 1973 National Symposium on Fracture Mechanics, Part II, ASTM Special Technical Publication 560 (1973) 2–28.
[8] K. Palaniswamy and W.G. Knauss, in *Mechanics Today*, ed. S. Nemat-Nasser, 4, Pergamon Press (1978) 87–148.
[9] G.C. Sih, *Engineering Fracture Mechanics* 5 (1973) 365–377.
[10] G.C. Sih, *A Special Theory of Crack Propagation in Mechanics of Fracture, Methods, Analysis and Solutions of Crack Problems*, Noordhoff (1973).
[11] G.C. Sih, *International Journal of Fracture* 10 (1974) 305–321.
[12] B. Cotterell and J.R. Rice, *International Journal of Fracture* 16 (1980) 155–169.
[13] L.P. Pook, *Engineering Fracture Mechanics* 3 (1971) 205–218.
[14] D.L. Jones and D.B. Chisholm, *Engineering Fracture Mechanics* 7 (1975) 261–270.
[15] D.B. Chisholm and D.L. Jones, *Proceedings of the ASCE Convention*, San Francisco (1977).
[16] J.A. Begley and J.D. Landes, in *Fracture Toughness*, ASTM Special Publication 514 (1972) 1–23.
[17] R. Hill, *Journal of the Mechanics and Physics of Solids* 1 (1953) 271–276.
[18] B. Cotterell and Y.W. Mai, to be published in *Proceedings 5th International Fracture Conference*, Cannes (1981).
[19] B. Cotterell and J.K. Reddel, *International Journal of Fracture* 13 (1977) 267–277.
[20] B. Cotterell, in *Fracture Mechanics and Technology*, eds. G.C. Sih and C.L. Chow, 2, Sijthoff and Noordhoff Alphen aan den Rijn (1977) 785–795.
[21] P.C. Paris, H. Tada, A. Zahoor and H. Ernst, in *Elastic-Plastic Fracture*, ASTM Special Publication 668 (1979) 5–36.
[22] J.W. Hutchinson and P.C. Paris, in *Elastic-Plastic Fracture*, ASTM Special Publication 668 (1979) 37–64.
[23] R. Hill, *Journal of the Mechanics and Physics of Solids* 1 (1953) 19–30.
[24] H. Ishikawa, H. Kitigawa and H. Okamura, in *Proceedings ICM 3*, Cambridge (1979) 447–455.

RÉSUMÉ

On montre que des ruptures mixtes de mode ductile dans des feuillards minces sont possibles. En utilisant une éprouvette de traction comportant deux entailles latérales, profondes et décalées, on peut mesurer J_p composante de propagation de l'intégrale J en état plan de tension et dJ/d_a, le taux d'accroissement de J en fonction de la croissance de la fissure à partir du travail spécifique de rupture. L'intégrale J peut également être séparée en ses deux composantes des modes J_I et J_{II}.

Dans le cas de l'acier faiblement allié particulier qui a été traité, J_p est virtuellement indépendant du mode de rupture; cependant, pour d'autres matériaux, J_p peut dépendre du mode de rupture.